Herbert Scarf (1930-2015)

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Herbert Eli Scarf died on November 15, 2015. The obituary in the *New York Times* was titled "Herbert Scarf, 85, Professor Who Shaped Economic Policy," (Roberts, 2015). This title is pretty remarkable, really, when you consider that Herb Scarf never took an economics class in his life. Both his undergraduate and graduate training were in mathematics. And yet, the *New York Times* title was completely appropriate. Herb Scarf became a remarkable economist and his work enabled the whole field of applied general equilibrium (AGE) economics to develop. The *Times* article quoted Glenn Hubbard, former Chairman of the President's Council of Economic Advisors, as saying that "the Scarf Algorithm was a revolutionary advance in economic analysis that has helped shape policies affecting every American."

We might add that is not just every American who may have been affected by this policy evaluation tool and note that AGE models are in use the world over. These models have helped evaluate government policies in the U.K., Canada, Mexico, Japan, China, Australia, Vietnam, the Philippines, and Cote d'Ivoire, just to name some of the more prominent examples. Glenn Hubbard was further quoted as saying "the Scarf Algorithm made much of modern policy analysis possible." The same *Times* obituary quoted Larry Summers, former U.S. Treasury Secretary and Head of the National Economic Council, as saying "No economist has done more to import math (to economics) over the last half-century than Herbert Scarf."

Scarf made many seminal contributions to economics. His work was brilliantly summarized by Kenneth J. Arrow and Timothy J. Kehoe (1994). A more recent and comprehensive presentation of his work is contained in the four-volume set of books edited by Zaifu Yang, *Herbert Scarf's Contributions to Economics, Game Theory, and Operations Research*, (2013). These publications cover his early and pioneering work in inventory theory, which has important practical applications today in supply chain management, his work on the correspondence of competitive equilibria and the core of the economy, his work on the possibility of multiple equilibria in general equilibrium models, his breakthrough algorithm for computing an approximate equilibrium of a fully specified competitive economy,

and his work on indivisibilities and increasing returns to scale. We are going to focus on what we know best and what we think of as his greatest accomplishment, turning the general equilibrium model into a powerful tool for economic policy analysis.

The general equilibrium model, with all prices affecting the demand and supply of each and every good was formulated by Leon Walras in 1874 in his *Elements of Pure Economics*. If there are *n* goods in the economic model, there are *n* prices. By equilibrium, we mean a set of prices for which there is no pressure for them to change. This is the same concept of equilibrium used throughout the sciences. In economic terms this means a set of prices where the aggregate demand is less than or equal to the total supply for all goods, with equality for goods with a non-zero price. While the general equilibrium framework was recognized as the ultimate model of the market by economists, even a proof that such an equilibrium set of prices exists proved tremendously challenging. Walras noted that the model had as many equations as unknowns, but recognized that this was not a sufficient condition to guarantee that a set of equilibrium prices exists. It took the profession more than seventy-five years to prove even the existence of an equilibrium (i.e. market clearing) set of prices. Kenneth Arrow of Stanford and Gerard Debreu of Berkeley, in communication but working separately, accomplished this monumental task using developments in mathematical topology (Arrow and Debreu, 1954). Lionel McKenzie, who was an assistant professor at Duke, independently solved the existence problem at almost exactly the same time (McKenzie, 1954). Interestingly, their proofs of existence did not lead directly or simply to a method for calculating the set of market clearing prices.

The task of computing the market clearing set of prices took another fifteen years or so and was accomplished by Herbert Scarf. It was a monumental accomplishment, at least as important as the existence proofs of the 1950s. We are not going to go through the beautiful logic of Scarf's algorithm since that is covered in many publications including his own 1973 book, *The Computation of Economic Equilibria*. Walras' general equilibrium structure was labeled by Joseph Schumpeter (1954) as the "magna carta of economics" but several leading economists including Barone (1908), Robbins (1934) and Hayek (1940) doubted it could be used to address actual policy questions of the day. There simply was no

way to compute the equilibrium solution of the model. For Scarf, that was the challenge. In the preface to his 1973 book, Herb states that the "ultimate purpose of general equilibrium analysis is as a method for the evaluation of economic policy."

Herb Scarf was a mathematical economist and theorist and did not engage in policy evaluation himself. But, he certainly supported those of us who wanted to turn his algorithm into a tool for policy evaluation. We were fortunate to be in the right place at the right time, a point made to us by Peter Mieszkowski at Yale. Peter had just taught us the Harberger two-sector model used in public finance and we were quite aware of other small-scale general equilibrium models of James Meade and Harry Johnson used in international trade. These models had to be kept incredibly simple and small in order to be analytically tractable. At the same time, Scarf had taught us his just-developed method for computing economic equilibria for arbitrarily large Walrasian general equilibrium models. Computing power was quickly ramping up so as to make Scarf's algorithm truly computable. There was a tremendous opportunity to take what he had created and develop a toolkit for applying Scarf's algorithm to real world policy issues.

