# Supplements to “Demographic Transition, Industrial Policies, and Chinese Economic Growth”

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# A Appendix

## A1.1 First Order Conditions with Households' Fertility Decision and The Construction of Fertility Wedges

At the fertility age $J_f$, the household solves the following problem,

$$V_B(\Lambda, j, a, h, h_c) = \max_{c,a',n}\{\frac{c^{1-\sigma}}{1-\sigma} + \nu_B log(n_B) + \beta EV_B(\Lambda, j+1, a', h, h_c')\}$$

$$s.t.\ c + a' + T_{B,j} \leq a + y_{\Lambda,B,j}.$$

The first-order condition associated with its fertility decision is,

$$\sum_{j=J_f}^{J_B} \frac{\beta^{j-J_1}\nu_B}{n_B} = LM\{\sum_{j=J_f}^{J_f+J_1} \frac{\phi_1 E_{B,j}}{\prod_{i=J_1+1}^{j}(1+r_i)} + \sum_{j=J_f+7}^{J_f+J_1} \frac{i_{h,j} w_{B+j}}{\prod_{i=J_1+1}^{j}(1+r_i)} - \sum_{j=J_r}^{J_B} \frac{\mu_0\mu_1 n_B^{\mu_1-1} h_{c,J_1} e_{j-J_f} w_{B+j}}{\prod_{i=J_1+1}^{j}(1+r_i)}\}, \tag{33}$$

where $LM$ is the Lagrange multiplier associated with the budget constraint the household faces. To obtain the Lagrange multiplier, we use the first-order condition on consumption,

$$\beta^{j-J_1} c_j^{-\alpha} = \frac{LM}{\prod_{i=J_1+1}^{j}(1+r_i)}.$$

Combining those two equations above gives us a simplified equation that captures the determinants of fertility decisions,

$$\sum_{j=J_f}^{J_B} \frac{\beta^{j-J_1}\nu_B}{n_B} = c_{J_1}^{-\alpha}\{\sum_{j=J_f}^{J_f+J_1} \frac{\phi_1 E_{B,j}}{\prod_{i=J_1+1}^{j}(1+r_i)} + \sum_{j=J_f+7}^{J_f+J_1} \frac{i_{h,j} w_{B+j}}{\prod_{i=J_1+1}^{j}(1+r_i)} - \sum_{j=J_r}^{J_B} \frac{\mu_0\mu_1 n_B^{\mu_1-1} h_{c,J_1} e_{j-J_f} w_{B+j}}{\prod_{i=J_1+1}^{j}(1+r_i)}\}. \tag{34}$$

When the fertility control policy in China is binding, the above question does not hold, allowing us to measure the distortion on fertility outcome as the fertility wedge. More specifically, in the benchmark economy, we add a wedge on the transfer received by elderly parents so that the amount received after tax is:

$$(1 - wedge) \sum_{j=J_r}^{J_B} \frac{\mu_0 \mu_1 n_B^{\mu_1 - 1} h_{c,J_1} e_{j-J_f} w_{B+j}}{\prod_{i=J_1+1}^{j} (1 + r_i)}.$$

We then calculate the wedge to make the FOC hold. In other words, the wedge is the implicate tax rate so that the government-imposed fertility control policy is optimal fertility under adjusted transfer. We present the time path of the fertility wedges in SA Figure A4. Note that the wedge is huge initially because of the steep drops in fertility rates and remains at very high levels until 1995. The wedge turns slightly negative after the mid-2000s.

