Communication Technology Advance and Consequences: Using Two-sided Search Model*

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Abstract

Do advances in communication technology, such as online dating sites and social networking services, change our perception of the value of being in a relationship? In this paper, I construct a non-stationary two-sided search market equilibrium model. Using it, I analyze the quantitative effects of advances in communication technology on individuals' marital behavior and welfare. The model includes cohabitation as well as marriage as individual choices. I provide a new proof of the existence of the non-stationary market equilibrium. Using the model's equilibrium condition, I develop a new identification argument to separately identify parameters that have been considered important but difficult to identify. I estimate the structural model with indirect inference, using the National Longitudinal Study of the High School Class of 1972 and the National Longitudinal Survey of Youth 1997. I quantify the effects of advances in communication technology on society and reveal which types of individuals benefit from these changes.

Keywords: Marriage, divorce, cohabitation, two-sided search model, market equilibrium, structural estimation.

JEL: C57, C62

1 Introduction

Communication technology has significantly changed our lives during the past decades. Consequently, people today use texting message services for daily communication, and even many couples meet online. Just as Amazon has simplified shopping, finding and communicating with potential partners has become easier. You can now swipe and like profile pictures on your phone whenever you have a free moment.

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What kind of changes have happened due to the ease of communication brought about by the technology advances at equilibrium? How would people change their marital behavior at equilibrium? In this research, I present an empirical non-stationary two-sided search market equilibrium model for analysing marital relationship formation including cohabitation. Using the model, I quantify the effects of the technology change on relationship formation patterns and welfare.

Advances in communication technology have made it easier to connect with potential partners through texting, social media, and online dating platforms. These advances also expand a choice set. At first glance, these changes appear beneficial to all. More choices can lead to higher overall welfare, which might be unattainable with fewer options (see, for example, Mas-Colell et al. (1995)). However, while we can theoretically predict some increase in overall welfare, certain types of individuals may face disadvantages, which are important to quantify.[1]

Beyond evaluating the overall impact of technological changes, it is important to understand the specific channels through which society is affected; for example, individuals' preference change, stocks of individuals change and/or matching technology change. Depending on the channels, implications of the model and, therefore, policy implications would change.[2] I reveal which parts of my marital behavioral model change by the advent of communication technology advances.

This paper makes two main contributions: The first one is to quantitively analyze the complicated impacts of the advances in communication technology on cohabitation and marriage patterns, and on welfare. The second contribution is about identification: In the search-matching literature, identifying parameters of the model is challenging (see, for example, Flinn and Heckman (1982)). Suppose we have observed more matches between men and women with particular characteristics (for example, a highly educated man and a highly educated woman). This might happen because (i) they like each other more (a deterministic part of a match value, or preference); (ii) there are more opportunities for them to meet (meeting technology); (iii) there are just more people with the particular characteristics (stocks of singles issue) and/or (iv) a variance of a stochastic part of a match value is larger.[3] However, typically, we can observe only data about "duration of being single," "who matches with whom" and "how long they have been matched." This data limitation is common in structural estimation for marriage matching studies, which this paper addresses.[4]

Identification often relies on specific functional forms for meeting probability in search-matching models or, in some cases, just ignore the possible aspects of a model (for example, as an extreme example, a frictionless marriage market assumption (Choo and Siow (2006)). However, due to complicated interactions caused by the technology advances, it seems somehow problematic to justify assuming a particular parametric func-

[1]Theoretically, technological advances can be seen as facilitating competition by expanding choice sets for individuals and reducing search costs and information friction in the economy. The situation of the marriage market becomes closer to a sort of competitive equilibrium situation, which may increase inequality. Typically, market completeness ensures efficient allocation, but whether the equilibrium allocation is fair remains ambiguous (see, for example, Debreu (1959)). Depending on initial endowments, an economy might reach an unfair allocation, even if it is still efficient. Therefore, this research can also be framed as a quantitative evaluation of the trade-off between efficiency and inequality due to technological advances in equilibrium.

[2]For example, if facilitated mating search causes the economy to be worse, government intervention is required: Because I employ a two-sided search model framework, I need to pay attention to a Hosios-like inefficiency argument with congestion and thick market externalities (Hosios (1990)). An optimal search friction level exists which leads to an efficient allocation in the economy.

[3]Note that preference (i) includes separation costs also. About (iv), see, for example, Weizman's Pandora's Box argument (Weizman (1979)). I will discuss in more detail in Section 6.

[4]As I mentioned above, for policy implications, distinguishing sources of welfare reduction is important: Whether reduction in welfare comes from facilitated matching technology or from changes of preference gives us different policy implications.

tional forms to each primitive beforehand. Needless to say, ignoring a possible channel is also problematic. So, I include all of the above four possibilities in the model. I present a method for identifying parameters without relying fully on functional form assumptions, using a market equilibrium concept. Through this model framework, I can identify the four factors above. Separating these factors is key to highlighting the effects of technological advances, which I discuss in more detail later. This identification method is widely applicable to other search studies. Also, related to the second contribution, I provide sufficient conditions for existence of equilibrium under non-stationarity and limited commitment assumptions. Providing the sufficient conditions is worthwhile on its own.

This study uses the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97) as two different cohorts. The NLS 72 is assumed to represent the cohort under the situation before the advances in the communication technology occur, and the NLSY 97 is assumed to represent the cohort under the situation after the advances of the communication technology occur. In addition to detailed demographic information, both data sets have relationship type information including cohabitation from early ages of respondents.

I estimate a dynamic discrete choice model with three alternatives: single, cohabitation, and marriage. In this equilibrium model, stocks of individuals and meeting probabilities are also equilibrium objects. Individuals optimally decide single, cohabitation, or marriage, while taking into account dynamics of the endogenously determined aggregate stocks of individuals and meeting probabilities. The structural parameters are estimated through indirect inference. In matching moments, I use a *consistency belief* (rational expectations) condition associated with the dynamics of equilibrium stock, providing new sources of identification for parameters of the meeting probabilities. These appear as new moment conditions within the market equilibrium framework.

Following model estimation, I perform a series of counterfactual experiments. In these experiments, I assess welfare changes caused by technological advancements and decompose through which channels the marital behaviors change, incorporating *equilibrium effects*. The estimation results indicate that individuals in the NLSY 97 cohort enjoy more efficient meeting technology. However, welfare—defined as an ex-ante expected lifetime payoff in the marital game—is lower in the NLSY 97 cohort when equilibrium effects are considered.[5] Second, associated with the changes in communication technology, the counterfactual experiments reveal the primary driver of changes in marital behavior between the NLS 72 and NLSY 97 cohorts is not the change of the meeting technology itself. Instead, the changes of the distribution of the stochastic component of a match value, which is also considered caused by the advance in the communication technology. Third, I change the meeting technology parameters to those of the NLS 72, while holding all other parameters constant at NLSY 97 values, and recalculate the equilibrium. Then, the counterfactual results suggest that the ex-ante expected lifetime payoff is higher at equilibrium. At the same time, I find that most reduction in the ex-ante expected lifetime payoff between the NLS 72 and NLSY 97 cohorts comes from changes in preferences.

Note that the technological changes are assumed to have occurred between the two cohorts. To highlight the impact of these technological advances on marital behaviors, it is necessary to control for other changes that also took place during this period. Previous literature mentions changes along several dimensions.[6] How-

[5] More precisely, individuals in the NLSY 97 cohort find less value in being matched. I will discuss this point in more detail in Section 10.

[6] Various changes occurred between the two cohorts, as documented in previous literature. These include alterations in divorce costs

ever, I emphasize that most of the changes pointed out by previous literature can be reinterpreted roughly as a change in preferences, separation costs, non-stationary distributions of individual types (stocks), and attitudes toward cohabitation. By extending a model used in previous studies and developing the rigorous identification argument, I can control for other changes.[7] This is another reason why I need to be serious about the identification argument to isolate the effects of the technology changes. In other words, this paper provides an approach to extract the effects of the technology changes through using microeconomics under data limitations without explicit technology variations.

Broadly, this study can be characterized from the following three perspectives: first, the use of a non-stationary market equilibrium model; second, the incorporation of search friction; and third, the inclusion of both cohabitation and marriage. In these aspects, this study differs from previous studies. I introduce some of the previous works related to my research.

First, the recent marriage search models developed by, for example, Akın and Platt (2016), Goussé et al. (2017), Beauchamp et al. (2018) and Shepard (2019) are most closely related. The main differences are that they do not consider cohabitation. Cohabitation has substantially increased in the U.S. and holds similar significance to marriage (Stevenson and Wolfers (2007)). Excluding it could lead to misleading results. One significant change resulting from technological advances is the advent of online dating sites. However, marriage may not be directly influenced by the new style of dating using online dating sites. Modeling cohabitation allows indirect capture of online dating's effects, which may be overlooked by focusing solely on marriage.[8]

Second, this model relates to matching studies assuming a large market with perfect information, such as Chiappori (1988) and Choo and Siow (2006) in labor economics, as well as recent studies in industrial organization, including Lee (2020). Although a large-market, perfect-information approach simplifies identification and estimation, its assumption of no search friction remains controversial. Incorporating meeting probability, conceptually representing search friction, is central to this research. Thus, this study is closer to works that include search friction, such as Goussé et al. (2017) and Shepard (2019).

Third, this model contributes to literature dealing with both cohabitation and marriage, including works by Brien et al. (2006), Matouschek and Imran (2008), and Blasutto (2023). Brien et al. (2006) and Blasutto (2023) incorporate cohabitation with marriage through a learning structure but focus only on a woman's side. Matouschek and Imran (2008) employ a basic non-cooperative game framework, incorporating cohabitation as a choice.[9] Unlike Brien et al. (2006), Matouschek and Imran (2008), and Blasutto (2023), this study explicitly employs a market equilibrium framework to capture strategic interactions and marriage market dynamics.[10] This study accommodates non-stationarity of the economy to explicitly control for the variations of the stocks

(Voena (2015)), a decline in the gender wage gap (Blau and Kahn (2000)), advancements in household technology (Greenwood and Vandenbroucke, 2005), shifts in the gender ratio in colleges (Goldin et al. (2006)), changing perceptions toward cohabitation (Stevenson and Wolfers (2007)), adjustments in the minimum wage (Flinn (2006)), and evolving social norms (Fernández (2013)).

[7] For example, changes in divorce laws between the two cohorts are captured as changes in separation costs. Changes in wage differences are captured through changes in preferences. Changes in the gender ratio are captured through changes in the distribution of individual types (stocks). Changes in attitudes toward cohabitation are incorporated by explicitly modeling cohabitation in the model. Changes in social norms are captured through changes in preferences.

[8] Another reason is, as I mentioned above, I want to isolate the technology effects by explicitly endogenizing a cohabitation choice.

[9] Matouschek and Imran (2008) uses a two-players simultaneous non-cooperative game setting. However, they assume a symmetric game where a husband and a wife have the same payoff. Therefore, essentially, the game can be considered as a single-agent model.

[10] Note that, only after employing a market equilibrium framework, the stocks of individuals show up as an important aspect in the model. The stocks should not play any role in a partial equilibrium framework only with an individual optimaization. This point is a key for the identification argument in this research. I will discuss in more detail in Section 6.

of singles.[11]

1.1 Roadmap

Section 2 documents the trend of relationship formulation in the US. Section 3 describes the basic environment of the model. Section 4 describes how a player decides their optimal behavior under the game rule I assume. In Section 5, I discuss the equilibrium concept employed in this paper. Section 6 explains the identification strategy. In Section 7, I provide the description of data sets I use. Section 8 provides the estimation method to estimate the model primitives. Section 9 provides the estimation results. In Section 10, I introduce several counterfactual experiments. Section 11 gives concluding remarks.

2 Empirical trend about relationship formation

One of the most striking changes US society has experienced over the last 40 years is the change in ways of communication and dating styles. People did not even have a cell phone 40 years ago. So, if a person wanted to communicate with or ask for a date from someone, he/she needed to do it in person. Even in a relationship, mostly, daily communication within a couple was done only through home phones. However, gradually, our lifestyles have changed alongside the advent of new technology. People started using messaging services that made communication among people much easier. Today, with the advent of the internet, almost everyone uses their own smartphone with several messaging services, social networking services, and even online dating apps.[12] This phenomenon has definitely induced changes with respect to relationship formation.

Along with the advent of new technology, one surprising fact we have seen associated with relationship formation is that today, most couples meet their partners online. In the past, the typical ways of meeting a partner were within their network, for example, through friends of friends, their colleagues, their religious group or at a bar near where they lived. In this sense, their choice set was restricted. However, the rise of the Internet has allowed individuals to use it even in choosing their partners.

As shown in Figure 1, for example, Rosenfeld et al. (2019) document that the percentage of couples who met online had risen from 0 percent before 1995. About 22 percentage of couple who meets in 2009 meets online has increased even further to 39 percent by 2017.[13] Figure 1 provides solid evidence of how much our society accepts online dating. It also indicates that we now have access to numerous potential partners simply by swiping photos on our phones. This phenomenon could lead to changes in cohabitation and marriage patterns.

[11] Even though many previous studies assume stationarity of an economy, individuals' dynamic behaviors naturally should cause non-stationarity of an economy which should be captured (Manea (2017)).

[12] According to a study from Pew Research Center (2024), 95 percent of U.S. people have their own smartphone in 2023.

[13] The paper uses data from the How Couples Meet and Stay Together (HCMST) survey. The HCMST is a nationally representative longitudinal survey of adults in the US with a spouse or partner, conducted in 2009 and 2017. In each wave, the sample answers a "how did you meet"question. It is retrospective because the question can be asked only about relationships that have already formed. See, Rosenfeld et al. (2019) for more detailed information.

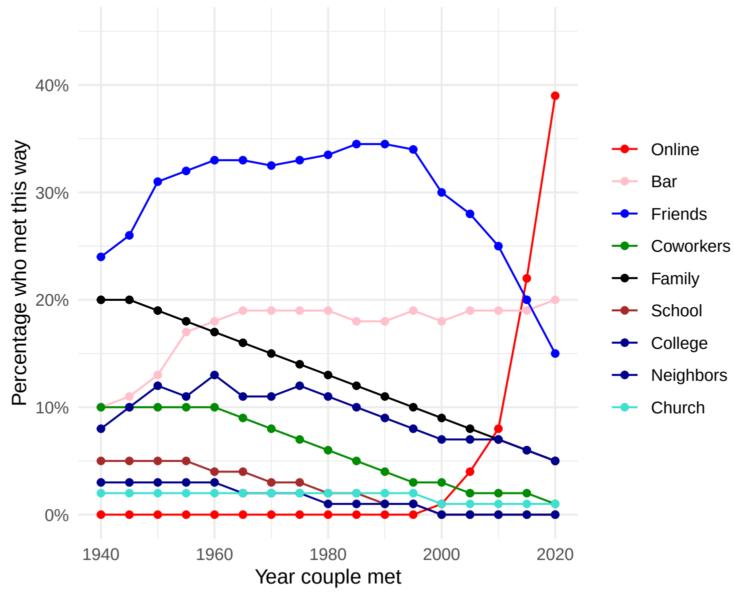

Figure 1: How couples met

3 Model

The model builds on the work of, for example, Drewianka (2006), Brien et al. (2006), Saez (2009), Akın and Platt (2016), Manea (2017), Beauchamp et al. (2018) and Shephard (2019). In this research, I present a finite horizon non-stationary two-sided search market equilibrium model for analysing marital relationship formulation including cohabitation. The environment of the game is theoretically described in the following section, while its detailed empirical specification is provided in Section 8.

3.1 Environment

The economy consists of continua of men and women with observable finite discrete types. The type of an individual is specified by various dimensions. A man's type is indexed by $i \in \mathcal{I} = \{1, 2, ..., I\}$, and a woman's type is indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$. The discrete type consists of time-variant and time-invariant characteristics. For example, $\mathcal{I}$ and $\mathcal{J}$ include race and education. I explain the specific types in more detail later in Section 8.

I confine my attention to a non-stationary economy, where situations an individual faces change through time periods: Especially, stocks of individuals in the economy change through time periods. For the discussion under a non-stationary assumption, I explicitly introduce an additional dimension, $t = \{1, 2, .., T\}$, which implies a time period or age.

Also, the technology exposure level is denoted as $\kappa \in \mathcal{K} = \{1, 2, ..., K\}$. It represents, for example, whether people can use texting services, social networking services, or whether there are online dating sites in the economy. This technology index κ is determined when an individual enters into the marriage market. It does not evolve during the game. The whole economy is assumed to be completely divided into sub-economies denoted by κ. This corresponds to segregated sub-marriage markets for individuals. This point is explained in more detail in Section 5.

Let $\Lambda^{Sm}_{it\kappa}$ be the stock of type i single men with κ at time t, $\Lambda^{Sw}_{it\kappa}$ be for women, $\Lambda^{C}_{ijt\kappa}$ be the stock of cohabitating couples by a type i man and a type j woman with κ at time t, and $\Lambda^{M}_{ijt\kappa}$ be the stock for married couples. I assume a bounded support for each stock. Let $\mathbf{\Lambda}_t$ be a vector of stocks of individuals in the economy given time t, $\mathbf{\Lambda}_t = \{\Lambda^{Sm}_{it\kappa}, \Lambda^{Sw}_{jt\kappa}, \Lambda^{C}_{ijt\kappa}, \Lambda^{M}_{ijt\kappa}\}^{IJK}_{ij\kappa}$, and assume that it is defined on the topological vector space, $\mathbb{R}^{4IJK}_{+}$. Let $\mathbf{\Lambda}$ be a vector of stocks of individuals in the economy, $\mathbf{\Lambda} = \{\mathbf{\Lambda}_t\}^{T}_{t}$, and assume it is defined on a space of the topological vector space, $\mathbb{R}^{4IJTK}_{+}$. The stocks of individuals, $\mathbf{\Lambda}$, are determined endogenously in the model, which I discuss in more detail in Section 5.

3.2 Decision timing, game type and information assumption

The decision timing, game type and information structure are specified next. Let $\alpha^{m}_{ijt\kappa}(\mathbf{\Lambda}_t) \in [0,1]$ be a type i man's probability of meeting a type j woman under a level of technology κ at time t. Similarly, let $\alpha^{w}_{ijt\kappa}(\mathbf{\Lambda}_t) \in [0,1]$ be a type j woman's probability of meeting a type i man under a level of technology κ at time t. A meeting probability is a continuous function mapped from stocks of individuals in the economy at time t, $\mathbf{\Lambda}_t$.

At the beginning of each period, all of the following happen: A type i man and a type j woman meet based on their meeting probability if they are single. If they meet, they draw a flow match value (match surplus), $s_{ijt\kappa} \in \mathbb{R}$, from the conditional distribution $F_{s|ijt\kappa}$ conditional on observable types, i, j, t and κ, which is introduced in more detail later. If they are already matched, they redraw a new match value at every period.

Utility flows from a match are perfectly transferable with side payments within the couple. After meeting, singles divide the value of their match under a Nash bargaining procedure based on their bargaining weight, $\phi \in [0,1]$, which is denoted as a woman's bargaining weight. They jointly decide whether to remain single, cohabit or get married. If they already are matched, they jointly decide whether to continue being in the same relationship or dissolve at every decision time (limited commitment). If a single does not meet with anyone, the individual stays single in the time period. Note that separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. Namely, under the limited commitment setting, a couple commits to shares within their match at the time of cohabitation and marriage with an option to renegotiate if their situation changes. All individuals play this game until they reach the terminal time period, T.

I assume that an individual has perfect information only about his potential partner after meeting. However, an individual is assumed to know about his/her potential partners the individual has not met yet in the economy with information friction (search friction). It means that a player knows distributions of others in the economy.

There are many theoretical studies and several empirical studies about the marriage market using non-cooperative game frameworks also (for example, Wong (2003), Del Boca and Flinn (2012), Friedberg and Stern (2014) and Del Boca and Flinn (2014)). However, in this research, I employ a generalized Nash bargaining approach, which is frequently used in cooperative games. Under a Nash bargaining framework, side payments between players can be incorporated into the model, which makes each player's decision process more straightforward. I return to this point in more detail in Section 4.

3.3 Flow match value

Let $d_{ijt\kappa} = 1, 2$, or 3 be a mutually exclusive choice at time t by a decision unit denoted by i, j, t and κ: $d_{ijt\kappa} = 1$ represents staying single, $d_{ijt\kappa} = 2$ represents cohabiting, and $d_{ijt\kappa} = 3$ represents being married.[14]

Let $u^m_{ijt\kappa} + u^w_{ijt\kappa} \in \mathbb{R}$ be a deterministic part of the flow match value conditioned on observables, i, j, t and κ. It represents the deterministic part of the flow match value caused by a match itself between an i man and a j woman under κ at time t. Note that κ in the subscript of $u^m_{ijt\kappa} + u^w_{ijt\kappa}$ captures an additional benefit/loss caused by a level of technology κ. We can think of the effect of technology on a match itself in the following way: People would, *on average*, match with more/less compatible partners due to technology advance, even conditioned on the same observables, i, j and t. There is a marriage bonus, $\mathbb{M}_{ijt\kappa} \in \mathbb{R}$, which a couple by a type i man and a type j woman gets under κ at time t when $d_{ijt\kappa} = 3$. There are separation costs included in this research. A couple incurs separation costs when going back to single from a match. Let $\mathbb{C}^C_{ijt\kappa} \in \mathbb{R}$ be a cohabitation separation cost between a type i man and a type j woman at time t with κ. Similarly, let $\mathbb{C}^M_{ijt\kappa} \in \mathbb{R}$ be a divorce cost between i and j at time t with κ. Due to the cooperative game setting, we do not have to explicitly mention how to split separation costs when they separate, if we focus only on relationship dynamics. This is because, in a typical cooperative game setting with perfectly transferable utilities, a couple decides their action based only on their total match value. I describe the dynamics in more detail in Section 3.4.

