A Search and Matching Model of Housing and Rental Market Interactions

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Motivation

- Housing market subject to search frictions
  - ➔ Time to buy/sell $\sim$ 5-6 months
  - ➔ Costly $\sim$ realtor fee, closing costs, flow costs

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  - ➔ Time to rent $\sim$ 1-2 months
  - ➔ Costly $\sim$ realtor fee, flow costs, renovation costs
  - ➔ Separations $\sim$ 1-2 years

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  - ➔ Costly $\sim$ realtor fee, flow costs, renovation costs
  - ➔ Separations $\sim$ 1-2 years
- Previous studies fail to address connection with rental market
  - ➔ Average home-ownership rate $\sim$ 70%
  - ➔ Policy spillovers- Han, Ngai and Sheedy (2023, WP)

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  - ➔ Previous home-owner flows $\sim$ 55%
  - ➔ Transitioning from rental flows $\sim$ 45%

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- Bachmann and Cooper (2014, CEPR) show buyers are
  - ➔ Previous home-owner flows $\sim$ 55%
  - ➔ Transitioning from rental flows $\sim$ 45%
- Existing models
  - ➔ Fixed measure of buyers
  - ➔ Free entry of buyers

This Paper

- Study joint behavior of housing and rental market

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- Are frictions and prices in housing market correlated to those in rental market?
  - ➔ Price to rent ratio, sales, time on market, housing vacancies, rental vacancies, buyers, rental seekers

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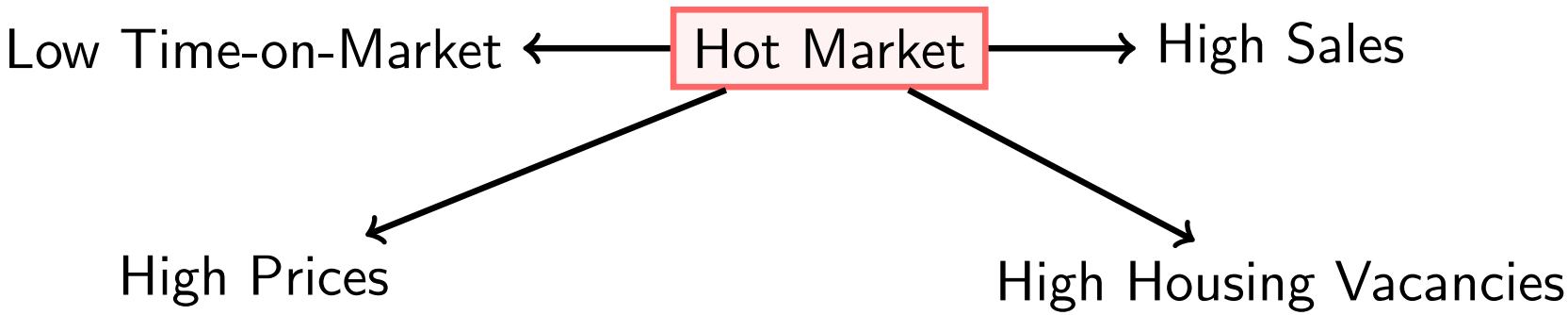
- Are frictions and prices in housing market correlated to those in rental market?
  - ➔ Price to rent ratio, sales, time on market, housing vacancies, rental vacancies, buyers, rental seekers
- How do these frictions impact household movement?
  - ➔ Within each market
  - ➔ Transitions from rental to homeownership

Existing Empirical Facts



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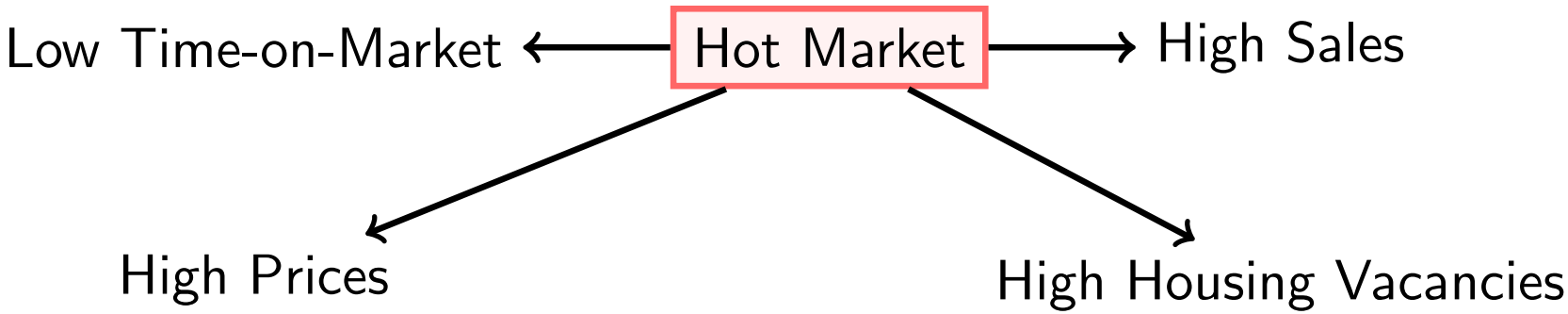
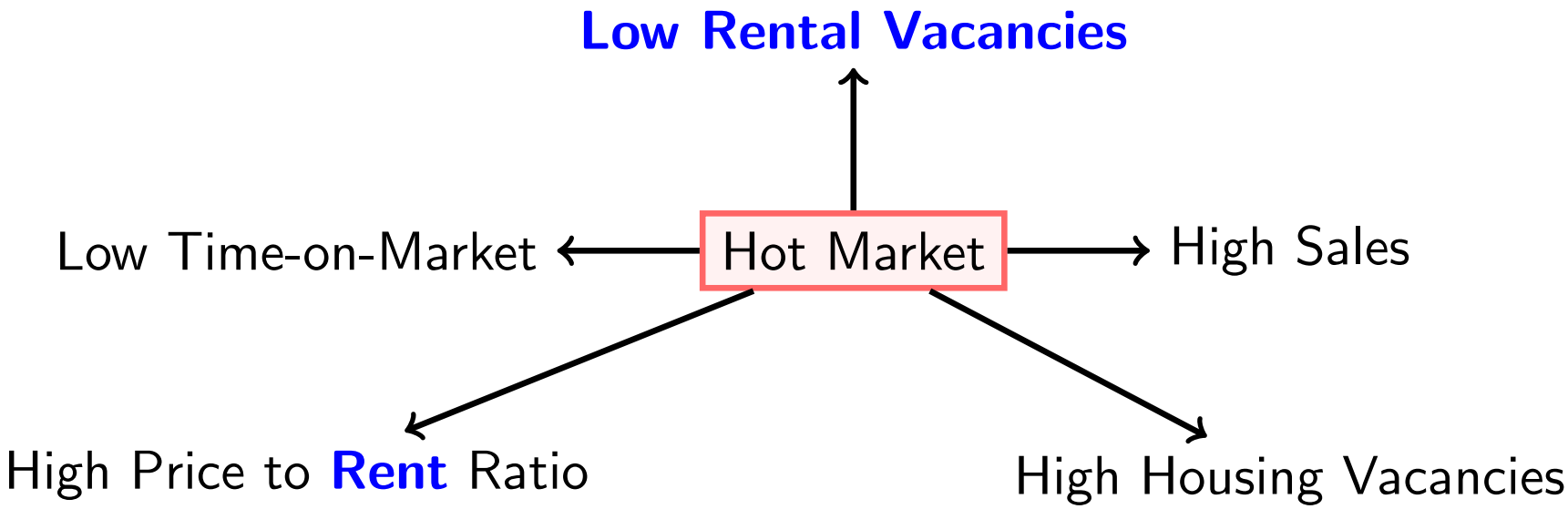


- Upward sloping Beveridge Curve in both markets
  Gabrovski and Ortego-Marti (2019, JET)
  Badarinza et al. (2024, WP)

Results


Low Rental Vacancies
Low Time-on-Market
Hot Market
High Sales
High Price to Rent Ratio
High Housing Vacancies

Results

- Demand and supply shocks used to match stylized facts

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- Demand and supply shocks used to match stylized facts
- Match upward sloping Beveridge Curve in both markets

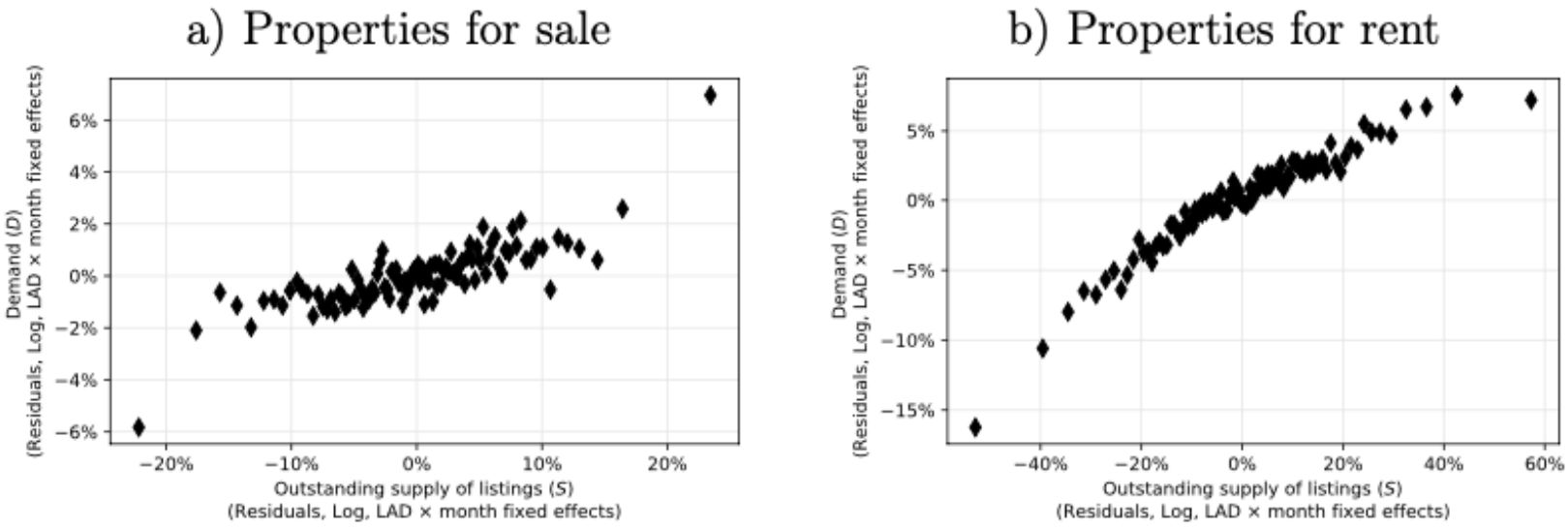
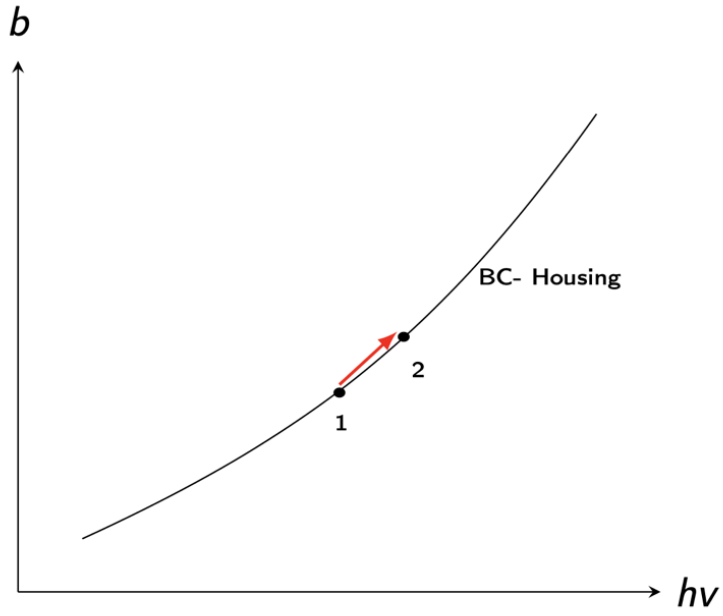
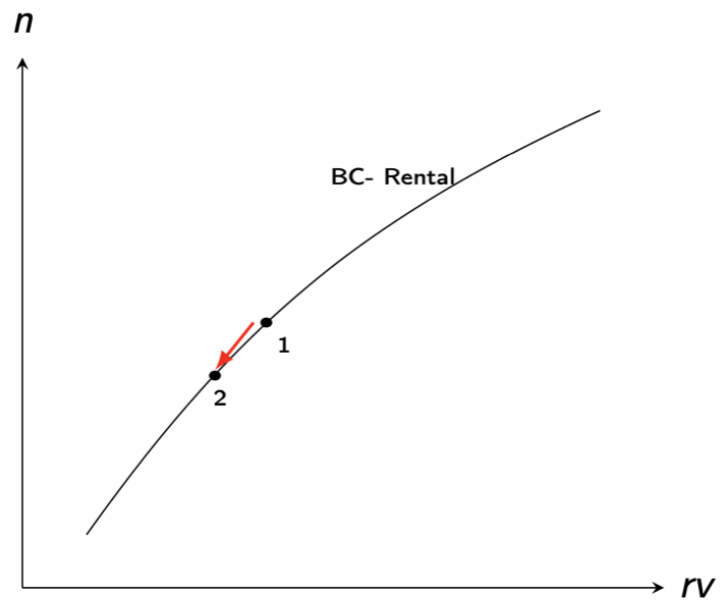