## A1.2 Additional Figures

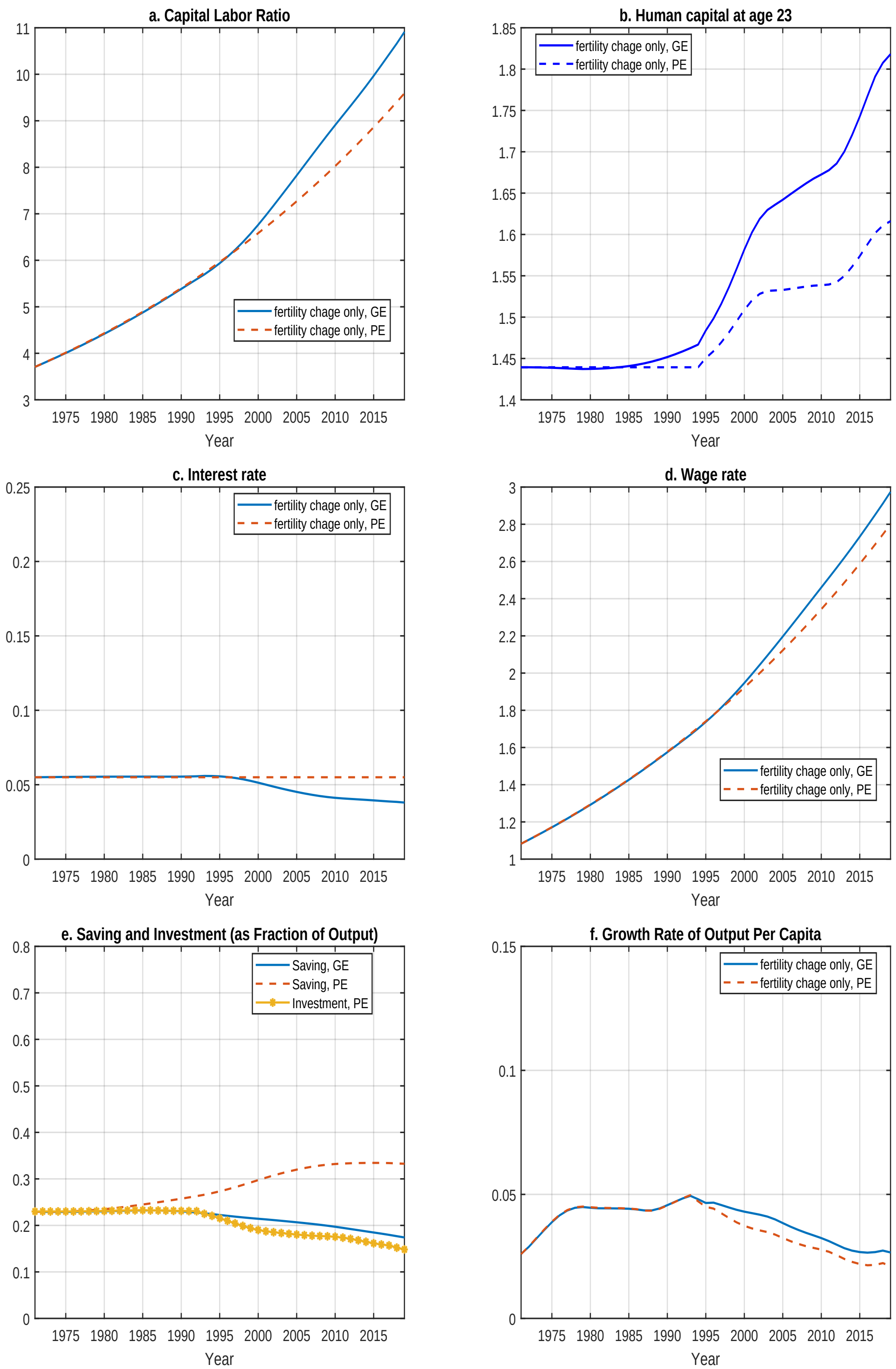


Figure A1: **Selected Statistics with Fertility Changes Only: General Equilibrium vs Partial Equilibrium**