I suppress the notation for an individual in the following discussion. Given a man i and a woman j with state variables, t and κ, the flow match value, $s_{ijt\kappa} \in \mathbb{R}$, is

$$\begin{aligned} s_{ijt\kappa} = {} & u^m_{ijt\kappa} + u^w_{ijt\kappa} + \mathbb{M}_{ijt\kappa}\mathbb{I}[d_{ijt\kappa} = 3] + (\mu^{Cm}_\kappa + \mu^{Cw}_\kappa)\mathbb{I}[d_{ijt\kappa} = 2] + (\mu^{Mm}_\kappa + \mu^{Mw}_\kappa)\mathbb{I}[d_{ijt\kappa} = 3] + \epsilon_{ijt} \\ & - \mathbb{C}^C_{ijt\kappa}\mathbb{I}[d_{ijt-1\kappa} = 2, d_{ijt\kappa} = 1] - \mathbb{C}^M_{ijt\kappa}\mathbb{I}[d_{ijt-1\kappa} = 3, d_{ijt\kappa} = 1], \end{aligned} \tag{1}$$

where $\mathbb{I}[\cdot]$ is an indicator function. In equation (1), $\epsilon_{ijt} \in \mathcal{E} = \mathbb{R}$ is a stochastic part of the flow match value, which is match-specific. The stochastic component, ϵ_{ijt}, represents a match value shock independent of technology after conditioning on observables i, j, t and κ. I assume that, after a match, it starts exhibiting serial correlation. I provide a detailed specification of the dynamics of ϵ_{ijt} in Section 8. Additionally, there are two persistent unobserved heterogeneity terms: Let $\mu^{Cm}_\kappa \sim iidN(0, \sigma^2_{\mu^C}(\kappa))$ and $\mu^{Mm}_\kappa \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ be cohabitation-specific unobserved heterogeneity and marriage-specific unobserved heterogeneity for a man, and $\mu^{Cw}_\kappa \sim iidN(0, \sigma^2_{\mu^C}(\kappa))$ and $\mu^{Mw}_\kappa \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ for a woman, which an individual draws when the individual enters into the marriage game.They are observed by a couple but not by econometricians.

Note that the technology effect on the stochastic part of the flow match value is captured by allowing the variances, $\sigma^2_{\mu^C}(\kappa)$ and $\sigma^2_{\mu^M}(\kappa)$, to differ depending on technology level κ. Intuitively, even conditional on the same observables, i, j, t and t, an individual might match with an extremely good/bad partner based on the technology level κ.[15]

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[14]Note that, about a "decision unit," if a single does not meet with anyone, the individual cannot *jointly* decide and needs to stay single. For example, if an i man does not meet with any woman ($j = \emptyset$), then $d_{i\emptyset\kappa t} = 1$. Otherwise, a couple is assumed to jointly decide as in typical cooperative game literature.

[15]I accommodate another possibility: There is increased sorting based on observable characteristics, which is reflected in smaller variances. When meeting potential partners, there is an incentive to sort based on these observable traits. This occurs across various meeting platforms.

3.4 Value functions

In the following sections, I suppress the notation κ for simpler notation.[16] With the above flow match value, I can write down the value functions of single, cohabitation and marriage in the game. Let $U_{it}^{Sm} \in \mathbb{R}$ be a type i man's value function of staying single under at time t. Similarly, let $U_{jt}^{Sw} \in \mathbb{R}$ be for a j woman. Denote $W_{ijt}^{C}(\epsilon_{ijt}) \in \mathbb{R}$ as a match value caused by cohabitation between a type i man and a type j woman at time t with a realization of the stochastic part of the flow match value, ϵ_{ijt}. Similarly, denote $W_{ijt}^{M}(\epsilon_{ijt}) \in \mathbb{R}$ for marriage.[17] Let $i' \in \mathcal{I}$ and $j' \in \mathcal{J}$ be the next period's type of a man and the next period's type of a woman respectively.

I denote a sum of the amounts of marriage surpluses caused by marriage between a type i man and a type j woman at time t as $Z_{ijt}^{SM}(\epsilon_t) \in \mathbb{R}$. Namely,

$$Z_{ijt}^{SM}(\epsilon_{ijt}) = W_{ijt}^{M}(\epsilon_{ijt}) - U_{it}^{Sm} - U_{jt}^{Sw}. \tag{2}$$

Similarly, a sum of the amounts of cohabitation surpluses, $Z_{ijt}^{SC}(\epsilon_t) \in \mathbb{R}$, is denoted as

$$Z_{ijt}^{SC}(\epsilon_{ijt}) = W_{ijt}^{C}(\epsilon_{ijt}) - U_{it}^{Sm} - U_{jt}^{Sw}. \tag{3}$$

Let $\phi \in [0,1]$ be a woman's bargaining weight. Let $\varsigma \in (0,1)$ be a discount factor. Let the deterministic part of the flow utility of being single be normalized to 0. The value functions for being single, cohabiting and getting married are

$$\begin{aligned} U_{it}^{Sm} =& \varsigma \sum_j \mathbb{E}_{t+1|t}[\alpha_{i'jt+1}^{m}(\mathbf{\Lambda}_{t+1}) \max\{U_{i't+1}^{Sm}, U_{i't+1}^{Sm} + (1-\phi) Z_{i'jt+1}^{SC}(\epsilon_{i'jt+1}), U_{i't+1}^{Sm} + (1-\phi) Z_{i'jt+1}^{SM}(\epsilon_{i'jt+1})\} \\ &+ \varsigma(1 - \sum_j \alpha_{i'jt+1}^{m}(\mathbf{\Lambda}_{t+1})) U_{i't+1}^{Sm}]; \end{aligned} \tag{4}$$

$$\begin{aligned} U_{jt}^{Sw} =& \varsigma \sum_i \mathbb{E}_{t+1|t}[\alpha_{ij't+1}^{w}(\mathbf{\Lambda}_{t+1}) \max\{U_{j't+1}^{Sw}, U_{j't+1}^{Sw} + \phi Z_{ij't+1}^{SC}(\epsilon_{t+1}), U_{j't+1}^{Sw} + \phi Z_{ij't+1}^{SM}(\epsilon_{ij't+1})\} \\ &+ \varsigma(1 - \sum_i \alpha_{ij't+1}^{w}(\mathbf{\Lambda}_{t+1})) U_{j't+1}^{Sw}]; \end{aligned} \tag{5}$$

$$W_{ijt}^{C}(\epsilon_{ijt}) = s_{ijt} + \varsigma \mathbb{E}_{t+1|t} \max\{U_{i't+1}^{Sm} + U_{j't+1}^{Sw}, W_{i'j't+1}^{C}(\epsilon_{i'j't+1}), W_{i'j't+1}^{M}(\epsilon_{i'j't+1})\}; \tag{6}$$

$$W_{ijt}^{M}(\epsilon_{ijt}) = s_{ijt} + \varsigma \mathbb{E}_{t+1|t} \max\{U_{i't+1}^{Sm} + U_{j't+1}^{Sw}, W_{i'j't+1}^{M}(\epsilon_{i'j't+1})\}. \tag{7}$$

In equations (4) and (5), the surplus division occurs through a generalized Nash bargaining over potential gains represented by spouses' value functions. I assume that the bargaining weight, ϕ, is the same in each bargaining

[16] As I described in Section 3.1, κ is fixed when an individual enters into the game. Therefore, it does not play a role when we consider the dynamics of the game.

[17] Note that, for simpler notation, I deliberately represent the value functions without the notion of μ^C and μ^M.

case, cohabitation and marriage.[18,19] Individuals in the economy are assumed to have rational expectations on stocks of individuals and meeting probabilities in the future marriage market. Following typical search literature with a continuum of agents, in equation (4) and (5), we can treat the future meeting probabilities as exogenous given for each individual.[20] The duration dependence, for example, caused by the increased household tightness or the investment between spouses, is captured through the evolution of ϵ_t in s_{ijt} and the change in the length of a relationship which is captured by changes in i and j. Because the law of motion for ϵ_{ijt} exhibits serial correlation, I need to take the conditional expectation of $\epsilon_{i'j't+1}$ conditional on its previous realization, ϵ_{ijt} after a match. The separation costs in s_{ijt}, $\mathbb{C}^C_{ijt}$ and $\mathbb{C}^M_{ijt}$, would ensure a long-term relationship somewhat. I include them for the model to explain some typical household behaviours that require long-term commitment.

Separation is endogenous and occurs when the value of a match is too low for a couple to remain mutually beneficial. When a couple separates, they move to single. Note that, in equation (7), transitions from marriage to cohabitation are assumed not to happen.[21] In the model, the ideal of the search cost is captured by the discount factor.[22]

In this research, the terminal period for all individuals, T, is assumed to be 45. I treat the state of an individual at time T as an absorbing state, which means that, after the period, an individual does not change their marital status, and I give an *arbitrary* value to it.[23]

3.5 Law of motions of stocks

Because my model is a market equilibrium model, to close the model, I need to describe the law of motions associated with the aggregate stocks, which are endogenous objects. Let $\boldsymbol{D}_t$ be a vector of mappings associated with the law of motion of aggregate stocks at time t, which is decided by the model in a complicated way. The law of motions of the stocks in this economy is described as, at t,

$$\boldsymbol{\Lambda}_{t+1} = \boldsymbol{D}_t \cdot \boldsymbol{\Lambda}_t. \tag{8}$$

[18] Note that, in this research, I take ϕ as exogenously given. The value of the bargaining weight ϕ decides the distribution of assets or resources between spouses after their match. Therefore, a different value of ϕ might lead to a big difference in their lifetime utility. However, in focusing only on the dynamics of the marital formulation/dissolution, the value of ϕ does not matter by the assumption of the cooperative game with transferable utilities employed in this research. I give 0.5 to ϕ. However, this is an issue because my research question emphasizes *welfare*. Depending on a value of ϕ, an individual lifetime welfare changes. I describe this point in more detail in Section 10.

[19] Note that I can identify ϕ by using a "happy variable" in the spirit of, for example, Byrne et al. (2009) and Friedberg and Stern (2014). Of course, we can think of ϕ as an endogenous equilibrium object also. However, this extension is beyond the scope of this research.

[20] An individual needs to take expectations on the aggregate future stocks and meeting probabilities only. A player does not need to take expectations on other players' actions in the economy. It is theoretically equivalent to say an each player is not assumed to observe other players' action. The aggregate stocks of individuals and meeting probabilities work as a sufficient statistics for each individual in making their decision in this model. See, for example, Dubey and Kaneko (1984) and Stokey and Lucas (1989) about exogenous treatment of endogenous whole-market-related objects under general/market equilibrium frameworks.

[21] We can say this without loss of generality because I can model the separation cost from marriage happens in going single *as well as going back to cohabitation*. Then, the transitions from marriage to cohabitation do not happen.

[22] Note that implicitly the periods left for a searcher is also one of search costs (see, for example, Rust and Hall (2003)).

[23] I use a set of moments to match for 30 time periods. As Wolpin (1992) points out, if we have a discount factor in a model, the information about utilities difference of the distant future state is not so important in deciding a *current* decision.

4 Decision process

Under a setting with transferable utilities with side payments between spouses, we can simplify the decision process of a decision unit after meeting and during a match. Namely, we focus only on the sum of the match surpluses of a couple.

In this section, I explain why the sum of the amounts of the match surpluses plays an important role as a decision criteria under a Nash bargaining framework setting even with three-alternatives with a market equilibrium, single, cohabitation and marriage, which is considered in this research.[24]

4.1 Relationship decision

Remember that, as I described in Section 3.4, I denote a sum of the amounts of marriage surpluses caused by a transition from single to marriage between a type i man and a type j woman at time t as $Z_{ijt}^{SM}(\epsilon_{ijt})$. Similarly, the sum of cohabitation surpluses is denoted as $Z_{ijt}^{SC}(\epsilon_{ijt})$.

For convenience of explanation for the case with three-alternatives, I first focus on the commonly used two-alternatives case; deciding single or marriage. As in standard search-matching-bargaining studies (see, Shimer and Smith (2000)), after meeting, a type i man and a type j woman compare their profit from getting married with their value of remaining single under a generalized Nash bargaining framework. Namely, with a woman's bargaining weight, ϕ, a type i man and a type j woman compare

$$U_{it}^{Sm} + (1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqgtr U_{it}^{Sm}; \tag{9}$$

$$U_{jt}^{Sw} + \phi Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqgtr U_{jt}^{Sw}. \tag{10}$$

The left-hand side of equation (9) represents the (potential) gain that a type i man would get from marriage with a type j woman through the generalized Nash bargaining framework. On the other hand, the right-hand side of equation (9) means the value that a type i man would get if he selects to remain single at time t. I can similarly describe a type j woman's situation , which is equation (10).

By modifying equations (9) and (10), for a type i man and a type j woman, their critical value (net value) of whether they decide to get married is described as

$$(1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqgtr 0; \tag{11}$$

$$\phi Z_{ijt}^{SM}(\epsilon_{ijt}) \lesseqgtr 0. \tag{12}$$

Here, I assume that the woman's bargaining weight, ϕ, is $\phi \in [0,1]$. If $Z_{ijt}^{SM}(\epsilon_{ijt}) \geq 0$, both agree on marriage because, for both, their net value of their match, $(1-\phi)Z_{ijt}^{SM}(\epsilon_{ijt})$ and $\phi Z_{ijt}^{SM}(\epsilon_{ijt})$, are positive. Therefore, if $Z_{ijt}^{SM}(\epsilon_{ijt}) \geq 0$, getting married is better than remaining single for both of them. In this sense, the sum of the marriage surpluses between spouses, $Z_{ijt}^{SM}(\epsilon_{ijt})$, determines their single to marriage transition. See, for example, Shimer and Smith (2000) for a more formal discussion.

[24]See, for example, Shimer and Smith (2000) for a basic search-matching-bargaining model with the two-alternatives, single and marriage.

Now, focus on the three-alternatives case; single, cohabitation and marriage. In this case, a couple compares the sum of each match surplus, $Z^{SM}_{ijt}(\epsilon_{ijt})$ and $Z^{SC}_{ijt}(\epsilon_{ijt})$, as follows. As in the marriage surplus case, the net value of cohabitation from single for a type i man and a type j woman at time t is defined as $(1-\phi)Z^{SC}_{ijt}(\epsilon_{ijt})$ and $\phi Z^{SC}_{ijt}(\epsilon_{ijt})$ respectively.

With the three alternatives, a couple compares the net values of cohabitation with the net values of marriage presented above. If $Z^{SC}_{ijt}(\epsilon_{ijt}) \geq Z^{SM}_{ijt}(\epsilon_{ijt})$, a type i man and a type j woman jointly decide to cohabit. This is because, in the case,

$$U^{Sm}_{it} + (1-\phi)Z^{SC}_{ijt}(\epsilon_{ijt}) \geq U^{Sm}_{it} + (1-\phi)Z^{SM}_{ijt}(\epsilon_{ijt}) \tag{13}$$

for a type i man, and

$$U^{Sw}_{jt} + \phi Z^{SC}_{ijt}(\epsilon_{ijt}) \geq U^{Sw}_{jt} + \phi Z^{SM}_{ijt}(\epsilon_{ijt}) \tag{14}$$

for a type j woman hold.[25] Therefore, they both agree on cohabitation from single rather than on marriage. On the other hand, if $Z^{SC}_{ijt}(\epsilon_{ijt}) \leq Z^{SM}_{ijt}(\epsilon_{ijt})$, they move to marriage from single. Accordingly, only the amounts, $Z^{SC}_{ijt}(\epsilon_{ijt})$ and $Z^{SM}_{ijt}(\epsilon_{ijt})$, matter, when they decide their decision.

4.2 Existence and uniqueness of reservation match values

Given the decision process provided above, I move to properties of reservation match values. I first give definitions of the reservation match values in the game. Then, I provide a single-crossing property of the value functions and a proof of existence of the reservation match values.[26]

Let $\boldsymbol{\epsilon}^*$ be a vector of reservation match values, $\boldsymbol{\epsilon}^* = \{\epsilon^{*SC}_{ijt}, \epsilon^{*SM}_{ijt}, \epsilon^{*Si}_{ijt}, \epsilon^{*CS}_{ijt}, \epsilon^{*MS}_{ijt}\}^{IJT}_{ijt}$, where ϵ^{*SC}_{ijt} is a reservation match value such that $Z^{SC}_{ijt}(\epsilon^{*SC}_{ijt}) = 0$. Namely, ϵ^{*SC}_{ijt} is a reservation match value with which a couple indexed by i and j at time t is indifferent between staying single and moving to cohabitating. Similarly, ϵ^{*SM}_{ijt} is defined for single to marriage. Let ϵ^{*Si}_{ijt} be a reservation match value such that $Z^{SC}_{ijt}(\epsilon^{*Si}_{ijt}) = Z^{SM}_{ijt}(\epsilon^{*Si}_{ijt})$. Namely, ϵ^{*Si}_{ijt} is a reservation match value with which a couple indexed by i and j at time t is indifferent between moving cohabitation or marriage from single. Hereafter, I refer to ϵ^{*Si}_{ijt} as a *single-crossing point*. Let ϵ^{*CS}_{ijt} be a reservation match value such that $Z^{CS}_{ijt}(\epsilon^{*CS}_{ijt}) = 0$. It means that ϵ^{*CS}_{ijt} is a reservation match value with which a couple indexed by i and j at time t is indifferent between staying in cohabitation and going back to single. Similarly, ϵ^{*MS}_{ijt} is defined from marriage to single.

In the following, I propose two propositions: First, there exists the *single-crossing point*, ϵ^{*Si}_{ijt}. Second, there exists the reservation match values, $\epsilon^{*SC}_{ijt}, \epsilon^{*SM}_{ijt}, \epsilon^{*CS}_{ijt}, \epsilon^{*MS}_{ijt}$ in a finite range, $[\underline{\epsilon}, \bar{\epsilon}]$, and they are unique given arbitrary values of stocks.

Proposition 1. *Under the two-sided market equilibrium setting, with $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$ and $\mathbb{C}^M_{ijt} > \mathbb{C}^C_{ijt} > 0$, $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}^M_{i'j't+1} - \mathbb{C}^C_{i'j't+1})$ for all, i, j and t, there exists a unique ϵ^{*Si}_{ijt} given arbitrary values of stocks.*

[25] Two important assumptions to conclude this are that the bargaining weights for cohabitation and marriage are assumed to be exogenously given and they are the same between cohabitation and marriage. We can extend this setting to a more general one. For example, we can think of the bargaining weights as an endogenous equilibrium object. However, this extension is beyond the scope of this research.

[26] The existence of the reservation values and their uniqueness immediately follow from the argument of the existence of the single-crossing point.

Proposition 2. *The reservation values, $\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}$, $\epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS} \in [\underline{\epsilon}, \bar{\epsilon}]$ for all, i, j and t exist and are unique given arbitrary values of stocks.*

Proof. Proof of Propositions 1 and 2: The proof proceeds in the following way: I first prove that $W_{ijt}^{M}(\epsilon_{ijt}) - W_{ijt}^{C}(\epsilon_{ijt})$ is increasing in ϵ_{ijt} for all t.[27] Second, under the sufficient conditions, I prove that the case where both functions, $W_{ijt}^{M}(\epsilon_{ijt})$ and $W_{ijt}^{C}(\epsilon_{ijt})$, are parallel or overlap with measure 0.

Focus on the last period T, and $W_{ijT}^{M}(\epsilon_{ijT}) - W_{ij)T}^{C}(\epsilon_{ijT^*})$ is written as

$$W_{ijT}^{M}(\epsilon_{ijT}) - W_{ijT}^{C}(\epsilon_{ijT}) = \mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}).$$

The last period's continuation value is assumed to be 0. Because the right-hand side of the equation is independent of ϵ_{ijT}, so it is increasing in ϵ_{ijT} , but not *strictly* increasing in ϵ_{ijT}.

Going back one time period before, $W_{ijT-1}^{M}(\epsilon_{ijT-1}) - W_{ijT-1}^{C}(\epsilon_{ijT-1})$ is written as

$$\begin{aligned}W_{ijT-1}^{M}(\epsilon_{ijT-1}) - W_{ijT-1}^{C}(\epsilon_{ijT-1}) &= \mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) \\ &+ \varsigma \mathbb{E} \max_{T|T-1} \{U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{M}, W_{i'j'T}^{M}(\epsilon_{i'j'T})\} \\ &- \varsigma \mathbb{E} \max_{T|T-1} \{U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{C}, W_{i'j'T}^{C}(\epsilon_{i'j'T}), W_{i'j'T}^{M}(\epsilon_{i'j'T})\}.\end{aligned}$$

This is further modified to

$$\begin{aligned}W_{ijT-1}^{M}(\epsilon_{ijT-1}) - W_{ijT-1}^{C}(\epsilon_{ijT-1}) &= \mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) \\ &+ \varsigma \mathbb{E}_{T|T-1}[\max\{U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T'}^{M}, W_{i'j'T}^{M}(\epsilon_{i'j'T})\} \\ &- \max\{U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{C}, W_{i'j'T}^{C}(\epsilon_{i'j'T}), W_{i'j'T}^{M}(\epsilon_{i'j'T})\}].\end{aligned}$$

The property of increasing in ϵ_{ijt} is preserved by the following operations; maximization, integration over an increasing transition function, which is equivalent to say the law of motion of $\epsilon_{i'j't+1}$ based on a realization of ϵ_{ijt} is first-order stochastically dominant in ϵ_{ijt}, and summation (Hopenhayan and Prescott (1992) and Milgrom and Shannon (1994)).