Figure: Taken from Badarinza et al. (2024)

Results

- Opposite movements along Beveridge curve over business cycle





Literature Review

- Empirical evidence on frictions in the housing market
  - ➔ Diaz and Jerez (2013, IER); Ngai and Tenreyro (2014, AER); Halket and Custoza (2015, JME); Badarinza et al. (2024, WP)
- Search and Matching models in the housing market
  - ➔ Han and Strange (2015, Handbook of RUE); Gabrovski and Ortego-Marti (2019 JET); Han, Ngai and Sheedy (2023, WP)
- Transition of households from rental to housing
  - ➔ Gyourko and Linneman (1997, EJ); Ortalo-Magne and Rady (2006, REStud); Andrews and Sanchez (2011, OECD)

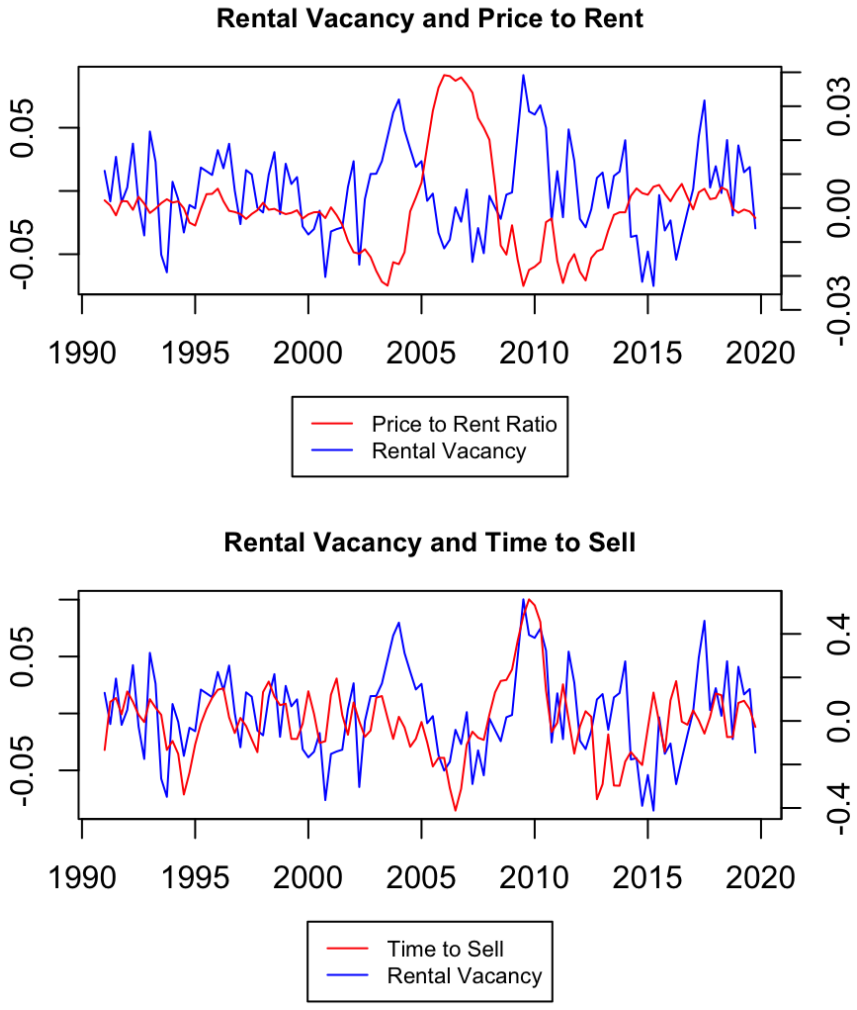
Empirical Analysis

Data Sources

- **Rents**- Consumer Price Index for All Urban Consumers: Rent of Primary Residence in U.S. City Average (US BLS)
- **Rental Vacancy rate**- United States Census Bureau (Housing Vacancy Survey)
- **Housing Vacancies**- United States Census Bureau
- **Prices**- All Transactions HPI from FRED (FHFA)
- **Time on Market**- Median Number of Months on Sales Market for Newly Completed Homes from FRED (US Census)
- **Sales**- New One Family Houses Sold (US Census)

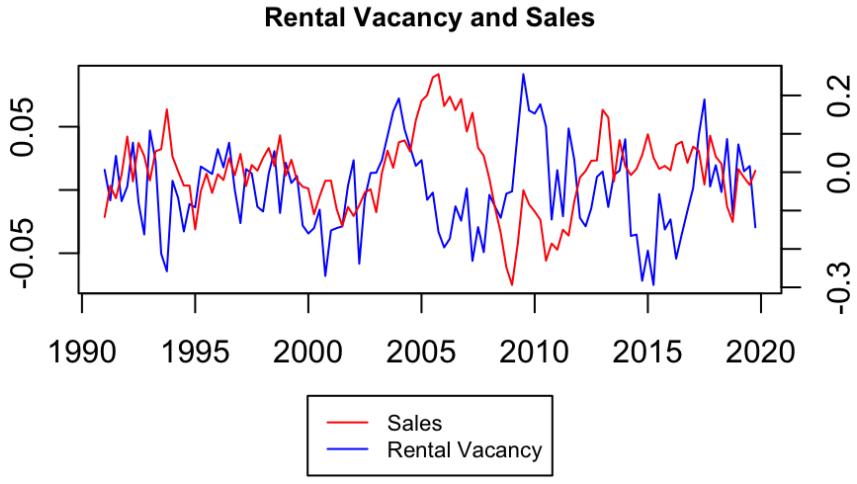
*Data is from 1991 to 2019- Ngai and Sheedy (2024, IER)

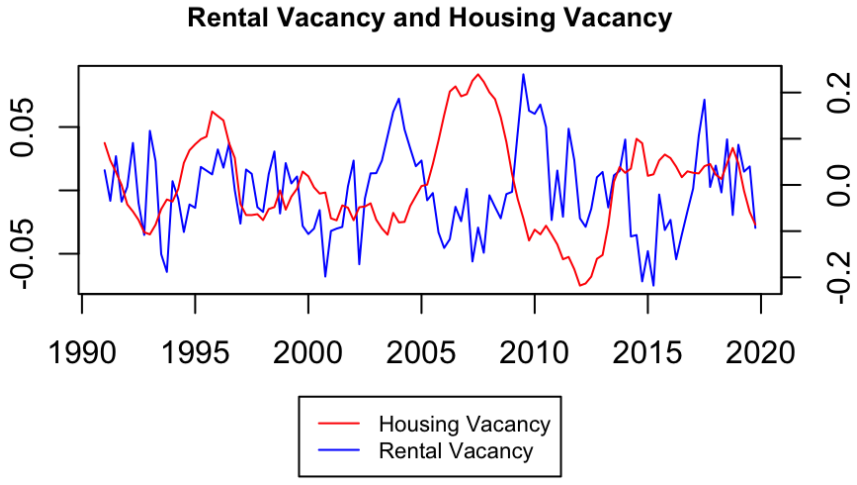
Stylized Facts



- The elasticity of Price to Rent ratio and Time to sell with respect to Rental Vacancy rate is -0.15 and 1.48

Stylized Facts





- The elasticity of Sales and Housing Vacancies with respect to Rental Vacancy rate is -0.66 and -0.82

Model

Environment

- Discrete time, discount factor β
- Agents are risk neutral and die at rate d
- Households are in one of these stages: Homeowners, Home-buyers, Tenants, Rental-seekers or idle
- Segmented properties, destroyed at rate δ

Matching

Rental Market

- Matching function: $M_r(n, r_v)$
- Rental market tightness: ϕ
- Rent Seekers meet landlords: $m(\phi) = M_r(n, r_v)/\mathrm{n}$
- Landlords meet Rent Seekers: $\phi m(\phi) = M_r(n, r_v)/r_v$