Several steps were necessary to turn the computational algorithm into a useful policy analysis tool. Real world policy instruments such as taxes, tariffs, government expenditures and transfer programs had to be added to the Walrasian model. Economy-wide data have to be organized into what we refer to as "general equilibrium data sets." The model has to be parameterized in such a way that it can reproduce the observed equilibria, a process that we dubbed "calibration." This same calibration technique is now widely used by macroeconomists. Policy issues, such as corporate tax reform or the NAFTA free trade agreement have to be captured in model equivalent form. Then, the counter-factual equilibrium is calculated and comparisons with the existing equilibrium are made. We published a series of articles on these various tools for applying the general equilibrium model for policy analysis. Much of our work in this regard is summarized in our 1992 book, *Applying General Equilibrium* (Shoven and Whalley, 1992).

The application of the general equilibrium model mushroomed after Scarf's algorithm made computing equilibria possible. Both of our 1973 dissertations

featured initial applications of the applied general equilibrium approach to tax policy, in the U.S. and the U.K., respectively. The American application was an evaluation of the differential taxation of capital income in the U.S. (Shoven and Whalley, 1972), a topic that had been earlier addressed by Harberger (1959) with the two-sector analytic model. But, with the Scarf algorithm, the results were much richer. For instance, the model would predict how changes in the corporate income tax rate would affect the proceeds of the personal tax system. We were able to address both the incidence and the efficiency cost of the existing corporate tax design relative to alternative designs. More comprehensive analyses followed in subsequent publications. The initial U.K. work was a comprehensive evaluation of the 1973 U.K. tax reform published in 1975 (Whalley, 1975).

These early tax policy evaluations stimulated the U.S. Treasury to undertake a major effort to develop an applied general equilibrium model of the American economy to be used for its tax policy evaluations. The U.S. tax model was laid out in full in Ballard, Fullerton, Shoven and Whalley (1985). This model and its successors were used to evaluate all sorts of tax reform proposals, including the landmark 1986 tax reform, possible schemes to integrate the U.S. personal and corporate tax systems and the possible adoption of a Value Added tax in the U.S.

Tax policy was only the first application. The evaluation of trade policy was equally important. Some of the relatively early contributions are presented in Srinivasan and Whalley (1986). Both AGE techniques and a summary of early tax and international trade applications are contained in Shoven and Whalley (1984). Development applications were constructed at the World Bank and by many academics. One presentation of the World Bank research agenda is contained in Dervis, De Melo, and Robinson (1982).

Herb Scarf was both a consumer and an advisor on all of these applications and extensions of the general equilibrium toolkit. He co-organized a conference and co-edited a book that contained advancements in computational algorithms, state of the art work on the numerical specification of general equilibrium models, the addition of money and bonds into the model, and applications to development, energy economics, the Mexican and Australian economies, and U.S. tax policy evaluations (Scarf and Shoven, 1984). By the mid-1980s, the applied general

equilibrium models were workhorse tools for economic policy evaluation. What Barone, Robbins and Hayek had said was impossible had been accomplished. As Glenn Hubbard said, the Scarf algorithm made much of modern economic policy analysis possible. A more recent compendium of applied general equilibrium contributions appears in Kehoe, Srinivasan and Whalley (2005). Among other chapters it includes an evaluation of global climate change and an assessment of the influence of computable general equilibrium models on policy. Scarf allowed the general equilibrium model to transition from pure theory to a powerful policy-relevant tool.

What we have tried to do in this brief essay is put Scarf's contributions to the development of the general equilibrium model in context. The model was formulated by Leon Walras, who laid out the whole structure of individual consumers optimizing their participation in the economy taking prices as given, of producers likewise optimizing their behavior given prevailing prices. Next, three generations later, we have the breakthrough existence proofs of Arrow, Debreu and McKenzie in 1954. Scarf then makes the model applicable to policy for the first time with his computational algorithm laid out in 1967 and 1973. And, finally, the two of us along with Kehoe and other Scarf students developed the tools necessary to use the model and a whole host of researchers used the model in a large number of important applications. In our opinion, the Mount Rushmore of general equilibrium has to include Walras, Arrow, Debreu, McKenzie and, most definitely, Herbert Eli Scarf.

On a final and more personal note, Herb Scarf was a magnificent teacher, a great thesis advisor, a constant supporter of our careers and a very good friend. We will miss him greatly.

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