The following procedure of the proof uses the above properties explicitly: The above $[\max\{\cdot\} - \max\{\cdot\}]$ part is rewritten with division into cases, depending on a realization of $\epsilon_{i'j'T}$. Accordingly, the overall $[\max\{\cdot\} - \max\{\cdot\}]$ part can be rewritten in one of the following cases:

- 1. $[W_{i'j'T}^{M}(\epsilon_{i'j'T}) - (U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{C})]$,
- 2. $[W_{i'j'T}^{M}(\epsilon_{i'j'T}) - W_{i'j'T}^{C}(\epsilon_{i'j'T})]$,
- 3. 0,
- 4. $[U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{C} - (U_{i'T}^{Sm} + U_{j'T}^{Sw} - \mathbb{C}_{i'j'T}^{M})]$.

[27] Note that the difference between *strictly* increasing and increasing is the following: Let $f(x) \in \mathbb{R}$ be a function mapped by $x \in \mathbb{R}$, and let a be a strictly positive value, $a \in \mathbb{R}_{++}$. The statement, $f(x)$ is *strictly* increasing in x, means that, $\forall a$ and x, $f(x+a) > f(x)$. The statement, $f(x)$ is increasing in x, means, $\forall a$ and x, $f(x+a) \geq f(x)$.

Note that, under the region of $\epsilon_{i'j'T}$ where going back to single from marriage is better than staying in marriage, going back to single from cohabitation is better than staying in cohabitation or moving to marriage. This is because we assume that a marriage bonus is $\mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$ for all t, and that the separation cost structure is $\mathbb{C}_{ijt}^{M} > \mathbb{C}_{ijt}^{C} > 0$ for all t. It follows $\epsilon_{ijt}^{*MS} < \epsilon_{ijt}^{*CS}$. See, Brien et al. (2006) for a similar discussion. This is why we need to consider only the above 4 cases in total when considering all possible cases of $[\max\{\cdot\} - \max\{\cdot\}]$.

So, we can ensure that $W_{ijT-1}^{M}(\epsilon_{ijT-1}) - W_{ijT-1}^{C}(\epsilon_{ijT-1})$ is also increasing in ϵ_{ijT-1}. This is because, in either case (case 1 - 4), we can say $\mathbb{E}[\max\{\cdot\} - \max\{\cdot\}]$ part is increasing in ϵ_{ijT-1}: The integrant is increasing in $\epsilon_{i'j'T}$ (case 1 and 2), a constant value (case 3) and independent of ϵ_{ijT-1}(case 4). Note that, again, the flow part, $\mathbb{M}_{ijT-1} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw})$, is independent of ϵ_{ijT-1}, and, therefore, it is also increasing in $\epsilon_{i'j'T-1}$. So, overall, $W_{ijT-1}^{M}(\epsilon_{ijT-1}) - W_{ijT-1}^{C}(\epsilon_{ijT-1})$ is increasing in ϵ_{ijT-1}. I do the same procedure backward until $t = 1$. So, for all t, $W_{ijt}^{M}(\epsilon_{ijt}) - W_{ijt}^{C}(\epsilon_{ijt})$ is increasing in ϵ_{ijt}. Note that, even if I employ a market equilibrium setting, at this point of describing an individual behavior, the endogenous stocks and meeting probabilities are taken as given for an individual.

The increasing property of $W_{ijt}^{M}(\epsilon_{ijt}) - W_{ijt}^{C}(\epsilon_{ijt})$ in ϵ_{ijt} cannot get rid of the case where two functions, $W_{ijt}^{M}(\epsilon_{ijt})$ and $W_{ijt}^{C}(\epsilon_{ijt})$ are parallel or perfectly overlap. In the following, I show that I can get rid of the cases under the sufficient condition in Proposition 1.

I first investigate a condition ensuring that, at the far right part of ϵ_{ijt} support, $W_{ijt}^{M}(\epsilon_{ijt}) > W_{ijt}^{C}(\epsilon_{ijt})$ holds. Second, I provide a condition ensuring that, at far left part of ϵ_{ijt} support, $W_{ijt}^{M}(\epsilon_{ijt}) < W_{ijt}^{C}(\epsilon_{ijt})$ holds. Then, I show that they are actually the sufficient condition for the single-crossing property.

Remember that the law of motion of $\epsilon_{i'j't+1}$ based on a realization of ϵ_{ijt} is first-order stochastically dominant in ϵ_{ijt}. So, when $\epsilon_{ijt} \to +\infty$, the probability of $\epsilon_{i'j't+1}$ becoming below than any value of $\epsilon_{i'j't+1}$, $\bar{\epsilon}_{i'j't+1}$, converges to 0, $f(\bar{\epsilon}_{i'j't+1} > \epsilon_{i'j't+1} | \epsilon_{ijt}) \to 0$. Therefore, as long as $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$, at far right part of ϵ_{ijt} support (equivalent to say $\epsilon_{ijt} \to +\infty$), $W_{ijt}^{M}(\epsilon_{ijt})$ is, at least, $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw})$ greater W_{ijt}^{C}. This is because, at the extreme case ($\epsilon_{ijt} \to +\infty$), the probability of drawing a negative value of $\epsilon_{i'j't+1}$ small enough that going back to single is the best choice converges to 0. It is equivalent to say that, at the extreme case, a couple only enjoys a marriage bonus without their risk of separations in the next period.[28]

On the other hand, let us focus on another extreme case, $\epsilon_{ijt} \to -\infty$. As long as $\mathbb{M}_{ijT} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}_{i'j't+1}^{M} - \mathbb{C}_{i'j't+1}^{C})$ holds, at far left part of ϵ_{ijt} support, $W_{ijt}^{M}(\epsilon_{ijt}) < W_{ijt}^{C}(\epsilon_{ijt})$ holds. This is from the similar logic: When $\epsilon_{ijt} \to -\infty$, a probability of drawing a relatively high value of $\epsilon_{i'j't+1}$ converges to 0. So, the probability of going back to single at time $t+1$ converges to 1 when $\epsilon_{ijt} \to -\infty$. So, a couple with the extreme law ϵ_{ijt} will incur separation costs with probability 1 at time $t+1$, even if the couple enjoys a marriage bonus at time t. A couple thinks about a trade-off between a current marriage bonus and the next period's bigger separation cost. Therefore, As long as $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) < \varsigma(\mathbb{C}_{i'j't+1}^{M} - \mathbb{C}_{i'j't+1}^{C})$ holds, at the extreme ($\epsilon_{ijt} \to -\infty$), $W_{ijt}^{M}(\epsilon_{ijt}) < W_{ijt}^{C}(\epsilon_{ijt})$ holds.

Now, we focus on the middle part of the support of ϵ_{ijt}. At the region, it might be the case, mathematically, the parallel situation happens or the two curves might overlap perfectly. It is equivalent to say multiple values (actually, infinite number) of ϵ_{ijt} make $W_{ijt}^{M}(\epsilon_{ijt}) - W_{ijt}^{C}(\epsilon_{ijt}) = 0$ hold.

[28] Note that, mathematically, it is enough to focus on the extreme case ($\epsilon_{ijt} \to +\infty$) to check whether it is a sufficient condition.

Define ϵ^0_{ijt} such that $W^M_{ijt}(\epsilon^0_{ijt}) - W^C_{ijt}(\epsilon^0_{ijt}) = 0$. Focus on $\mathbb{E}[\max\{\cdot\} - \max\{\cdot\}]$ part, which is the same as above (case 1-4) for all $t < T$. In case 1 and 2, there are no multiple values of ϵ^0_{ijt} happening. This is because the integrant is strictly increasing in $\epsilon_{i'j't+1}$ in case 1. In case 2, the integrant is increasing in $\epsilon_{i'j't+1}$, but the transition function is strictly increasing in ϵ_{ijt}. Therefore, in either case (case 1 and 2), the continuation part is strictly increasing in ϵ_{ijt}.[29] So, $W^M_{ijt}(\epsilon_{ijt}) - W^C_{ijt}(\epsilon_{ijt})$ itself is strictly increasing in ϵ_{ijt}. In case 3, the integrand is a constant value. In Case 3,

$$\begin{aligned} 0 &= \mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) + \int 0 f(\epsilon_{i'j't+1}|\epsilon^0_{ijt}) d\epsilon_{i'j't+1} \\ &= \mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}). \end{aligned}$$

However, under the condition I consider now, $\mathbb{M}_{ijt} + \mu^{Mm} + \mu^{Mw} - (\mu^{Cm} + \mu^{Cw}) > 0$, the above equation does not hold. Therefore, it is contradiction. Similarly, in Case 4, the logic is the same as case 3. Under the condition, $\mathbb{C}^M_{ijt} > \mathbb{C}^C_{ijt}$, in Case 4, there should not the single-crossing point exist. So, we can restrict Case 1 and Case 2 only. In both cases, we can say strongly increasing property of $W^M_{ijt}(\epsilon_{ijt}) - W^C_{ijt}(\epsilon_{ijt})$. Therefore, we can have a unique single-crossing point.

The proof of Proposition 2 directly follows from the fact that $W^C_{ijt}(\epsilon_{ijt})$ and $W^M_{ijt}(\epsilon_{ijt})$ are strictly increasing on the unbounded support, ϵ_{ijt}. See, Brien et al, (2006) and Drewianka (2006) for more detail. □

I provide how a couple decides their martial choice based on the a joint decision structure.

Theorem 1. *Under the situation where Proposition 1 holds, a couple decision $d_{ijt} = \{1, 2, 3\}$ shows the property of increasing in ϵ_{ijt} under a market equilibrium setting.*

Proof. This directly comes from the discussion about existence of the single-crossing point. □

This discussion about the existence of a single-crossing point ensures cohabitation and marriage coexist. This coexistence plays a key role in the following identification discussion.[30]

5 Equilibrium

In this section, I characterize the equilibrium concept in this research. I first provide the formal definition of the equilibrium. Next, I give a proof of the existence of the equilibrium.

5.1 Technology-dependent sub-economies setting

The whole economy is assumed to be completely divided into several non-stationary sub-economies, depending on their level of technology, κ. Namely, there are multiple segregated non-stationary economies existing simultaneously at different technology levels κ, and each sub-economy has its own equilibrium.

[29] Mathematically, it is not a correct statement. In case 2, the integrand might be just a constant along with ϵ_{ijt} because the integrand is just increasing in ϵ_{ijt}. In this case, the basic structure is the same as the following case 3 below.

[30] Remember that an identification argument starts assuming the number of observations goes to infinity. This means $\exists$ individuals with their value of μ^{Cm} or μ^{Cw}, and μ^{Mm} or μ^{Mw}, that satisfy the sufficient condition. I will discuss this in more detail in Section 6.

Associated with the segregated technology-dependent sub-economies, two important assumptions are also imposed: An individual's technology level, κ, does not evolve during the dynamic marriage game, and people who live in different κ worlds cannot meet with each other.[31] At first glance, restricting possible meetings seems somehow a strong assumption. It is true that, theoretically, the model does not have to restrict possible meetings between singles to happen only in the same technology κ world. Neither, do we need to divide the whole economy into its sub-economies. However, restricting possible meetings is not a such strong assumption in this research. This is because, eventually, I compare the cohort without the communication technology advance (NLS 72), which corresponds to $\kappa = 1$, and the cohort with the communication technology advance (NLYS 97), which corresponds to $\kappa = 2$. Although I mention in more detail in Section 8, the two groups are separated by a significant amount of time.[32] So, it seems reasonable to put the restriction because people in different cohorts rarely match with each other. I will give more details about how to construct the marriage market in Section 8. [33]

5.2 Requirements, definition and existence of equilibrium

The equilibrium conditions consist of two requirements; an *optimality condition* and a *consistency belief condition*. The *optimality condition* requires, at equilibrium, each individual behaves optimally given their perception (*belief*) about the future dynamics of the marriage market (stocks of individuals in the economy and meeting probabilities).[34] The *consistency belief condition* requires, at equilibrium, their given perception about the dynamics of the marriage market should match with the actual aggregate dynamics of the marriage market derived by aggregating each individuals' decision in the economy.

I propose the formal definition of the market equilibrium in the model, and state that, under my construction of the model, the equilibrium exists.

Definition 1. *Denote* $\boldsymbol{\epsilon}^{**} = \{\epsilon_{ijt}^{**SC}, \epsilon_{ijt}^{**SM}, \epsilon_{ijt}^{**Si8}, \epsilon_{ijt}^{**CM}, \epsilon_{ijt}^{**CS}, \epsilon_{ijt}^{**MS}\}_{ijt}^{IJT}$ *as a vector of equilibrium reservation match values,* $\boldsymbol{\Lambda}^{**} = \{\Lambda_{it}^{**Sm}, \Lambda_{jt}^{**Sw}, \Lambda_{ijt}^{**C}, \Lambda_{ijt}^{**M}\}_{ijt}^{IJT}$ *as a vector of equilibrium stocks of individuals in the economy and* $\boldsymbol{\alpha}^{**} = \{\alpha_{ijt}^{m^{**}}, \alpha_{ijt}^{w^{**}}\}_{ijt}^{IJT}$ *as a vector of equilibrium meeting probabilities.*

Definition 2. *A non-stationary market equilibrium is defined by a triple* $(\boldsymbol{\epsilon}^{**}, \boldsymbol{\Lambda}^{**}, \boldsymbol{\alpha}^{**})$ *such that:*

- *Each individual optimizes their behavior, given their own perception about future stocks of individuals and meeting probabilities (Optimality condition);*
- *The given perception of the stocks of individuals and meeting probabilities are consistent with the actual aggregate dynamics of the economy (Consistency belief condition).*

Theorem 2. *There exists the market equilibrium.*

[31] Accordingly, we can think of κ in the following way: The technology level index, κ, is not about the attribute that an individual has, but rather about the segregated world that the individual enters into.

[32] I implicitly assume that people at $\kappa = 1$ cohort cannot enjoy the technology advance, even though in reality it is not true.

[33] Empirically, as Shepherd (2019) and Beauchamp et al. (2018) restrict possible meetings between players for computational tractability, I also need to put some restrictions eventually.

[34] This is equivalent to say that, given their belief, the equilibrium concept is a subgame perfect equilibrium.

Proof. Now, I prove the existence of the market equilibrium defined above. Accordingly, first, I introduce the main mathematical objects which are used in the proof although some of them have been already provided in previous sections.

Let $\boldsymbol{\Lambda} = \{\Lambda_{it}^{Sm}, \Lambda_{jt}^{Sw}, \Lambda_{ijt}^{C}, \Lambda_{ijt}^{M}\}_{ijt}^{IJT}$ be a vector of stocks of individuals in the economy. Each stock is assumed to be located in a closed and bounded region between 0 and $\overline{\Lambda}$, which is denoted as an upper bound. Let $\mathcal{M}$ be a convex Euclidean product space for stocks of individuals in the economy, $\mathcal{M} = \{\boldsymbol{\Lambda}\}$. The space, $\mathcal{M}$, is compact as well because, by Tychonoff's theorem, a finite product of compact spaces is compact. Next, let $\boldsymbol{\alpha} = \{\alpha_{ijt}^{m}, \alpha_{ijt}^{w}\}_{ijt}^{IJT}$ be a vector of meeting probabilities. Each meeting probability is bounded between 0 and 1. Let $\mathcal{P}$ be a convex and compact Euclidean product space for meeting probabilities, $\mathcal{P} = \{\boldsymbol{\alpha}\}$.[35] Moreover, let $\boldsymbol{U} = \{U_{it}^{Sm}, U_{jt}^{Sw}\}_{ijt}^{IJT}$ be a vector of value functions for single. Define the space for $\boldsymbol{U}$ as $\mathcal{U} = \{\boldsymbol{U}\}$. Let $\boldsymbol{W} = \{W_{ijt}^{C}(\epsilon), W_{ijt}^{M}(\epsilon)\}$ be a vector of value functions for a match, where ϵ is a shorthand notation for a realization of ϵ_{ijt}. Define a functional space $\mathcal{W}$ which is for continuously differentiable functions, $\mathcal{W} = \{\boldsymbol{W}\}$. [36] Each value of a value function is assumed to be located in a closed and bounded region. This is because I assume that every parameter set is compact, and, by the structure of the value functions defined in Section 3.4, each value function is continuously mapped from the compact parameter spaces. As a result, each value function is defined also on a compact set. Let $\boldsymbol{V} = \{\boldsymbol{U}, \boldsymbol{W}\} \in \mathcal{C}^1$, and let $\mathcal{V}$ be a measurable convex and compact functional space for value functions, $\mathcal{V} = \{\boldsymbol{V}\}$. Furthermore, let $\boldsymbol{\epsilon}^* = \{\epsilon_{ijt}^{*SC}, \epsilon_{ijt}^{*SM}, \epsilon_{ijt}^{*Si}, \epsilon_{ijt}^{*CS}, \epsilon_{ijt}^{*MS}\}_{ijt}^{IJT}$ be a vector of reservation match values. I can say that each reservation match value is located in a closed and bounded region between $\underline{\epsilon}$ and $\overline{\epsilon}$, which is a lower bound and an upper bound respectively. This is because, as I have proved in Section 4.2, each reservation match value does not go to $-\infty$ or $+\infty$. Let $\mathcal{E}^*$ be a convex and compact Euclidean product space for reservation match values, $\mathcal{E}^* = \{\boldsymbol{\epsilon}^*\}$. Finally, let $\boldsymbol{\tau} = \{\tau_{ijt}^{SS(m)}, \tau_{ijt}^{SS(w)}, \tau_{ijt}^{SC(m)}, \tau_{ijt}^{SC(w)}, \tau_{ijt}^{SM(m)}, \tau_{ijt}^{SM(w)}, \tau_{ijt\epsilon}^{CS}, \tau_{ijt\epsilon}^{CC}, \tau_{ijt\epsilon}^{Si}, \tau_{ijt\epsilon}^{MS}, \tau_{ijt\epsilon}^{MC}, \tau_{ijt\epsilon}^{MM}\}$ be a vector of transition probabilities, where ϵ is a shorthand notation for a realization of ϵ_{ijt-1}. Each transition probability is located in a closed and bounded region between 0 and 1. Let $\mathcal{T}$ be a convex and compact Euclidean product space for marital status transition probabilities, $\mathcal{T} = \{\boldsymbol{\tau}\}$.

I construct a mapping $\Phi : \mathcal{M} \to \mathcal{M}$ composing the paths of the whole endogenous interactions of the model: The map Φ is decomposed as,

$$\mathcal{M} \overbrace{\to}^{\varpi} \mathcal{P} \overbrace{\to}^{\iota} \mathcal{V} \overbrace{\to}^{\psi} \mathcal{E}^* \overbrace{\to}^{\xi} \mathcal{T} \overbrace{\to}^{\varrho} \mathcal{M}. \tag{15}$$

These mappings are specified as:

- $\varpi : \mathcal{M} \to \mathcal{P}$ describes a mapping from stocks of individuals to meeting probabilities. Given a structure about a meeting probability as I mentioned in Section 8, it is assumed to be a continuous and bounded function in $\boldsymbol{\Lambda}$.

- $\iota : \mathcal{P} \to \mathcal{V}$ describes a mapping from meeting probabilities to the space for the value functions. Each 7 value function is uniquely mapped given a set of meeting probabilities. The structure of a value function

[35] Its compactness comes also from Tychonoff's theorem.

[36] The values of value functions and the reservation values, which will be introduced below, depend also on $\mu_i^C, \mu_i^M, \mu_j^C$ and μ_j^M. However, for simple notation, I suppress the notation of them in this section.

described in Section 3.4 ensures that value functions continually changes in $\boldsymbol{\alpha}$, and therefore the space $\mathcal{V}$ is compact.

- $\psi : \mathcal{V} \to \mathcal{E}^*$ describes a mapping from value functions to reservation match values. Note that, again, this ψ is also a function. This is because, remember, given $\boldsymbol{V} \in \mathcal{V}$, the reservation match values, ϵ_{ijt}^{*SC}, ϵ_{ijt}^{*SM}, ϵ_{ijt}^{*Si}, ϵ_{ijt}^{*CS}, ϵ_{ijt}^{*MS}, are uniquely decided, as I have proven in Section 4.2. From the Implicit Function Theorem, $\boldsymbol{\epsilon}^*$ is continuous in $\boldsymbol{V}$.

- $\xi : \mathcal{P} \times \mathcal{E}^* \to \mathcal{T}$ denotes a mapping from meeting probabilities and reservation match values to transition probabilities. This is a continuous function in $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}^*$ given a structure of a transition probability described in Section 6.

