Supplement to "Incentive contracts when agents distort probabilities"

(*Quantitative Economics*, Vol. 15, No. 3, July 2024, 607–653)

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Appendix B: Theoretical extensions

B.1 *Deterministic output*

Assume that y is deterministic. Specifically, let $y = e$. In that case, both agent and principal know that a higher output is due to higher effort. The question is whether higher effort can be elicited when the principal chooses a stochastic contract instead of a standard deterministic one. As in main body of the paper, denote the deterministic contract by $t(y)$.

Given the deterministic nature of output, Assumption 2 and Assumption 3 are no longer necessary. Moreover, it is assumed that the agent's cost of effort and preferences are still governed by Assumption 1 and Assumption 4, respectively. All in all, the agent's preferences are given by

$$U\big(t(y), e\big) = u\big(t(y)\big) - c(e). \tag{37}$$

The objective function of the principal is given by

$$\Pi(t, e) = S(y) - t(y) \tag{38}$$

where $S(y)$ is a continuous and strictly concave function.

The principal could also choose to introduce risk in this otherwise deterministic setting by paying with a stochastic contract $L = (T(y), r; 0, 1-r)$. In that case, the preferences of the decision maker are given by

$$\mathrm{RDU}(L, e) = w(r)u\big(T(y)\big) - c(e), \tag{39}$$

where w adopts the properties of Assumption 5. The principal's objective function becomes

$$\mathbb{E}\big(\Pi(t, e)\big) = S(y) - pT(y). \tag{40}$$

Finally, I assume that the equivalence in Assumption 7 also holds in this setting.

The following result presents the condition under which the stochastic contract outperforms standard contracts $t(y)$. It turns out that the condition under which stochastic

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Available at http://qeconomics.org. https://doi.org/10.3982/QE2275

contracts outperform deterministic contracts is comparable to that presented in Proposition 2.

PROPOSITION 5. *Let Assumptions* 1, 4, *and* 7 *hold. The stochastic contract L improves upon any deterministic contract $t(y)$ if $w(r) > u(r)$.*

PROOF. First, I show that L leads to higher incentives as compared to t. Use Assumption 7 to rewrite equation (39) as follows:

$$\mathrm{RDU}(L,e) = w(r)u\left(\frac{t(y)}{r}\right) - c(y). \tag{41}$$

The optimal output level chosen by the agent with RDU preferences when working under L, y^{**}, satisfies the following first-order condition:

$$\frac{w(r)}{r}u'\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{dt\big(y^{**}\big)}{dy} - c'\big(y^{**}\big) = 0. \tag{42}$$

That y^{**} is a maximum requires the second-order condition to be negative. Formally,

$$\frac{w(r)}{r^2}u''\left(\frac{t\big(y^{**}\big)}{r}\right)\left(\frac{dt\big(y^{**}\big)}{dy}\right)^2 + \frac{w(r)}{r}u'\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{d^2t\big(y^{**}\big)}{dy^2} - c''\big(y^{**}\big) < 0. \tag{43}$$

Instead, if the principal chooses to implement $t(y)$, the agent would choose to produce, y^*, which is the output level that satisfies the condition:

$$u'\big(t\big(y^*\big)\big)\frac{dt\big(y^*\big)}{dy} - c'\big(y^*\big) = 0. \tag{44}$$

That y^* is a maximum requires the second-order condition to be negative. Formally,

$$u''\big(t\big(y^*\big)\big)\left(\frac{dt\big(y^*\big)}{dy}\right)^2 + u'\big(t\big(y^*\big)\big)\frac{d^2t\big(y^*\big)}{dy^2} - c''\big(y^*\big) < 0. \tag{45}$$

Note that when $r = 1$, then $y^{**}|_{r=1} = y^*$; in the absence of risk t and L yield the same performance. Therefore, my analysis focuses on how changes in r alter y^{**}. To that end, implicitly differentiate (42) with respect to y^{**} and r to obtain

$$\frac{\mathrm{d}y^{**}}{\mathrm{d}p} = \frac{\left(\frac{w'(r)}{r} - \frac{w(r)}{r^2}\right)u'\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{dt\big(y^{**}\big)}{dy} - \frac{w(r)}{r^3}u''\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{dt\big(y^{**}\big)}{dy}}{c''\big(y^{**}\big) - \frac{w(r)}{r^2}u''\left(\frac{t\big(y^{**}\big)}{r}\right)\left(\frac{dt\big(y^{**}\big)}{dy}\right)^2 - \frac{w(r)}{r}u'\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{d^2t\big(y^{**}\big)}{dy^2}}. \tag{46}$$

A necessary condition for equation (46) to be negative at the optimum, so that choosing $r = 1$ is not optimal, is

$$\frac{w'(r)}{w(r)} \le \frac{1}{r}\left(1 - \frac{u''\left(\frac{t\big(y^{**}\big)}{r}\right)\frac{t\big(y^{**}\big)}{r}}{u'\left(\frac{t\big(y^{**}\big)}{r}\right)}\right). \tag{47}$$

The class of probability weighting functions $w(r)$ that satisfy (47) are found using Grönwall's lemma. First, find the solution to

$$\frac{v'(r)}{v(r)} = \frac{1}{r}\left(1 - \frac{u''\left(\frac{t(y^{**})}{r}\right)\frac{t(y^{**})}{r}}{u'\left(\frac{t(y^{**})}{r}\right)}\right), \tag{48}$$

where $v(r)$ is a weighting function with the properties of Assumption 5, that is, it has the same properties as $w(p)$. The solution to (48) is given by

$$\begin{aligned}
\int \frac{v'(r)}{v(r)}\,\mathrm{d}r &= \int \frac{1}{r}\left(1 - \frac{u''\left(\frac{t(y^{**})}{r}\right)\frac{t(y^{**})}{r}}{u'\left(\frac{t(y^{**})}{r}\right)}\right)\mathrm{d}r \\
\Leftrightarrow \quad \ln\big(v(r)\big) &= \ln(r) - \ln\left(u'\left(\frac{t}{r}\right)\right) + \ln\big(u'(t)\big) \\
\Leftrightarrow \quad \frac{v(r)}{r} &= \frac{u'(t)}{u'\left(\frac{t}{r}\right)}.
\end{aligned} \tag{49}$$

Solving for t in (49) gives

$$\begin{aligned}
\frac{v(r)}{r}u'\left(\frac{t}{r}\right) = u'(t) \quad \Leftrightarrow \quad & \int \frac{v(r)}{r}u'\left(\frac{t}{r}\right)\mathrm{d}t = \int u'(t)\,\mathrm{d}t \\
\Leftrightarrow \quad & v(r) = \frac{u(t)}{u\left(\frac{t}{r}\right)}.
\end{aligned} \tag{50}$$

Second, the way in which $w(r)$ and $v(r)$ relate is investigated by computing the derivative of the ratio $\frac{w(r)}{v(r)}$:

$$\begin{aligned}
\frac{\mathrm{d}\left(\frac{w(r)}{v(r)}\right)}{\mathrm{d}p} &= \frac{\big(v(r)w'(r) - w(r)v'(r)\big)}{v(r)^2} \\
&= \frac{v(r)\left(w'(r) - \frac{w(r)}{r}\left(1 - \frac{u''\left(\frac{t(y^{**})}{r}\right)\frac{t(y^{**})}{r}}{u'\left(\frac{t(y^{**})}{r}\right)}\right)\right)}{v(r)^2},
\end{aligned} \tag{51}$$

where the second equality results from replacing $v'(r)$ with (48). Using the last equality of the above equation together with (47), it can be established that $\frac{\mathrm{d}(\frac{w(r)}{v(r)})}{\mathrm{d}r} \leq 0$. Thus, the minimum of $\frac{w(r)}{v(r)}$ is attained at $r = 1$ and it must be that for any $r \in (0, 1]$:

$$\frac{w(r)}{v(r)} \geq \frac{w(1)}{v(1)} = 1. \tag{52}$$

Equation (52) implies that the solution to (47) must be bounded by (50) in the following way:

$$w(r) \geq \frac{u(t)}{u\left(\frac{t}{r}\right)}. \tag{53}$$

Using the fact that $T(y) = \frac{t(y)}{r}$, the above equation can be rewritten as

$$w(r) \geq \frac{u\big(rT(y)\big)}{u\big(T(y)\big)}. \tag{54}$$

The above equation is similar to that given in (16), so I can proceed as in Proposition 1; the condition $w(r) > u(r)$ implies the inequality in (10). Hence, if $w(r) > u(r)$ holds for some nonempty interval in $r \in (0, 1)$, then $\frac{\mathrm{d}y^{**}}{\mathrm{d}r} \leq 0$ and the principal derives higher incentives from offering L.