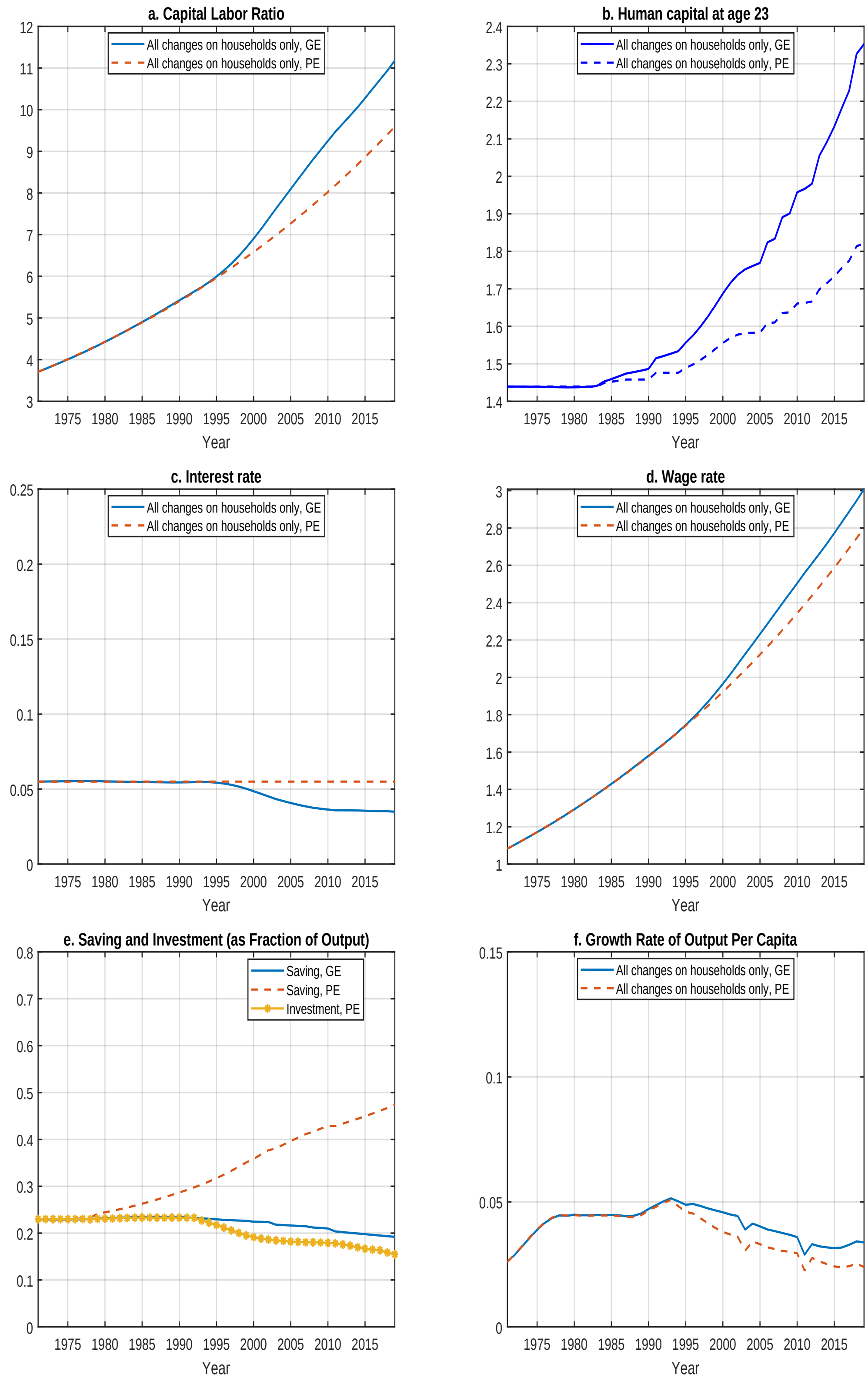


Figure A2: **Selected Statistics with Household Side Changes Only: General Equilibrium vs Partial Equilibrium**