- $\varrho : \mathcal{T} \times \mathcal{M} \to \mathcal{M}$ describes a function from transition probabilities and the stocks of individuals to stocks of individuals in the economy. This mapping, conceptually, corresponds to a mapping from *stocks of individuals (belief)* to *stocks of individuals (actual)*.

By Tychonoff's theorem, the space $\mathcal{M}$ is compact. It is also convex. The function $\varpi(\boldsymbol{\Lambda})$ is continuous in $\boldsymbol{\Lambda}$, the function $\iota(\boldsymbol{\alpha})$ is continuous in $\boldsymbol{\alpha}$, the function $\psi(\boldsymbol{V})$ is continuous in $\boldsymbol{V}$, the function $\xi(\boldsymbol{\alpha}, \boldsymbol{\epsilon^*})$ is continuous in $\boldsymbol{\alpha}$ and $\boldsymbol{\epsilon}^*$. The function $\varrho(\boldsymbol{\tau}, \boldsymbol{\Lambda})$ is continuous in $\boldsymbol{\tau}$ and $\boldsymbol{\Lambda}$. In sum, the whole mapping Φ is a function and continuous in $\boldsymbol{\Lambda}$. So, by Brouwer's fixed point theorem, there exists $\boldsymbol{\Lambda}^{**}$, which is a vector of equilibrium stocks satisfying the equilibrium conditions. Therefore, there also exist $\boldsymbol{\alpha}^{**} = \varpi(\boldsymbol{\Lambda}^{**})$, $\boldsymbol{V}^{**} = \iota(\boldsymbol{\alpha}^{**})$, $\boldsymbol{\epsilon}^{**} = \psi(\boldsymbol{V}^{**})$ and $\boldsymbol{\tau}^{**} = \xi(\boldsymbol{\alpha}^{**}, \boldsymbol{\epsilon}^{**})$. $\square$

5.3 Uniqueness

It is possible that, given a set of parameter values, the model has multiple equilibrium, meaning multiple equilibrium stocks, and, therefore, multiple equilibrium reservation values and multiple equilibrium meeting probabilities. I cannot give mathematical proof of the uniqueness of the equilibrium. However, given a set of parameter values, numerically I have checked that the equilibrium stocks converge to the same values with different initial starting values of stocks.[37] Theoretically multiple equilibrium would be possible given a set of parameter values, however, I can say that empirically it is unimportant.[38] .

6 Identification

In this section, I explain my identification strategy and how it works in detail. The identification argument uses the similar idea to Friedberg and Stern (2014). Friedberg and Stern (2014) conceptually divide their identification argument into parts to provide intuitively clear identification sources for each parameter to readers.[39]

[37] To be more precise, equilibrium is *locally* unique near the estimated parameter values.
[38] Proving theoretically the uniqueness of the equilibrium with this level of generality is obviously a future direction we need to pursue.
[39] Intuitively, the key idea of identification in this paper is that the moments which show up in this section have different information respectively to identify each parameter.

Before jumping into the identification argument, remember that the main primitives in the model, which are represented as a function with parameters, are

$$\{\{s_{ijt}, \mathbb{M}_{ijt}, \mathbb{C}^{M}_{ijt}, \mathbb{C}^{C}_{ijt}\}^{IJT}_{ijt}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\}, \tag{16}$$

where s_{ijt} is the deterministic part of the flow match value which a type i man and a type j woman jointly get at time t, $\mathbb{M}_{ijt}$, $\mathbb{C}^{M}_{ijt}$ and $\mathbb{C}^{C}_{ijt}$ are a set of model primitives associated with the marriage bonus, the divorce cost and the separation cost from cohabitation respectively. The distribution of the stochastic part in the flow match value is denoted as F_{ϵ} for an initial draw, and $F_{\tilde{\epsilon}|\epsilon}$ is the conditional distribution of ϵ_{ijt+1} conditional on ϵ_{ijt}. The joint distribution associated with cohabitation- and marriage-specific unobserved heterogeneity are denoted as F_{μ^C,μ^M}. Denote an exogenously given discount factor by ς and a woman's bargaining weight by ϕ.

Let $\boldsymbol{\beta}^s$ be a vector of the parameters associated with the flow match value, $\boldsymbol{\beta}^M$ be a vector of the parameters associated with the marriage bonus, $\boldsymbol{\beta}^{C^C}$ be a vector of parameters associated with the cohabitation separation cost and $\boldsymbol{\beta}^{C^M}$ be a vector of the parameters associated with the marriage cost. We can rewrite the set of the primitives (1) more explicitly, with $\boldsymbol{\beta}^s$, $\boldsymbol{\beta}^M$, $\boldsymbol{\beta}^{C^C}$ and $\boldsymbol{\beta}^{C^M}$, as

$$\{\{s_{ijt}(\boldsymbol{\beta}^s), \mathbb{M}_{ijt}(\boldsymbol{\beta}^M), \mathbb{C}^{M}_{ijt}(\boldsymbol{\beta}^{C^M}), \mathbb{C}^{C}_{ijt}(\boldsymbol{\beta}^{C^C})\}^{IJT}_{ijt}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\}. \tag{17}$$

Additionally, the meeting probabilities are *endogenously* decided in the model, which are also represented as a function having primitives as its arguments, $\boldsymbol{\beta}^{\alpha^m}$ and $\boldsymbol{\beta}^{\alpha^w}$. Namely, the meeting probabilities are

$$\{\alpha^m_{ijt}(\boldsymbol{\beta}^{\alpha^m}), \alpha^w_{ijt}(\boldsymbol{\beta}^{\alpha^w})\}^{IJT}_{ijt}. \tag{18}$$

Let Ω be the parameter set in the model, and it is an union of the sets,

$$\Omega = \{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M, \boldsymbol{\beta}^{C^M}, \boldsymbol{\beta}^{C^C}, F_{\epsilon}, F_{\tilde{\epsilon}|\epsilon}, F_{\mu^C,\mu^M}, \varsigma, \phi\} \cup \{\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}\}. \tag{19}$$

For the following argument, it is also important to remember, again, that each equilibrium reservation match value is represented as a function of all parameters in the economy (equilibrium meeting probabilities also). A vector of equilibrium reservation values, $\boldsymbol{\epsilon}^{**} = \{\epsilon^{**SC}_{ijt}, \epsilon^{**SM}_{ijt}, \epsilon^{**Si}_{ijt}, \epsilon^{**CS}_{ijt}, \epsilon^{**MS}_{ijt}\}^{IJT}_{ijt}$ is explicitly represented as $\boldsymbol{\epsilon}^{**}(\Omega) = \{\epsilon^{**SC}_{ijt}(\Omega), \epsilon^{**SM}_{ijt}(\Omega), \epsilon^{**Si}_{ijt}(\Omega), \epsilon^{**CS}_{ijt}(\Omega), \epsilon^{**MS}_{ijt}(\Omega)\}^{IJT}_{ijt}$ in this section.

6.1 Parameters associated with mating preference, s_{ijt}, and marriage bonus, $\mathbb{M}_{ijt}$

From this subsection, I start to provide sources of identification for each parameter. In this subsection, I explain how to separately identify the parameters associated with the mating preference and the marriage bonus while treating all of the other parameters as hypothetically fixed.

Assume the existence of equilibrium reservation match values and an equilibrium single-crossing point, and assume to satisfy the conditions for the coexistence of cohabitation and marriage discussed in Section 4.2.[40] I can neatly divide the support of ϵ_{ijt} into three parts with the thresholds, the equilibrium reservation

[40] Under some values of the parameter vector and realizations of persistent unobserved heterogeneity terms, there would be some cases

match value from single to cohabitation, ϵ_{ijt}^{**SC}, and the equilibrium single-crossing point, ϵ_{ijt}^{**Si}.

Let $h_{ijt}^{SM(m)}$ be a type i man's hazard rate out of being single to being married with a type j woman at time t. It is given by

$$h_{ijt}^{SM(m)} = \alpha_{ijt}^{m^{**}}(1 - F_\epsilon(\epsilon_{ijt}^{**Si})). \tag{20}$$

Similarly, the single to cohabitation hazard is

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{m^{**}}\left[F_\epsilon(\epsilon_{ijt}^{**Si}) - F_\epsilon(\epsilon_{ijt}^{**SC})\right]. \tag{21}$$

Equations (20) and (21) are originally with the unknown parameters, Ω. However, at this point of discussion, ϵ_{ijt}^{**SC} and ϵ_{ijt}^{**Si} are assumed hypothetically represented by a function of the following unknown parameters associated with the mating preference and marriage bonus, $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$ only.

So, in the end, at this point of discussion, only the parameters associated with the mating preference, $\boldsymbol{\beta}^s$, and marriage bonus, $\boldsymbol{\beta}^M$, are left. Accordingly, I can rewrite equations (20) and (21) as

$$h_{ij(c_t)\kappa t}^{SM(m)} = \alpha_{ijt}^{m^{**}}(1 - F_\epsilon(\epsilon_{ijt}^{**Si}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M\})), \tag{22}$$

$$h_{ijt}^{SC(m)} = \alpha_{ijt}^{m^{**}}\left[F_\epsilon(\epsilon_{ijt}^{**Si}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M)\}) - F_\epsilon(\epsilon_{ijt}^{**SC}(\{\boldsymbol{\beta}^s, \boldsymbol{\beta}^M)\}))\right]. \tag{23}$$

First, remember that the equilibrium meeting probability, $\alpha_{ijt}^{m^{**}}$, is treated as given at this point of the argument. I have $2 \times IJT$ equations because I get two equations (22) and (23) for each i, j and t. Note that we can use a woman side's information in the same way. In sum, the parameters associated with the mating preference and the marriage bonus are mainly identified by matching the sample moments of $h_{ijt}^{SC(m)}$ and $h_{ijt}^{SM(m)}$, $\hat{h}_{ijt}^{SC(m)}$ and $\hat{h}_{ijt}^{SM(m)}$ for men, and $h_{ijt}^{SC(w)}$ and $h_{ijt}^{SM(w)}$, $\hat{h}_{ijt}^{SC(w)}$ and $\hat{h}_{ijt}^{SM(w)}$ for women to the corresponding theoretical moments. We can put it in a traditional way of explanation of identification: The *ceteris paribus* effect by changing the explanatory variable associated with $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$ on $\hat{h}_{ijt}^{SC(w)}$ and $\hat{h}_{ijt}^{SM(w)}$ mainly identify $\boldsymbol{\beta}^s$ and $\boldsymbol{\beta}^M$.

6.2 Parameters associated with cohabitation separation cost, $\mathbb{C}_{ijt}^C$, and divorce cost, $\mathbb{C}_{ijt}^M$

Next, I move to the discussion of how to pin down the parameters associated with separation costs, $\mathbb{C}_{ijt}^C$ and $\mathbb{C}_{ijt}^M$. As in the previous procedure where I have focused only on the parameters associated with the mating preference and the marriage bonus, I hypothetically fix parameters other than the parameters associated with $\mathbb{C}_{ijt}^C$ and $\mathbb{C}_{ijt}^M$. To identify the parameters associated with the separation costs, I use different moments, a sample cohabitation to single hazard rate and a sample marriage to single hazard rate.

6.2.1 Parameters associated with cohabitation separation cost

Let h_{ijt}^{CS} be a hazard rate out of cohabitation by a type i man and a type j woman to single at time t. Remember ϵ_{ijt}^{**CS} is an equilibrium reservation match value with which a couple is indifferent between continuing to cohabit and returning to single.

where, for some men and women, cohabitation is not their best choice. Nor the single-crossing condition is not be satisfied. However, if the number of observations goes infinity, the number of observations which satisfies the single-crossing condition goes to infinity also.

The hazard rate is represented as, using the notion of $\boldsymbol{\beta}^{C^C}$,

$$h_{ijt}^{CS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**CS}(\{\boldsymbol{\beta}^{C^C}\}) \right) dF_{\epsilon_{t-1}}. \tag{24}$$

In equation (9), I emphasize that the equilibrium reservation match values, ϵ_{ijt}^{**CS}, depends on the parameters associated with the cohabitation separation cost, $\boldsymbol{\beta}^{C^C}$. Remember there is serial correlation in the law of motion of ϵ_{ijt}. Therefore, in taking the integral, I take into account the previous draw, ϵ_{ijt-1}, which is written as ϵ_{t-1} in equation (9) for shorter notation. The parameters, $\boldsymbol{\beta}^{C^C}$, is mainly identified by matching the sample moment constructed from the data, $\hat{h}_{ijt}^{CS}$, to the corresponding theoretical moments in the same way as the parameters associated with the mating preference and the marriage bonus case.

6.2.2 Parameter associated with divorce cost

Similarly, let h_{ijt}^{MS} be a hazard rate out of marriage by a type i man and a type j woman to single at time t. Similarly, define ϵ_{ijt}^{**MS} as an equilibrium reservation match value with which a couple is indifferent between remaining married and returning to single. The hazard rate is

$$h_{ijt}^{MS} = \int_{\epsilon_{t-1}} F_{\epsilon|\epsilon_{t-1}} \left(\epsilon_{ijt}^{**MS}(\{\boldsymbol{\beta}^{C^M}\}) \right) dF_{\epsilon_{t-1}}. \tag{25}$$

I emphasize that ϵ_{ijt}^{**MS} depends on $\boldsymbol{\beta}^{C^M}$. It is mainly identified by matching the sample moment constructed from the data, $\hat{h}_{ijt}^{MS}$, to the corresponding theoretical moment in the same way as before.

6.3 Parameters associated with second moments

The variance, covariance and the law of motion terms, F_ϵ, F_{μ^C,μ^M} and $F_{\tilde{\epsilon}|\epsilon}$, are identified through second sample moments calculated by generalized residuals. Note that, for example, in a standard ordinary least squares model, we can easily calculate its residuals to construct its second sample moments. However, in a discrete choice model, it is not so straightforward to calculate its generalized residuals due to the unobservable latent variable. See, for example, Gourieroux et al. (1987), Goeree (2008) and Friedberg and Stern (2014) for more detail about how to construct them.

6.4 Parameters associated with meeting probability, α_{ijt}^m and α_{ijt}^w

In the last step, I pin down the parameters associated with α_{ijt}^m and α_{ijt}^w. So far, they have been taken as given. They are identified by using the equilibrium condition, the *consistency belief* condition, which we need to satisfy at equilibrium discussed in Section 5. Namely, the law of motions for the stocks in the economy mainly backs up the meeting probabilities. Let $\boldsymbol{D}_t^{**}$ be a vector of equilibrium operators associated with the law of motion for equilibrium stocks at time t, which is complicated functions of the model primitives. We have, given t,

$$\hat{\boldsymbol{\Lambda}}_{t+1}^{**} = \boldsymbol{D}_t^{**}(\Omega)\hat{\boldsymbol{\Lambda}}_t^{**}. \tag{26}$$

I treat the observed stocks as equilibrium stocks, $\hat{\boldsymbol{\Lambda}}_t^{**}$, assuming that the economy is in equilibrium. I have explicitly proven the existence of the equilibrium stocks in the economy.[41] It is, theoretically, ensured that there are the equilibrium stocks satisfying equation (26). Therefore, we can use the equations as a set of new moments. The same as the previous arguments, I emphasize the dependence of the meeting technology parameters, $\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}$, and rewrite it,

$$\hat{\boldsymbol{\Lambda}}_{t+1}^{**} = \boldsymbol{D}_t^{**}(\{\boldsymbol{\beta}^{\alpha^m}, \boldsymbol{\beta}^{\alpha^w}\})\hat{\boldsymbol{\Lambda}}_t^{**}. \tag{27}$$

The set of moments mainly identifies $\boldsymbol{\beta}^{\alpha^m}$ and $\boldsymbol{\beta}^{\alpha^w}$. Additionally, a point which should be emphasized is that *market thickness* affects the meeting probabilities through an aggregate matching function but does not affect other model primitives. See, Section 8.4 for more detail.[42]

7 Data

In this research, I use the two data sets, the National Longitudinal Study of High School Class of 1972 (NLS 72) and the National Longitudinal Survey of Youth 1997 (NLSY 97). The NLS 72 is assumed to represent the cohort under the situation before the advances in the communication technology occurs, and the NLSY 97 is assumed to represent the cohort under the situation after the advances in the communication technology occurs. In addition to detailed demographic information, both data sets have relationship type information including cohabitation from early ages of respondents.[43] Compared with other national representative surveys, the two data sets are unique in that they contain detailed retrospective marital history information during sample periods. Capturing shorter spells of cohabitation than marriage, I track individuals' marital status transitions every six months.

In the NLS 72, around 22,000 students are first interviewed when they are leaving a high school in the spring of 1972 with follow-up interviews in 1973, 1974, 1976 and 1979, and, for a limited group, 1986. I use the subset of the whole sample which answers the 1986 follow-up interview. As Brien et al. (2006) and Lillard et al. (1995) mention, the limited group of the whole sample, which answers the 1986 interview, does not represent the whole population composition. Therefore, I give appropriate weights on the observations to adjust a choice-based sampling problem (see, Manski and Lerman, 1977, Hellerstein and Imbens, 1999, and Nevo, 2003). As used in Weiss and Willis (1997), I use the weights discussed in Tourangeau (1987).

Particularly, on each cohort, I focus on the data associated with martial status transitions of an individual, conditioning on an individual's gender, race, educational level as well as a partner's educational level if matched, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.[44] For detailed empirical specification for how the marriage market with the observable data is formed, see, Section 8.

[41] Note that, *theoretically*, the equality needs to hold exactly. However, in constructing moments associated with the consistency belief condition, I add a sampling error to make the moment conditions work properly in estimation.

[42] To strengthen the identification argument, I also include *exclusion restrictions*: To separately identify the parameters associated with the preference (match surplus) and those of the meeting technology, there are explanatory variables which only belong to either of the two parameter specifications. See, Section 8.3, for more detailed discussion.

[43] This solves an initial conditions issue.

[44] The NLS 72 is basically a single age cohort in which every sample has the same age. So, the effects of respondents' age and the calendar time on marital behaviors cannot be separately identified. I basically interprets the date of cohabitation/marriage as a measure of age at cohabitation/marriage.

7.1 Detailed sample selection criteria

Tables 1 and 2 show the sample selection criteria in this research. Originally, I have 22,650 observations in the NLS72.[45] I use respondents who answer the fifth follow-up interview, 12,840. I further restrict the sample left to black and white individuals, 10,790. I delete observations for who I cannot identify their basic demographic variables, 10,720. I drop individuals for who I cannot identify their marital history, 10,400. Because a partner's information is important in this research, I drop respondents who do not report their partner's eduction level or whether their partner has children from their previous relationship, 9,920.[46] I drop observations who serve the military, 9,160.[47] Lastly, I confine sample who has not experienced cohabitation before age 18, 7,410.[48]

Table 1: Sample selection criteria(NLS 72)

Selection criteria	Observations left
Whole Population	22,650
Fifth follow-up	12,840
Black or white	10,790
With basic demographic variables	10,720
With marital history	10,400
With partner's information	9,920
Not military	9,160
Cohabitation starts after age 18	7,410
Number of observations	7,410

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 2: Sample selection criteria (NLSY 97)

Selection criteria	Observations left
Whole Population	8,984
Black or white	7,622
With basic demographic variables	6,776
With marital history	6,776
With partner's information	6,776
Not military	6,348
Number of observations	6,348

On the other hand, in the NLSY 97, originally, I have 8,984 observations. I restrict the sample to black and white individuals, 7,622. I delete observations for who I cannot identify their basic demographic variables, 6,766. For the NLSY 97 sample, even if a respondent misses an annual interview about his/her relationship, I impute the missing answer consistent with his/her answer for the periods before and after the period.[49] For a partner's education level, not a few responses do not annually report the information. In this research, as long as a partner has the same unique id, I fill in the missing partner's information during the corresponding spells

[45]For the NLS 72 data, the U.S. Department of Education's National Center for Education Statistics requires rounding numbers to the nearest 10 to ensure disclosure protection.

[46]Since my main objective in this research is to investigate individual marital dynamics, I remove individuals who miss or refuse to answer questions associated with their marital history during the survey periods. Even though there are several imputation methods to deal with missing data issues (for example, Lavy et al. 1998; Van der Klaauw and Wolpin 2008; and Keane and Sauer 2010), I remove them for simplification of the analysis. This is partly justified because I still have enough observations left for estimation, 210,630 person-period observations in the NLS 72.

[47]Individuals who serve in the military because their marital behavior differs from that of the rest of population.

[48]There are some observations who have cohabited before age 18. However, their marital behavior differs from that of the rest of population. As Kaplan (2012) does, I need to check how estimates differ using two alternative samples: One that contains individuals who have cohabited before age 18 and one without the individuals.