Next, I show that L can generate higher utility than $t(y)$. So, the agent is more willing to work under L than under $t(y)$. This happens whenever

$$\mathrm{RDU}(L, e) > U\big(t(y), e\big) \quad \Leftrightarrow \quad w(r)u\big(T(y)\big) \geq u\big(t(y)\big) \quad \Leftrightarrow \quad \frac{u\big(rT(y)\big)}{u\big(T(y)\big)}. \tag{55}$$

Again, this condition is similar to that given in (16), so I can proceed as in Proposition 1. Namely it can be shown that $w(r) > u(r)$ implies the inequality in (55).

All in all, if $w(r) > u(r)$, then L is implemented by the principal instead of $t(y)$; it generates stronger incentives and motivates participation more often at a lower cost for the principal. □

The principal is better off introducing risk in the agent's environment when his risk-probabilistic–risk-seeking preferences outweigh the risk aversion emerging from utility curvature. That is, when $w(r) > u(r)$. In that case, the agent is globally risk seeking, has a preference for risky contracts, and would thus prefer to face risk in a situation in which he would otherwise not face any.

B.2 *Binary effort*

This subsection considers the special case in which effort is binary. Formally, the agent's action consists on exerting a high effort level or a low effort level $e = \{e_L, e_H\}$. In this setting, only high effort is costly.

Assumption 8 (Cost of effort).

$$c(e) = \begin{cases} c & \textit{if } e_H, \\ 0 & \textit{if } e_L, \end{cases}$$

where $c > 0$.

Output is still assumed to be a random variable taking any value in the interval $y \in [\underline{y}, \bar{y}]$. The simplifying assumption that effort is binary leads to less stringent assumptions on the density function of output. The following assumption relaxes Assumption 2.

Assumption 9 (Output distribution). $F(y|e) : [\underline{y}, \bar{y}] \to [0, 1]$ *admits a density function* $f(y|e)$.

The relationship between effort and output is still given by the monotone likelihood ratio property, which is now captured by the following assumption.

Assumption 10 (Monotone likelihood ratio property). *Effort and output relate according to* $\frac{\partial}{\partial y}(\frac{f(y|e_H)}{f(y|e_L)}) \geq 0$.

In the considered setting, the principal's program is

$$\begin{aligned} \max_{\{t(y)\}} \quad & \int_{\underline{y}}^{\bar{y}} \big(S(y) - t(y)\big) f(y|e_H)\, \mathrm{d}y \\ \text{s.t.} \quad & \int_{\underline{y}}^{\bar{y}} u(t) w'\big(1 - F(y|e_H)\big) f(y|e_H)\, \mathrm{d}y - c \geq \bar{U}, \\ & \int_{\underline{y}}^{\bar{y}} u(t)\big(w'\big(1 - F(y|e_H)\big) f(y|e_H) - w'\big(1 - F(y|e_L)\big) f(y|e_L)\big)\, \mathrm{d}y - c. \end{aligned} \tag{56}$$

The following proposition presents the condition under which the stochastic contract outperforms standard contracts $t(y)$.

Proposition 6. *Let Assumptions* 4–10 *hold. The stochastic contract* L *improves upon any deterministic contract* $t(y)$ *if* $w(r) > u(r)$.

Proof. Denote by t^{fo} the standard solution to the principal's problem (equation (56)) derived from the first-order approach. For the optimality of $L = (r, T(y); 1 - r, 0)$, it suffices that for any generic t^{fo} the following inequality holds:

$$\begin{aligned} & \mathrm{RDU}(L, e) > \mathrm{RDU}\big(t(y), e\big) \\ \Leftrightarrow \quad & w(r) \int_{\underline{y}}^{\bar{y}} u\big(T(y)\big) w'\big(1 - F(y|e)\big) f(y|e)\, \mathrm{d}y > \int_{\underline{y}}^{\bar{y}} u\big(t^{\text{fo}}(y)\big) w'\big(1 - F(y|e)\big) f(y|e)\, \mathrm{d}y. \end{aligned} \tag{57}$$

Equation (57) holds if the following inequality holds for any output realization $y^* \in [\underline{y}, \bar{y}]$:

$$w(r)u\big(T(y^*)\big) > u\big(t^{\text{fo}}(y^*)\big) \quad \Leftrightarrow \quad w(r) > \frac{u\big(t^{\text{fo}}(y^*)\big)}{u\big(T(y^*)\big)}. \tag{58}$$

Using Assumption 7, the inequality above can be rewritten as

$$w(r) > \frac{u\big(rT(y^*)\big)}{u\big(T(y^*)\big)} \quad \Rightarrow \quad w(r) > u(r)\frac{u\big(T(y^*)\big)}{u\big(T(y^*)\big)} \quad \Leftrightarrow \quad w(r) > r, \tag{59}$$

where the second implication is due to the concavity of u (Assumption 4). Note that $u(r) > r$ due to the concavity of u, $u(0) = 0$, and the fact that $u(M_2) = 1 > u(1)$. As such, $w(r) > u(r)$ immediately implies $w(r) > r$. □

Proposition 6 shows that the result from Proposition 2 holds when the effort space is binarized. This modeling approach is often followed in the literature to avoid making strong assumptions on the shape of the distribution function (See Assumption 2). For instance, because some distributions cannot comply with the convexity requirement $F_{ee}(y|e) > 0$. The result in Proposition 6 shows that the effectiveness of stochastic contracts go beyond imposing that assumption. Hence, the result from Proposition 2 is not an artifact of the assumptions on the density function, but a consequence of the agent's risk preferences.

Appendix C: Agents with CPT preferences

In this Supplemental Appendix, I analyze the incentives generated by stochastic contracts when agents have risk preferences characterized by CPT (Tversky and Kahneman (1992)). I find that under mild additional conditions, the result stated in Proposition 2 holds: stochastic contracts that expose the agent to large amounts of risk can generate higher output than linear piece-rate contracts. This finding is not surprising since CPT incorporates probability distortions in the same way as RDU.

Agents with CPT preferences evaluate possible outcomes in the stochastic contract relative to a reference point $R \geq 0$. Outcomes below the reference point are coined *losses* and outcomes above it are *gains*. In the original formulation of CPT, R represents the status quo, or the monetary amount that the agent owns and is thus exogenous to the principal's choice. In the following, I adopt the assumption that the reference point is exogenous to the principal's offer.

The main difference of CPT with respect to RDU is that the agent can exhibit different risk preferences in the domain of gains and the domain of losses. This is partly because outcomes are evaluated with a value function that exhibits the following properties.

Assumption 11. *$V(t(y), R)$ is the piecewise function,*

$$V(t, R) = \begin{cases} u\big(t(y) - R\big) & \text{if } t(y) \geq R, \\ -\lambda u\big(R - t(y)\big) & \text{if } t(y) < R \end{cases}$$

with $R \geq 0$, $\lambda > 1$, $u(0) = 0$, $u(M_2) = 1$, *and* u *is strictly increasing in* t, $u_t > 0$, *and concave in* t, $u_{tt} \leq 0$.

In words, the value function, V, is an increasing function that is concave in the domain of gains and convex in the domain of losses. This S-shaped utility function generates risk averse and risk seeking attitudes in gains and losses, respectively. Additionally, the worker is loss averse, that is, losses loom larger than equally-sized gains. This property is captured by the parameter $\lambda > 1$, which only enters the value function for the domain of losses.

Like the RDU agent, a CPT agent transforms probabilities using a probability weighting function. However, transformations of probability can be different for gains and losses. Let $w(p)$ with the properties from Assumption 5 be the probability weighting function used to transform probabilities in the domain of gains.