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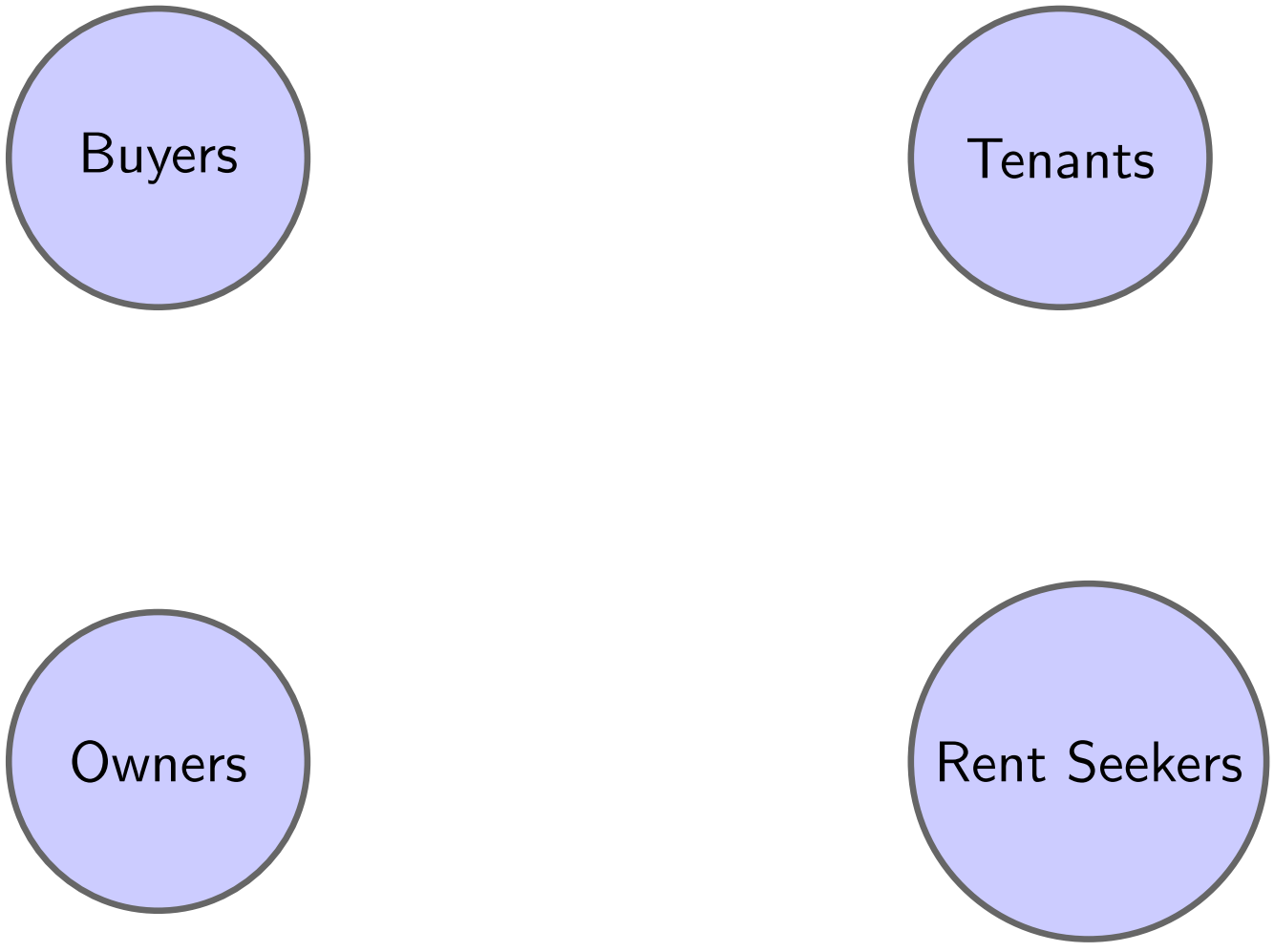
Housing Market

- Matching function: $M_h(b, h_v)$
- Housing market tightness: θ
- Buyers meet sellers: $m(\theta) = M_h(b, h_v)/\mathrm{b}$
- Sellers meet buyers: $\theta m(\theta) = M_h(b, h_v)/h_v$

Transition to Housing

- ϵ: idiosyncratic utility of being a home-owner
- Separated tenants draw from G(ϵ)
- If idiosyncratic utility $\epsilon \geq \epsilon^R$ become home-buyers

Model- Agents



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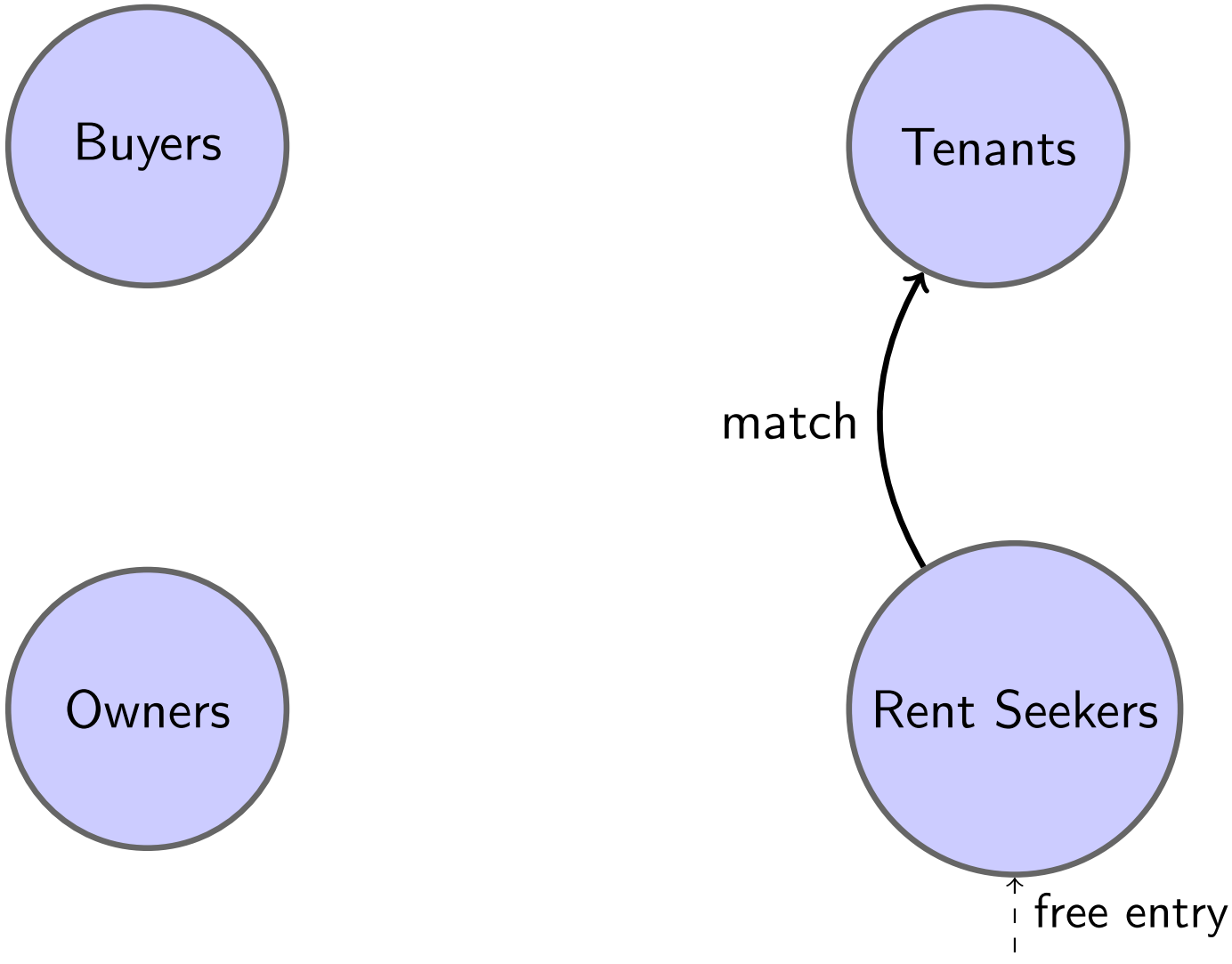

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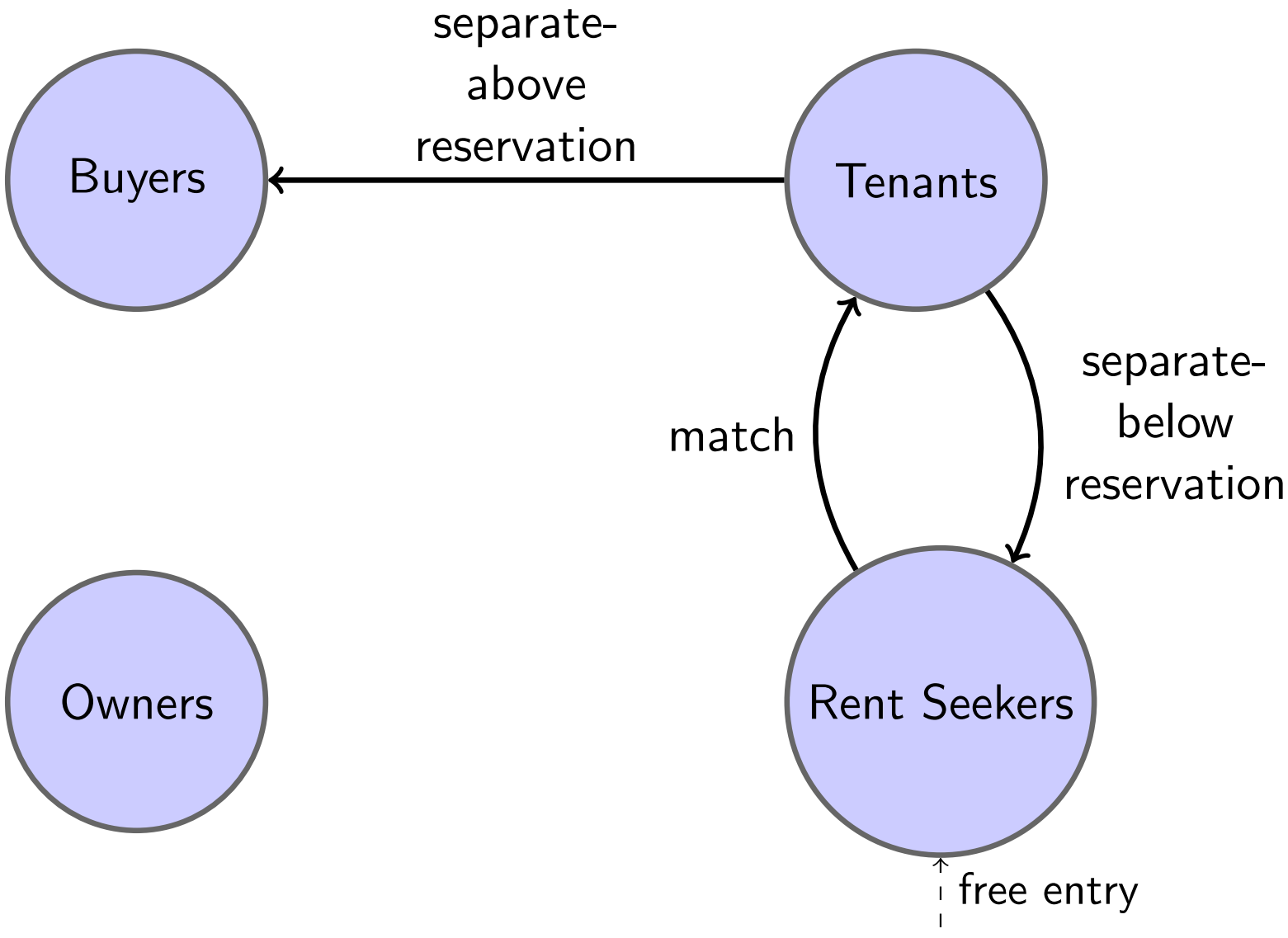