[49]Suppose an individual misses or refuses to answer an annual interview. However, if he reports that he cohabits with the partner with the same unique id for periods before and after the period. I assume that he also cohabits with the same person during the missing period. I can do this imputation only for the NLSY 97 sample because, for the NLSY 97, the sampling period is annual. On the other hand, the NLS 72 only ask the retrospective questions once at 1986.

with the partner's lowest level of education.[50] I drop observations who serve the military, 6348.[51]

7.2 Descriptive statistics of initial (time-0) distribution and marital status dynamics

For a non-stationary model, the initial (time-0) distribution of individuals in the economy is also an exogenous model primitive.[52] Tables 3 through 6 show the composition of individuals in the economy, which comes from the NLS 72 and the NLSY 97 with proper sampling weights. Note that the information is represented as stocks because I use a special case of a Pissarides' style matching function whose arguments are stocks not proportions, as discussed in Section 8.4.[53]

Table 3: Population Statistics aged 14-18 in 1980 (NLS 72)

Exogenous characteristics	Number/Percentage
Whole Population	12,716 (in thousands)
Women	50 %
Black	11 %
Without High School Degree	26 %
High School Degree	52 %
Above College	21 %

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 4: Population Statistics aged 14-18 in 2000 (NLSY 97)

Exogenous characteristics	Number/Percentage
Whole Population	16,131 (in thousands)
Women	50 %
Black	12 %
Without High School Degree	13 %
High School Degree	57 %
Above College	29 %

Describing the descriptive statistics and emphasizing remarkable changes is quite important in this research: I want to quantify how much of the changes in marital patterns come from the changes of the composition of the economy (Tables 3-6) or the changes of the model primitives.

In terms of the exogenous characteristics shown in Tables 3-6, one remarkable change is that, in the NLSY 97 cohort, a smaller proportion is without high school degree. The percentage is 26% in the NLS 72 and it reduces to 13% in the NLSY 97. Other than the educational attainment difference, a remarkable change is the total population, which changes from $12,716$ thousands to $16,131$ thousands. As shown in Tables 5 and 6, educational attainment by gender and race changes significantly during the two cohorts.

Next, we show simple descriptive statistics of marital dynamics to point out that there quantitative differences exist at an aggregate level before and after the advances in the communication technology: Certainly, we can observe, to non-negligible extent, their marital behaviors differ during the two cohorts, which is shown in Figures 2 - 9. Aggregate stocks evolve in a different way in each cohort. One of notable points I need to mention

[50] As Guvenen and Smith (2014) do, I need to check the robustness of this imputing filling-in method on the estimates.

[51] Note that, as Fiorini and Keane (2011) do, I need to check whether my attrition, missing data and sample selection criteria lead to a sample selection issue with respect to the original data sets: I run a probit model where the dependent variable is equal to 1 if the respondent is in my sample and zero otherwise. The explanatory variables are reported demographic characteristics in the original data sets. I can implement a statistically significant test, for example, by seeing the values of the coefficients.

[52] In a non-stationary model, the initial (time 0) distribution is also an exogenous model primitive: If we observe the difference in equilibrium outputs of the two cohorts, it might come just from the difference of the initial distributions, not from the difference of the other model primitives. However, in my model setting, I can overcome this issue by controlling the stocks in estimation. See, Section 8.4 for the empirical specification of meeting probabilities.

[53] Remember that Pissarides' style matching function is assumed to constant return to scale. In this sense, not only the ratio but also the amount of stocks matters (Petrongolo and Pissarides (2001)).

Table 5: Educational attainment by gender and race (NLS 72)

Male	White	Black
Without High School Degree	10.9 %	25.3 %
High School Degree	62.3 %	64.2 %
College or above	26.8 %	10.5 %
Female	White	Black
Without High School Degree	10.8 %	21.7 %
High School Degree	66 %	65.9 %
College or above	23.2 %	12.4 %

SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 6: Educational attainment by gender and race (NLSY 97)

Male	White	Black
Without High School Degree	7.9 %	8.8 %
High School Degree	60.7 %	77.7 %
College or above	31.4 %	13.5 %
Female	White	Black
Without High School Degree	5.5 %	13.4 %
High School Degree	57 %	66.6 %
College or above	37.5 %	20 %

is that, compared with the NLS 72 cohort, people in the NLSY 97 cohort get married less and cohabit more. As shown in Figure 5, during the sample periods of the NLS 72, the proportion of the stock of cohabitation is at most 10%. For people in the NLSY 97, cohabitation is a more popular choice. When the sample in the NLS 72 reaches age 32, which is 1986 as shown in Figure 4, more than half of them get married (over 60 percent). In the NLSY 97 cohort, the proportion of the stock of marriage is much lower than in the NLS 72 cohort throughout the sampling period.

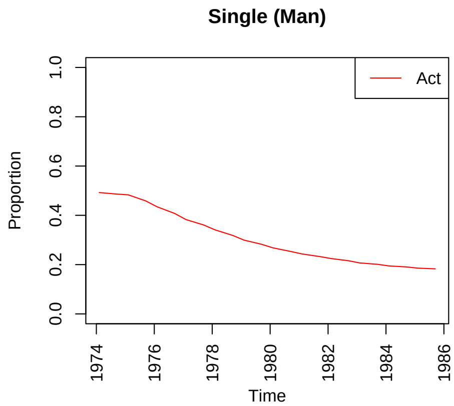


Figure 2: Stock of single men (NLS 72)

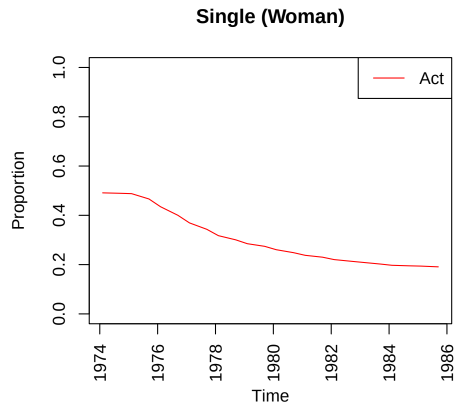


Figure 3: Stock of single women (NLS 72)

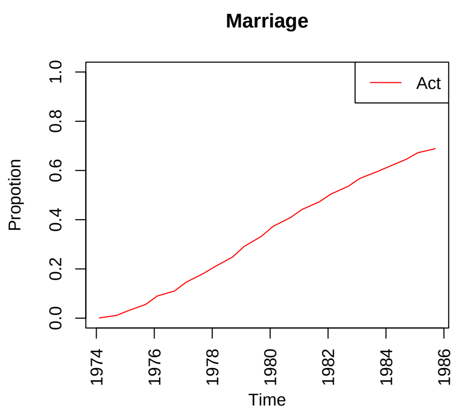


Figure 4: NLS 72 Stock of marriage actual

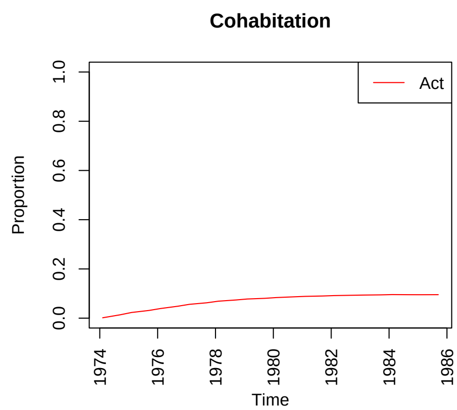


Figure 5: Stock of Cohabitation (NLS 72)

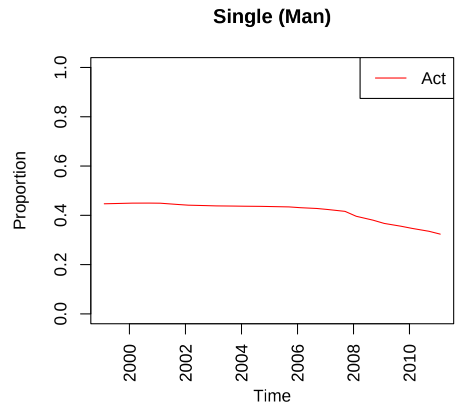


Figure 6: Stock of single men (NLSY 97)

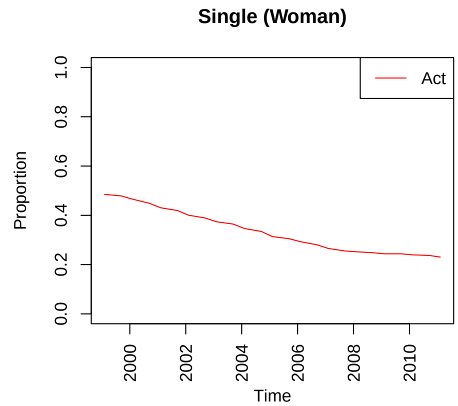


Figure 7: Stock of single women (NLSY 97)

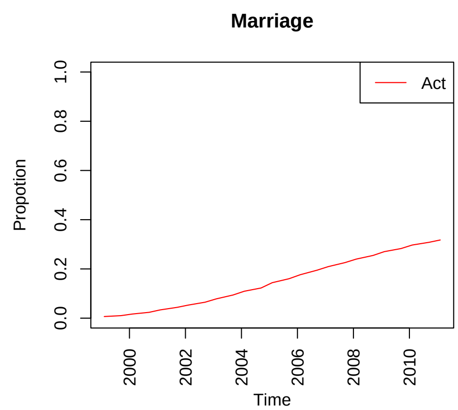


Figure 8: Stock of marriage (NLSY 97)

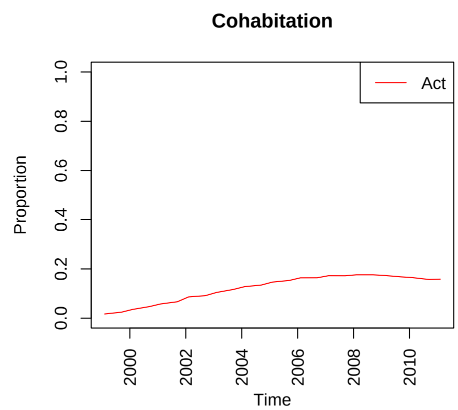


Figure 9: Stock of Cohabitation (NLSY 97)

7.3 Non-structural analysis of two cohorts difference

In addition to the changes in the descriptive patterns of the aggregate exogenous characteristics and the aggregate endogenous marital behaviors, I measure the distance of the two date sets, using auxiliary statistics, to get more sense of the importance of the technology advances: If the two data sets can be considered as close measured by a certain norm, for example, Euclidean norm or Manhattan distance, it implies that there are no changes during the two cohorts. I use the coefficients of the following non-structural model as a set of auxiliary statistics.[54]

In the following, I suppress the notation representing an individual for simpler notation. Consider a type i man and a type j woman at time t under technology level κ represented by a vector of demographic and endogenous state variables, $X^m_{it\kappa}$ and $X^w_{jt\kappa}$ with $X_\kappa = [X^m_{it\kappa}, X^w_{jt\kappa}]$. The vector of variables, X_κ, includes gender,

[54]We do not have to have a correctly-specified model. Therefore, the estimates would be bias. The interpretation of the estimates does not have much meaning. The estimates are just for measuring the distance between the two data sets: If two data sets are considered as the same, auxiliary statistics, for example, regression coefficients are the same across the regressions. The idea is the same as indirect inference (Gourieroux et al, (1993)).

race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.

Let $y^S_{t\kappa} = \{1,2,3\}$ be a dependent variable representing that a *single* individual in selects single, $y^S_{t\kappa} = 1$, cohabitation, $y^S_{t\kappa} = 2$, and marriage, $y^S_{t\kappa} = 3$ at time t under technology κ. Let $y^C_{t\kappa} = \{1,2,3\}$ be a dependent variable representing that a *cohabiting* individual selects single, $y^C_{t\kappa} = 1$, cohabitation, $y^C_{t\kappa} = 2$, and marriage, $y^C_{t\kappa} = 3$ at time t under κ. Similarly, $y^M_{t\kappa}$ is defined for transitions from *marriage*.

Then, the non-structural ordered linear probability models are

$$y^S_{t\kappa} = \boldsymbol{X_\kappa \gamma^S_\kappa} + e^S; \tag{28}$$

$$y^C_{t\kappa} = \boldsymbol{X_\kappa \gamma^C_\kappa} + e^C; \tag{29}$$

$$y^M_{t\kappa} = \boldsymbol{X_\kappa \gamma^M_\kappa} + e^M; \tag{30}$$

where $\boldsymbol{\gamma^S_\kappa}$, $\boldsymbol{\gamma^C_\kappa}$ and $\boldsymbol{\gamma^M_\kappa}$ are vectors of coefficients for the non-structural ordered linear probability models about a transition from single, cohabitation and marriage under technology level κ respectively, and e^S, e^C and e^M are specified as $e^S, e^C, e^M \sim iidN(0,1)$ for simplicity.[55]

The difference (distance) of the two cohorts is summarised by using the auxiliary statistics in Tables 7-9. I implement a F-statistics test where the null hypothesis are the coefficients of those of the NLSY 97 cohort is the same as those of the NLS 72 cohort.[56] In the case of the transition from single (eq (28)), I reject the hypothesis. In the case of the transition from cohabitation (eq (29)) , I reject the hypothesis. In the case of the transition from marriage (eq (30)), I reject the hypothesis. These results also support the different marital trends exist between the NLS 72 and the NLSY 97.

Table 7: Coefficients associated with Transition from single

Parameter	NLS 72[a]	NLSY 97
Constant	1.005	0.013
Age (time)	0.006	0.000
Age spline $\leq$ 5 years	0.003	-0.002
Age spline $\geq$ 10 years	-0.008	-0.009
Female	0.000	-0.000
Respondent education	0.139	0.170
Partner education	0.139	0.171
Education difference	-0.140	-0.174
Kid from previous relationship	0.104	0.002
Black	-0.005	-0.003

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[55] Note that we can assume a different distribution. This is because I do not use directly the information of the shape of the distribution of e^S, e^C, e^M in estimation.

[56] I use the NLSY 97 data set with the coefficients of the NLS 72 cohort in the hypothesis testing. Though, if the number of observations is large enough, any hypothetical testing does not work properly.

Table 8: Coefficients associated with Transition from cohabitation

Parameter	NLS 72[a]	NLSY 97
Constant	2.893	0.931
Age (time)	0.009	-0.002
Age spline $\leq$ 5 years	0.004	0.002
Age spline $\geq$ 10 years	0.000	0.000
Female	-0.020	0.000
Respondent education	0.125	0.002
Partner education	0.123	0.004
Education difference	-0.136	-0.008
Match duration	-0.051	0.000
Match duration spline $\leq$ 2 years	-0.005	0.000
Black	0.003	-0.005

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

Table 9: Coefficients associated with Transition from marriage

Parameter	NLS 72 [a]	NLSY 97
Constant	3.141	1.905
Age (time)	0.000	-0.001
Age spline $\leq$ 5 years	0.000	0.000
Age spline $\geq$ 10 years	0.000	0.000
Female	-0.238	-0.001
Respondent education	-0.192	0.014
Partner education	-0.196	0.018
Education difference	0.194	-0.110
Kid from previous relationship	-0.005	-0.007
Kid from current relationship	0.006	0.027
Match duration	0.000	-0.001
Match duration spline $\leq$ 2 years	0.000	0.004
Black	-0.001	0.000

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

8 Estimation

In this section, I explain the estimation process. Given a set of the parameters, Ω, there exist equilibrium reservation values in this game as proven in Section 5. Therefore, through the equilibrium reservation values calculated, I calculate the objective function, which conceptually corresponds to how much my model explains actual data patterns. I will introduce how to construct the objective function below. I keep doing the above procedure with different values of the parameters until the numerically calculated objective function reaches its minimum. Particularly, I look for a set of values of the parameters in the parameter space which minimizes the objective function with the idea of indirect inference, which will be introduced briefly below.

So far, particular functional forms for model primitives have not been explicitly specified. I need to specify the functional forms of the model primitives before I start solving the model. So, in the end of this section, I give a detailed specification of the economy for estimation purposes. Meanwhile, assume that it is appropriately specified according to the theoretical requirements discussed in Section 5.

8.1 Indirect Inference

I use indirect inference in estimation (Gourieroux et al. (1993)). The underlying concept of indirect inference involves three steps: first, I select a set of moments, which is also called auxiliary statistics. They are assumed to represent the characteristics of the real data (observed moments); second, simulating the structural economic model and calculating the corresponding simulated moments several times with different values of parameters; third, picking parameter values such that the simulated moments closely replicate the observed moments. When the number of observations becomes infinite, indirect inference gives us consistent estimates of the parameters (Gourieroux et al. (1993)).

The reason why I use indirect inference in this research is that the NLS 72 data set is highly confidential. Its discloser policy does not allow me to use any estimation method which others might be able to identify an individual in the sample. Indirect inference only requires aggregate moments information in a particular data set.

Following previous studies with indirect inference, I select a set of auxiliary statistics which are easy to compute and are able to capture a variety of patterns in the data. Also, the set of moments should be informative for the underlying structural model.[57] We evaluate the distance between the two data sets using the Euclidian distance constructed by the following auxiliary statistics.

As a part of auxiliary statistics, I focus on a simple non-structural ordered linear probability model for a single-single, single-cohabitation and single-marriage transition. Similarly, I also focus on a transition from cohabitation and marriage using a non-structural ordered linear probability model. They are introduced in more detail below.

In the following, I suppress the notation representing an individual. Consider a type i man and a type j woman at time t represented by a vector of demographic and endogenous state variables, $\boldsymbol{X}_{it}^{m}$ and $\boldsymbol{X}_{jt}^{w}$ with $\boldsymbol{X} = [\boldsymbol{X}_{it}^{m}, \boldsymbol{X}_{jt}^{w}]$. The vector of variables, $\boldsymbol{X}$, includes gender, race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.[58]

[57]As, for example, van der Klaauw and Wolpin (2008) and Collard-Wexler (2013) say, auxiliary statistics do not need to have any interpretation. They are just required to describe the characteristics of data as much as possible. As Hall and Rust (2020) mention, a variety of moments can be used as a possible set of auxiliary statistics (first, second, third and fourth moments, covariance and quantiles, etc.). In general, it does not matter which auxiliary statistics I use as long as I can properly measure the distance between the simulated and actual data through the "lens" of the statistics (for example, see, Gourieroux et al. (1993) and Alan (2006)), as long as the moments are related to the underlying structural model (*invertibility or identifiability*).

[58]The NLS 72 is basically a single age cohort in which every sample has the same age. So, the effects of respondents' age and the calendar time on marital behaviors cannot be separately identified. On the other hand, the NLSY 97 has the variation on the respondents' age at a given time. However, I assume that the effects on marital behaviors from the age difference are not significant. This assumption is partly justified because the variation of ages is small in the NLSY 97 cohort as I wrote above. So, it is not such strange to assume similar

Let $y_t^S = \{1, 2, 3\}$ be a dependent variable representing that a *single* individual selects single, $y_t^S = 1$, cohabitation, $y_t^S = 2$, and marriage, $y_t^S = 3$, at time t. Let $y_t^C = \{1, 2, 3\}$ be a dependent variable representing that a *cohabiting* individual selects single, $y_t^C = 1$, cohabitation, $y_t^C = 2$, and marriage, $y_t^C = 3$, at time t. Similarly, y_t^M is defined for transitions from *marriage*. Then, the non-structural ordered linear probability models are

$$y_t^S = \boldsymbol{X}\boldsymbol{\gamma^S} + e^S; \tag{31}$$

$$y_t^C = \boldsymbol{X}\boldsymbol{\gamma^C} + e^C; \tag{32}$$

$$y_t^M = \boldsymbol{X}\boldsymbol{\gamma^M} + e^M; \tag{33}$$

where $\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}$ and $\boldsymbol{\gamma^M}$ are vectors of coefficients for the non-structural ordered linear probability models about a transition from single, cohabitation and marriage respectively, and e^S, e^C and e^M are specified as $e^S, e^C, e^M \sim N(0, 1)$.[59] Particularly, as a set of auxiliary statistics, I focus on the partial derivative of relationship status transitions of individuals with respective to a set of explanatory variables, which are captured by the coefficients of the linear probability models, $\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}$ and $\boldsymbol{\gamma^M}$.

Let y_t be an individual choice at time t observable in the data, $y_t = \{Single, Cohabitation, Marriage\}$, and let $\boldsymbol{y}$ be a vector of an individual observable relationship choices throughout his/her life, $\boldsymbol{y} = \{y_1, y_2, ..., y_T\}$. I construct the sample covariance matrix of the vector of an individual's relationship choices, $\boldsymbol{y}$, using the information of $\boldsymbol{y}$ for all individuals in the data. I use the sample covariance matrix as a part of auxiliary statistics. The set of auxiliary statistics associated with the covariance matrix are denoted as $\boldsymbol{\vartheta}$.

Aggregate stocks of single individuals, cohabitating individuals and individuals in marriage also work as a part of auxiliary statistics. I denote $\boldsymbol{q^S}, \boldsymbol{q^C}$ and $\boldsymbol{q^M}$ as vectors of auxiliary statistics associated with aggregate stocks of individuals of single, cohabitation and marriage at equilibrium respectively. Let Ξ be a set of auxiliary statistics. Particularly,

$$\Xi = \{\boldsymbol{\gamma^S}, \boldsymbol{\gamma^C}, \boldsymbol{\gamma^M}, \boldsymbol{q^S}, \boldsymbol{q^C}, \boldsymbol{q^M}, \boldsymbol{\vartheta}\}.$$

Table 10 summarizes the auxiliary statistics used in this research.

preference on marital behaviors within respondents within the age variations. Practically, with respect to computation issues as well, I do not want to deal with age and the calendar time as different state variables.

[59]Note that I can assume a different distribution. This is because I do not use directly the information of the shape of the distribution of e^S, e^C, e^M in estimation.

