

Higher-Order Forward Guidance

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Motivation

Forward guidance — How does it work, exactly?

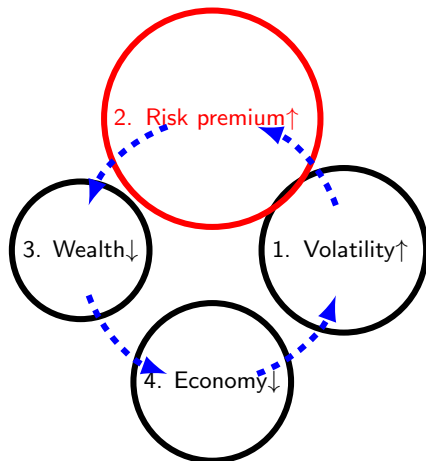
- First-order effects (level): “Interest rates will stay low” → **intertemporal substitution channel** (aggregate demand↑): e.g., Eggertsson et al. (2003), McKay et al. (2016)
- Second-order effects (volatility): reduce uncertainty, avoid worst-case scenarios, “whatever it takes” → **precautionary savings channel** (aggregate demand↑)

This paper: focus on central bank's strategic uncertainty management and coordination. Possible for central banks to pick an equilibrium where:

- During the ZLB (**now**): reduce aggregate volatility (and risk premium). Then aggregate demand↑
- But central banks **now** create uncertainty about where the economy ends up after the ZLB (**future**): commit less stabilization
- Welfare-enhancing overall

Non-linear Two-Agent New Keynesian (TANK) model with nominal rigidities

- With an aggregate stock market + (standard) portfolio choice problem



Output and asset price gaps

A non-linear IS equation (in contrast to textbook linearized one)

$$d\hat{Q}_t = \left(i_t - \left(r^n - \frac{1}{2} \underbrace{(\sigma + \sigma_t^q)^2}_{\text{Fundamental volatility}} + \frac{1}{2} \underbrace{\sigma^2}_{\text{risk premium}} \right) \right) dt + \sigma_t^q dZ_t$$
$$= (i_t - r_t^T) dt + \sigma_t^q dZ_t$$

The diagram shows the derivation of the risk-adjusted rate r_t^T . The term $(\sigma + \sigma_t^q)^2$ is highlighted in a red box and labeled "Fundamental volatility". The term σ^2 is also highlighted in a red box and labeled "risk premium". A red arrow points from the "Fundamental volatility" box to the term σ_t^q in the drift term of the second equation. A red arrow also points from the "risk premium" box to the term σ_t^q in the drift term of the second equation. A red arrow points from the σ_t^q term in the drift term of the second equation to the σ_t^q term in the diffusion term of the second equation.

$$\sigma_t^q \uparrow \rightarrow rp_t \uparrow \rightarrow \hat{Q}_t \downarrow \rightarrow \hat{Y}_t \downarrow$$

What is r_t^T ?: a risk-adjusted natural rate of interest ($\sigma_t^q \uparrow \rightarrow r_t^T \downarrow$)

$$r_t^T \equiv r^n - \frac{1}{2} \hat{r}p_t, \quad \hat{r}p_t = \underbrace{rp_t - rp_t^n}_{\text{risk-premium gap}}$$

Monetary policy outside the ZLB

Outside the ZLB: can we stabilize the business cycle? Can we prevent the volatility feedback loop?

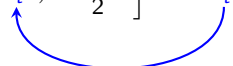
- **Yes:** Lee and Dordal i Carreras (2024, Job Market Paper)

- Under a risk-premium targeting rule:

$$i_t = r_t^T + \phi_q \hat{Q}_t$$

With $\phi_q > 0$ (i.e., Taylor principle) $\rightarrow \hat{Q}_t = 0$ for $\forall t$ (unique equilibrium)

At the ZLB, the volatility feedback loop reappears:

$$\begin{aligned} d\hat{Q}_t &= -r_t^T dt + \sigma_t^q dZ_t \\ &= -\left[r^n - \frac{1}{2}(\sigma + \sigma_t^q)^2 + \frac{1}{2}\sigma^2 \right] dt + \sigma_t^q dZ_t \end{aligned}$$


