

Dealer costs and trading choices

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What we do

Observations

- Three observations about structural changes in the corporate bond market
1. Changes in the composition of trades (Choi, Huh, and Shin 2024)
 - Dealers facilitate matching of customers with other customers (risk-free trades or agency trades)
 - Increase in customer liquidity provision
 - Risk-free trades are more common relative to inventory trades (customer-dealer).
Introduction of direct customer-customer trades
 2. Decrease in measures of illiquidity (γ and Amihud): paradox
 3. Increase in the liquidity premium

What we do

- We propose a model to explain these three observations
- Summary of the observations
 - Market participants indicate that trading is more difficult
 - But illiquidity measures do not indicate worsening liquidity
 - Matchmaking more frequent
- Model
 - Decision of customers on the trading counterparty: trade with other customers (matchmaking) \times trade with dealers
 - Propose new measure of illiquidity

What we do

- Search frictions affect trading decisions. Model of over-the-counter markets
- We show that high intermediation costs is consistent with decreasing illiquidity (decreasing bid-ask spreads)
 - Bid-ask spreads of costumers are small
 - Change in the composition of trades from dealer-customer to customer-customer
- Results consistent with trends on liquidity, turnover, and liquidity premium
- Prediction: high intermediation costs associated with financial instability

Observations

- Measures of illiquidity
 - γ — Bao, Pan, and Wang (2011) and Roll (1984)
 - AMD — Amihud (2002)
 - An increase in γ or in AMD indicates that the market is more illiquid
- Indication of illiquidity: deviation of the price of an asset from its fundamental value
 - Market frictions, difficulty to trade
- Decreases in γ or AMD associated with decreases in the bid-ask spread

- Let $\Delta p_{it} = p_{it} - p_{it-1}$
- Define γ_i for asset i as

$$\gamma_i = -\text{Cov}(\Delta p_{it}, \Delta p_{it+1})$$

- Efficient markets: the covariance of consecutive price changes (returns) should be small
- High γ : some form of frictions allow predictability of returns

- The Amihud measure is given by the average of absolute returns divided by the volume of trades,

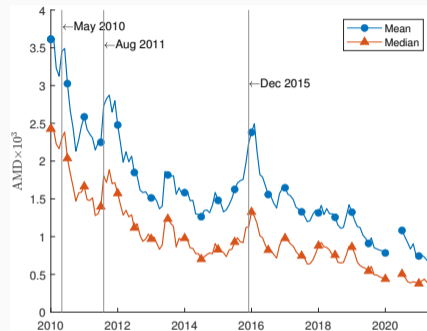
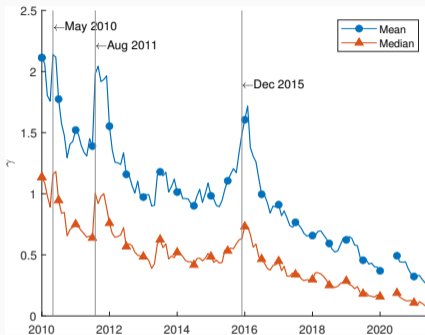
$$AMD_{id} = \frac{1}{N_{id}} \sum_{t=1}^{N_{id}} \frac{|r_{it}|}{V_t} = \frac{1}{N_{id}} \sum_{t=1}^{N_{id}} \left| \frac{P_{it} - P_{it-1}}{P_{it-1}} \right| \frac{1}{V_t},$$

- N_{id} : number of available returns in day d , r_{it} returns of bond i during day t
- V_t : volume of trade t
- High AMD
 - The price impact is high relative to volume: illiquidity

Empirical procedure

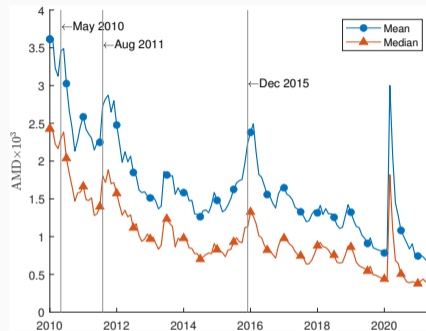
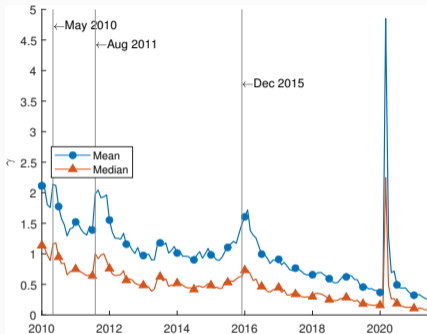
- Data
 - TRACE Enhanced (ETTRACE) database from January 2005 to June 2021
 - Bloomberg (equity data)
 - Refinitiv/Eikon (CDS5Y)
- Regressions of corporate yield spreads on γ and AMD