Let $z(p)$ be the probability weighting function used to transform probabilities in the domain of losses. To simplify matters, I assume that $w(p)$ and $z(p)$ relate through the duality $z(p) = 1 - w(1 - p)$. Hence, the weighting function for losses adopts the same properties as that for gains, but differs in that probability transformations are applied to loss ranks, or a ranking of outcomes from least-desirable to most-desirable, rather than to gain ranks.[33]

All in all, the utility of the agent with CPT preferences when offered $t_s(y)$ is equal to

$$\mathrm{CPT}\big(t_s(y), t\big) = \begin{cases} w(p)u\big(t(y) - R\big) - c(e) & \text{if } t \geq R \geq 0, \\ -z(p)\lambda u\big(R - t(y)\big) - c(e) & \text{if } R > t > 0. \end{cases} \tag{60}$$

I am in a position to compare the two contracts with respect to the output that they deliver. Proposition 7 provides the conditions under which the principal is better off exposing the agent to large amounts of risk with the stochastic contract.

Proposition 7. *Let Assumptions* 1–7, *and* 11 *hold. Contract* L *improves upon any contract* $t(y)$ *if and only if* $w(r) > u(r)$.

Proof. Let $w(r) > u(r)$ and $t \geq R$. For the optimality of $L = (r, t(y); 1 - r, 0)$, it suffices that for any generic contract $t(y)$,

$$\begin{aligned} & \mathrm{CPT}(L, e) > \mathrm{CPT}\big(t(y), e\big) \\ \Leftrightarrow \quad & w(r) \int_{\underline{y}}^{\bar{y}} u\big(T(y) - R\big) w'\big(1 - F(y|e)\big) f(y|e)\, \mathrm{d}y \\ & \quad > \int_{\underline{y}}^{\bar{y}} u\big(t(y) - R\big) w'\big(1 - F(y|e)\big) f(y|e)\, \mathrm{d}y. \end{aligned} \tag{61}$$

[33]Formally, an agent with CPT preferences facing a lottery $(x_1, p_1; x_2, p_2; \ldots; x_n, p_n)$ ranks the outcomes using an increasing arrangement $x_1 < x_2 < \cdots x_{r-1} < r < x_{r+1} \cdots < x_n$ and evaluates the outcomes of the lottery relative to r through the function $v(y, r)$. The lottery outcomes $x_{r+1}, \ldots x_n$ are gains and the outcomes $x_1, \ldots x_{r-1}$ are losses. The individual assigns decision weights to gains in the following way: $\pi_n = w(p_n)$, $\pi_{n-1} = w(p_{n-1} + p_n) - w(p_n)), \ldots, \pi_{r+1} = 1 - \sum_{j=r+1}^{n} w(p_j)$, and assigns decision weights to losses in the following way: $\pi_1 = z(p_1)$, $\pi_2 = z(p_1 + p_2) - z(p_1)), \ldots, \pi_{r-1} = 1 - \sum_{j=r-1}^{n} z(p_j)$.

the condition in (61) holds if for any realization $\tilde{y} \in [\underline{y}, \bar{y}]$ the following inequality holds:

$$w(r)u\big(T(\tilde{y}) - R\big) > u\big(t(\tilde{y}) - R\big) \quad \Leftrightarrow \quad w(r) > \frac{u\big(t(\tilde{y}) - R\big)}{u\big(T(\tilde{y}) - R\big)}. \tag{62}$$

Using Assumption 7, the above inequality can be rewritten as

$$w(r) > \frac{u\big(rT(\tilde{y}) - R\big)}{u\big(T(\tilde{y}) - R\big)} \quad \Rightarrow \quad w(r) > r\frac{u\left(T(\tilde{y}) - \frac{R}{r}\right)}{u\big(T(\tilde{y}) - R\big)}, \tag{63}$$

where the second equivalence is due to the concavity of u (Assumption 11). Since $\frac{R}{r} > R$ and $u_t > 0$ (Assumption 11), then $0 < \frac{u(T(\tilde{y}) - \frac{R}{r})}{u(T(\tilde{y}) - R)} < 1$. Hence, $w(r) > u(r)$, which in turn implies $w(r) > r$, suffices for the inequality in equation (63) to hold.

Let now $T < R$. In that case, the stochastic contract yields higher utility if

$$\begin{aligned}
&\mathrm{CPT}(L, e) > \mathrm{CPT}\big(t(y), e\big) \\
\Leftrightarrow \quad & -z(r)\lambda \int_{\underline{y}}^{\bar{y}} u\big(R - T(y)\big)w'\big(1 - F(y|e)\big)f(y|e)\,\mathrm{d}y \\
& > -\lambda \int_{\underline{y}}^{\bar{y}} u\big(R - t(y)\big)w'\big(1 - F(y|e)\big)f(y|e)\,\mathrm{d}y.
\end{aligned} \tag{64}$$

The condition in (64) holds if for any given realization $\tilde{y} \in [\underline{y}, \bar{y}]$ the following inequality holds:

$$-z(r)u\big(R - T(\tilde{y})\big) > -u\big(R - t(\tilde{y})\big) \Leftrightarrow z(r) < \frac{u\big(R - t(\tilde{y})\big)}{u\big(R - T(\tilde{y})\big)}. \tag{65}$$

Since $t(\tilde{y}) < T(\tilde{y})$ and $u_t > 0$ (Assumption 11), then $\frac{u(R - t(\tilde{y}))}{u(R - T(\tilde{y}))} > 1$. Therefore, the inequality in equation (65) holds for any r.

Consequently, contract L leaves the participation and incentive compatibility constraint unchanged as compared to offering a generic contract $t(y)$. It provides the same utility and incentives (equation (63)) at the same costs for the principal (Assumption 7).

Let L be optimal. This part of the proof is identical to that presented in Proposition 1; see equations (18)–(22). □

As with RDU preferences, the principal derives greater motivation using the stochastic contract. This result emerges when the agent sufficiently overweights the probability specified by the principal, so that the probabilistic risk seeking attitude of the agent outweighs the risk averse attitudes stemming from his value function. When L is evaluated as a gain, the lower bound of probability overweighting to be attained is $u(r)$. This is the same requirement as in the RDU case presented in Section 2. Instead, when L is a loss, that lower bound is no longer necessary but suffices to implement risk seeking.

To conclude this Supplemental Appendix, I comment on the role of loss aversion and diminishing sensitivity, two factors that determine risk attitude under CPT preference and that are absent under RDU. The coefficient of loss aversion, λ, does not enter in the requirement of Proposition 5. Therefore, that the agent exhibits strong or moderate loss aversion is immaterial to the effectiveness of the stochastic contract.

The second component is diminishing sensitivity, that is, that the value function is concave for gains and convex for losses. This property implies that the requirement on probability overweighting to ensure risk seeking is less stringent in losses. Intuitively, the convexity of the value function in the domain of losses generates risk seeking and facilitates that the agent becomes more motivated with a contract that introduces risk.

Appendix D: Utility functions

This Supplemental Appendix investigates the properties of the elicited utility functions. Decision sets 1 to 6 of the second part of the experiment are designed to elicit the sequence of outcomes $\{x_1, x_2, x_3, x_4, x_5, x_6\}$ for each subject. This elicited sequence has the relevant property that it ensures equally-spaced utility values, that is, $u(x_j) - u(x_{j-1}) = u(x_{j-1}) - u(x_{j-2})$, allowing me to characterize a subject's preference over monetary outcomes by mapping each utility value, $u(x_j)$ to the subject's stated preference x_j.

I focus on two properties of the utility function: the sign of the slope and the curvature. To that end, I construct two variables. The first variable is $\Delta'_i := x_j - x_{j-1}$, for $j = 1, \dots, 6$ and the second is $\Delta''_j := \Delta'_j - \Delta'_{j-1}$ for $i = 2, \dots, 6$. The sign of Δ'_j as j increases determines the sign of the slope, that is, whether a subject prefers larger monetary outcomes to smaller monetary outcomes. Similarly, the sign of Δ''_j as j increases determines the utility curvature. For example, a subject with $\Delta'_j > 0$ and $\Delta''_j > 0$ for all j exhibits a preference for larger monetary outcomes and experiences smaller utility increments with larger monetary outcomes. This is equivalent to say that this subject has an increasing and concave utility function.