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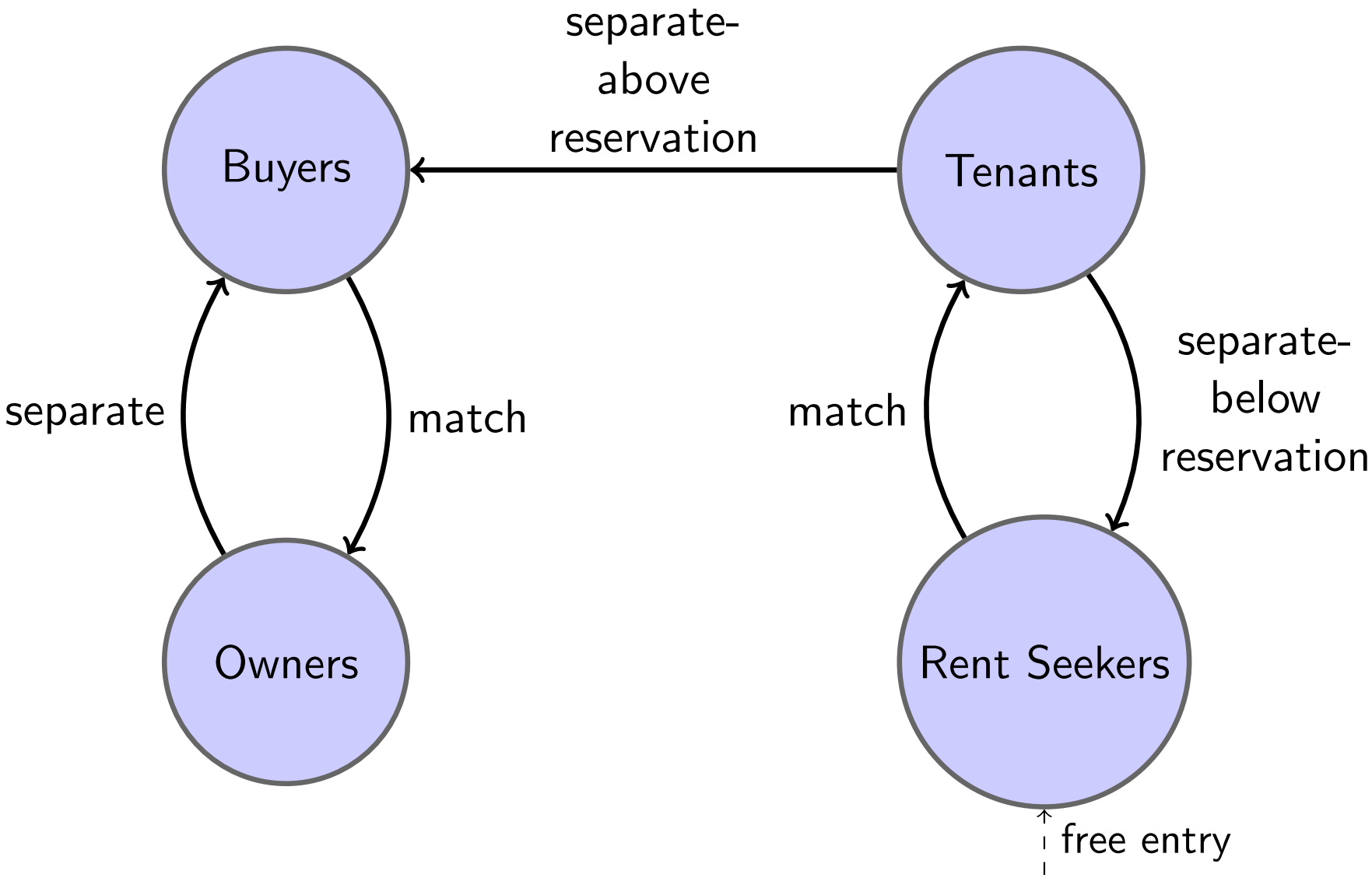

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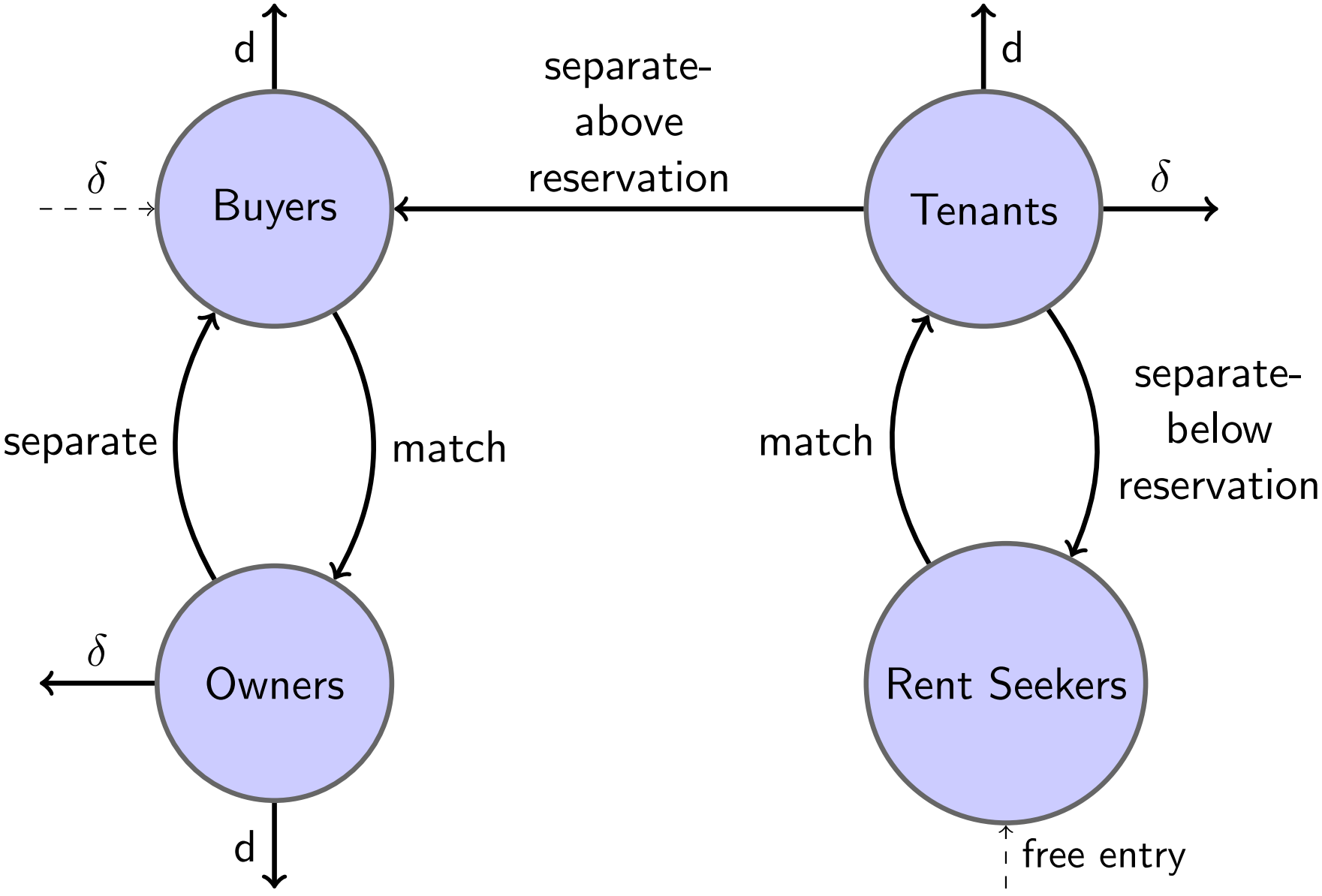
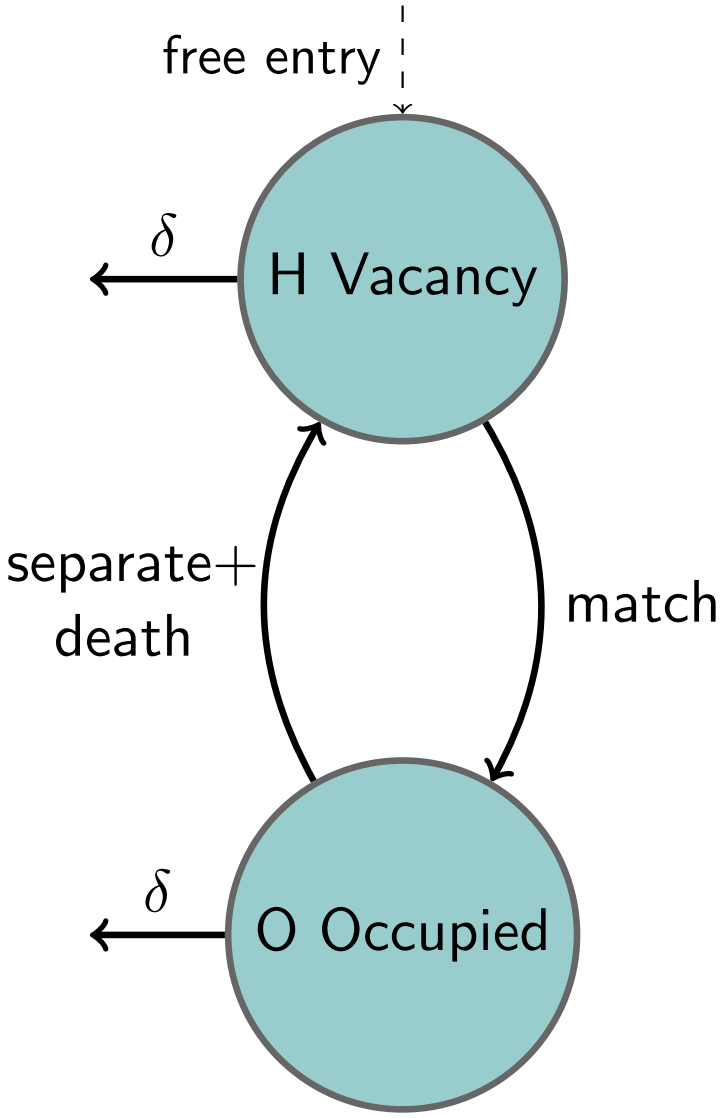


Model- Properties



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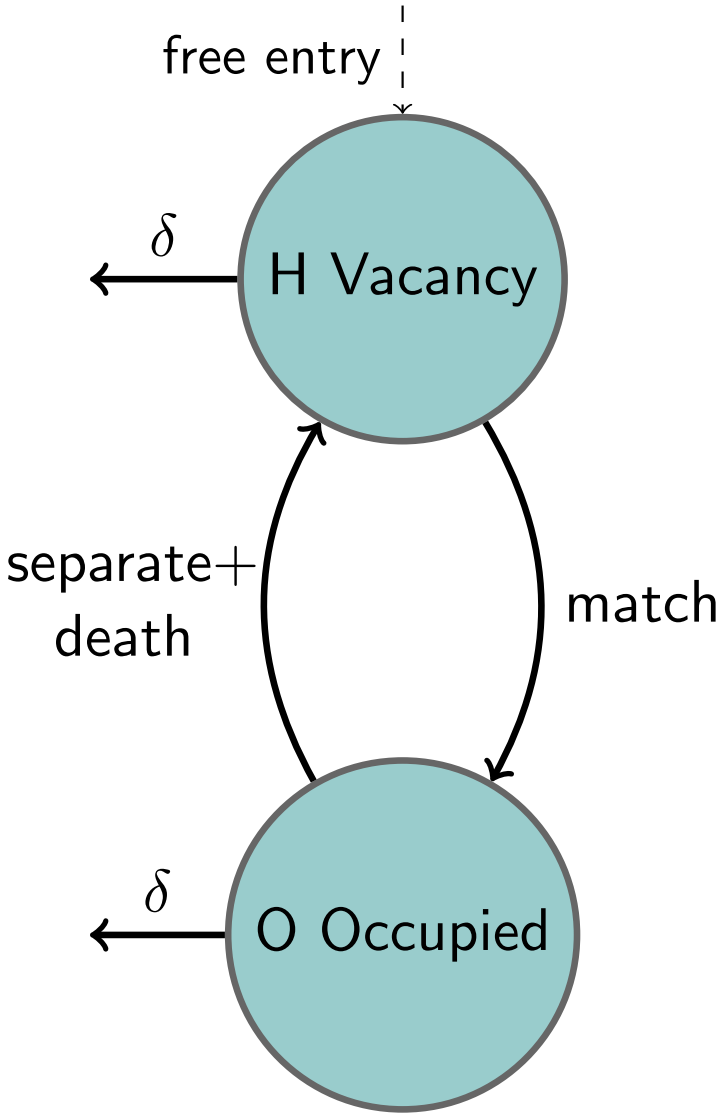