Table 10: Auxiliary statistics

#	Auxiliary statistics
1	Coefficients of ordinal linear probability of transitions from single to cohabitation/marriage conditional on gender, race, educational level, partner's educational level, children from previous relationship, age and age spline modification
2	Coefficients of ordinal linear probability of transitions from cohabitation to single/marriage conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration and duration spline modification
3	Coefficients of ordinal linear probability of transitions from marriage to single conditional on gender, race, educational level, partner's educational level, children from previous relationship, age, age spline modification, children from current relationship, duration and duration spline modification
4	Across-person variation in length of single
5	Across-person variation in length of cohabitation
6	Across-person variation in length of marriage
7	Across-person variation in length of a match (cohabitation or marriage)
8	Across-person correlation between lengths of cohabitation and marriage
9	Calculated Stocks (stock of single, cohabitation and marriage individuals)

8.2 Objective function

With the set of auxiliary statistics, I can construct the objective function I need to minimize. Let Ω be a vector of the structural parameters. Let R be the number of simulations. Let $\hat{\Xi}(\Omega)$ be a vector of auxiliary statistics calculated by using R simulated data sets with Ω. Let $\bar{\Xi}$ be a vector of auxiliary statistics calculated by the actual data. Let Γ be a weighting matrix. Let $\hat{\Omega}$ be a vector of estimates of the structural parameters, and it is written as

$$\hat{\Omega} = \arg\min_{\Omega}[[\hat{\Xi}(\Omega) - \bar{\Xi}]'\Gamma[\hat{\Xi}(\Omega) - \bar{\Xi}]]. \tag{34}$$

Following much of the literature (for example, Altonji and Segal (1996)), I do not use the optimal weighting matrix. Instead, I use a *diagonal* weighting matrix to adjust scales of each moment.

In estimation, the equilibrium consistency belief condition is not explicitly imposed because of extremely large computational burdens, as in Seitz (2009) and Beauchamp et al. (2018). I assume that the economy is already on the equilibrium mapped by the underlying structural parameters. Therefore, I regard that the observed stocks in the data as the equilibrium stocks in the economy. In solving an individual dynamic decision process, all players are assumed to have the consistent belief already. However, as I discuss in Section 10, when I do counterfactual experiments with different values of parameters, I iterate the fix-point algorithm and derive the corresponding equilibrium stocks.[60]

In constructing the simulated data, I perform simulation as follows: Given a set of values of the structural parameters, I solve the dynamic programming problem. With an individual's observable initial exogenous characteristics and the permanent unobserved heterogeneity terms drawn, I simulate an individual decision by drawing shocks, and update the individual's state variables using the decision rules driven by the dynamic programming. I keep doing this procedure until the terminal period.

[60]Given parameter values, I need to recalculate a fixed-point algorithm so that players' beliefs converge to the actual aggregate dynamics of stocks. For example, see, Rust (2008).

In this research, I employ a moment-based estimation method (indirect inference), which creates non-smoothness of the objective function on the parameter space, given the finite number of simulated draws. In this research, I set $R = 450$ which makes the number of simulated draws at each iteration approximately 100 million. This number is sufficient to consider the objective function as smooth enough on the parameter space (see, for example, Pakes (1986)). In this research, additionally, accommodating that the possibility of my objective function is non-smooth, I use a derivative-based optimization method for a non-smooth function (Lewis and Oberton (2013)).

8.3 Empirical Specification

Before estimating the model, I need to give specific functional forms to the model primitives described in Section 3. Also, I impose some practical restrictions on dynamics of explanatory variables and marriage markets feasible for players. In this section, I provide parsimonious functional forms of the model primitives and some restrictions imposed on dynamics of a part of explanatory variables and marriage markets in more detail. Particular focuses of this section are the following model primitives; flow match value, $s_{ijt\kappa}$, marriage bonus, $\mathbb{M}_{ijt\kappa}$, cohabitation separation cost, $\mathbb{C}^{C}_{ijt\kappa}$, divorce cost, $\mathbb{C}^{M}_{ijt\kappa}$, meeting probability, $\alpha^{m}_{ijt\kappa}$ and $\alpha^{w}_{ijt\kappa}$, childbearing probability, $P^{b}_{t\kappa}$, distributions associated with cohabitation and marriage unobserved heterogeneity terms and the stochastic part of a match value, F_{μ^C,μ^M}, F_{ϵ} and $F_{\tilde{\epsilon}|\epsilon}$ respectively. I also introduce restrictions on dynamics of stocks of children, evaluation of match durations and possible marriage markets.

8.3.1 Flow match value, marriage bonus, separation costs and unobserved terms

Most of the model primitives are assumed to be approximated by a function of a linear index in parameters. Consider an observable type for an i man and an observable type for a j woman at time t represented by a vector of demographic and endogenous state variables, $\boldsymbol{X}^{m}_{it}$ and $\boldsymbol{X}^{w}_{jt}$, respectively, and denote $\boldsymbol{X} = [\boldsymbol{X}^{m}_{it}, \boldsymbol{X}^{w}_{jt}]$. The matrix of the variables, $\boldsymbol{X}$, includes gender, race, educational level, whether there is a child from a previous relationship, whether there is a child from a current relationship, match durations, and age which is assumed equivalent to time effects.

Let $\boldsymbol{\beta}^{m}_{\kappa}$ and $\boldsymbol{\beta}^{w}_{\kappa}$ be a vector of coefficients for the flow match value associated with a man's observable type and a woman's observable type, and denote $\boldsymbol{\beta}_{\kappa} = [\boldsymbol{\beta}^{m}_{\kappa}, \boldsymbol{\beta}^{w}_{\kappa}]$. The variable κ emphasizes that the parameter values change depending on the communication technology level.

The match surplus between a type i man and a type j woman at time t under κ presented in equation (1) in Section 3 is specified as [61]

$$
\begin{aligned}
s_{ijt\kappa} = {} & \boldsymbol{X}\boldsymbol{\beta}_{\kappa} + \mathbb{M}_{ijt\kappa}\mathbb{I}[d_{ijt} = 3] + (\mu^{Cm}_{\kappa} + \mu^{Cw}_{\kappa})\mathbb{I}[d_{ijt} = 2] + (\mu^{Mm}_{\kappa} + \mu^{Mw}_{\kappa})\mathbb{I}[d_{ijt} = 3] + \epsilon_{ijt\kappa} \\
& - \mathbb{C}^{C}_{ijt\kappa}\mathbb{I}[d_{ijt-1} = 2, d_{ijt} = 1] - \mathbb{C}^{M}_{ijt\kappa}\mathbb{I}[d_{ijt-1} = 3, d_{ijt} = 1],
\end{aligned}
$$

where, letting $\boldsymbol{m}^{m}_{\kappa}$ and $\boldsymbol{m}^{w}_{\kappa}$ be a vector of marriage bonus coefficients associated with a man's type and a

[61] In the following discussion, I suppress the notation which represents an individual.

woman's type with $\boldsymbol{m}_\kappa = [\boldsymbol{m}^m_\kappa, \boldsymbol{m}^w_\kappa]$, $\mathbb{M}_{ijt\kappa}$ is specified as

$$\mathbb{M}_{ijt\kappa} = \boldsymbol{X}\boldsymbol{m}_\kappa, \tag{35}$$

and, the costs, $\mathbb{C}^C_{ijt\kappa}$ and $\mathbb{C}^M_{ijt\kappa}$, are specified in a similar fashion as,

$$\mathbb{C}^C_{ijt\kappa} = \boldsymbol{X}\boldsymbol{c}^C_\kappa; \tag{36}$$

$$\mathbb{C}^M_{ijt\kappa} = \boldsymbol{X}\boldsymbol{c}^M_\kappa, \tag{37}$$

and

$$\epsilon_{ijt\kappa} \sim N(0, \sigma^2_f(\kappa)) \text{ if a couple first meets;} \tag{38}$$

$$\epsilon_{ijt\kappa} = \rho_\kappa \epsilon_{ijt-1\kappa} + \eta_t \text{ after a match;}$$
$$\eta_t \sim iidN(0,1), \tag{39}$$

and μ^{Cm}_κ and μ^{Cw}_κ are cohabitation-specific unobserved heterogeneity terms for a man and a woman respectively. Their variance depends on the technology level κ. Similarly, μ^{Mm}_κ and $\mu^{Mw}_\kappa \sim iidN(0, \sigma^2_{\mu^M}(\kappa))$ are marriage-specific unobserved heterogeneity terms for a man and a woman respectively. Let $\boldsymbol{\mu}^m_\kappa = [\mu^{Cm}_\kappa, \mu^{Mm}_\kappa]$ and $\boldsymbol{\mu}^w_\kappa = [\mu^{Cw}_\kappa, \mu^{Mw}_\kappa]$ with their joint distribution

$$\boldsymbol{\mu}^m_\kappa \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}); \tag{40}$$

$$\boldsymbol{\mu}^w_\kappa \sim iidN(0, \boldsymbol{\Gamma}_{\mu(\kappa)}). \tag{41}$$

For identification of $\boldsymbol{\Gamma}_{\mu(\kappa)}$, I impose the restriction on the realizations of cohabitation and marriage specific unobserved terms, $\mu^{Cm}_\kappa = \mu^{Cw}_\kappa$ and $\mu^{Mm}_\kappa = \mu^{Mw}_\kappa$.[62] Because I assume the continuous support for $\mu^{Cm}_\kappa, \mu^{Cw}_\kappa, \mu^{Cm}_\kappa$ and μ^{Cm}_κ, for numerically solving the model recursively, I discretize each support with 3 grid points, and interpolate each support with a B-spline method.[63]

8.4 Meeting probability

In this subsection, I provide function specifications for the meeting probabilities. Let $\Lambda^{Sm}_{t\kappa}$ and $\Lambda^{Sw}_{t\kappa}$ be an aggregate stock of single men and an aggregate stock of single women at time t under κ in the economy respectively, and $\Lambda^{Sm}_{t\kappa} = \sum_i^I \Lambda^{Sm}_{it\kappa}$ and $\Lambda^{Sw}_{t\kappa} = \sum_j^J \Lambda^{Sw}_{jt\kappa}$. Let $M_{t\kappa}$ be the total number of meetings happening at time t under κ. Let $z(\cdot)$ be an aggregate matching function, and $M_{t\kappa} = z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa})$ (See, for example, Petrongolo and Pissarides (2001)).[64] Let $\alpha^m_{ijt\kappa}$ be a meeting probability of a type i man of meeting a type j

[62] Since my game setting is a cooperative one, only the sum is identified, and this assumption corresponds to a woman's bargaining weight is 0.5.

[63] For detailed discussion about how to make the discrete grid points, see, Brien et al. (2006).

[64] The aggregate matching function, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa})$, should have the aggregate stocks, $\Lambda^{Sm}_{t\kappa}$ and $\Lambda^{Sw}_{t\kappa}$, as its arguments to capture aggregate externalities on meetings caused by actions of other types. See, for example, Petrongolo and Pissarides (2001).

woman at time t under κ from a man's side and $\alpha^{w}_{ijt\kappa}$ be a meeting probability from a woman's side. Let $\delta_{ijt\kappa}(\cdot)$ be a meeting efficiency between a type i man and a type j woman at time t under κ. The meeting efficiency is eventually mapped from a vector of observable characteristics for a type i man at time t, $\boldsymbol{X}_{it}$, and a vector of observable characteristics for a type j woman at time t, $\boldsymbol{X}_{jt}$, as I will show soon. Let $\boldsymbol{\beta}^{\alpha^m}_{\kappa}$ and $\boldsymbol{\beta}^{\alpha^w}_{\kappa}$ be a vector of coefficients for the meeting efficiency associated with a man's observable type and a woman's observable type, and denote $\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\kappa} = [\boldsymbol{\beta}^{\alpha^m}_{\kappa}, \boldsymbol{\beta}^{\alpha^w}_{\kappa}]$. I specify the meeting efficiency as

$$\delta_{ijt\kappa}(\boldsymbol{X}) = \frac{exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\kappa})}{1 + exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\kappa})}. \tag{42}$$

Then, the meeting probability, $\alpha^{m}_{ijt\kappa}$ and $\alpha^{w}_{ijt\kappa}$, is specified as

$$\alpha^{m}_{ijt\kappa} = \left[\delta_{ijt\kappa}(\boldsymbol{X}) z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa}) \frac{\Lambda^{Sm}_{it\kappa}}{\Lambda^{Sm}_{t\kappa}} \frac{\Lambda^{Sw}_{jt\kappa}}{\Lambda^{Sw}_{t\kappa}}\right] / \Lambda^{Sm}_{it\kappa}; \tag{43}$$

$$\alpha^{w}_{ijt\kappa} = \left[\delta_{ijt\kappa}(\boldsymbol{X}) z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa}) \frac{\Lambda^{Sm}_{it\kappa}}{\Lambda^{Sm}_{t\kappa}} \frac{\Lambda^{Sw}_{jt\kappa}}{\Lambda^{Sw}_{t\kappa}}\right] / \Lambda^{Sw}_{jt\kappa}. \tag{44}$$

The specific reason why I explicitly model $\delta_{ijt\kappa}(\cdot)$ is that it would be the case that, even if the stocks are the same, an individual with a certain type tends to meet with an individual with a certain type more or less. To capture this, I explicitly include $\delta_{ijt\kappa}(\cdot)$. Following much of previous studies, the aggregate matching technology, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa})$, is assumed to have the functional form, $z(\Lambda^{Sm}_{t\kappa}, \Lambda^{Sw}_{t\kappa}) = \Lambda^{Sm^{0.5}}_{t\kappa} \Lambda^{Sw^{0.5}}_{t\kappa}$.

Remember that, because I assume the non-stationarity of the economy, I can explicitly include the effects of the stocks on the meeting probabilities and, through them, on marital behaviors. If I assume a stationarity of the economy used commonly in previous studies, I cannot control for the effects of changes of stocks. Therefore, the estimates are contaminated.

8.4.1 Childbearing probability

A part of uncertainty of the model also arises from the imperfect control women have over childbirth. Let $\boldsymbol{p}^{m}_{\kappa}$ and $\boldsymbol{p}^{w}_{\kappa}$ be a vector of childbearing coefficients associated with a man's type and a woman's type, and denote $\boldsymbol{p}^{\boldsymbol{b}}_{\kappa} = [\boldsymbol{p}^{m}_{\kappa}, \boldsymbol{p}^{w}_{\kappa}]$. The probability of giving a birth at time $t+1$, $P^{b}_{t\kappa}$, is specified as,

$$P^{b}_{t\kappa} = \frac{exp^{\boldsymbol{X}\boldsymbol{p}^{b}_{\kappa}}}{1 + exp^{\boldsymbol{X}\boldsymbol{p}^{b}_{\kappa}}}. \tag{45}$$

The parameters of this function are estimated outside of the main structural model to reduce the computational cost and treated as given in the estimation of the main structural estimation. It implicitly indicates that childbearing probabilities is treated as given by players and that birth probability errors are independent of other errors in the model.

8.5 Law of motion of explanatory variables

8.5.1 Children

Women are assumed to inherit children from their current marriage, L_t^c, and from their previous relationship, L_t^{pr}. I denote $b_{t\kappa}$ as whether to have a birth at time t. So, $b_{t\kappa} = 1$ with probability $P_{t\kappa}^b$. The law of motion of the stock of children evolves as,

$$L_{t+1}^{pr} = \begin{cases} L_t^{pr} + b_{t\kappa} & \text{if } d_{ijt} = 2,3 \textit{ and } d_{ijt-1} = 1 \\ L_t^{pr} + L_t^c + b_{t\kappa} & \text{if } d_{ijt} = 1, \end{cases} \tag{46}$$

and

$$L_{t+1}^{c} = \begin{cases} L_t^{c} + b_{t\kappa} & \text{if } d_{ijt} = 2,3 \textit{ and } d_{ijt-1} = 2,3 \\ 0 & \text{if } d_{ijt} = 1. \end{cases} \tag{47}$$

Note that the above law of motion implies that if a couple gets separated at time t, their children from the match become the stock of children from a previous relationship upon entering time $t+1$. If a birth happens at time t and the individual is single, the child becomes part of the stock of the previous children even if the individual gets matched at the end of time t.

8.5.2 Match Duration evolution

The state space for the dynamic programming of the model is too large without imposing some restrictions on evolutions of some part of state variables. Particularly, I put an upper bound for the match durations at time t, $duration_t$. It evolves following the following law of motion,

$$duration_{t+1} = \begin{cases} 8 & \text{if } duration_t = 8 \textit{ and } d_{ijt} = 2,3 \\ duration_t + 1 & \text{if } duration_t \leq 7 \textit{ and } d_{ijt} = 2,3 \\ 0 & \text{if } d_{ijt} = 1. \end{cases} \tag{48}$$

Note that the implication of this limit is not that individuals cannot experience the duration effects more than 8 sampling periods. Instead, it means that the extra years beyond do not have any marginal effects on individual behavior.

8.6 Marriage market

An individual's marriage market is assumed limited to same race individuals, individuals with the same time period and individuals in the same technology level cohort. The same race marriage market assumption is motivated by computational burden and the low rates of interracial cohabitation and marriage. The same time period assumption is motivated by the fact that almost all individuals get matched with an individual within the similar age range. This same time period restriction is used also in, for example, Eckstein et al. (2019).

9 Estimation results

I take the following two steps to estimate the structural model: First, I estimate the non-structural childbirth probability function outside of the main structural model to reduce the computational costs. Then, taking the estimates of the childbirth probability function as given, I estimate the main structural model.

The age and duration variables in the following tables are measure in 6-month time periods. In this section, I discuss the parameter estimates, especially, highlighting those associated with the meeting technology.

9.1 Estimates of outside of main structural model

Tables 11 and 12 represent the estimates of the parameters of the non-structural childbirth probability function, which is parsimoniously specified and their standard errors.[65] Table 11 gives the coefficients of the logistic childbearing probability function when a woman is single. As expected, with more education, a woman has a smaller probability of giving birth. If an individual's race is black, she is more likely to give birth. Table 12 shows the coefficients of the logistic childbearing probability function when a woman is matched with a man. Overall, if women are matched rather than single, the probability of giving birth is higher. The results in Tables 11 and 12 are consistent with Seitz (2006) and Sheran (2007).

Table 11: Parameters associated with the logistic function for childbirth (single woman)

#	Parameter	NLS 72	NLSY 97 [a]
1	Constant	-3.878	-3.822
		(0.226)	(0.167)
2	Age (time)	-0.036	0.060
		(0.012)	(0.008)
3	Age spline $\leq$ 5 years (school effect)	0.028	0.010
		(0.020)	(0.015)
4	Age spline $\geq$ 10	-0.087	-0.045
		(0.013)	(0.006)
5	Education	-0.596	-0.834
		(0.067)	(0.054)
6	Black	2.108	1.334
		(0.111)	(0.081)

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[65] I can specify the functional form in a more flexible way with more explanatory variables. However, this part is not my main focus of this research.

Table 12: Parameters estimates associated with the logistic function for childbirth (matched woman)

#	Parameter	NLS 72	NLSY 97 [a]
1	Constant	-2.691	-1.284
		(0.155)	(0.175)
2	Age (time)	0.042	-0.064
		(0.008)	(0.009)
3	Age spline $\leq$ 5 years (school effect)	0.024	-0.004
		(0.010)	(0.013)
4	Age spline $\geq$ 10	-0.006	0.018
		(0.003)	(0.004)
5	Education	-0.060	-0.148
		(0.035)	(0.037)
6	Education partner	-0.012	0.011
		(0.015)	(0.008)
7	Match duration	-0.036	0.026
		(0.006)	(0.002)
8	Match duration spline $\leq$ 2	-0.143	-0.122
		(0.024)	(0.034)
9	Kid from previous relationship	-1.371	0.523
		(0.131)	(0.072)
10	Black	0.180	-0.080
		(0.081)	(0.064)

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

9.2 Estimates of main structural model

Tables 13-18 show the main structural parameter estimates and their standard errors estimated with a bootstrap procedure. First, Table 13 provides the coefficient estimates of a flow value of cohabitation relative to being single. Table 14 represents coefficients for the marriage bonus.[66] The estimates in Table 13 is interpreted as an incremental value by cohabitation by changing its corresponding explanatory variable. For example, the coefficient on *Black* in the NLS 72 cohort, which is -0.087, means that, holding other variables constant, black people get less utility from cohabitation relative to being single, compared with white people. The interpretation of the coefficient estimates in Table 14 is as follows: If a couple selects marriage, they can get their corresponding marriage bonus part in addition to the corresponding cohabitation flow match value based on their characteristics.[67]

[66] I can have symmetric explanatory variables between the cohabitation flow value and the marriage bonus specifications. Compared with Brien et al. (2006), they consider the utilities explicitly as a cohabitation flow value and a marriage flow value. Under the interpretation, their model is a multinomial (ordered) choice model. Therefore, it is more natural to have symmetric explanatory variables. I interpret the marriage flow value as an additional marriage bonus. It means an additional utility a couple gets by selecting marriage (See, for example, Drewianka (2006)). There is no reason why I am forced to employ symmetric explanation variables used in a typical multinomial choices model setting. Children are typically considered to induce a match itself (cohabitation and marriage). However, at the same time, children are also considered as *investments* within a household. So, it seems true that utilities from children are different between cohabitation and marriage because a married couple are willing to invest more on their household capital. However, in my explanatory variables, the duration also capture the degree of household investments (#12 $-$ 13 and #24 $-$ 25). So, if I add children in the marriage bonus specification to try to capture the incentive difference on a relation-specific capital, it is weakly identified.