The first analysis focuses on classifications at the individual level. I classify subjects according to the curvature of their utility function. Since I have multiple observations for each subject and it was possible that subjects made mistakes, this classification is based on the sign of Δ''_j with the most occurrence. Specifically, a subject with at least three negative Δ''_js was classified as having a convex utility, a subject with at least three positive Δ''_js had a concave utility and subject with three or more Δ''_is had a linear utility. A subject with a utility function that cannot be classified as concave, convex, or linear, had a mixed utility. Furthermore, to statistically assess the sign of a Δ''_j, I construct confidence intervals around zero. In particular, I multiply the standard deviation of each Δ''_j by the factors 0.64 and -0.64. Thus, if Δ''_j follows a normal distribution, 50% of the data should lie within the confidence interval.[34]

[34] More stringent confidence intervals were also used for the analysis. These confidence intervals were also constructed using the standard deviation of a Δ''_j, which was multiplied by different factors, such as 1 and -1, 1.64 and -1.64, and 2 and -2. The qualitative results of these analyses are not different from the main result presented here that the majority of subjects exhibit a linear utility function. This is not surprising inasmuch as these confidence intervals are more stringent and yield less subjects classified as having a mixed utility function and more subjects exhibiting a linear utility function.

TABLE 7. Classification of subjects according to utility curvature.

Shape	Convex	Concave	Linear	Mixed	Total
Observations	3	13	133	23	172

Note: This table presents the subjects classification according to the shape of their utility function. Subjects are classified as having a convex, concave, linear, or mixed utility function based on the sign of $\Delta''_j := (x_j - x_{j-1}) - (x_{j-1} - x_{j-2})$ for $j = 2, 3, 4, 5, 6$. A subject has linear utility if most values of Δ''_j are close to zero, concave utility if most values of Δ''_j are positive, convex utility if most values Δ''_j are negative, and mixed utility otherwise.

The data suggest that all subjects in the experiment exhibit an increasing sequence $\{x_1, \dots, x_6\}$, which denotes, not surprisingly, a generalized preference for larger amounts of money. Table 7 presents the classification of subjects according to the curvature of their utility function. The data suggest that the majority of subjects exhibit linear utility functions. Specifically, 77% of the subjects have linear utility, while the rest of the subjects have mixed utility (13% of the subjects), and concave utility (7% of the subjects). A proportions test suggest that the proportion of subjects with linear utility is significantly larger than 50% ($p < 0.001$). Moreover, this test also yields that the proportion of subjects having linear functions is significantly larger than the proportion of subjects with mixed ($p < 0.001$) and concave utility ($p < 0.001$).

The result that more than two-thirds of the subjects exhibit linear utility is at odds with the principle of diminishing sensitivity, a key property of cumulative prospect theory (CPT). However, disregarding CPT as a possible representation for the subjects' preferences for money on the basis of this classification may be incorrect. As pointed out by Wakker and Deneffe (1996), their trade-off method, used to elicit $\{x_1, x_2, x_3, x_4, x_5\}$, requires lotteries with large monetary outcomes in order to obtain utility functions with curvature. Therefore, one of the advantages of the experimental design, that it elicits the utility function and the probability weighting function of subjects using monetary stakes that reflect the monetary incentives in the first part of the experiment, is also the reason that diminishing sensitivity is not be observed.

To understand how the aforementioned results aggregate, I analyze the sequence $\{x_1, \dots, x_6\}$ when each outcome x_j is averaged for all subject. Table 8 presents the descriptive statistics of the resulting outcomes. I find that the average outcome x_j is increasing with j, implying that on average subjects exhibit a taste for larger monetary outcomes. Moreover, the column displaying the average values of the variable Δ'_j shows that as j increases, increments of x_j become larger. Thus, while on average subjects exhibit linear utility, this tendency ceases as monetary outcomes in the lotteries become larger. In fact, for large values of x_j the average utility function displays concavity. This result is also found by Abdellaoui (2000).

The last analysis of the data consists on fitting well-known parametric families of utility functions. Specifically, I assume a power utility, belonging to the CRRA family of utility functions, and an exponential function, belonging to the CARA family of utility functions. Table 9 the regression estimates when nonlinear least squares is used to fit the data to the assumed utility function. For the two parametric specifications, I find that the average utility function of the subjects is approximately linear. For instance,

TABLE 8. Aggregate results x_1, x_2, x_3, x_4, x_5, and x_6.

Variable	Mean	Std. dev.	Mean Δ'_j
x_1	2.580	1.996	1.580
x_2	4.574	4.446	1.994
x_3	6.684	6.792	2.110
x_4	9.179	9.421	2.495
x_5	11.774	11.881	2.595
x_6	14.380	14.419	2.606

Note: This table presents the mean and standard deviation of each variable included in the sequence $\{x_1, x_2, x_3, x_4, x_5, x_6\}$. It also presents the average difference $\Delta'_j = x_j - x_{j-1}$ for each $j = 1, 2, 3, 4, 5, 6$.

when the power utility function $u(x) = x^k$ is assumed, the parameter attains a value of 0.995. This finding is consistent with the large proportion of subjects that were classified as having a linear utility function in the individual analysis and the modest increments that the averaged outcomes x_j exhibit as j increases presented in Table 8.

To ensure robustness, I perform the above analyses assuming that subjects' have CPT preferences. Their reference point was assumed to be the monetary equivalent of their performance belief. This analysis is performed to account for the possibility that subjects form a belief about how much they think they will earn in the first part of the experiment and evaluate potential earnings in the lotteries of the second part of the experiment relative to that amount. The experiment elicited the subject's performance beliefs in the first part of the experiment by asking them to provide an estimate of the amount of correct tasks they solved. If that belief was correct, they were given a small bonus otherwise they received nothing. Naturally, this question was asked before subjects received feedback about the performance on the task. I find that the result that

TABLE 9. Parametric estimates of average utility function.

Panel 1. $u(x) = \frac{1-\exp(-\gamma x)}{\gamma}$	
$\hat{\gamma}$	0.000 (0.000)
Adj. R^2	0.928
Observations	1032
Panel 2. $u(x) = (x)^k$	
$\hat{k}$	0.996 (0.002)
Adj. R^2	0.926
Observations	1032

Note: This table presents the estimates of nonlinear least squares regressions. The upper panel assumes the parametric form $u(x) = \frac{1-\exp(-\gamma x)}{\gamma}$ belonging to the CARA utility family. The lower panel assumes the parametric form $u(x) = (x)^k$ belonging to the CRRA utility family. Robust standard errors are presented in parentheses.

subjects have linear utility is robust to incorporating that reference point. Previous versions of the paper provided these analyses in more detail (see Gonzalez-Jimenez (2021)).

All in all, the data suggest that subjects have linear utility functions. This is not a surprising finding given the magnitude of the stakes used to elicit the subject's risk preferences. Furthermore, the conclusion that the utility function is linear implies that probability risk attitudes fully determine the risk attitudes. Implying that performance differences across treatments must be explained by probability distortions.

Appendix E: Individual analysis of probability weighting functions

This Supplemental Appendix presents alternative analyses of the probability weighting functions. Decision sets 7 to 11 included in the second part of the experiment were designed to elicit the subjects' weighting functions. In the main body of the paper, I present parametric analyses of the average data. In this Supplemental Appendix, I present non-parametric analyses of these data performed at the individual level.

The first analysis classifies each subject according to the shape of the elicited probability weighting function and is based on Bleichrodt and Pinto (2000). There were five possible shapes of the probability weighting function. A subject could display a weighting function with either lower subadditivity (LS), upper subadditivity (US), or both properties. These classifications result from comparing the behavior of the probability weighting function at extreme probabilities to the behavior of the same function at intermediate probabilities. Moreover, a subject could display a concave or a convex probability weighting function.

To classify a subject into one of these five categories, I created the variable $\partial^{j}_{j-1} := \frac{w(p_j)-w(p_{j-1})}{w^{-1}(p_j)-w^{-1}(p_{j-1})}$, which captures the average slope of the probability weighting function between probabilities j and $j-1$. I also created the variable $\triangledown^{j}_{j-1} \equiv \partial^{j}_{j-1} - \partial^{j-1}_{j-2}$, which represents the change of the average slope of the weighting function between successive probabilities.