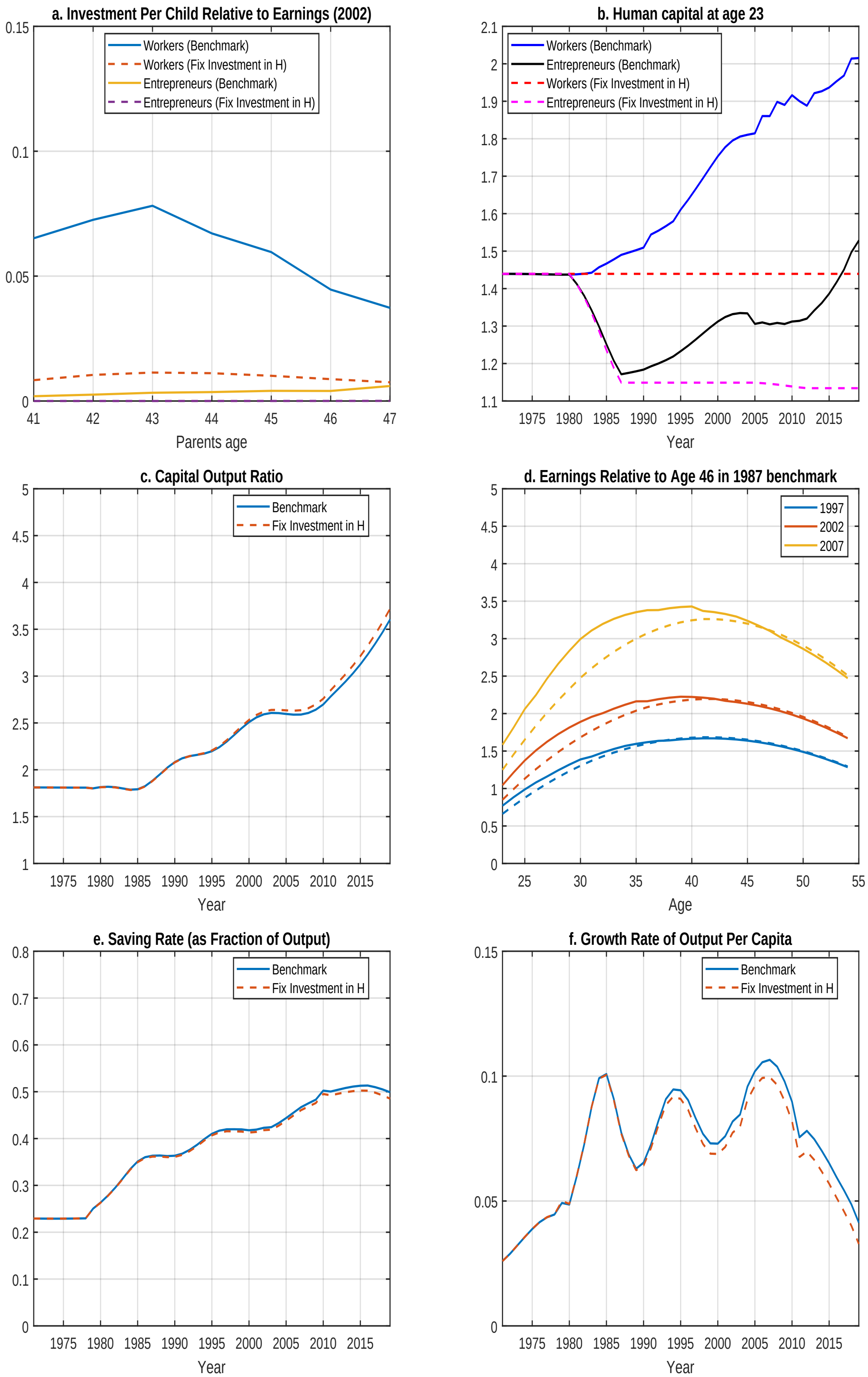


Figure A3: **Selected Economic Statistics: Benchmark versus Exogenous Human Capital Investment** In the exogenous human capital investment experiment, we keep optional education expense per child as a fraction of parent earnings the same as in the initial balanced growth path. The solid lines depict the benchmark economy and the dashed lines depict the experiment.

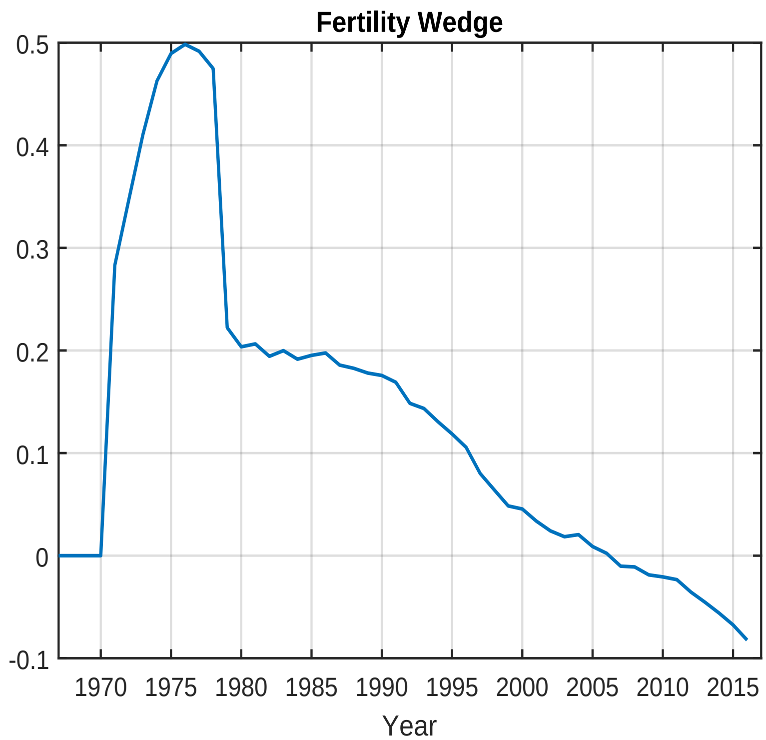


Figure A4: **Fertility Wedge** Fertility wedge is the implicate tax rate so that the government-imposed fertility control policy coincides with the optimal fertility given the after-tax transfers to the parents. See Section 8 and Appendix A1.1 for more details.