ZLB from fundamental volatility shock

Thought experiment: fundamental volatility $\sigma \uparrow$: $\bar{\sigma}$ on $[0, T]$ (e.g., [Werning \(2012\)](#)) and comes back to $\underline{\sigma}$ with $\bar{\sigma} > \underline{\sigma}$

- $\bar{r} \equiv r^n(\underline{\sigma}) = \rho + g - \underline{\sigma}^2 > 0$: no ZLB before, $t < 0$, or after, $t > T$
- $\underline{r} \equiv r^n(\bar{\sigma}) = \rho + g - \bar{\sigma}^2 < 0$: ZLB binds for $0 \leq t \leq T$

Assume: perfect stabilization (i.e., $\hat{Q}_t = 0$) is achievable outside ZLB, i.e.,

$$i_t = \bar{r} - \frac{1}{2} \hat{r}p_t + \phi_q \hat{Q}_t, \quad \text{with } \phi_q > 0$$

Result: perfect stabilization of risk-premia gap (i.e., excess uncertainty) inside the ZLB

- Recursive argument: full stabilization at T implies $\hat{Q}_T = 0 \rightarrow \sigma_{T-dt}^q = 0$, and so on (so $\hat{r}p_t = 0$ for $\forall t$)

ZLB path (full stabilization after T)

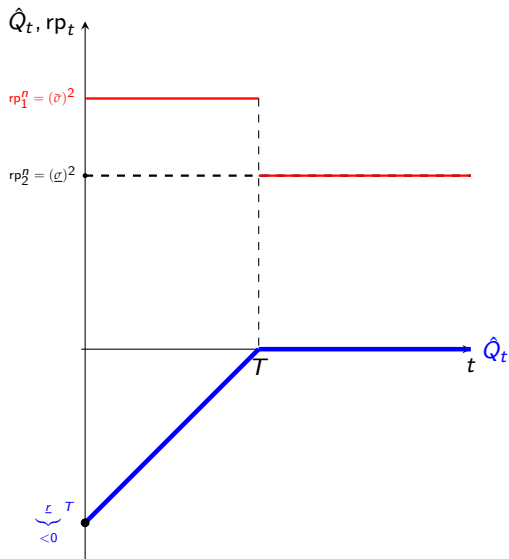


Figure: ZLB dynamics (Benchmark)

Traditional forward guidance (keep $i_t = 0$ until $\hat{T}^{\text{TFG}} > T$) [» Details](#)

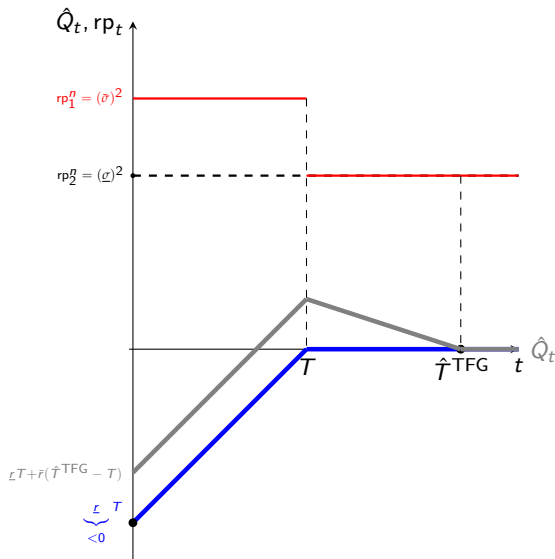


Figure: ZLB dynamics with forward guidance until $\hat{T}^{\text{TFG}} > T$

Alternative forward guidance policies

Big Question

Can we do even better than the traditional forward guidance?