To understand the subjects' behavior at extreme and intermediate probabilities, I focus on the sign of the variables $\triangledown^{0.33}_{0.16}$ and $\triangledown^{1}_{0.83}$. If a subject exhibits $\triangledown^{0.33}_{0.16} < 0$, his probability weighting exhibits LS. In other words, his probability weighting function assigns larger weights to small probabilities than to medium-ranged probabilities. Moreover, if a subject has $\triangledown^{1}_{0.83} > 0$, then his probability weighting function exhibits the property of US. That is, his weighting function assigns larger weights to large probabilities than to medium-ranged probabilities.

In addition, I examine the sign of $\triangledown^{j}_{j-1}$ as j increases to determine the shape of the weighting function of each subject over the whole probability interval. A subject was classified as having a concave weighting function if at least three (out of five) $\triangledown^{j}_{j-1}$ had a negative sign and he did not exhibit US. Alternatively, a subject had a convex probability weighting function if at least three (out of five) $\triangledown^{j}_{j-1}$ were positive and he did not exhibit LS. Note that these classifications allow for the possibility of response error.

Table 10. Classification of subjects according to the shape of their weighting function.

Shape	Convex	Concave	LS	US	LS and US
Observations	68	27	98	43	76

Note: This table presents the classification of subjects according to the shape of their probability weighting function. Subjects are classified as having a probability weighting function with upper subadditivity (US), lower subadditivity (LS), or both. Also, subjects are classified as having a convex or concave probability weighting function if they do not exhibit LS and US. This classification depends on the sign of ∇_{j-1}^{j} as explained in the text.

Table 10 presents the results of the individual classification. I find that 57% of subjects exhibit LS, 25% of subjects exhibit US, and 44% of subjects display probability weighting functions with both LS and US. Therefore, most subjects in the experiment had weighting functions that yield overweighting of small probabilities or underweighting of large probabilities. Also, almost half of subjects exhibit probability weighting functions that assign large weights to small and large probabilities. These proportions are however considerably lower than those reported by Bleichrodt and Pinto (2000). Moreover, I find that 39% of the subjects exhibit convex weighting functions and only 13% of the subjects exhibit concave weighting functions. Thus, more subjects in the experiment exhibit pessimism. Furthermore, the proportion of subjects in the experiment with either concave or convex probability weighting functions is higher than that reported by Bleichrodt and Pinto (2000), who finds that only 15% of the subjects have probability weighting functions with either of these shapes.

For the sake of robustness, I perform an alternative classification of LS and US also proposed by Bleichrodt and Pinto (2000). In comparison to the above classification, weights given to extreme probabilities are contrasted to the corresponding objective probability. In particular, a subject has a weighting function with LS if $w^{-1}(\frac{1}{6}) < 0.16$. Similarly, a subject has a weighting function with US if $1 - w^{-1}(\frac{5}{6}) < 0.16$. The resulting dummy variables LS and US or both were used in the main body of the paper to investigate the effect of these properties of the weighting function on the treatment effects.

The results of the alternative classification are presented in Table 11. I find that a similar proportion of subjects exhibit US and LS. Specifically, 40.12% of subjects exhibit LS and 38.37% subjects exhibit US. Also, only 20% of subjects exhibit both LS and US. These proportions are considerably lower than those obtained with the initial classification and are also smaller to those reported by Bleichrodt and Pinto (2000).

The last classification of probability weighting functions, evaluates the strength of the possibility effect relative to the certainty effect. A subject exhibits a weighting function with a possibility effect that is stronger than the certainty effect when $1 - w^{-1}(\frac{5}{6}) > w^{-1}(\frac{1}{6})$. Table 12 shows that the majority of subjects in the experiment have probability weighting functions with the certainty effect exceeding the possibility effect. This result

TABLE 11. Classification of subjects according to LS, US, or both.

Shape	LS	US	Both
Observations	55	89	25

Note: This table presents an alternative classification of subjects according to the shape of their weighting functions. Subjects are classified as having weighting functions with lower subadditivity (LS) if $w^{-1}(\frac{1}{6}) < w^{-1}(\frac{2}{6}) - w^{-1}(\frac{1}{6})$. Subjects have weighting functions with upper subadditivity (US) if $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{5}{6}) - w^{-1}(\frac{4}{6})$. When these two properties hold, subjects are classified in both.

is in line with the findings of Tversky and Fox (1995). Nevertheless, the proportion of subjects for which Certainty exceeds Possibility is not negligible as it constitutes close to 32% of subjects.

I consider the possibility that subjects have CPT preferences with a reference point equal to their earnings expectation in the first part of the experiment. The idea behind this robustness check is to account for the possibility that subjects evaluate lotteries in the second part of the experiment relative to their expected earnings in the first part of the experiment. Supplemental Appendix D provides a detailed explanation of how these beliefs were elicited in the experiment. All in all, I find that the aforementioned results are robust to subjects having CPT preferences. In particular, for both domains there is a large proportion of subjects with US and/or LS. Also, regardless of the domain, more subjects exhibit weighting functions with the certainty effect being stronger than the possibility effect. Previous versions of the paper provided these analyses in more detail (see Gonzalez-Jimenez (2021)).

In conclusion, the analyses of the data at the individual level suggest that the majority of subjects have weighting functions with US or LS. Moreover, I find that less than half of subjects exhibit both properties at the same time, which is a remarkable difference with respect to Bleichrodt and Pinto (2000). Finally, as in Abdellaoui (2000) and Tversky and Fox (1995), I find that the certainty effect is stronger than the possibility effect for a larger share of individuals.

TABLE 12. Classification of subjects according to strength of possibility effect.

Shape	Certainty	Possibility	Equal
Observations	107	55	10

Note: This table presents an alternative classification of subjects according to the shape of their probability weighting function. Subjects classified into Possibility have a weighting function with the property $1 - w^{-1}(\frac{5}{6}) > w^{-1}(\frac{1}{6})$. That is, the possibility effect is stronger than the certainty effect. Instead, if $1 - w^{-1}(\frac{5}{6}) < w^{-1}(\frac{1}{6})$ subjects were classified into Certainty. Their weighting function exhibits a certainty effect that is stronger than the possibility effect. Finally, subjects with $1 - w^{-1}(\frac{5}{6}) = w^{-1}(\frac{1}{6})$ were included in Equal.

Appendix F: Additional analyses

F.1 *Robustness of main results*

Table 13. Regression of performance by round on treatments.

	(1) Performance	(2) Performance	(3) Performance	(4) Performance
LowPr	1.674	1.656	1.937	1.600
	(0.705)	(0.741)	(0.749)	(0.739)
MePr	0.652	0.671	0.565	0.634
	(0.645)	(0.652)	(0.642)	(0.655)
HiPr	0.237	0.168	0.184	0.162
	(0.595)	(0.581)	(0.582)	(0.619)
Mixed U.		0.386	0.350	0.370
		(0.654)	(0.684)	(0.657)
Convex U.		0.762	0.258	0.853
		(0.997)	(1.108)	(1.027)
Concave U.		1.436	1.390	1.507
		(0.928)	(0.917)	(0.935)
Overw. 1/6			0.852	
			(0.641)	
Overw. 2/3			−2.013	
			(1.192)	
Overw. 1/2			0.528	
			(1.285)	
US				0.090
				(0.509)
LS				0.292
				(0.496)
Constant	6.369	6.213	6.153	6.081
	(0.465)	(0.484)	(0.513)	(0.502)
Round fixed effects	Yes	Yes	Yes	Yes
R^2	0.070	0.083	0.103	0.084
Observations	1720	1720	1720	1720

Note: This table presents Random Effects estimates of model $\text{Performance}_{it} = \beta_0 + \beta_1 \text{LowPr} + \beta_2 \text{MePr} + \beta_3 \text{HiPr} + \text{Controls}'\Lambda + \sum_{t=1}^{10} \gamma_t + \alpha_i + \varepsilon_{it}$, with $E(\varepsilon_{it} | \text{MepR, LowPr, HiPr, Controls}) = 0$. "Performance" is the number of calculations correctly solved by a subject i in each round t in the first part of the experiment. "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering stochastic contracts implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. Robust standard errors are presented in parentheses.

TABLE 14. Regression of performance on treatments with continuous variables.