[67] For example, in the NLS 72 cohort, if a black couple selects cohabitation, they get -2.294 as their constant part and -0.087 for their match. However, if they select marriage, they get additionally 0.190 as their constant part for their match and -0.038 caused by the *Black*

Table 13: Parameters associated with cohabitation flow match value

#	Parameter	NLS 72 [a]	NLSY 97 [b]
1	Constant	-2.294*	-3.824*
2	Age (time)	-0.366*	-0.091*
3	Age spline $\leq$ 5 years (school effect)	-1.048*	-2.287*
4	Age spline $\geq$ 10	0.269*	0.052*
5	Man education (High school)	0.033*	0.048*
6	Man education (College degree)	0.015*	0.321*
7	Woman education (High school)	0.074*	0.024*
8	Woman education (College degree)	-0.011*	-0.036*
9	Education difference	-0.653*	-0.660*
10	Kid from previous relationship	-0.283*	-0.688*
11	Kid from current relationship	0.756*	0.742*
12	Match duration	0.482*	0.113*
13	Match duration spline $\leq$ 2	-1.532*	-0.104*
14	Black	-0.087*	-0.035*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 14: Parameters associated with marriage bonus

#	Parameter	NLS 72[a]	NLSY97 [b]
15	Constant	0.190*	-0.113*
16	Age (time)	-0.029*	-0.005*
17	Age spline $\leq$ 5	0.010*	-2.073*
18	Age spline $\geq$ 10	0.011*	0.005*
19	Man education (High school)	0.108*	0.115*
20	Man education (College degree)	-0.416*	-0.451*
21	Woman education (High school)	-0.040*	-0.135*
22	Woman education (College degree)	-0.277*	-0.334*
23	Education difference	-0.335*	-0.341*
24	Match duration	0.627*	0.1440*
25	Match duration $\leq$ 2 spline	-0.235*	-0.003*
26	Black	-0.038*	-0.001*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

These estimation results are caused by choosing parameter values to match data features. For example, a result that black people get a lower utility flow both from marriage and cohabitation than white people is reflected that the transition rates from single to cohabitation and marriage of black people are lower than white people after controlling for other observable characteristics. This is consistent with Seitz (2006), Sheran (2007), Keane and Wolpin (2010) and Beauchamp et al. (2018).

term. Therefore, their flow match value of marriage is the sum of these two constant terms, $-2.294 + 0.190$. As well, they get -0.089 (*Black*) for cohabitation and -0.038 for marriage. In sum, a black married couple gets $-2.294 + 0.190 + (-0.087 - 0.038)$, holding constant other characteristics.

Next, consider the estimates of the effects of age on the flow match values are negative ($\sharp 2-4$ and $\sharp 16-18$), but, with the spline modification after ten years, the negative effects are slightly mitigated. It leads to the positive relationship observed in the data between marriage (cohabitation) and age.

In both cohabitation and marriage, the structural model estimates indicate positive effects of duration of a match. Marriage has a stronger positive effect than cohabitation ($\sharp 12-13$ and $\sharp 24-25$). The positive effects imply more investment or accumulation of match-specific capital for a couple in marriage than in cohabitation. The results of the sign of duration effects are consistent with Brien et al, (2006). However, Sheran (2007) and Beauchamp et al. (2018).

We see that women with more education get lower flow match utility in marriage ($\sharp 8$ and $\sharp 21-22$). This may occur because there might be better employment opportunities for singles or they just prefer being single. It directly affects the value of their outside options of being single. Remember that I do not include a mechanism for how an individual wage is decided. Nor do I include wages as a part of state variables. I assume that the level of education can partly capture the difference of wages. However, note that Eckstein et al. (2019) point out that married women earn 18% more than single women these days, which I cannot find in this research. This is because, in the NLSY 97 cohort, the education effects are more negative. They discuss that controlling changing labor market opportunities and a mother's education matters. The difference of the estimates between Eckstein et al. (2019) and mine comes partly from the fact that I do not control for them.

Thinking that getting higher education is disadvantage for women is misleading ($\sharp 8$ and $\sharp 21-22$). As I will show below, women with higher education get more chances to meet their potential partner as pointed out in Eckstein et al. (2019) and and Ge (2011).

The estimates indicate that couples with children are less likely to separate and male divorcees are more likely to get remarried than female, as other empirical studies suggests (Guvenen and Rendall (2015)): The estimates of having a kid within the relationship causes a positive effect ($\sharp 10$), but having a kid outside of the relationship causes a negative effect on the relationship ($\sharp 11$). This might be because, if people are assumed to gain happiness from emotional satisfaction (Becker (1973)), having children increases emotional attachment and satisfaction between the biological father and mother.However, if children are stepchildren, we do not expect to have an increase in emotional attachment; rather it reduces. This might be because children from previous relationship would be a potential source of conflict within the new relationship, as pointed out in Beauchamp et al. (2018). As written in Section 8, it is assumed that a custodial parent is the mother after their separation. So, the negative coefficient of the existence of a kid from previous relationship indicates that a woman is less likely to get remarried if she has a kid.

Compared with previous research with cohabitation, as shown in Brien et al. (2006), constant terms for the flow values for cohabitation and marriage are negative ($\sharp 1$ and $\sharp 15$). However, the value (magnitude) of the estimates in this research are larger than those in Brien et al. (2006). One possible reason for this might be that this research explicitly includes meeting probabilities which are strictly less than 1. This mechanism causes the flow value of single to further go down. If observed data patterns are similar between Brien et al. (2006) and this research, to explain the similar data pattern, the flow value of a match should also go down.

When we look at the NLS 72 cohort, the value for the separation costs ($\sharp 27$ and $\sharp 30$) are similar to those of Brien et al. (2006). The divorce cost is higher than the cohabitation separation cost. However, when we focus on the NLSY 97 cohort, the difference of the two separation costs is smaller. This comes from the observed data pattern: In the NLSY 97 cohort, more married couples get divorced and go back to single. As discussed in Matouschek and Rasul (2008), the lower divorce cost induce higher turnover of relationship. This is also consistent with the data pattens we observe.

Table 15: Parameters associated with cohabitation separation cost

#	Parameter	NLS72 [a]	NLSY 97 [b]
27	Constant	2.568*	2.547*
28	Existence of kid	0.351*	0.347*
29	Black	0.203*	0.104*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 16: Parameters associated with marriage separation cost

#	Parameter	NLS 72[a]	NLSY 97 [b]
30	Constant	4.681*	2.885*
31	Existence of kid	0.560*	0.568*
32	Black	0.216*	0.070*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 17: Parameters associated with second moments

#	Parameter	NLS 72 [a]	NLSY 97 [b]
33	ρ (Coef AR1)	0.706*	0.697*
34	Standard deviation of cohabitation unobserved heterogeneity	1.559*	1.678*
35	Standard deviation of marriage unobserved heterogeneity	2.345*	1.704*
36	Covariance of cohabitation and marriage unobserved heterogeneity	1.524*	1.328*
37	Standard deviation of of match value when single	2.021*	2.013*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Table 17 provides the parameters associated with second moments. Compared with Brien et al. (2006), where the specification of variance of the stochastic part of a match value of a first meet is given as $\frac{1}{1-\rho^2}$. Instead, I estimate the term from the data without the restriction (see, the empirical specification described in Section 8.3). The standard deviation of the stochastic part of the flow match value when a couple first meets, which is around 2 in both cohorts (♯37).

Compared with Keane and Wolpin (2010) which reports putting an unobserved type to a marriage utility specification is redundant, the unobserved heterogenous types are important in both cohabitation and marriage (♯$34-35$). This may be because, in this research, I do not control, for example, detailed labor market information or tax system changes for computational reason. As a result, the information may be captured by the cohabitation and marriage specific unobserved heterogeneity parts.

The standard errors are calculated by 20 nonparametric block bootstrap replications, using individual level clusters (Kaplan (2012), and Baum-Snow and Pavan (2012)). I re-estimate the parameters with bootstrapped sample and do the same procedure 20 times. The standard errors are quite small partly because the number of observations for the NLS 72 cohort is 7410×30 and, for the NLSY 97 cohort, is 6348×30. The observations used in estimation are quite large.[68]

9.3 Meeting technology estimates

Table 18 shows the estimates of the parameters associated with the meeting technology. Remember that we allow the meeting probabilities of individuals to differ from what is implied by purely random meeting models. The meeting probabilities are specified as a product of population frictions (frictions associated with stocks of singles) times a scaling factor depending on a man's and woman's characteristics which is called *meeting efficiency*.

Remember that the specification of the meeting efficiency (eq (42)) is

$$\delta_{ijt\kappa}(\boldsymbol{X}) = \frac{exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\boldsymbol{\kappa}})}{1 + exp(\boldsymbol{X}\boldsymbol{\beta}^{\boldsymbol{\alpha}}_{\boldsymbol{\kappa}})},$$

which is a key part constructing the meeting probabilities.

Table 18: Parameters associated with meeting technology

#	Parameter	NLS 72 [a]	NLSY 97 [b]
38	Constant	0.653*	2.04*
39	Age (time)	-0.012*	-0.013*
40	Age spline $\leq$ 5 years (school effect)	0.688*	0.684*
41	Age spline $\geq$ 10	-0.019*	-0.001*
42	Man education (High school)	0.225*	0.121*
43	Man education (College degree)	0.728*	0.429*
44	Woman education (High school)	0.429*	0.322*
45	Woman education (College degree)	0.672*	0.577*
46	Education difference	-0.147*	-0.341*
47	Black	0.203*	0.204*

[a] SOURCE: U.S. Department of Education, National Center for Education Statistics, National Longitudinal Study of the High School Class of 1972 (NLS 72)

[b] $*$ represents that the standard error is less than 10^{-3}.

Holding other things constant, black people are more efficient meeters compared with white people in Table 18 (♯47). If individuals are in a school, they have more opportunities to meet. After 10 years, people suffer a bit less meeting opportunities (♯41).

One of main focuses of this research is how the meeting technology changes during the two cohorts. Remarkably, there is a huge increase in the constant term during the two cohorts (♯38). It means, *on average*, people in the NLSY 97 have more opportunities to meet their potential partner. However, there is less premium of getting higher education associated with getting more meeting opportunities ($\sharp 42 - 45$).

A interesting point to be emphasized is the following: In interpreting the meeting technology parameter estimates, we cannot immediately conclude that the bigger constant value in the NLSY 97 is beneficial for people in the NLSY 97. Remember that we need to focus on what happens at equilibrium. Therefore, we

[68] Suppose there is no bootstrap error, the variance of estimates by indirect inference is $\mathcal{O}(\frac{1}{\sharp\text{Observations}})$ (Gourieroux et al. (1993)).

need to focus rather on the behavioral marital patterns' changes driven by all factors in the model taking into account *equilibrium effects*, not focus only on each coefficient. It might be the case that people or some types of people in the NLSY 97 might be worse off in the end at equilibrium, even if the meeting technology is more efficient.[69] It is further discussed in the following counterfactual and policy implication section.

9.4 Goodness of fit

Figures 10 - 17 compare the simulated and actual proportions of stocks of individuals of single, cohabitation and marriage in the NLS 72 cohort and the NLSY 97 cohort, as a function of a year. My model predicts the change of each proportion relatively well. Some parts of the simulated stocks have kink points. They would happen because my functional specification would have non-smoothness at the places with the spline modification. I would get better fit by abandoning the parsimonious specification and employing more sophisticated and flexible functional specification with more explanatory variables.

[69]Note that, somehow, we can guess this conclusion *before running experiences discussed below in Section 10*: Some type of people lose their comparative (relative) advantage associated with the meeting technology because, in the NLSY 97 cohort, everyone enjoys more efficient meeting technology (♯38). The comparative advantage some type of people enjoy in the NLS 72 cohort, for example, people who have high education, is lost in a relative sense. So these type of people may end up with worse equilibrium in the NLSY 97. This, conceptually, leads to a Hosios-type thick market and congestion externalities argument (Hosios (1980)). Even if the model employed is not exactly the same as mine, but we can see the similar conjecture more theoretically in Balasko and Shell (1981) and empirically in Shepard (2017).

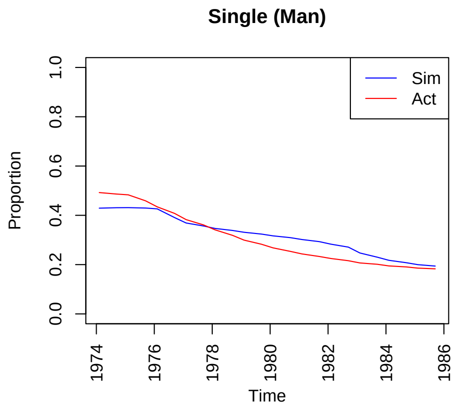


Figure 10: Stock of single men (NLS 72)

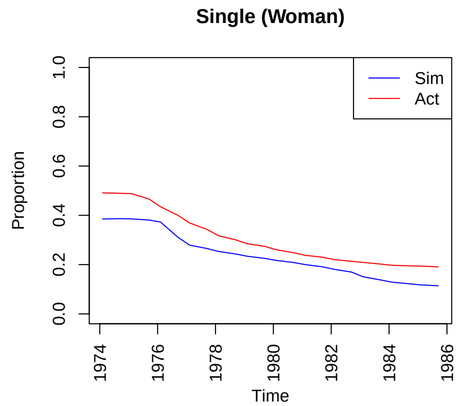


Figure 11: Stock of single women (NLS 72)

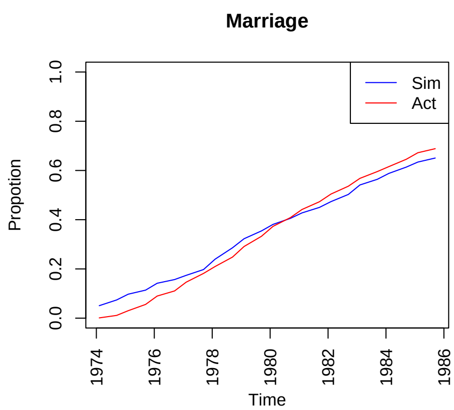


Figure 12: Stock of marriage (NLS 72)

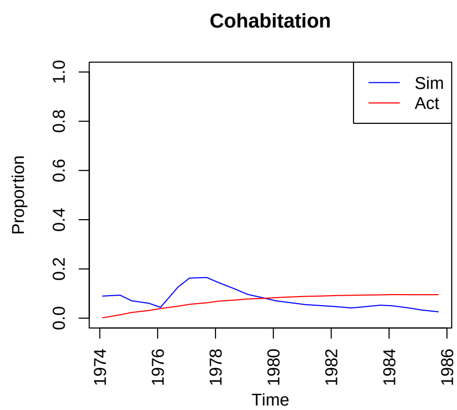


Figure 13: Stock of Cohabitation (NLS 72)

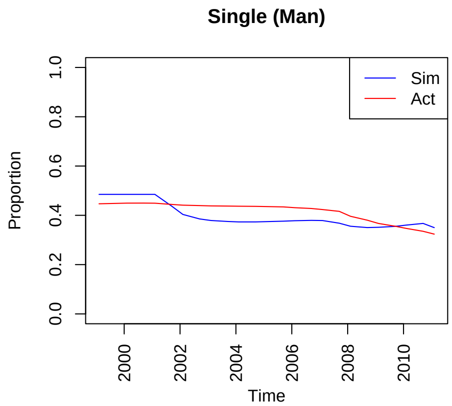


Figure 14: Stock of single men (NLSY 97)

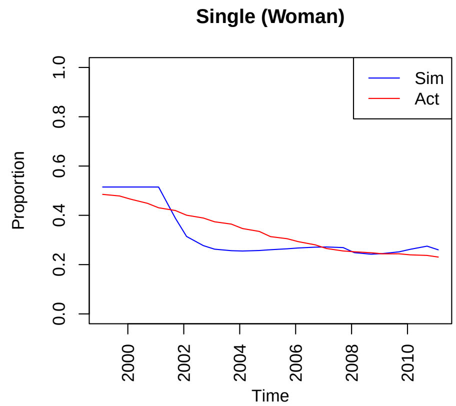


Figure 15: Stock of single women (NLSY 97)

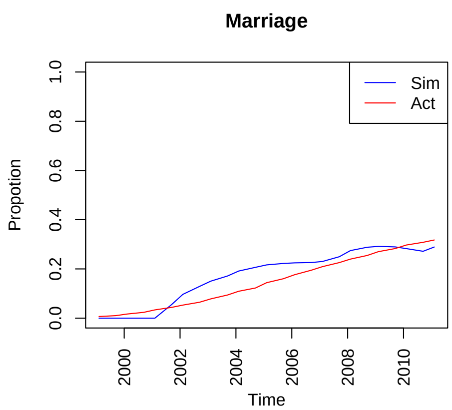


Figure 16: Stock of marriage (NLSY 97)

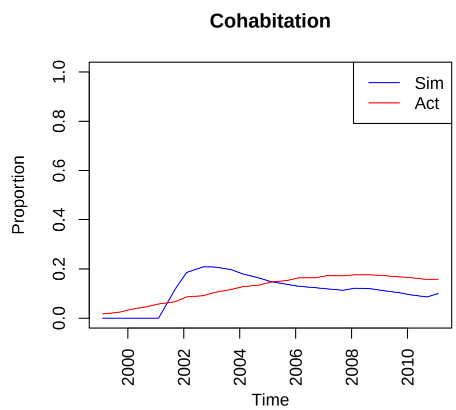


Figure 17: Stock of Cohabitation (NLSY 97)

10 Counterfactuals

Based on the parameter estimates above, the natural questions we want to ask are "Do changes in communication technology impact marital behaviors and welfare?" and "To what extent can changes in marital behavior be explained only by the change in the communication technology?" This section addresses these questions.

The primary advantage of estimating structural parameters is their assumed representation of the true underlying model primitives, provided the model is correctly specified. These parameter estimates are assumed invariant to changes in policy environments. As demonstrated in Section 9.4, the estimates fit the data relatively well and, therefore, they seem credible to use for the counterfactual experiments discussed below.

Guideline

The structural approach taken in this research enables us to evaluate welfare implications of the communication technology advances.[70] I conduct three types of experiments. First, I assess welfare changes across the two cohorts. This is done by comparing the welfare of the NLS 72 cohort and that of the NLSY 97 cohort. Second, I decompose the channels driving changes in marital behaviors between the NLS 72 and NLSY 97 cohorts. I implement several counterfactuals by altering an exogenous environment.[71] I conduct a series of counterfactuals, isolating the effects of each specific channel. These counterfactual experiments assess the contributions of specific parameters to observed marital patterns, accounting for equilibrium effects. This approach reveals how much of the marital behaviors in the NLSY 97 cohort are left to be explained by the NLS 72 parameters. Third, I evaluate whether the technology advances are beneficial or not. Focusing on the NLSY 97 cohort, I change parameters related to technological advancements—specifically, meeting technology and second moments parameters—to their corresponding NLS 72 values, while holding all other parameters constant at NLSY 97 estimates. Then, I simulate the model. This is exactly "What would happen if meeting technology goes back to 1970s levels, while other factors remained constant at 2010s levels?"

To measure welfare, I calculate an individual's expected lifetime welfare beyond the veil of ignorance-before Nature assigns exogenous characteristics and realizations of marriage-cohabitation specific unobserved heterogeneity to a player (Low and Pistaferri (2015) and Abbott et al, (2019)).[72] This is equivalent to welfare from an ex-ant perspective.

Before implementing counterfactuals, childbearing probabilities are estimated outside the main structural model. Childbearing probability parameters are assumed invariant throughout these experiments. Following most of bargaining literature, I assume that the bargaining parameter of a woman is taken as given, 0.5, and it is also policy invariant.[73]

10.1 Experiment 1: Total welfare comparison

In calculating welfare, I apply adjusted weights to each individual in the datasets to accurately represent the true composition of the economy. I calculate the ex-ante lifetime welfare for individuals in both cohorts.[74] The procedure to generate the ex-ante lifetime welfare begins by fixing values of all structural parameters of interest. For each cohort, sample histories are generated by simulated error terms given individuals' initial exogenous characteristics and cohabitation- and marriage-specific unobserved heterogeneity. Each sample history is assigned a value by summing up the discounted flow value of all marital choices until the terminal period. I then average the calculated lifetime welfare across individuals to obtain an ex-ante lifetime welfare.The ex-ante

[70] Note that I do not explicitly introduce consumption or wage into the model. However, I assume the transferable utility, which leads that an individual utility is implicitly assumed as a monetary utility. I assume individuals' utility is comparable between individuals (See, Harsanyi (1955)).

[71] Under each experiment below with different values of parameters, I recalculate the equilibrium stocks based on the parameter values, using a fixed point algorithm: In Section 5.2, I prove the existence of the equilibrium, which, while *assuming uniqueness of the equilibrium*, guarantees me to use a direct method of solving the equilibrium; the method of successive approximation (fixed point iteration), originally proposed by Rust (1994). I iteratively solve the model with a guess of stocks and get the stocks calculated by the model until the guess and calculated stocks are close enough.