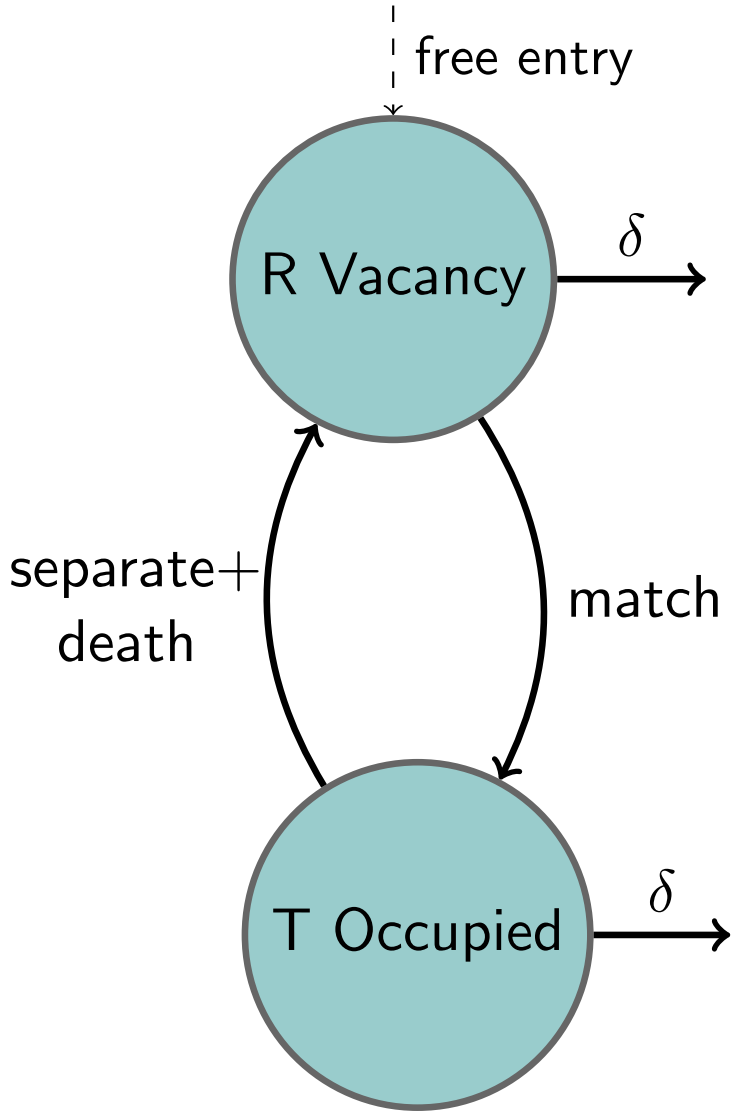

Bellman Equations- Agents- Housing

Home-owners (H) receive separation (s) shock and become home-buyers (B) creating a housing vacancy (V^H)

$$H(\epsilon) = \underbrace{\epsilon\chi^H}_{\text{utility}} + \underbrace{(1-d)\Big((1-\delta)}_{\text{no death or destruction}} \Big(\overbrace{(1-s)\beta H(\epsilon)}^{\text{unseparated}} + \overbrace{s\beta(B(\epsilon)+V^H)}^{\text{separated}} \Big) + \underbrace{\delta\beta B(\epsilon)}_{\text{house destroyed}} \Big) + \underbrace{d\beta V^H(1-\delta)}_{\text{death}}$$

$$B(\epsilon) = \underbrace{-c^B}_{\text{flow cost}} + \underbrace{(1-d)}_{\text{no death}} \Bigg(\underbrace{m(\theta)\Big(\beta H(\epsilon) - p(\epsilon)\Big)}_{\text{matched}} + \underbrace{(1-m(\theta))\beta B(\epsilon)}_{\text{unmatched}} \Bigg)$$

Bellman Equations- Agents- Rental

Tenants (T) receive separation (σ) shock and become home-buyers (B) or rental seekers (R) depending on their idiosyncratic draw of utility (ϵ)

$$T = \underbrace{\chi^T}_{\text{utility}} - \underbrace{\rho}_{\text{rent}} + \underbrace{(1-d)\Big((1-\delta)}_{\text{no death or destruction}} \Big(\overbrace{(1-\sigma)\beta T}^{\text{unseparated}} + \underbrace{\sigma\beta(1-G(\epsilon^R))\,\mathbb{E}(B)}_{\text{separated}} \Big) \Big)$$

$$R = \underbrace{-c(n)}_{\text{flow cost}} + \underbrace{m(\phi)\beta T}_{\text{matched}} + \underbrace{(1-m(\phi))\beta R}_{\text{unmatched}}$$

Bellman Equations- Properties

$$V^H = \underbrace{-c^S}_{\text{flow cost}} + \underbrace{(1-\delta)}_{\text{no destruction}} \left(\underbrace{\theta m(\theta)\, \mathbb{E}(p)}_{\text{matched}} + \underbrace{(1-\theta m(\theta))\beta V^H}_{\text{unmatched}} \right)$$

$$V^R = \underbrace{-c^R}_{\text{flow cost}} + \underbrace{(1-\delta)}_{\text{no destruction}} \left(\underbrace{\phi m(\phi)\beta L}_{\text{matched}} + \underbrace{(1-\phi m(\phi))\beta V^R}_{\text{unmatched}} \right)$$

$$L = \underbrace{\rho}_{\text{rent}} + \underbrace{(1-d)\Big((1-\delta)}_{\text{no death or destruction}} \left(\underbrace{(1-\sigma)\beta L}_{\text{unseparated}} + \underbrace{\sigma\beta V^R}_{\text{separated}} \right)\Big) + \underbrace{d\beta V^R(1-\delta)}_{\text{tenant death}}$$

Nash Bargaining

- Prices are,

$$p(\epsilon) = \underset{p(\epsilon)}{\text{argmax}} \left(\underbrace{\beta H(\epsilon) - p(\epsilon) - \beta B(\epsilon)}_{\text{Buyer surplus}} \right)^{\eta} \left(\underbrace{p(\epsilon) - \beta V^H}_{\text{Seller surplus}} \right)^{1-\eta},$$

$$\forall \epsilon \geq \epsilon^R$$

- Rent is,

$$\rho = \underset{\rho}{\text{argmax}} \quad \beta \left(\underbrace{T - R}_{\text{Rent seeker surplus}} \right)^{\alpha} \left(\underbrace{L - V^R}_{\text{Landlord surplus}} \right)^{1-\alpha}$$

Equilibrium

Equilibrium-Rental Market

- Free entry of rental seekers $\Longrightarrow$ $R = 0$,

$$(RE): \quad \underbrace{\frac{c(n)}{m(\phi)}}_{\text{expected cost}} = \underbrace{\beta T}_{\text{surplus}}$$

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- Free entry of landlords $\Longrightarrow$ $V^R = k^R$,

$$(LE): \quad \underbrace{\frac{k^R(1-\beta(1-\delta))+c^R}{\beta(1-\delta)\phi m(\phi)}}_{\text{expected cost}} = \underbrace{\frac{\rho - k^R(1-\beta(1-\delta))}{1-\beta(1-\delta)(1-\sigma)(1-d)}}_{\text{surplus}}$$

Equilibrium-Rental Market

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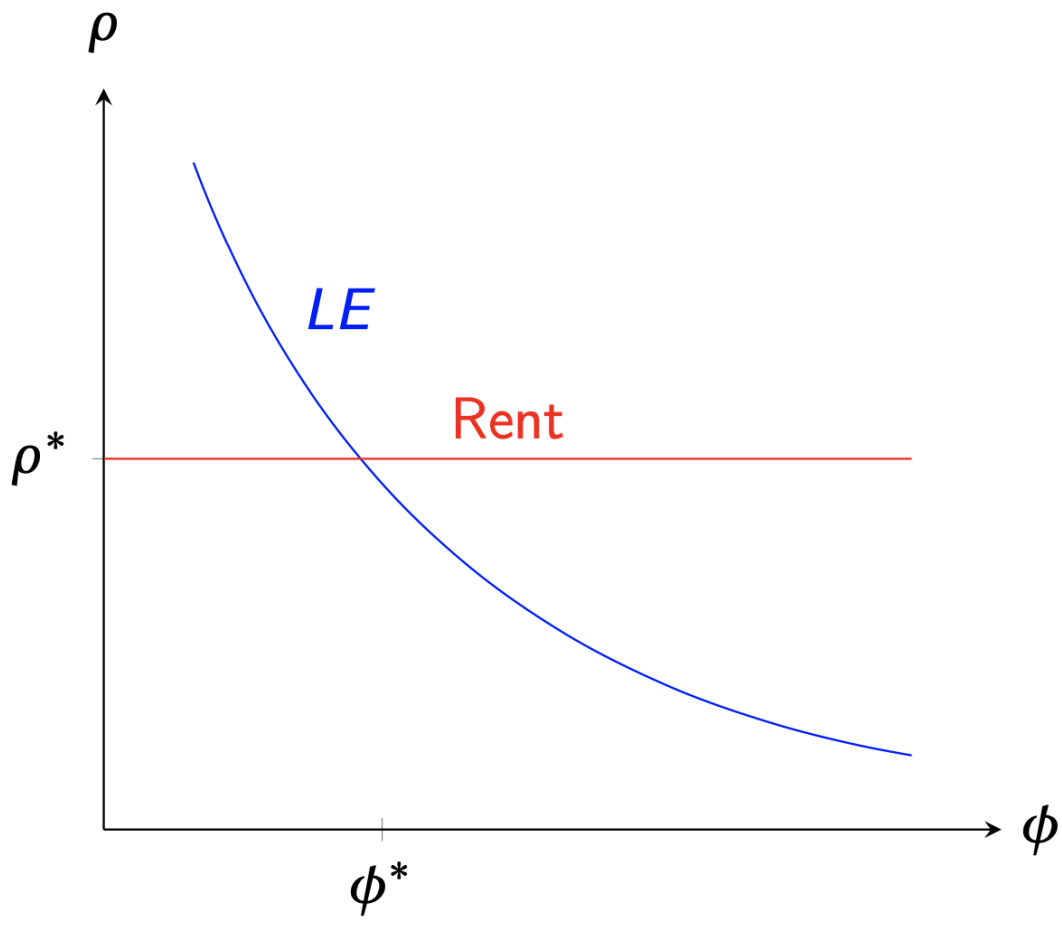
- Nash bargaining gives (Rent),

$$\rho = (1-\alpha)\left[\chi^T + (1-d)(1-\delta)\sigma\beta(1-G(\epsilon^R))\,\mathbb{E}(B)\right] + \alpha[k^R(1-\beta(1-\delta))]$$