Results



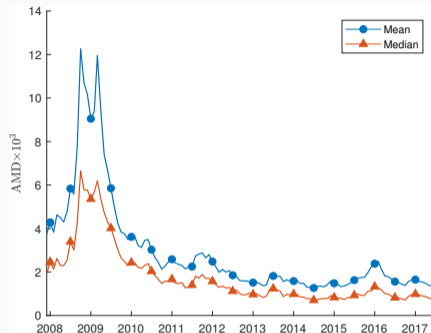
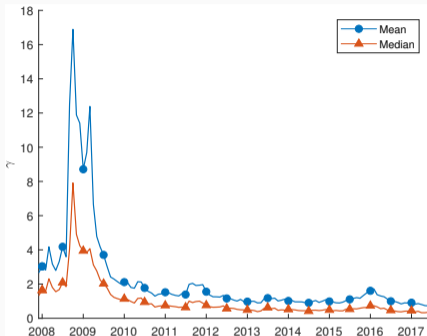
May 2010: Dodd-Frank bill passed the U.S. Senate; Greece's bailout. **August 2011:** US credit rating downgraded. **December 2015:** Fed interest rate increase since financial crisis. **Covid:** February–June 2020

Results



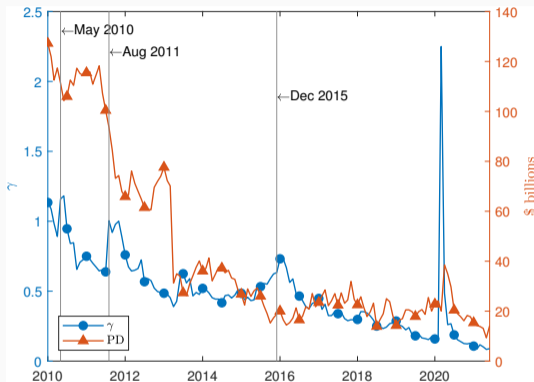
May 2010: Dodd-Frank bill passed the U.S. Senate; Greece's bailout. **August 2011:** US credit rating downgraded. **December 2015:** Fed interest rate increase since financial crisis. **Covid:** February–June 2020

Results



- Market illiquidity does not seem to increase
- Dealer positions decreased and liquidity premium increased

Illiquidity and inventory of bonds

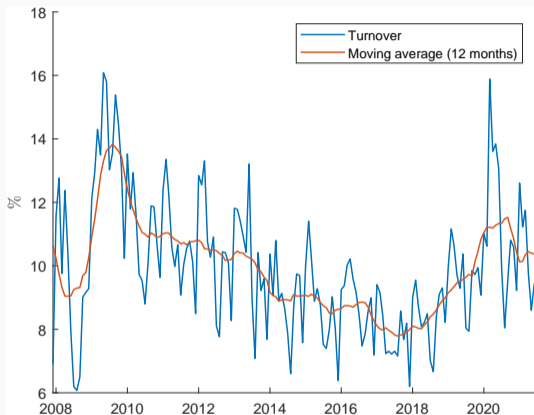


Median γ (left)

Median Primary Dealers monthly net positions in corporate debt instruments (right)