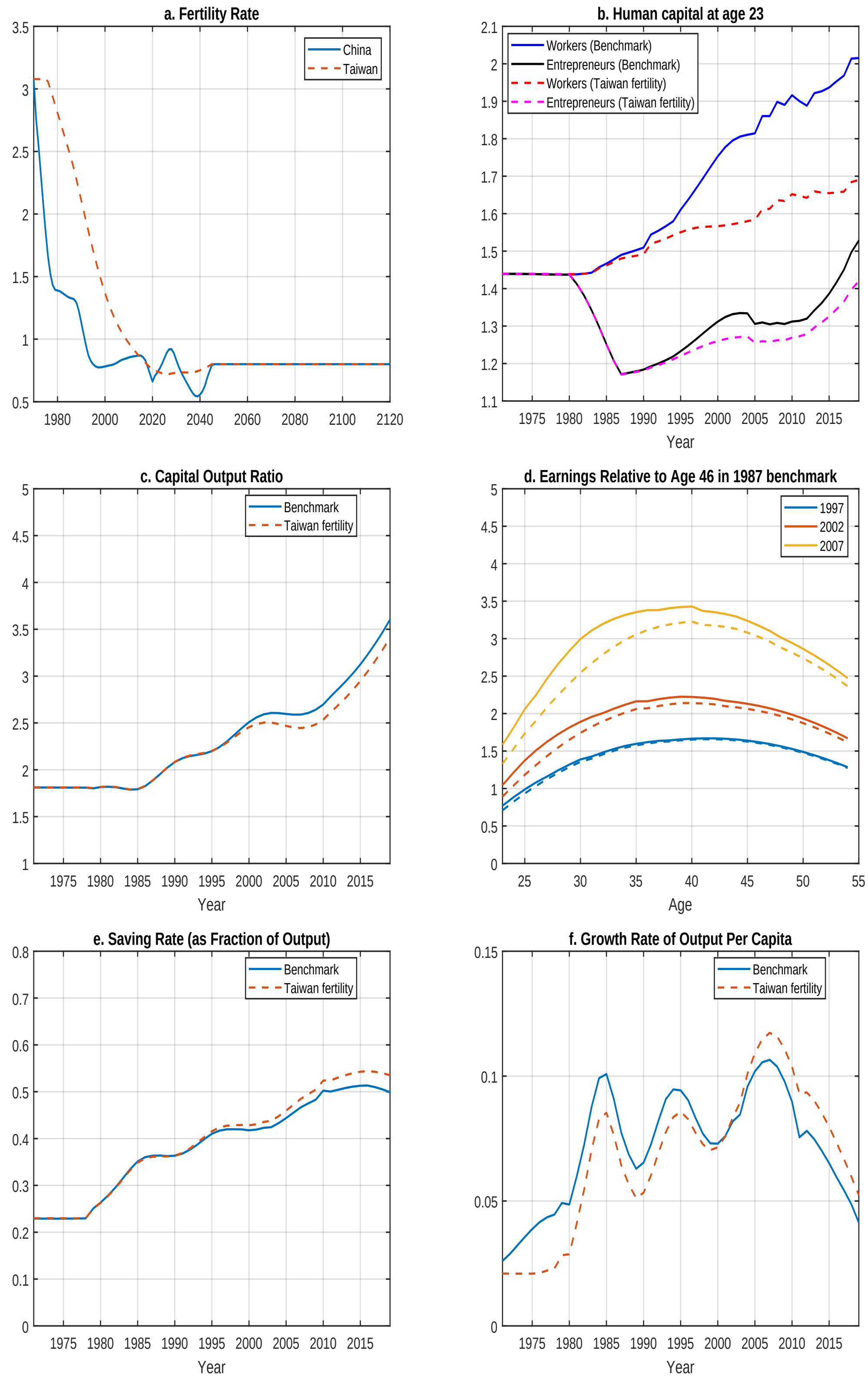


Figure A5: **Selected Economic Statistics: Benchmark versus Endogenous Fertility Decision** In the experiment, we endogenize fertility decision so that it matches fertility rates of Taiwan at similar levels of per capita GDP as detailed in Section 8. The solid lines depict the benchmark economy and the dashed lines depict the experiment.