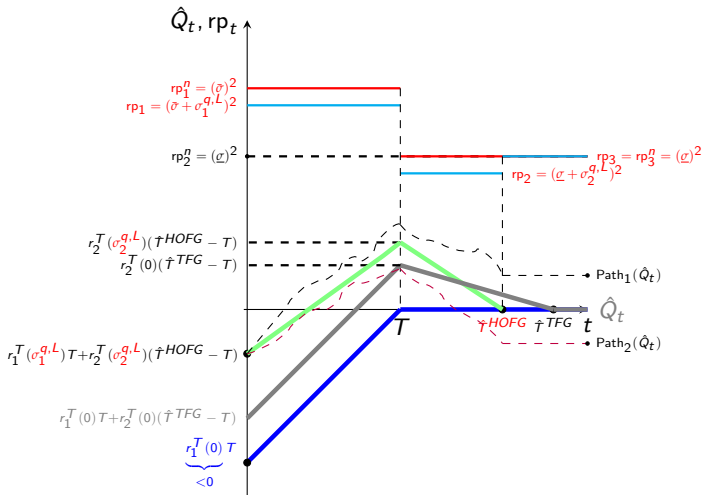
What if we reduce aggregate uncertainty via $\sigma_t^q < 0$?

- Then $rp_t = (\bar{\sigma} + \sigma_t^q)^2 < rp_t^n$, raising stock prices and aggregate demand

But how?

- Nominal rigidities \rightarrow demand-determined production (and hence, wealth)
- Policy challenge: the central bank *must convince* households to “coordinate” on this particular equilibrium \rightarrow *higher-order forward guidance*
- Give up perfect stabilization in the future (no stabilization at all)

Central bank picks \hat{T}^{HOFG} and $\{\sigma_t^q\}$ ▶▶ Details



Proposition (Optimal commitment path)

At optimum, $\sigma_1^{q,L} < 0 = \sigma_1^{q,n}$, $\sigma_2^{q,L} < 0 = \sigma_2^{q,n}$, and $\hat{T}^{HOFG} < \hat{T}^{TFG}$

Optimal policy

Proposition (Optimal forward guidance policy)

Optimal higher-order forward guidance (HOFG) always results in an equal or lower expected quadratic loss than the traditional guidance policy

Proof

With $(\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{\tau}^{\text{HOFG}}) = (0, 0, \hat{\tau}^{\text{TFG}})$, solutions coincide

Remarks:

- Alternative higher-order forward guidance policy implementations **are** possible
- This paper shows **HOFG** dominates **TFG** in a simple setting

Optimal policy: extension

Extension: still higher-order forward guidance policy, now with stochastic stabilization after \hat{T}^{HOFG} . Return to stabilization with νdt probability after \hat{T}^{HOFG}

- Central bank commits to stabilizing the economy after \hat{T}^{HOFG} with some probability. Expected stabilization after $1/\nu$ quarters
- $\nu = 0$: the above higher-order forward guidance
- $\nu = \infty$: the traditional forward guidance policy

Big discontinuity:

$$\lim_{\nu \rightarrow +\infty^-} \mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu) < \underbrace{\mathbb{L}^{Q,*}(\{\hat{Q}_t\}_{t \geq 0}, \nu = \infty)}_{\text{Traditional forward guidance}}$$

- Slight probability that stabilization might not happen \rightarrow **HOFG** possible

HOFG equilibrium \rightarrow supported by fiscal policy as a unique equilibrium [▶ Details](#)

Welfare comparisons

$T = 20$ quarters ZLB spell

Loss function \mathbb{L} as the (conditional) quadratic output loss per quarter:

$$\mathbb{L}_{\text{Per-period}}^Y \equiv \rho \int_0^\infty e^{-\rho t} \mathbb{E}_0 \left(\hat{Y}_t^2 \right) \approx \zeta^2 \cdot \rho \int_0^\infty e^{-\rho t} \frac{1}{s} \sum_{i=1}^s \left(\hat{Q}_t^{(i)} \right)^2 dt$$

Policy	No guidance	Traditional	Higher-Order (no stochastic stabilization)	Higher-Order (with stoch. stab., $\nu = 1$)
$\sigma_1^{q,L}$	0	0	-1.27%	-4.13%
$\sigma_2^{q,L}$	0	0	-0.24%	-3.79%
\hat{T}	20	25.27	25.09	24.68
$\mathbb{L}_{\text{Per-period}}^Y$	7%	1.93%	1.81%	1.69%

- Still, traditional forward guidance too strong: e.g., [McKay et al. \(2016\)](#)
- **HOFG** with $\nu \rightarrow \infty$ but $\nu \neq \infty$ most effective