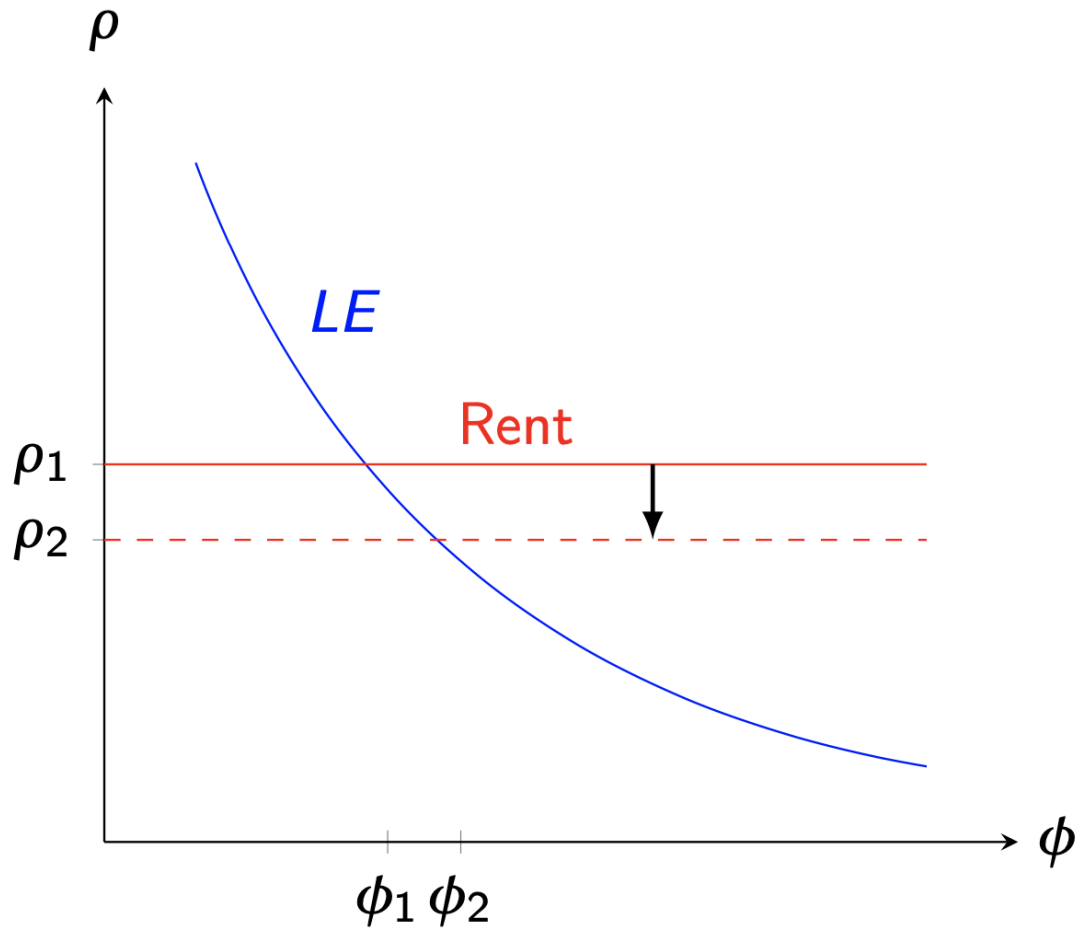
Equilibrium-Rental Market

- {rent, market tightness and number of rental seekers}



Comparative Statics- Rental Market

Demand Shock- χ^T decreases



Supply Shock

Equilibrium-Housing Market

- Free entry of sellers $\implies V_H = k^H$,

$$(HE): \quad \underbrace{\overline{p} - \beta k^H}_{\text{surplus}} = \underbrace{\frac{k^H(1-\beta(1-\delta)) + c^S}{\theta m(\theta)(1-\delta)}}_{\text{expected cost}}$$

- Equilibrium price from Nash Bargaining,

$$(PP): p(\epsilon) - \beta k^H = \frac{\beta(1-\eta)(\epsilon\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}$$

Reservation Utility

- For the marginal buyer, $T = B(\epsilon^R)$ i.e. $\epsilon = \epsilon^R$,

$$m(\theta)\eta\beta = \frac{(B(\epsilon^R)(1-\beta(1-d)) + c^B)(1-\beta(1-s)(1-\delta))}{(\epsilon^R\chi^H - k^H(1-\beta(1-\delta)) - B(\epsilon^R)(1-\beta(1-d))}$$

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- Assume $G(\epsilon)$ follows a Pareto distribution,

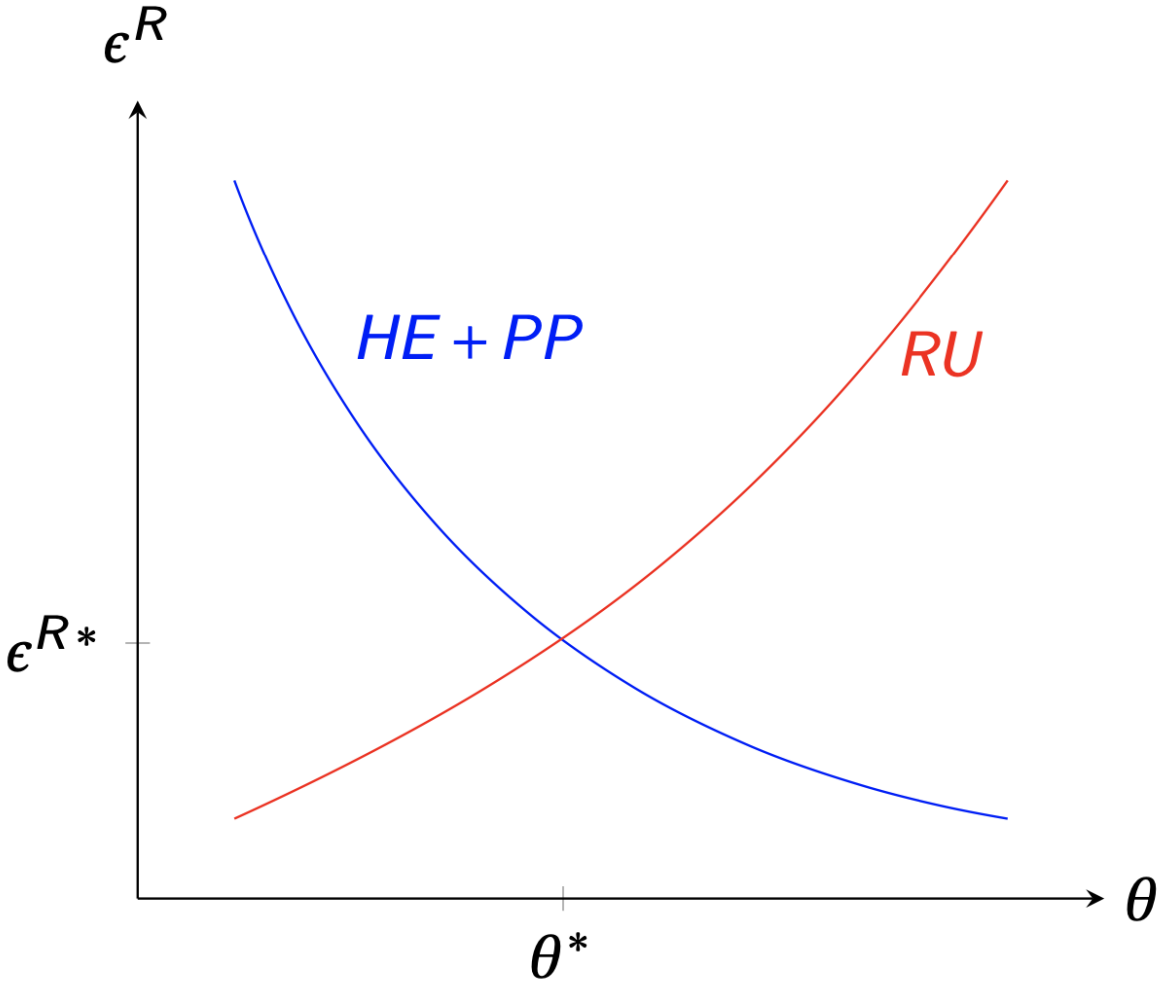
$$G(\epsilon) = 1 - (\frac{\epsilon_I}{\epsilon})^\lambda, \quad \lambda > 1$$

- HE and PP condition,

$$\frac{k^H(1-\beta(1-\delta)) + c^S}{\theta m(\theta)(1-\delta)} = \frac{\beta(1-\eta)(\frac{\lambda}{\lambda-1}\epsilon^R\chi^H + c^B - k^H(1-\beta(1-\delta)))}{1-(1-s)(1-d)(1-\delta)\beta + \beta(1-d)\eta m(\theta)}$$