	(1) Performance	(2) Performance	(3) Performance	(4) Performance	(5) Performance
LowPr	16.739	16.103	18.279	15.506	16.044
	(7.090)	(7.179)	(7.261)	(7.150)	(7.206)
MePr	6.522	6.730	5.574	6.339	6.979
	(6.487)	(6.444)	(6.358)	(6.475)	(6.345)
HiPr	2.372	1.953	2.204	1.904	2.294
	(5.985)	(5.842)	(5.925)	(6.317)	(5.800)
Curvature Utility		0.001	0.001	0.001	0.001
		(0.000)	(0.000)	(0.000)	(0.000)
Curvature Weighting					−3.198
					(2.654)
Overw. 1/6			9.989		
			(6.645)		
Overw. 2/3			−20.130		
			(12.202)		
Overw. 1/2			4.445		
			(13.246)		
US				0.953	
				(5.140)	
LS				3.288	
				(5.041)	
Constant	81.378	76.146	74.881	74.407	75.628
	(4.726)	(5.285)	(5.876)	(5.752)	(5.222)
R^2	0.045	0.066	0.097	0.070	0.074
Observations	172	172	172	172	172

Note: This table presents OLS estimates of the model $\text{Performance}_i = \beta_0 + \beta_1 \text{LowPr} + \beta_2 \text{MePr} + \beta_3 \text{HiPr} + \text{Controls}'\Lambda + \varepsilon_i$, with $E(\varepsilon|\text{MepR, LowPr, HiPr, Controls}) = 0$. "Performance" is the number of calculations correctly solved by a subject in the first part of the experiment, "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering stochastic contracts implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. "Curvature Utility" measures each subject's average deviation from the expected value of the lotteries presented in the second part of the experiment. "Curvature Weighting" measures each subject's deviation from the objective probabilities implied by the lotteries in the second part of the experiment. Robust standard errors are presented in parentheses.

Table 15. Nonparametric regression of performance.

	(1) Performance	(2) Performance	(3) Performance	(4) Marginal effects
LowPr	11.649	12.667	8.621	
	(5.143)	(5.938)	(3.920)	
LowPr				3.116
× Likelihood Ins.				(1.383)
LowPr				−0.388
* Optimism				(1.326)
MePr	4.573	4.604	3.243	
	(4.261)	(5.430)	(3.609)	
MePr				2.034
× Likelihood Ins.				(1.718)
MePr				−0.303
× Optimism				(0.984)
HiPr	1.684	0.301	0.009	
	(4.523)	(4.563)	(3.738)	
HiPr				−0.256
× Likelihood Ins.				(1.506)
HiPr				−1.290
× Optimism .				(0.937)
Likelihood Ins.			1.445	1.010
			(1.327)	(1.735)
Optimism			−0.797	−1.142
			(0.891)	(1.170)
Concave U.		1.219	1.413	
		(0.914)	(1.097)	
Convex U.		0.201	0.384	
		(0.198)	(0.325)	
Mixed U.		0.193	0.243	
		(0.444)	(0.677)	
Mean	87.665	87.241	87.380	
	(2.416)	(2.382)	(2.335)	
Method	Nonparametric	Nonparametric	Nonparametric	
N	172	172	172	172

Note: This table presents estimates of a nonparametric regression using a B-spline basis. The model to be estimated is: $\text{Performance}_i = \beta_0 + \beta_1 \text{LowPr} * \text{Likelihood ins.} + \beta_2 \text{LowPr} * \text{Optimism} + \beta_3 \text{LowPr} + \beta_4 \text{MePr} + \beta_5 \text{HiPr} + \beta_6 \text{Likelihood ins.} + \beta_7 \text{Optimism} + \text{Controls}'\Gamma + \varepsilon_i$, with $E(\varepsilon_i | \text{MePr, LowPr, HiPr, Piecerate, Optimism, Likelihood ins., Controls}) = 0$. "Performance" is the number of calculations correctly solved by a subject in the first part of the experiment, "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering a stochastic contract implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. "Likelihood ins." is a binary variable that takes a value of one if the subject is likelihood insensitive and zero otherwise. "Optimism" is a binary variable that takes a value of one if the subject displays optimism and zero otherwise. Columns (1)–(3) present estimates of different specifications. Column (4) presents average marginal effects that capture the effect of a subject becoming optimistic or likelihood insensitive for each of the treatments. Marginal effects are computed using the estimates presented in column (3). Robust standard errors are presented in parentheses.

TABLE 16. Parametric estimates of the probability weighting function using other functions.

Panel 1: $w(p) = \frac{p^{\psi}}{(p^{\psi}+(1-p)^{\psi})^{\frac{1}{\psi}}}$

	$\hat{\psi}$
	0.599
	(0.016)
Adj. R^2	0.838
N	860

Panel 2: $w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$

	$\hat{\gamma}$	$\hat{\delta}$
	0.281	0.922
	(0.025)	(0.021)
Adj. R^2		0.864
N		860

Note: This table presents the subject's average probability weighting function when different parametric forms are assumed. Panel 1 presents nonlinear least squares estimates of the model $w(p) = \frac{p^{\psi}}{(p^{\psi}+(1-p)^{\psi})^{\frac{1}{\psi}}}$. Panel 2 presents the nonlinear least squares estimates of the function $\frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$. Robust standard errors are presented in parentheses.

Table 17. The influence of continuous probability overweighting on treatment effects.

	(1) Performance	(2) Performance	(3) Performance	(4) Performance	(5) Performance	(6) Performance
LowPr × Curvature Weighting	9.299			10.906		
	(5.073)			(4.916)		
LowPr × Curvature WeightingS		18.700			17.776	
		(10.353)			(10.309)	
LowPr × Curvature WeightingM			6.090			7.750
			(11.944)			(12.453)
Curvature Weighting	−3.777			−14.348		
	(2.887)			(6.392)		
Curvature WeightingS		−14.606	−8.209		−16.209	−10.462
		(6.827)	(6.239)		(6.713)	(6.167)
Curvature WeightingM		5.014	2.845	16.714	4.887	2.352
		(6.908)	(7.636)	(11.138)	(6.921)	(7.490)
Curvature WeightingL		1.107	1.121	13.125	1.651	1.690
		(13.834)	(14.022)	(14.280)	(13.178)	(13.279)
LowPr	15.273	12.292	7.613	13.945	12.317	4.879
	(7.296)	(8.108)	(18.268)	(7.674)	(8.350)	(18.712)
MePr	6.770	6.982	6.756	7.546	7.411	7.219
	(6.449)	(6.603)	(6.654)	(6.717)	(6.707)	(6.756)
HiPr	2.869	3.893	3.190	3.185	3.280	2.582
	(6.015)	(6.499)	(6.496)	(6.299)	(6.316)	(6.311)
Convex U.				4.419	5.326	9.913
				(12.420)	(10.260)	(14.198)
Mixed U.				7.736	6.607	6.942
				(6.984)	(7.000)	(6.941)
Concave U.				15.476	14.819	15.630
				(8.724)	(8.994)	(9.137)
Constant	81.930	78.339	79.683	60.216	76.937	78.824
	(4.745)	(11.384)	(12.236)	(15.493)	(11.610)	(12.333)
Wald test of CurvatureW vs. LowPr * CurvatureW	$p = 0.068$	$p = 0.028$	$p = 0.840$	$p = 0.008$	$p = 0.023$	$p = 0.739$
R^2	0.060	0.069	0.056	0.092	0.088	0.078
Observations	172	172	172	172	172	172

Note: This table presents OLS estimates of the model $\text{Performance}_i = \beta_0 + \beta_1 \text{LowPr} + \beta_2 \text{Curv} * \text{LowPr} + \beta_3 \text{Curv} + \beta_4 \text{MePr} + \beta_5 \text{HiPr} + \text{Controls}' \Lambda + \varepsilon_i$, with $E(\varepsilon | \text{MePr, LowPr, HiPr, Controls}) = 0$. "Performance" is the number of calculations correctly solved by a subject in the first part of the experiment, "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to the treatment offering stochastic contracts implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. "Curvature Weighting" measures probability overweighting $w(p)_j - p_j$ for all five measurements of probability. "Curvature WeightingS" measures probability overweighting $w(p) - p$ for $p = \frac{1}{6}$ and $p = \frac{1}{3}$. "Curvature WeightingM" measures probability overweighting $w(p)_j - p_j$ for $p = \frac{1}{2}$ and $p = \frac{2}{3}$. Robust standard errors are presented in parentheses.