Thank you very much!
(Appendix)

Identical **capitalists** and **hand-to-mouth workers** (two types of agents)

- **Capitalists:** consumption - portfolio decision (between stock and bond)
- **Workers:** supply labor to firms (hand-to-mouth)

1. Technology

$$\frac{dA_t}{A_t} = \underbrace{g}_{\text{Growth}} \cdot dt + \underbrace{\sigma}_{\text{Aggregate shock}} \cdot \underbrace{dZ_t}_{\text{Aggregate shock}} \quad \begin{array}{l} \text{Fundamental risk} \\ \text{(Exogenous)} \end{array}$$

2. Hand-to-mouth workers: solves the following problem:

$$\max_{C_t^w, N_t^w} \frac{\left(\frac{C_t^w}{A_t}\right)^{1-\varphi}}{1-\varphi} - \frac{(N_t^w)^{1+\chi_0}}{1+\chi_0} \quad \text{s.t.} \quad \bar{p}C_t^w = w_t N_t^w$$

- Hand-to-mouth assumption can be relaxed, without changing implications

3. Firms: Dixit-Stiglitz production using labor + perfectly rigid prices ($\pi_t = 0$)

4. Financial market: zero net-supplied risk-free bond + stock (index) market

Capitalists: standard portfolio and consumption decisions (very simple)

1. Stock market valuation = $\bar{p}A_tQ_t$, where (real) stock price Q_t follows:

$$\frac{dQ_t}{Q_t} = \mu_t^q \cdot dt + \sigma_t^q \cdot dZ_t \quad \text{Financial risk (Endogenous)}$$

- μ_t^q and σ_t^q are both endogenous (to be determined)

2. Each solves the following optimization (standard)

$$\max_{C_t, \theta_t} \mathbb{E}_0 \int_0^\infty e^{-\rho t} \log C_t dt \quad \text{s.t.}$$

$$da_t = (a_t(i_t + \theta_t(i_t^m - i_t)) - \bar{p}C_t)dt + \theta_t a_t(\sigma + \sigma_t^q)dZ_t$$

- Aggregate consumption of capitalists \propto aggregate financial wealth

$$C_t = \rho A_t Q_t$$

- Equilibrium risk-premium is determined by the total risk

$$i_t^m - i_t \equiv rp_t = (\sigma + \sigma_t^q)^2$$

Equilibrium with rigid prices ($\pi_t = 0, \forall t$) [▶ Go back](#)

Flexible price economy as benchmark: 'natural' consumption of capitalists $C_t^n = \rho A_t Q_t^n$ follows

$$\begin{aligned}\frac{dC_t^n}{C_t^n} &\equiv \frac{d(A_t Q_t^n)}{A_t Q_t^n} = (r^n - \rho + \sigma^2) dt + \sigma dZ_t \\ &= gdt + \sigma dZ_t = \frac{dA_t}{A_t}\end{aligned}$$

where $r^n = \rho + g - \sigma^2$ is the 'natural' rate of interest

Define **asset price gap**

$$\hat{Q}_t = \ln \frac{Q_t}{Q_t^n}, \quad 0 = \underbrace{\text{Var}_t \left(\frac{dQ_t^n}{Q_t^n} \right)}_{\text{Benchmark volatility}}, \quad \underbrace{\left(\overset{\text{Endogenous}}{\sigma_t^q} \right)^2 dt}_{\text{Actual volatility}} = \text{Var}_t \left(\frac{dQ_t}{Q_t} \right)$$

which is proportional to **output gap**

$$\hat{Y}_t = \ln \left(\frac{Y_t}{Y_t^n} \right) \rightarrow \hat{Y}_t = \underbrace{\zeta}_{>0} \cdot \hat{Q}_t$$

Dividend yield: dividend yield = ρ , as in Caballero and Simsek (2020)

- A positive feedback loop between asset price \iff dividend (output)