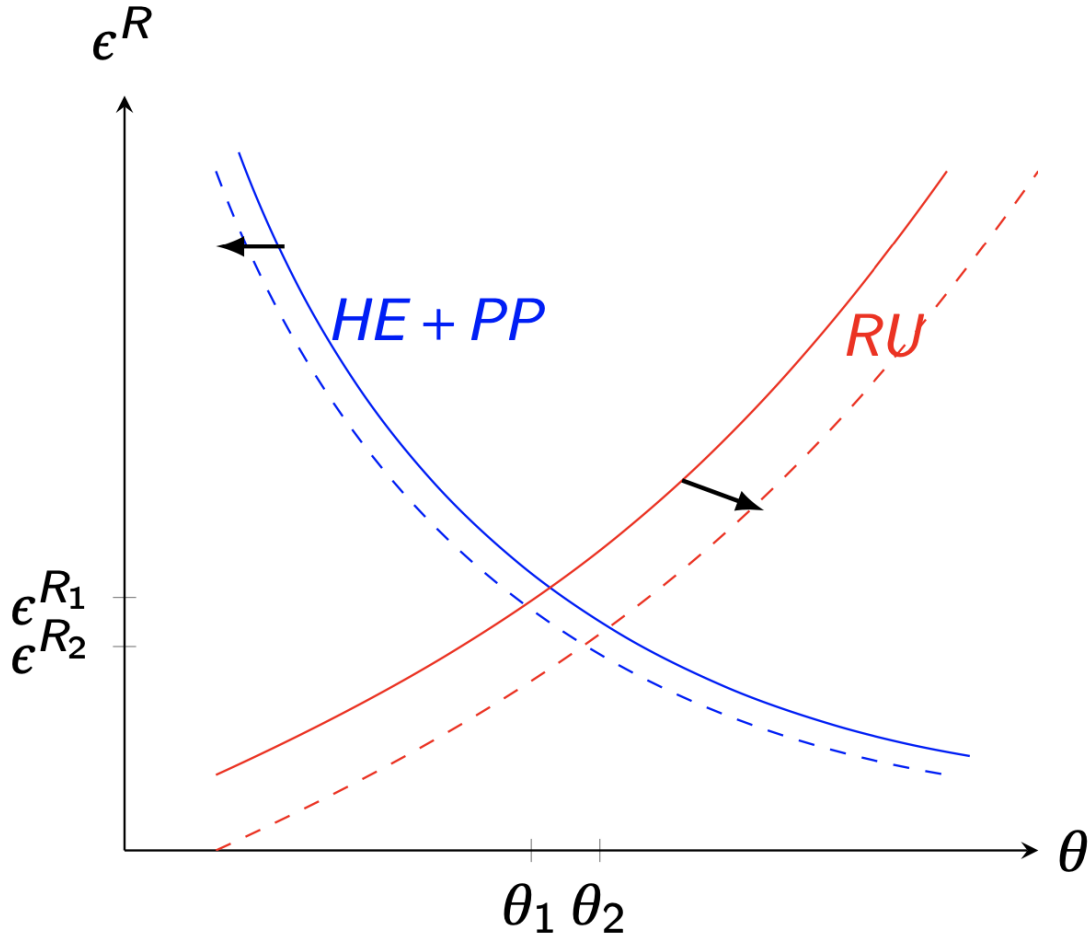
Equilibrium-Housing Market

- {prices, market tightness and reservation utility}



Comparative Statics- Housing Market

Demand Shock- χ^H increases



Supply Shock

Agent Flows

- For tenants,

$$\underbrace{m(\phi)n}_{\text{inflows}} = \underbrace{(\delta + d + \sigma(G(\epsilon^R) + (1 - G(\epsilon^R))))t}_{\text{outflows}}$$

- For buyers,

$$\underbrace{(s + \delta)h + \sigma(1 - G(\epsilon^R))t}_{\text{inflows}} = \underbrace{(m(\theta) + d)b}_{\text{outflows}}$$

- For homeowners,

$$\underbrace{m(\theta)b}_{\text{inflows}} = \underbrace{(s + \delta + d)h}_{\text{outflows}}$$

Calibration

Calibration

Preferences/Technology	Parameter	Value	Source/Target
Discount Factor	β	0.987	Interest rate= 5%
Elasticity of Matching function	$\psi_h = \psi_r$	0.16	Genesove and Han (2012)
Destruction rate	δ	0.004	Van Nieuwerburgh and Weill (2010)
Death rate	d	0.0044	Head, Lloyd and Stacey (2023)
Separation Rate Housing	s	0.022	Tenure= 9 years
Separation Rate Rental	σ	0.131	Tenure= 2 years
Housing Efficiency Matching Function	μ_h	0.75	TTB= 1.4625 quarters
Rental Efficiency Matching Function	μ_r	1.667	TTR = 0.65 quarters

Calibration

Preferences/Technology	Parameter	Value	Source/Target
Utility Scale Housing	χ^H	1	Normalization
Utility Scale Rental	χ^T	18.56	Equilibrium
Bargaining power Housing and Rental	$\eta = \alpha$	0.5	Han, Ngai and Sheedy (2022)
Seller cost	c^S	26.54	Average seller cost $= 5.4$ % of price
Buyer cost	c^B	28.37	Average buyer cost $= 5.7$ % of price
Rental seeker cost	$\overline{c}$	3.98	Equilibrium
Maintenance cost Landlord	c^R	0.18	Han, Ngai and Sheedy (2022)
Construction cost (H)	k^H	447.09	Equilibrium
Construction cost (R)	k^R	976.63	Equilibrium
Pareto Shape	λ	2.8927	Rent to Price

Results

Moment	Data/Source	Value
Average Price	Kotova and Zhang (2020)	491.2
Rent		17.2
Rent to Price	3.5%	3.5%
Housing vacancy	1.75%	4.49%
Rental vacancy	7.76%	7.75%

Business Cycle

Model with Business Cycles

- Solved same as before
- Perfectly correlated shocks along business cycle
  Shimer (2005, AER)
- All processes are AR(1) with same underlying shock u_t
- Approximate with Markov Chain- Rouwenhorst (1995)
- Discretize

$$ln(\chi_t^H) = \zeta_{\chi^H} + \nu ln(\chi_{t-1}^H) + u_t$$

$$ln(\chi_t^T) = \zeta_{\chi^T} + \nu ln(\chi_{t-1}^T) + xu_t$$

Results

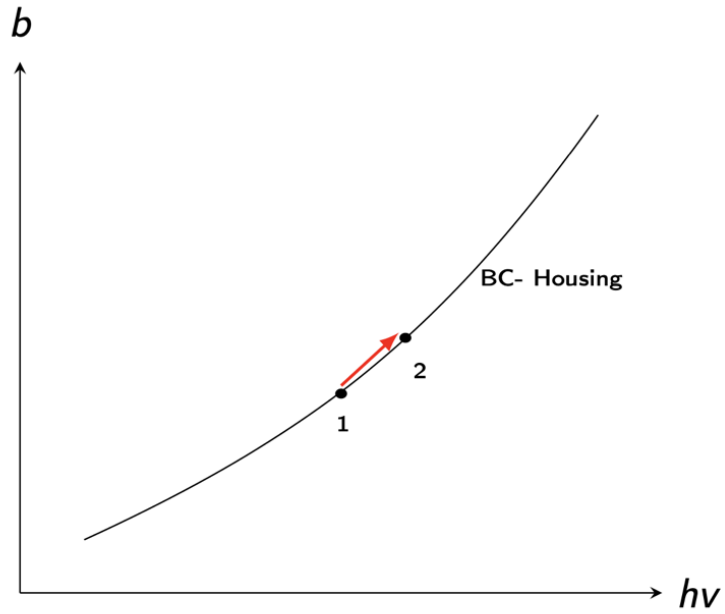
Moment	Data	Demand	Demand + Supply
Price to rent	-0.15	-0.12	-0.15
Time to Sell	1.48	0.03	0.09
Sales	-0.66	-0.60	-0.65
Housing vacancy	-0.82	-0.57	-0.56

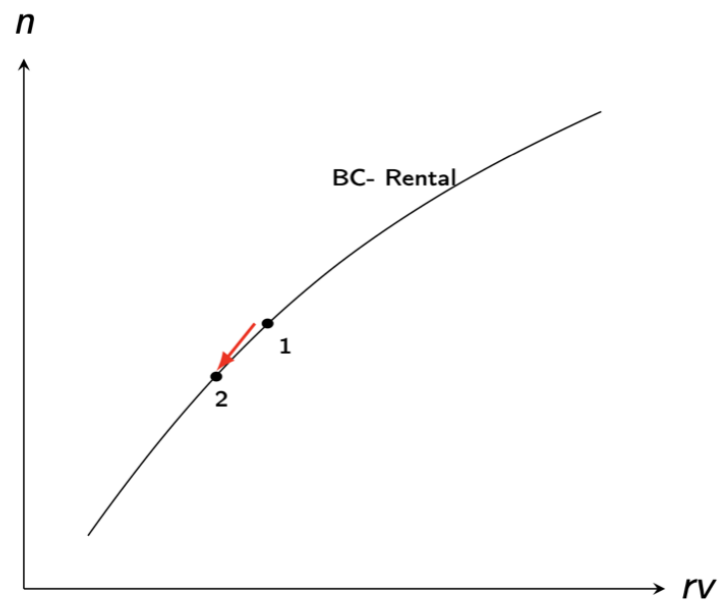
*With just demand shock, unable to match correlations in housing market

Demand+Supply Figure Investors

Beveridge Curve- Boom Cycle

- Matches upwards sloping BC
- Shows opposite movements along the BC





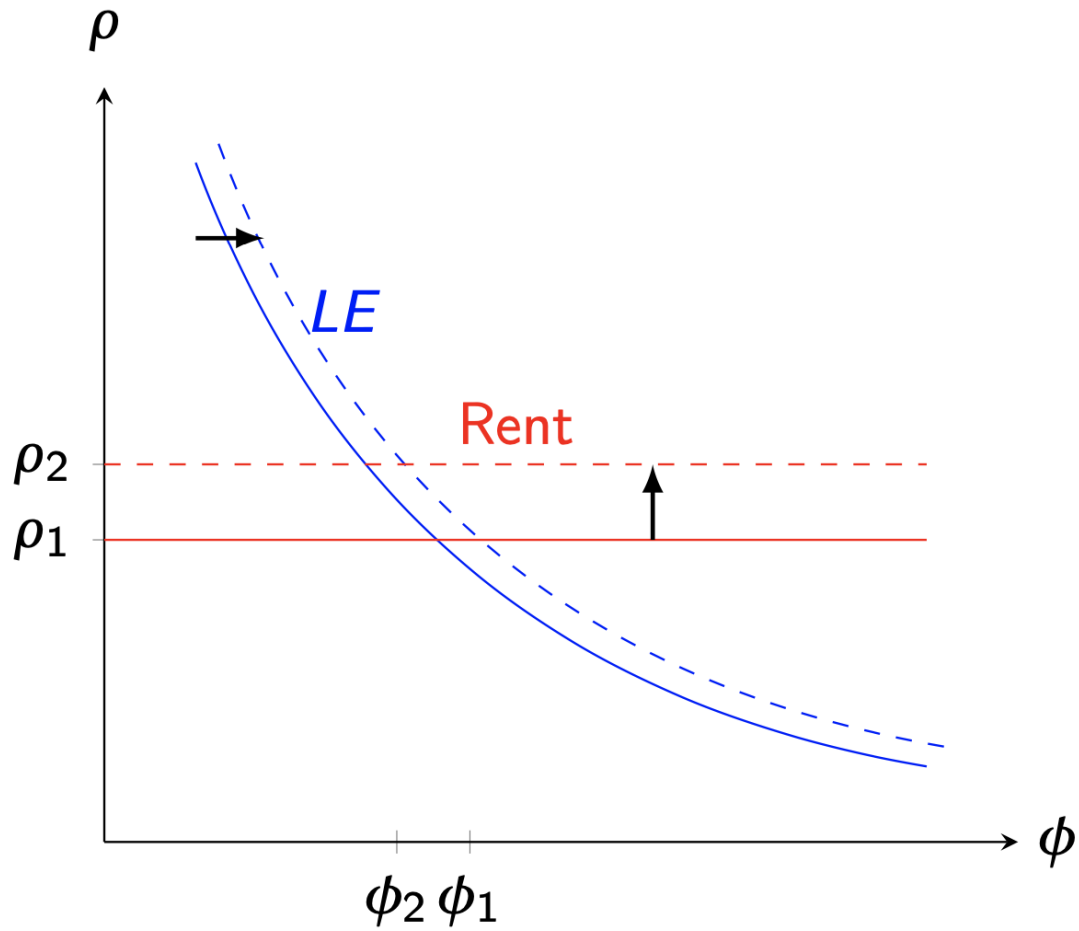
Conclusion

- Empirical evidence to show that frictions in both markets are correlated
- A model of housing and rental market
  - ➔ Search frictions in both markets
  - ➔ Heterogeneous buyers
  - ➔ Endogenous decision to move from rental to housing market
- Frictions in both markets affect the transition from renting to homeownership
- Model matches the observed elasticity

Thank You!