Liquidity not worsening, but less inventory of bonds with dealers

Turnover



Turnover: amount traded to amount outstanding

Median of daily turnover

The turnover rate has decreased since August 2009

Corporate yield spreads on illiquidity measures and controls, 2007–2021

Table 1: Corporate Yield Spreads on γ , AMD and controls

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R ²	Obs.
Panel A: December 2007–June 2021															
.429													1.41	.181	196,345
[11.01]													[7.86]		
	.278												1.37	.149	196,345
	[9.58]												[8.09]		
		.787											.664	.646	196,345
		[32.11]											[8.32]		
.143		.535	.548	.175	-.851	-.042	-.467	.423	.010	.004	.010	.370	.297	.791	196,345
[9.61]		[20.38]	[9.12]	[11.93]	[-9.20]	[-1.82]	[-4.39]	[8.68]	[2.01]	[.70]	[5.73]	[6.40]	[3.04]		
	.098	.537	.579	.179	-.862	-.058	-.410	.281	.015	-.005	.012	.603	.237	.792	196,345
	[8.70]	[19.59]	[9.71]	[11.33]	[-9.26]	[-2.11]	[-3.96]	[6.12]	[2.39]	[-1.05]	[6.79]	[6.66]	[2.49]		

Fama-MacBeth regression with Newey-West standard errors. T-statistics in square brackets. AMD multiplied by 10^3 . EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the volume divided by the amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted- R^2 's.

Corporate yield spreads on illiquidity measures and controls, 2007–2009

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R ²	Obs.
Panel B: December 2007–December 2009															
.151													3.33	.085	16,687
[4.56]													[6.19]		
	.102												3.28	.052	16,687
	[3.96]												[7.58]		
		.935											1.48	.744	16,687
		[20.33]											[5.75]		
.042	.758	.877	.290	−1.32	−.188	−.295	.009	−.046	.058	.009	.720	.638	.791	16,687	
[4.37]	[16.19]	[5.86]	[4.97]	[−3.07]	[−2.16]	[−1.02]	[.18]	[−3.13]	[3.09]	[1.84]	[3.74]	[1.28]			
	.036	.764	.876	.300	−1.32	−.233	−.196	−.062	−.048	.045	.010	1.28	.544	.792	16,687
	[3.62]	[10.42]	[6.26]	[5.77]	[−3.89]	[−2.42]	[−.56]	[−1.60]	[−2.88]	[4.27]	[3.06]	[3.56]	[1.40]		

Fama-MacBeth regression with Newey-West standard errors. T-statistics in square brackets. AMD multiplied by 10^3 . EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the volume divided by the amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted- R^2 's.

Corporate yield spreads on illiquidity measures and controls, 2010–2015

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R ²	Obs.
Panel C: January 2010–November 2015															
.327													1.20	.169	90,727
[10.37]													[18.13]		
	.211												1.21	.120	90,727
	[9.85]												[17.22]		
		.827											.446	.668	90,727
		[67.56]											[18.52]		
.098	.541	.392	.146	-.692	-.049	-.424	.460	.018	-.003	.008	.157	.228	.801	.801	90,727
[9.50]	[23.55]	[5.97]	[16.05]	[-14.86]	[-2.01]	[-4.51]	[13.67]	[7.43]	[-1.81]	[6.26]	[3.00]	[3.85]			
	.069	.547	.419	.148	-.709	-.065	-.411	.377	.023	-.010	.009	.317	.187	.800	90,727
	[8.64]	[22.93]	[6.19]	[16.83]	[-15.27]	[-2.86]	[-4.66]	[14.99]	[10.38]	[-4.22]	[8.30]	[6.78]	[3.12]		

Fama-MacBeth regression with Newey-West standard errors. T-statistics in square brackets. AMD multiplied by 10^3 . EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the volume divided by the amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted- R^2 's.

Corporate yield spreads on illiquidity measures and controls, 2015–2020

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R ²	Obs.
Panel D: December 2015–February 2020															
.606													.858	.225	68,997
[16.40]													[23.32]		
	.368												.818	.192	68,997
	[10.20]												[22.42]		
		.644											.592	.584	68,997
		[19.13]											[25.85]		
.225	.440	.569	.150	-.738	.010	-.393	.535	.026	-.011	.011	.390	.199	.776	68,997	
[13.06]	[16.52]	[5.63]	[19.55]	[-16.23]	[0.97]	[-1.75]	[6.15]	[23.51]	[-8.08]	[2.58]	[8.58]	[2.57]			
	.150	.432	.622	-.754	.008	-.328	.326	.033	-.021	.013	.585	.132	.778	68,997	
	[9.06]	[16.38]	[6.22]	[18.94]	[-16.70]	[0.76]	[-1.45]	[3.43]	[36.09]	[-7.77]	[3.19]	[10.27]	[1.74]		

Fama-MacBeth regression with Newey-West standard errors. T-statistics in square brackets. AMD multiplied by 10^3 . EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the volume divided by the amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted- R^2 's.