Determination of nominal stock return dI_t^m

$$\begin{aligned}
 dI_t^m &= \left[\underbrace{\rho}_{\text{Dividend yield}} + \underbrace{g + \mu_t^q + \overbrace{\sigma\sigma_t^q}^{\text{Covariance}}}_{\text{Capital gain}} \right] dt + \underbrace{(\sigma + \sigma_t^q)}_{\text{Risk term}} dZ_t \\
 &= \underbrace{i_t^m}_{\text{Drift}} = \underbrace{i_t}_{\text{Monetary policy}} + \underbrace{(\sigma + \sigma_t^q)^2}_{\text{Risk-premium}}
 \end{aligned}$$

Assume:

- Central bank commits to keep $i_t = 0$ until $\hat{T}^{\text{TFG}} \geq T$ (i.e., Odyssean guidance)
- Perfect stabilization (i.e., $\hat{Q}_t = 0$) afterwards, i.e., for $t > \hat{T}^{\text{TFG}}$
- From the same arguments, risk-premium gap stabilization beforehand, $t \leq \hat{T}^{\text{TFG}}$ (no excess volatility while $i_t = 0$)

Problem: minimize smooth quadratic welfare loss

$$\min_{\hat{T}^{\text{TFG}}} \mathbb{L}^Q(\{\hat{Q}\}_{t \geq 0}) \equiv \mathbb{E}_0 \int_0^{\infty} e^{-\rho t} (\hat{Q}_t)^2 dt$$

$$\text{s.t. } \hat{Q}_0 = \underbrace{r}_{<0} T + \underbrace{\bar{r}}_{>0} (\hat{T}^{\text{TFG}} - T)$$

- Smoothing the ZLB costs over time (i.e., welfare enhancing)

Assume:

- Central bank can commit to keep $i_t = 0$ until $\hat{T}^{HOFG} \geq T$
- No stabilization (i.e., $\hat{Q}_t = \hat{Q}_{\hat{T}^{HOFG}}$) guaranteed afterwards, $t \geq \hat{T}^{HOFG}$
- Pick $\{\sigma_t^q\}$ for $t < \hat{T}^{HOFG}$

Problem: minimize smooth quadratic welfare loss

$$\min_{\sigma_1^{q,L}, \sigma_2^{q,L}, \hat{T}^{HOFG}} \mathbb{L}^Q(\{\hat{Q}\}_{t \geq 0}) \equiv \mathbb{E}_0 \int_0^\infty e^{-\rho t} (\hat{Q}_t)^2 dt,$$

$$\text{s.t. } \begin{cases} d\hat{Q}_t = - \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} dt + \sigma_1^{q,L} dZ_t, & \text{for } t < T, \\ d\hat{Q}_t = - \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} dt + \sigma_2^{q,L} dZ_t, & \text{for } T \leq t < \hat{T}^{HOFG}, \\ d\hat{Q}_t = 0, & \text{for } t \geq \hat{T}^{HOFG}, \end{cases}$$

with

$$\hat{Q}_0 = \underbrace{r_1^T (\sigma_1^{q,L})}_{<0} T + \underbrace{r_2^T (\sigma_2^{q,L})}_{>0} (\hat{T}^{HOFG} - T)$$

Fiscal authority's monetary reserves F_t

$$dF_t = -\theta_t a_t \tau_t dZ_t, \quad \text{with: } F_0 = F_{0-} - \underbrace{\chi \theta_{0-} a_{0-}}_{\text{Instant subsidy}}, \quad (1)$$

Then capitalist's dynamic flow becomes:

$$da_t = (a_t (i_t + \theta_t (i_t^m - i_t)) - \bar{p} C_t) dt + \theta_t a_t [(\sigma_t + \sigma_t^q) + \tau_t] dZ_t, \quad (2)$$

with $\Delta a_0 \equiv a_0 - a_{0-} = \chi \theta_{0-} a_{0-} + \bar{p} A_{0-} \underbrace{\Delta Q_0}_{\text{Asset price change}}$

Proposition

HOFG equilibrium (with $\sigma_t^{q,*}$) becomes a unique equilibrium under the following rule:

$$\tau_t = (\sigma_t^{q,*} - \sigma_t^q), \quad \text{and} \quad \chi = \bar{p} A_{0-} \frac{Q_0^* - Q_0}{\theta_{0-} a_{0-}}, \quad (3)$$

In this case, $\tau_t = 0$, and $\chi = 0$ on the equilibrium path