TABLE 18. The influence of likelihood insensitivity and optimism on treatment effects.

	(1) Performance	(2) Performance	(3) Performance	(4) Performance	(5) Performance	(6) Performance
LowPr ×Likelihood Ins.		21.944			23.815	
		(9.031)			(8.992)	
LowPr ×Optimism			6.713			10.829
			(12.115)			(12.007)
LowPr	21.251	16.533	13.288	21.752	15.935	17.546
	(7.624)	(13.045)	(11.622)	(7.637)	(11.492)	(9.103)
MePr ×Likelihood Ins.		12.596		11.795		
		(9.693)		(9.221)		
MePr × Optimism			0.105			−0.344
			(10.773)			(9.956)
MePr	12.301	10.394	8.244	12.347	14.741	13.355
	(6.925)	(8.415)	(11.250)	(6.906)	(8.486)	(8.905)
HiPr ×Likelihood Ins.		3.807			2.260	
		(6.825)			(6.537)	
HiPr × Optimism			−8.421			−13.911
			(7.274)			(6.212)
HiPr	4.903	5.085	5.278	5.739	11.517	12.807
	(5.727)	(8.684)	(8.266)	(5.738)	(8.413)	(7.158)
Likelihood Ins.	0.964	−0.979	1.877	−1.469	0.934	−1.419
	(4.765)	(8.967)	(5.069)	(4.690)	(9.054)	(4.714)
Optimism	−2.336	−1.991	−6.969	−0.580	−0.635	2.166
	(4.852)	(5.113)	(9.549)	(4.851)	(4.897)	(9.872)
Mixed U.	6.489	6.825	5.872	6.683	7.726	6.311
	(7.396)	(7.578)	(7.221)	(7.362)	(7.554)	(6.972)
Convex U.	6.679	5.994	11.419	8.263	6.140	13.396
	(10.673)	(11.029)	(12.006)	(10.488)	(11.092)	(11.229)
Concave U.	16.562	16.359	14.646	16.035	14.600	14.601
	(9.368)	(9.493)	(10.075)	(9.135)	(8.999)	(8.924)
Constant	75.106	76.062	77.416	75.454	74.045	74.477
	(5.697)	(6.433)	(7.125)	(5.210)	(5.774)	(5.465)
Parametric family	Prelec (1998)	Prelec (1998)	Prelec (1998)	Goldstein and Einhorn (1987)	Goldstein and Einhorn (1987)	Goldstein and Einhorn (1987)
Likelihood ins.	$\hat{\alpha} < 1$	$\hat{\alpha} < 1$	$\hat{\alpha} < 1$	$\hat{g} < 1$	$\hat{g} < 1$	$\hat{g} < 1$
Optimist	$\hat{\beta} < 1$	$\hat{\beta} < 1$	$\hat{\beta} < 1$	$\hat{\delta} > 1$	$\hat{\delta} > 1$	$\hat{\delta} > 1$
R^2	0.099	0.100	0.108	0.095	0.104	0.118
Observations	156	156	156	157	157	157

Note: This table presents the OLS estimates of the model $\text{Performance}_i = \beta_0 + \beta_1\text{LowPr} * \text{Likelihood ins.} + \beta_2\text{LowPr} * \text{Optimism} + \beta_3\text{LowPr} + \beta_4\text{MePr} + \beta_5\text{HiPr} + \beta_6\text{Likelihood ins.} + \beta_7\text{Optimism} + \text{Controls}'\Gamma + \varepsilon_i$, with $E(\varepsilon_i|\text{MePr, LowPr, HiPr, Piecerate, Optimism, Likelihood ins., Controls}) = 0$. "Performance" is the number of calculations correctly solved by a subject in the first part of the experiment, "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering a stochastic contract implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. "Likelihood ins." is a binary variable that takes a value of one if the subject is classified as likelihood insensitive and zero otherwise. "Optimist" is a binary variable that takes a value of one if the subject displays optimism and zero otherwise. Robust standard errors in parenthesis. In columns (1)–(3), Likelihood ins. and Optimism are constructed with the nonlinear least squares estimates of the function $w(p_j) = \frac{\delta p_j^{\gamma}}{\delta p^{\gamma}+(1-p_j)^{\gamma}}$ estimated for each subject. In columns (4)–(6), likelihood insensitivity and optimism are constructed using the nonlinear least squares estimates of the function $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$ estimated for each subject. Robust standard errors are presented in parentheses.

TABLE 19. The influence of continuous likelihood insensitivity and optimism on treatment effects.

	(1) Performance	(2) Performance	(3) Performance	(4) Performance
LowP × $(1-\hat{s})$	−0.879	0.207		
	(3.950)	(3.804)		
MePr × $(1-\hat{s})$	−1.573	−1.015		
	(4.155)	(4.201)		
HiPr × $(1-\hat{s})$	−4.919	−5.197		
	(1.317)	(1.390)		
LowPr × $\frac{2\hat{c}+\hat{s}}{2}$			6.921	12.871
			(4.768)	(5.511)
MePr × $\frac{2\hat{c}+\hat{s}}{2}$			−22.138	−17.172
			(15.153)	(16.230)
HiPr × $\frac{2\hat{c}+\hat{s}}{2}$			−10.618	−7.920
			(11.773)	(9.710)
$1-\hat{s}$	−8.005	−7.911	−4.811	−4.806
	(2.572)	(2.590)	(1.332)	(1.310)
$\frac{2\hat{c}+\hat{s}}{2}$	−2.517	1.314	13.203	13.187
	(7.407)	(7.693)	(10.990)	(10.523)
LowPr	16.458	15.801	20.203	17.075
	(7.609)	(7.250)	(8.335)	(8.104)
MePr	4.944	4.893	21.359	18.721
	(6.924)	(6.746)	(11.732)	(11.740)
HiPr	0.180	0.302	11.409	10.279
	(5.795)	(5.701)	(8.732)	(7.936)
Optimism	−2.517	1.314		
	(7.407)	(7.693)		
Mixed U.	3.462		2.829	
	(6.636)		(6.826)	
Concave U.	13.550		14.001	
	(9.828)		(9.962)	
Convex U.	6.553		8.483	
	(10.675)		(10.753)	
Curvature U.		0.001		0.001
		(0.000)		(0.000)
Constant	82.323	76.349	74.696	70.845
	(6.070)	(6.766)	(6.689)	(6.613)
Wald test				
LowPr + LowPr ∗ $(1-\hat{s})$ vs. $1-\hat{s}$	$p=0.013$	$p=0.007$		
LowPr + LowPr ∗ $\frac{2c+s}{2}$ vs. $\frac{2c+s}{2}$			$p=0.227$	$p=0.138$
R^2	0.105	0.113	0.113	0.117
Observations	170	170	170	170

Note: This table presents the OLS estimates of a regression relating each subject's performance on the real-effort task, the assignment to the treatments, and indexes of optimism and likelihood insensitivity. "Performance" is the number of calculations correctly solved by a subject in the first part of the experiment, "LowPr," "MePr," and "HiPr" are binary variables that indicate if a subject was assigned to a treatment offering a stochastic contract implemented with low, medium, or high probability, respectively. "Piecerate" is the benchmark of the regression. $1-\hat{s}$ is an index of likelihood insensitivity. $\frac{2c+s}{2}$ is an index of optimism. Estimates c and s are obtained from a truncated regression of the model $w(p_j)=c+sp_j+\epsilon$ estimated for each subject. Robust standard errors are presented in parentheses.

F.2 *Descriptions and comparisons with previous studies*

Panel 1 in Table 4 presents the estimates of a truncated regression of the neo-additive function, $w(p) = c + sp$.[35] The resulting estimates display $\hat{c} > 0$ and $\hat{c} + \hat{s} < 1$, which imply that subjects on average overweighted small probabilities and underweighted large probabilities. Furthermore, $\hat{c}$ and $\hat{s}$ are larger and smaller, respectively, than the estimates reported in Abdellaoui, Baillon, Placido, and Wakker (2011), suggesting that subjects in my experiment exhibit higher degrees of optimism toward risk as well as higher degrees of likelihood insensitivity.