Corporate yield spreads on illiquidity measures and controls, 2020–2021

γ	AMD	CDS	EqVol.	Cpn	IG	Call	Volume	Freq.	Maturity	Age	Turnover	ZTD	Constant	Adj.R ²	Obs.
Panel E: March 2020–June 2021															
.748													.111	.242	19,934
[11.38]													[5.18]		
	.566												.878	.293	19,934
	[13.53]												[5.49]		
		.832											.592	.591	19,934
		[14.88]											[4.22]		
.239		.463	.658	.208	-1.19	.049	-1.17	.544	.015	-.003	.021	.708	.383	.799	19,934
[8.62]		[6.10]	[5.49]	[8.47]	[-10.97]	[1.30]	[-5.50]	[4.53]	[2.65]	[-.45]	[6.75]	[4.03]	[5.50]		
	.165	.474	.686	.203	-1.16	.038	-.999	.246	.020	-.010	.022	.870	.315	.801	19,934
	[4.54]	[4.99]	[5.32]	[7.82]	[-9.49]	[.89]	[-4.58]	[2.29]	[4.99]	[-1.82]	[8.00]	[5.22]	[3.66]		

Fama-MacBeth regression with Newey-West standard errors. T-statistics in square brackets. AMD multiplied by 10^3 . EqVol. is the annualized volatility of the issuer's equity returns and Cpn is the issue's coupon. IG is 1 if the bond is Investment Grade and 0 otherwise. Call is 1 if the bond is callable and 0 otherwise. Volume is calculated as the total \$ amount traded $\times 10^{-11}$. Frequency in thousands of trades. Maturity and Age calculated in years at the last business day of the month. Turnover is the volume divided by the amount outstanding and ZTD is the percentage of zero-trading days. Adj. R^2 is the time series average of cross-sectional adjusted- R^2 's.

Model

Model

- Lagos and Rocheteau (2009): customer-dealer trades
- Hugonnier, Lester, and Weill (2022): customer-customer trades
- Here: combination of the two models
- Search frictions applied to over-the-counter markets: Duffie, Gârleanu, and Pedersen (2005)
- Time is continuous, $t \geq 0$

Agents

- Infinitely-lived agents with discount rate $r > 0$
- Linear utility in the numeraire
 - Transferable utility technology
 - Agents can freely transfer payoffs between them at a constant rate
- Agents can be of two forms
 - Customers
 - Dealers

- There is an asset, which pays dividends over time
- Each unit of the asset pays a unit flow of dividend goods
- Goods cannot be traded
- Agents holding the asset consume its dividend good

Supply of assets

- The supply of assets is denoted by s
- At each time, some assets mature and new assets are issued
- Assets mature at Poisson rate $\mu > 0$
- Assets are issued at no cost at Poisson rate $\eta > 0$
 - A fund wants to create a bond with certain characteristics
 - An agent decides to sell an asset held for a long time, and a fund is contacted to sell the asset
- Stochastic process for the asset

Customers

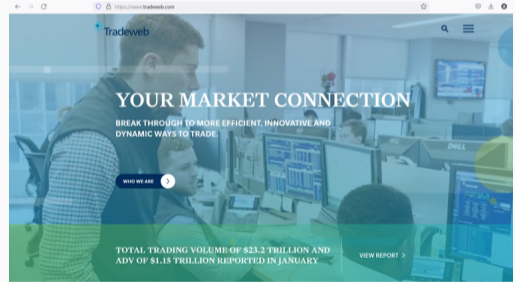
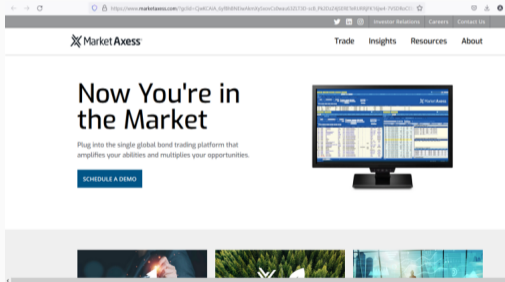
- Customers hold either 0 or 1 unit of the asset
 - If 0 the customer is called a **non-owner** (non-owners: potential buyers)
 - If 1 the customer is called an **owner** (owners: potential sellers)
- Each customer has a fixed type $\nu \in \mathbb{R}$
 - The types represent the value that the customers attribute to dividends
- Types have known distribution $F(\nu)$, density $f(\nu)$ and full support in \mathbb{R}