[72] The reason why I focus on "beyond the veil of ignorance" is that some policy experiments I want to focus on (changes of the second moments) hit individuals who do not even know their realization of persistent marriage-cohabitation specific unobserved heterogeneity.

[73] Note that they are non-negligible limitations. Especially, the fixed bargaining weight might be a strong assumption. This is, first, because, ideally, it should be modeled also as an endogenous object in the model, and, second, because, if so, the bargaining weight should change accordingly with the changes of the environment. We need to further extend part of the model in the future.

[74] To be more precise, "lifetime welfare" of an individual in this section means "lifetime (total) payoff" of an individual by this marriage game, while normalizing the flow value of being single to 0. I use these terms interchangeably in the following sections.

expected lifetime welfare is 0.1008 in the NLS 72 and 0.0801 in the NLSY 97 cohort. This result represents a reduction of $(0.0801 - 0.1008)/0.1008 = 0.2051$, or 20.51%.

Table 19: Welfare comparison: Ex-ante lifetime welfare

	NLS 72	NLSY 97	Lifetime welfare change
Expected total payoff	0.1008	0.0801	-20.51%

I now examine the dispersion in lifetime welfare outcomes resulting from the realization of cohabitation- and marriage-specific unobserved heterogeneity, which is shown in Figures 18 and 19.[75] The density function of lifetime welfare for individuals in the NLS 72 cohort exhibits fat tails. This occurs because, as shown in Section 9, the standard deviation of marriage-specific unobserved heterogeneity in the NLS 72 is larger than that in the NLSY 97.[76] Interestingly, the ranges of possible lifetime welfare for these two distributions (from -0.5 to 0.7) do not significantly differ, even though the marital patterns of the two cohorts vary considerably. One possible explanation is that during the early stages of their sampling periods, the marital behaviors of the NLS 72 and NLSY 97 cohorts are not significantly different. Although significant changes in marital behavior occur later in life, the discount factor mitigates the impact of these differences on overall lifetime welfare.[77]

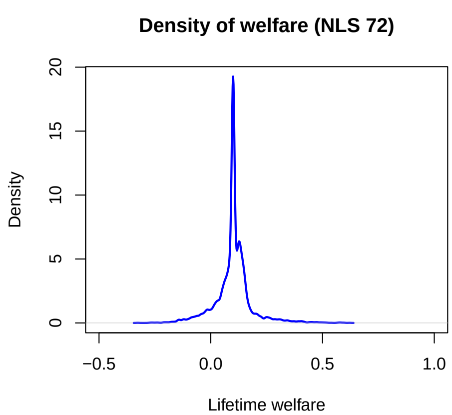


Figure 18: Welfare distribution NLS 72

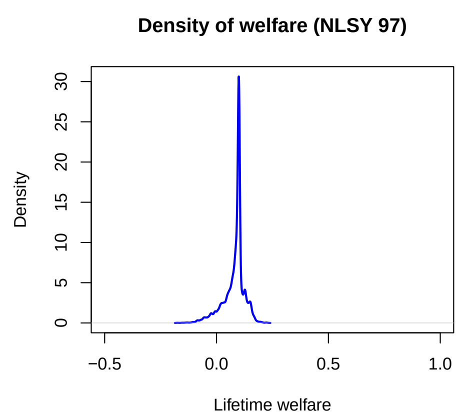


Figure 19: Welfare distribution NLSY 97

However, this result remains somewhat counterintuitive from a theoretical perspective: the economy itself becomes more like a complete market: As demonstrated in Section 9, meeting technology improves within the model. In this sense, the economy is facilitated to a complete market by the technology advances, while reducing search frictions. Consequently, individuals might achieve greater efficiency.[78] However, other model

[75]Exogenous individual demographics are also considered.

[76]Some individuals get negative lifetime welfare in Figures 18 and 19. This does not occur in theory when the number of simulations goes to infinity. However, the number of simulations is finite, 20 times, for an individual after his/her exogenous realization, in making the two figures.

[77]In this research, I set the discount factor as 0.9 for each period. The annual discount factor is 0.81. Note that the results depend on the discount factor, the terminal value of the value functions and the bargaining weight.

[78]"*Might*" represents a possibility of a Hosios-type inefficiency happening at equilibrium (Hosios (1990)).

primitives change across the two cohorts. This makes it challenging to identify the precise sources of welfare level changes across the two cohorts.[79] In the following sections, I decompose the channels through which marital behavior changes between the NLS 72 and NLSY 97 cohorts.

10.2 Experiment 2: Equilibrium impact and decomposition

In this section, I compare the simulated equilibrium marital patterns derived from the NLSY 97 model with those from the NLS 72 setting with one-by-one modification: First, I re-calculate the NLS 72 equilibrium marital patterns using the initial time-0 individual distribution from the NLSY 97, while keeping all other factors constant in the NLS 72 setting (Stage 1). This approach reveals how much the change of the economy's compositions contributes to differences in marital behaviors across the two cohorts. Next, I allow certain parameters to vary in the model: I sequentially change the meeting technology and second moments parts from those of the NLS 72 estimates to those of the NLSY 97 estimates (Stage 2 and Stage 3). This analysis quantifies the impact of technological advances on an individual's marital behavior. To isolate these effects of the changes and evaluate each contribution, I change the parameters sequentially. My goal is to identify which primitives critically influence changes in marital behavior between the NLS 72 and NLSY 97 cohorts.

10.2.1 Stage 1: Exogenous initial time-0 distribution change, Stage 2: Meeting technology change and Stage 3: Second moments change

Figures 20-23 illustrate the differences between the simulated marital behavior of the NLSY 97 cohort and the NLS 72 cohort, considering modifications of the initial time-0 distribution (Stage 1), meeting technology (Stage 2), and second moments (Stage 3).[80]

A key point of emphasis is that, in the time-0 and meeting technology experiments (Stages 1 and 2), there are fewer deviations from the original NLS 72 marital patterns, which is expected. Either the change in the time-0 distribution or the meeting technology induces changes in meeting probabilities. Suppose the Stage 1 and/or Stage 2 experiments induce an increase in meeting probabilities. Individuals in the economy have two possible responses. Individuals are more likely to meet, which induces more matches, but, at the same time, the value of being single also increases due to higher future meeting probabilities. These two effects offset each other, resulting in a relatively small overall effect of the marital patterns. A similar discussion can be found in other contexts in Santos and Weiss (2016) and Blasutto (2024).

Changes in second moments play a more critical role in determining changes in marital behavior (Stage 3). This aspect has received less empirical attention in previous literature. However, the results with larger changes are theoretically consistent with Weizman's Pandora's Box argument (Weizman, 1979) in that changes in the second moments induce changes in the optimal stopping rule.

[79] A similar discussion about difficulty of disentangling effects of each model primitive in an equilibrium model is found in Heathcote et al. (2014).

[80] I replace the childbearing probabilities also with those of the NLSY 97 cohort in implementing these experiments.

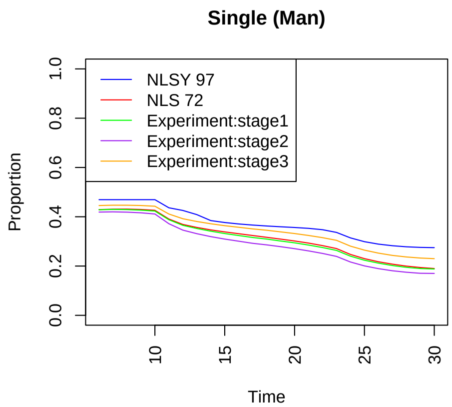


Figure 20: Stock of single men (Stage 1, 2 and 3)

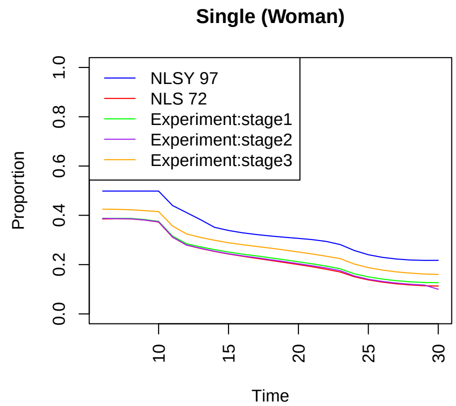


Figure 21: Stock of single women (Stage 1, 2 and 3)

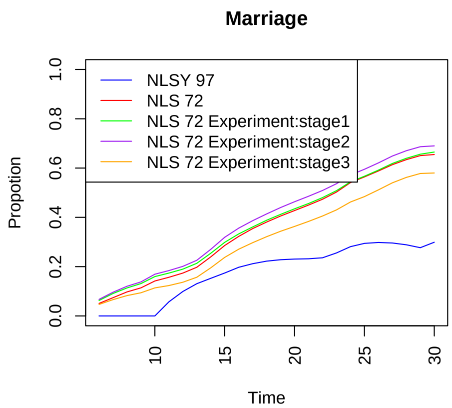


Figure 22: Stock of marriage (Stage 1, 2 and 3)

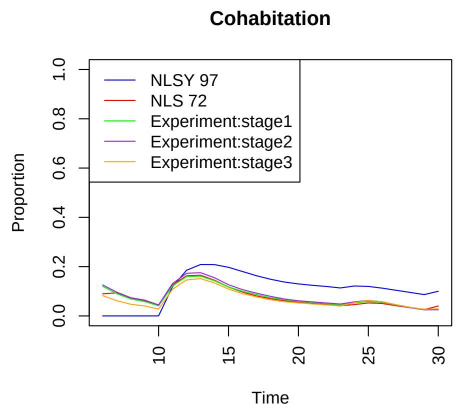


Figure 23: Stock of Cohabitation (Stage 1, 2 and 3)

The decomposition results indicate that changes in preferences (such as cohabitation match surplus and marriage bonus) and separation costs are also important factors in explaining the differences in marital behavior between the NLS 72 and NLSY 97 cohorts. This is because, even after the three experiments, deviations remain between the simulated aggregate stocks and those of the NLSY 97 cohort.

10.3 Experiment 3: Is the change of the meeting technology beneficial ?

In this section, using the structural estimates, I examine whether advances in communication technology are truly beneficial in terms of the total payoff from the marriage game.[81] First, I show how marital behavior changes when only the parameters associated with technological advances (meeting technology parameters and second-moment parameters) revert to the NLS 72 situation, while all else remains the same as in the NLSY 97 situation. Figures 24-27 illustrate changes in marital behavior.

[81]Sometimes, individuals can be better off with more restricted choices, as we see in some game settings.

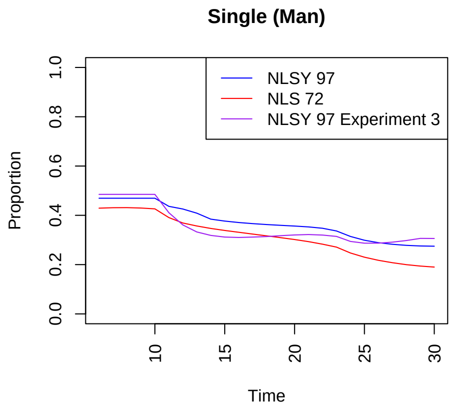


Figure 24: Stock of single men (Experiment 3)

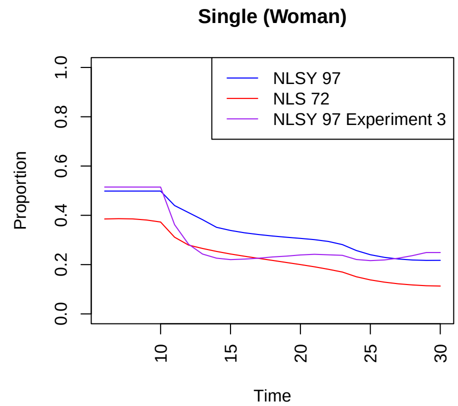


Figure 25: Stock of single women (Experiment 3)

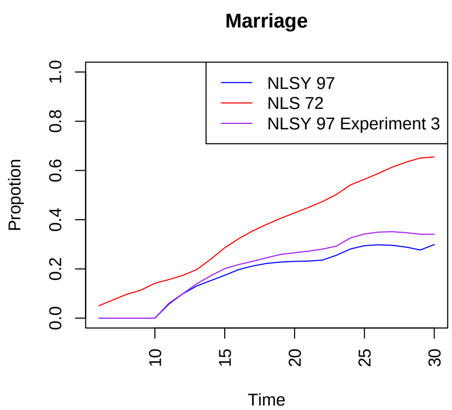


Figure 26: Stock of marriage (Experiment 3)

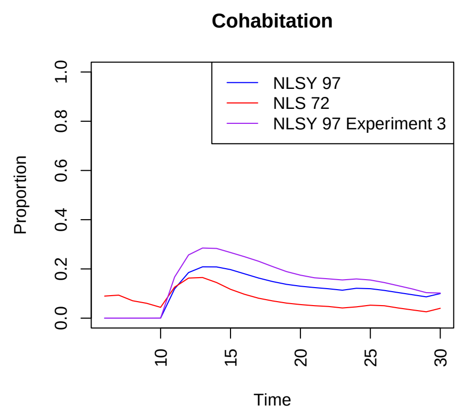


Figure 27: Stock of Cohabitation (Experiment 3)

Table 20 provides the ex-ante lifetime welfare under this experiment (Experiment 3). The increase is $(0.0851 - 0.0801)/0.0801 = 0.062$, or 6.2%. This suggests that if we revert to 1980s technology while keeping other factors the same as in the 2010s, the ex-ante lifetime welfare increases.

Table 20: Welfare comparison: NLSY 97 vs Experiment 3

	NLSY 97	Experiment 3	Lifetime welfare change
Expected total payoff	0.0801	0.0851	6.2%

At the same time, this indicates that most of the reduction shown in Table 19 is not due to technological change but rather to changes in preference components including separation costs. This implies that people's perception of the value of cohabiting and marriage has changed.[82] Interestingly, if so, it is questionable to conclude that individuals in the NLSY 97 cohort are worse off than those in the NLS 72 cohort. This is because

[82] In this research, the separation costs are assumed to include mental suffering, not only physical cost such as monetary cost.

the reduction comes from a change in their perception. We can not give any order in a change in preference. I provide a discussion about this issue in more detail in Section 11.

10.4 Marital Status turnover

In the above experiments, I focus on aggregate stocks change from an *aggregate* economy's perspective. Additionally, in this section, I focus more on changes from an *individual*'s perspective, which is marital relationship dynamics. The idea is simple: Only some parts of people might be able to enjoy this technology advance. They change their partners lots throughout their whole life, but the others might not be able to even get one partner during their whole life. I investigate how relationship turnover changes between the two cohorts. In Figures 28 and 29, I simulate how many individuals experience relationship in his/her whole life, either cohabitation or marriage in both cohorts.[83] Somehow, the results support my hypothesis in the sense that some type of people get married several times, but smaller proportion of people can get married in the NLSY 97 cohort.[84]

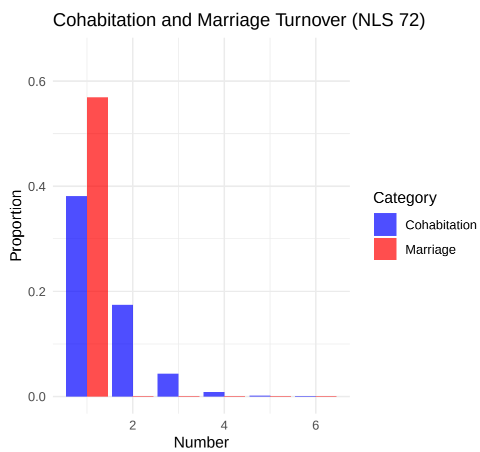


Figure 28: Marital status turnover (NLS 72)

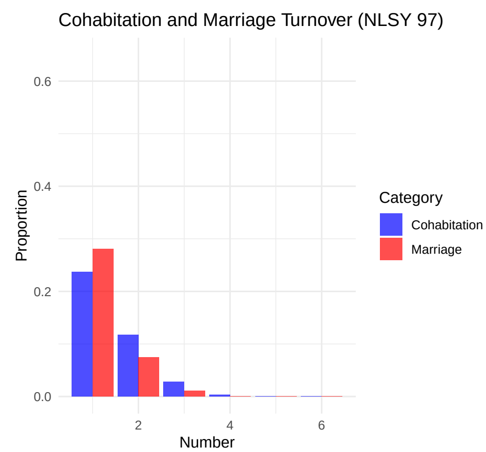
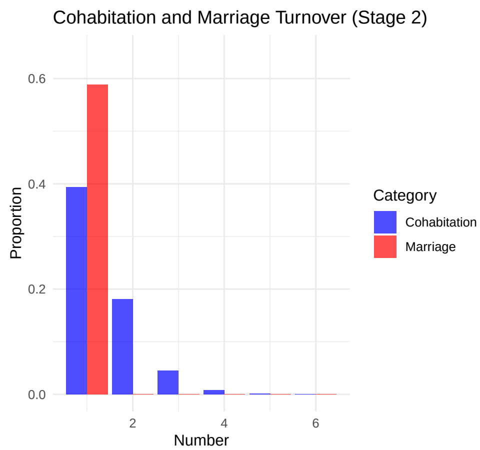


Figure 29: Marital status turnover (NLSY 97)

However, the question is "wether does this change come from the technology advance?" Next, as I did before, I focus on changes of the distribution of marital turnovers in the NLS 72 cohort by doing Experiment stage 2 and Experiment stage 3 (the change of the meeting technology and the change of the second moments respectively). The turnover of marital status changes little when we change the meeting technology parameters, as we can see in Figure 30. This may be explained by the similar argument discussed in Section 10.2.1. When we change the second moments parameters, we can see more changes shown in Figure 31. This is also similar to the discussion in Section 10.2.1.

[83] If an individual experiences both cohabitation and marriage during his lifetime, I add 1 for the number of individuals who experience both cohabitation and marriage in making Figures 28 and 29.

[84] Note that Figures 28 and 29 depend on the terminal period of the simulation. In the tables, I calculate marital history for 30 sampling periods. If I simulate individuals' behavior for longer horizons, the distributions would be different. This is consistent with, for example, Santos and Weiss (2016) where they point out that the decline and delay in first-time marriage. They say people settle down later these days.



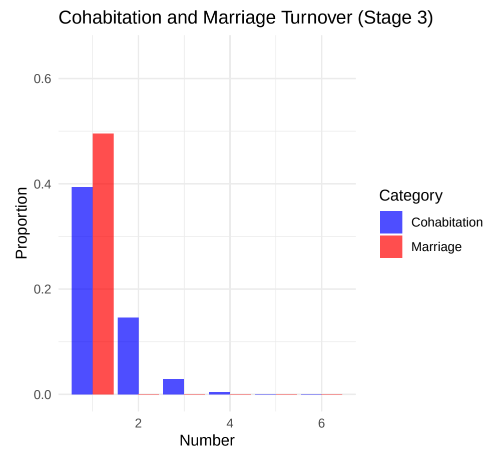


Figure 30: Marital status turnover (Experiment 2) Figure 31: Marital status turnover (Experiment 3)

11 Concluding Remarks

This paper introduces an empirical, non-stationary, two-sided search market equilibrium model with broad applicability to various contexts. Using the model, this paper evaluates the quantitative impacts of the advances in communication technology on marital behavior including cohabitation and on welfare. The findings reveal that advancements in technology facilitate partner search efficiency. However, the study finds that changes in the marriage patterns observed in the datasets and welfare are driven more by changes in preferences rather than technological improvements.

Despite its contributions, this research has several limitations that suggest directions for future work. First, due to data limitations, this paper introduces a new identification strategy to isolate the impacts of technological advances. This paper attempts to control for as many other changes occurring between the two cohorts as possible, using microeconomic theory and structural estimation. However, the estimators may still be contaminated and, thus, remains somewhat noisy. Access to richer data sets would allow us to directly measure the effects of the advances in the communication technology. This represents a promising direction for future research.

Second, the issue of multiple equilibria is non-negligible in this research. Although the existence of equilibrium is proven in the model, sufficient conditions for its uniqueness cannot be established, despite numerical evidence suggesting uniqueness. Given the level of generality of the model, proving uniqueness is quite challenging. This point is definitely worthy of further investigation.

Third, this research adopts a cooperative game setting to simplify the identification argument which is inherent in a non-cooperative framework. However, exploring a non-cooperative game setting with asymmetric information offers a promising direction for future research.

Fourth, the results are presented from a cardinal utility perspective. As discussed in Chiappori et al. (2018) and Del Boca and Flinn (2012), welfare analysis in this context depends critically on the specification of cardinal utility, or functional specification. Different cardinalizations of preferences can lead to opposite conclusions in welfare comparisons. The flow utility of being single is normalized to zero. Scaling both the flow utilities of being single and in a match by the same amount does not alter the relative utility differences, or the structure of the marriage game. This invariance arises because, in economics, preferences matter only as the order of choices rather than absolute values. Affine transformations of a payoff function preserve this order without changing the game's strategic dynamics. Therefore, our analysis measures the valuation of being in a match relative to being single. Our results do not exclude the possibility that individuals in the NLSY 97 cohort have higher overall welfare than those in the NLS 72 cohort, as both their flow utilities of being single and in a match may be higher. However, economic theory itself enables us to assess changes in the relative value of relationships but does not inform us about absolute welfare levels without further assumptions.

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