A more traditional parametric representation of the probability weighting function was proposed by Tversky and Kahneman (1992). Their proposal relates probabilities and their associated weights according to the following nonlinear function: $w(p) = \frac{p^{\psi}}{(p^{\psi}+(1-p)^{\psi})^{\frac{1}{\psi}}}$. The first panel of Table 16 shows that the nonlinear least squares method generates an estimate $\hat{\psi} = 0.59$, which is lower than those reported in previous studies. Specifically, classical experiments report estimates in the range 0.60–0.75 (Bleichrodt and Pinto (2000), Abdellaoui (2000), Wu and Gonzalez (1996), Tversky and Kahneman (1992)). Therefore, subjects in my experiment display a weighting function with more severe probability distortion.

A crucial disadvantage of Tversky and Kahneman's (1992) weighting function is that likelihood insensitivity and optimism/pessimism influence ψ, so their effect on probabilistic risk attitudes cannot be identified. To overcome such disadvantage, I also use the log-odds weighting function proposed by Goldstein and Einhorn (1987), $w(p) = \frac{\delta p^{\gamma}}{\delta p^{\gamma}+(1-p)^{\gamma}}$, which can, up to some extent, separate these two components. The estimates of a nonlinear least squares regression are presented in panel 2 of Table 16. The magnitude of $\hat{g}$ indicates that the average weighting function has a strong inverse-S shape and the magnitude of $\hat{\delta}$ an strikingly small degree of pessimism. These coefficients are lower and higher, respectively, than those found in previous studies (Bruhin, Fehr-Duda, and Epper (2010), Bleichrodt and Pinto (2000), Abdellaoui (2000), Gonzalez and Wu (1999), Wu and Gonzalez (1996), Tversky and Fox (1995)). Thus, subjects in the experiment had an average weighting function with more likelihood insensitivity and optimism than previously documented.

Lastly, I also estimate a regression assuming Prelec's (1998) probability weighting function with two parameters, $w(p) = \exp(-\beta(-\ln(p))^{\alpha})$. This parametric function also separates, up to some extent, optimism from likelihood insensitivity. Panel 3 in Table 4 presents the estimates of a nonlinear least squares regression. The estimate $\hat{\alpha}$, which is statistically lower than one, entails that the average probability function has a strong inverse-S shape. Moreover, the estimate $\hat{\beta}$, which is also statistically lower than one, entails that subjects on average display optimism. Previous estimations of this probability weighting function report larger values of α and β (Murphy and Brincke (2018), L'Haridon, Aycinena, Vieider, Belianin, and Bandur (2018), Fehr-Duda and Epper (2012), Abdellaoui et al. (2011), Bleichrodt and Pinto (2000)). Hence, these subjects display an

[35] The assumed truncation at the extremes, $w(0)$ and $w(1)$, provides the estimation with the flexibility to admit weighting functions with S-shape.

average probability weighting function with a stronger inverse-S shape and more optimism as compared to previous studies.

Appendix G: Instructions

This is an experiment in the economics of decision-making. The instructions are simple and if you follow them carefully and make certain decisions, you might earn a considerable amount of money, which will be paid to you via bank transfer at the end of the experiment. The amount of money that you earn will depend on your decisions and effort, and partly on chance. Once the experiment has started, no one is allowed to talk to anybody other than the experimenter. Anyone who violates this rule will lose his or her right to participate in this experiment. If you have further questions when reading these instructions, please do not hesitate to raise your hand and formulate the question to the experimenter.

The experiment consists of two parts. Your earnings in part 1 or part 2 of the experiment will be chosen at the end of the experiment and become your final earnings. Whether the earnings of part1 or the earnings of part 2 will be your final earnings will be established by roll of a die.

Part 1 of the experiment

In this part of the experiment, your task is to complete summations. Your earnings in this part of the experiment depend only on the number of correct summations that you deliver. You need to complete as many summations as you can in 10 rounds, each round lasts four minutes. In other words, you will have a total of 40 minutes to complete as many summations as you can.

Each summation consists of five two-digit numbers. For example, $11+22+33+44+55=$? Once you know the answer to the sum of these five two-digit numbers, input the answer in the interface, click OK, and a new set of numbers will appear on your screen.

For your better understanding, you will face with two examples next.

[Examples displayed]

The previous examples show what you have to do in this part of the experiment. The only thing left to be explained is to specify how you are going to earn money by completing the summations.

Piecerate treatment payment rule: In this part of the experiment, each correct summation will add 25 euro cents to your experimental earnings.

Remember: You have 40 minutes to complete summations, and only correct summations will count toward your earnings at a rate of 25 euro cents each. If you understood these instructions, press “OK.” When everyone is ready, we will start with this part of the experiment.

LowPr treatment payment rule: In this part of the experiment, 1 out of all the 10 rounds will be randomly chosen. The specific round is chosen at random by the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in a randomly chosen round will count toward your earnings at a rate of 250 euro cents per correct summation.

Remember: You have 40 minutes to complete summations, and only correct summations in 1 specific round, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 250 euro cents each. If you understood these instructions, press “OK.” When everyone is ready, we will start with this part of the experiment.

MePr Ttreatment payment rule: In this part of the experiment, 3 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at randomly by the computer at the end of this part of the experiment. This is, once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count toward your earnings at a rate of 85 euro cents per correct summation.

Remember: You have 40 minutes to complete summations, and only correct summations in 3 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 85 euro cents each. If you understood these instructions, press “OK.” When everyone is ready, we will start with this part of the experiment.

HiPr treatment payment rule: In this part of the experiment, 5 out of all the 10 rounds will be randomly chosen. The specific rounds are chosen at random by the computer at the end of this part of the experiment. This is once you completed summations in all the 10 rounds, only the correct summations in that randomly chosen round will count toward your earnings at a rate 50 euro cents per correct summation.

Remember: You have 40 minutes to complete summations, and only correct summations in 5 specific rounds, chosen randomly by the computer at the end of the experiment, will count toward your earnings at a rate of 50 euro cents each. If you understood these instructions, press “OK.” When everyone is ready, we will start with this part of the experiment.

[First part of the experiment taking place]

Belief elicitation

We are interested in knowing your belief about the amount of correct summations that you performed. Please enter that estimate below. If your estimate is correct, that is, it is exactly equal to the total amount of summations over the 10 rounds, you will gain 1 euro. Otherwise, you will receive nothing.

Part 2 of the experiment

In this part of the experiment, your task is to choose among two possible alternatives. Your earnings on this part of the experiment depend on how good your choices are. Particularly, you will face with 11 decision sets. In each of these sets, you need to choose between the option L, that delivers a fixed amount of money, and the option R that is a lottery between two monetary amounts. Each decision set contains six choices.

Be careful! Every time you make a choice between L and R, the monetary prizes of the options are going to change and you ought to make a choice again. One of your choices will be randomly picked by the computer, will be played, and its realization will count toward your earnings for this part of the experiment. You will be faced with one example next.

[Example displayed]

If it is clear what you have to do in this part of the experiment, press “OK” to start. Once everyone is ready, this part of the experiment will begin.

[Second part of the experiment taking place]

Survey

In the last part of the experiment, we would like to know some information about you. Please fill the following survey:

- Gender:
- Age:
- What is your education level? (Bachelor, Exchange, Pre-Master, Master, PhD):
- What is the name of your program of studies?
- How difficult did you find the task? (where 1 stands for easy and 5 for very difficult)
- Rate how confident you are that you can do the task good enough so you can be in the top half of performers in this group as of now. (1—Not confident, 10—Very confident)
- Are you any good at adding numbers? (1—Not good at all, 10—Very good)
- Are you generally a person who is fully prepared to take risks or do you try to avoid taking risks?
- Rate yourself from 0 to 10, where 0 means “unwilling to take any risks” and 10 means “fully prepared to take risks.”

- People can behave differently in different situations. How would you rate your willingness to take risks while driving? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks."
- How would you rate your willingness to take risks in financial matters? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks."
- How would you rate your willingness to take risks with your health? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks."
- How would you rate your willingness to take risks in your occupation? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks."
- How would you rate your willingness to take risks in your faith in other people? Rate yourself from 0 to 10, where 0 means "unwilling to take any risks" and 10 means "fully prepared to take risks."

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Co-editor Garance Genicot handled this manuscript.

Manuscript received 24 October, 2022; final version accepted 10 May, 2024; available online 20 May, 2024.