Customers and assets

- The utility types are constant
- To generate trade in steady state (Bethune, Sultanum, and Trachter 2022), assets are removed from the market and new assets are created
- There is no free disposal of assets
- In the steady state, $s = \frac{\eta}{\mu + \eta}$

Customers in practice

- Customers are large investors
 - Pension funds, managers of large portfolios
- They have access to new trading platforms



- Decentralized markets
 - Customers choose to meet a dealer
Customer-dealer trade; CD trade
 - Or another customer
Customer-customer trade; CC trade
- Matching with Poisson arrival rates given by
 - λ_D for meeting a dealer (Lagos and Rocheteau 2009)
 - $\lambda_C/2$ for meeting another customer (Hugonnier, Lester, and Weill 2022)

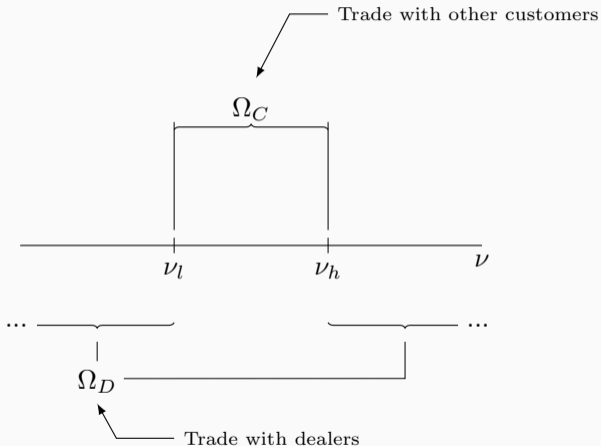
Dealers

- There is an inter-dealer market with endogenous price p
 - Centralized market among dealers
- Intermediation role of dealers
- Dealers intermediate transactions at a balance sheet cost τ
 - Sell to a customer at price equal or greater than $p + \tau$ (ask)
 - Buy asset from a customer at price equal or smaller than $p - \tau$ (bid)
- The final price depends on Nash bargaining
- We obtain the equilibrium final price

Bargaining power

- Bilateral trade (OTC markets)
- In a meeting with a dealer
 - Customers have bargaining power θ_D
- In a meeting with another customer
 - An owner has bargaining power θ_C^o
 - A non-owner has bargaining power $\theta_C^n = 1 - \theta_C^o$
- **Assumption:** $\lambda_D \theta_D > \lambda_C \max\{\theta_C^o, \theta_C^n\}$
 - When $\tau = 0$ (no balance-sheet costs), customers are better off searching for dealers than searching for other customers

Choice of counterparty



Customers will self-select to trade with dealers or with other customers

The decision will be guided by their utility type, ν

Let $\nu \in \mathbb{R}$

There will be an interval (ν_l, ν_h) such that customers choose to trade with other customers

More extreme types will trade with dealers (they can't wait!)

But we are getting ahead...

Future: from two to three trading choices

Here

DC trading (Ω_D): inventory trades

CC trading (Ω_C): matchmaking (agency)

Plan

P trading: inventory trades (principal)

DC trading: matchmaking (agency)

CC trading: direct electronic trading
(all-to-all)

Addition of Principal Trades

- Positive inventory, explanation for the small A2A market

Strategy of the exposition

1. Present the environment and define the participants
2. Obtain value functions
3. Define a reservation value, $\Delta(\nu)$
4. From the reservation value, obtain the partitions Ω_D, Ω_C
5. Obtain the distribution of trades for customer-dealers, $\nu \leq \nu_l$ and $\nu \geq \nu_h$
6. Obtain the distribution of trades for customer-customer, $\nu_l < \nu < \nu_h$
7. For a given exogenous τ , obtain an equilibrium ν_l and the other endogenous variables (prices, proportions of customers in CC or CD, and other variables)