Comparative Statics- Rental Market

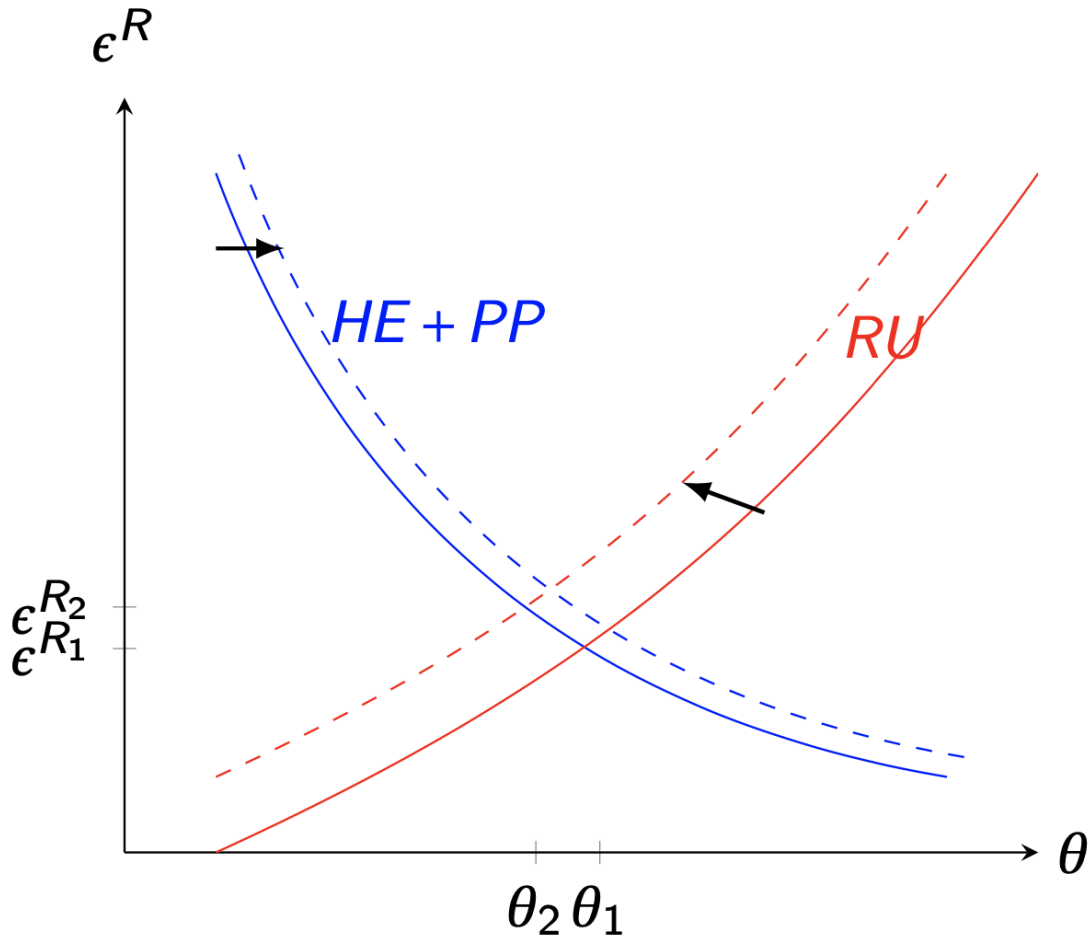
Supply Shock- k^R increases



Back

Comparative Statics- Housing Market

Supply Shock- k^H increases



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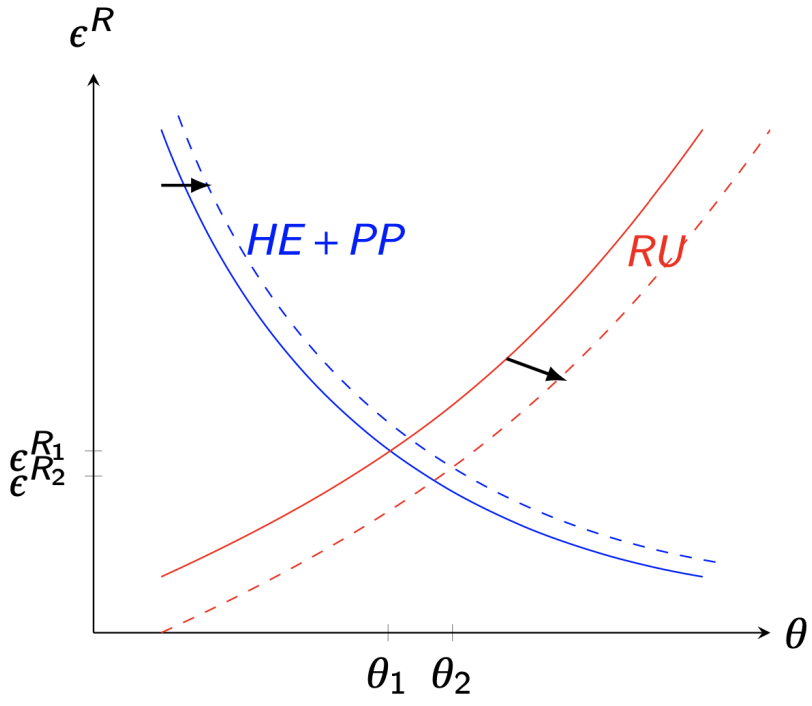
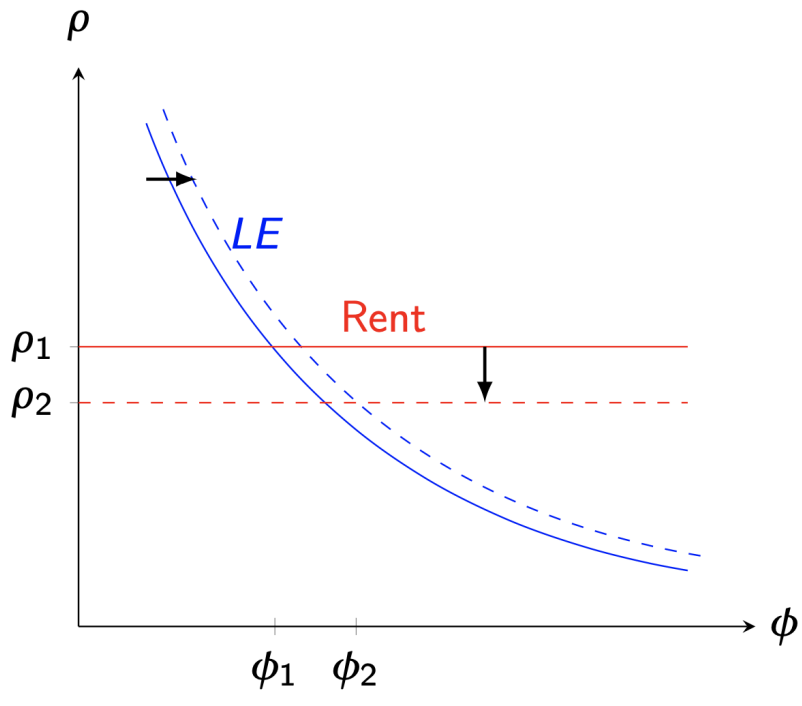
Example- Boom Cycle

Model matches the new and existing stylized facts





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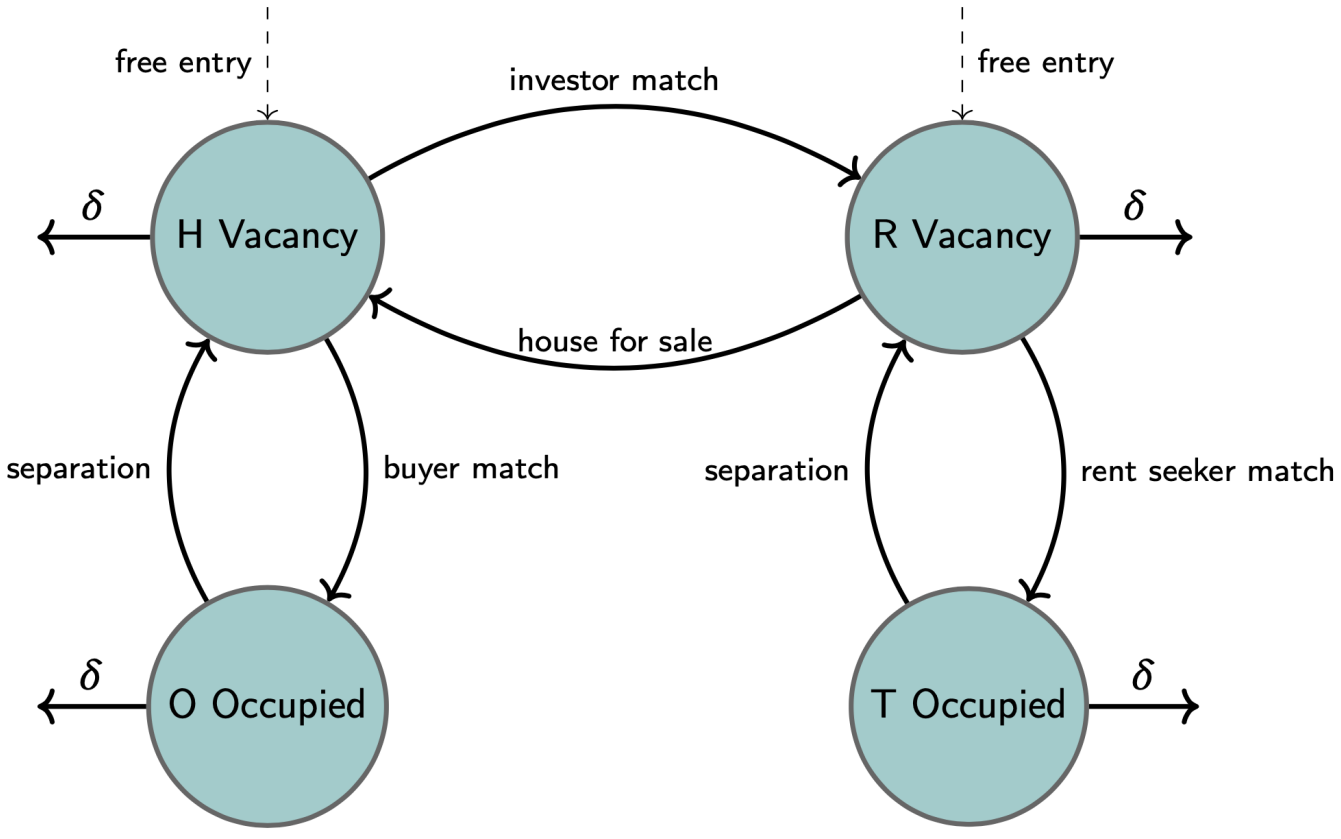
Model- Properties Move



Results

Moment	Data	Demand + Supply	Investors
Price to rent	-0.15	-0.15	-0.15
Time to Sell	1.48	0.09	0.08
Sales	-0.66	-0.65	-0.97
Housing vacancy	-0.82	-0.56	-1.60

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