Value functions

Distributions

- Cumulative distributions of customers
- Owners

$$\Phi^o(\nu)$$

- Non-owners

$$\Phi^n(\nu)$$

- The measure of assets is then given by

$$s = \int d\Phi^o$$

Sets for the decisions of customers

- Owners that search for dealers or for other customers

$$\Omega_D^o \quad \Omega_C^o$$

- Non-owners that search for dealers or for other customers

$$\Omega_D^n \quad \Omega_C^n$$

- Search partition

$$\mathcal{P} = \{\Omega_D^o, \Omega_C^o, \Omega_D^n, \Omega_C^n\}$$

Value functions

- Value functions of owners and non-owners when searching

- For dealers

$$V_D^o(\nu) \quad V_D^n(\nu)$$

- For customers

$$V_C^o(\nu) \quad V_C^n(\nu)$$

- The value functions of owners and non-owners satisfy

$$V^o(\nu) = \max\{V_D^o(\nu), V_C^o(\nu)\}$$

$$V^n(\nu) = \max\{V_D^n(\nu), V_C^n(\nu)\}$$

Value functions and the reservation value

- Value functions of owners and non-owners

$$V^o(\nu) = \max\{V_D^o(\nu), V_C^o(\nu)\}$$

$$V^n(\nu) = \max\{V_D^n(\nu), V_C^n(\nu)\}$$

- Reservation value

$$\Delta(\nu) \equiv V^o(\nu) - V^n(\nu) \implies V^o(\nu) = V^n(\nu) + \Delta(\nu)$$

Value functions, owners

- Owners searching for dealers

$$rV_D^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_D\theta_D \max\{(p - \tau) - \Delta(\nu), 0\}$$

ν : utility flow $-\mu\Delta(\nu)$: loss of reservation value caused by maturity

$\lambda_D\theta_D \max\{(p - \tau) - \Delta(\nu), 0\}$: profit of an owner when meeting a dealer

- Owners searching for customers

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C\theta_C^o \int_{\tilde{\nu} \in \Omega_C^n} [\Delta(\tilde{\nu}) - \Delta(\nu)] \mathbb{1}_{\{\Delta(\tilde{\nu}) > \Delta(\nu)\}} d\Phi^n(\tilde{\nu})$$

Last term: expected profits of an owner when meeting a non-owner

Value functions, non-owners

- Non-owners searching for dealers

$$rV_D^n(\nu) = \eta\Delta(\nu) + \lambda_D\theta_D \max\{\Delta(\nu) - (p + \tau), 0\}$$

$\eta\Delta(\nu)$: gain of reservation value caused by issuance

$\lambda_D\theta_D \max\{\Delta(\nu) - (p + \tau), 0\}$: profit of a non-owner when meeting a dealer

- Non-owners searching for customers

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C\theta_C^n \int_{\tilde{\nu} \in \Omega_C^o} [\Delta(\nu) - \Delta(\tilde{\nu})] \mathbb{1}_{\{\Delta(\nu) > \Delta(\tilde{\nu})\}} d\Phi^o(\tilde{\nu})$$

Last term: expected profits of a non-owner when meeting an owner

Counterparty choice

- Customers searching for dealers

$$\Omega_D^o = \{\nu \in \mathbb{R}; V_D^o(\nu) \geq V_C^o(\nu)\} \quad \Omega_D^n = \{\nu \in \mathbb{R}; V_D^n(\nu) \geq V_C^n(\nu)\}$$

- Customers searching for customers

$$\Omega_C^o = \{\nu \in \mathbb{R}; V_C^o(\nu) > V_D^o(\nu)\} \quad \Omega_C^n = \{\nu \in \mathbb{R}; V_C^n(\nu) > V_D^n(\nu)\}$$

- When indifferent, customers search for dealers

Inter-dealer market clearing

- Market clearing

$$\underbrace{\lambda_D \int_{\Omega_D^o} \mathbb{1}_{\{\Delta(\nu) < p - \tau\}} d\Phi^o(\nu)}_A = \underbrace{\lambda_D \int_{\Omega_D^n} \mathbb{1}_{\{\Delta(\nu) > p + \tau\}} d\Phi^n(\nu)}_B$$

- A: Measure of owners finding dealers to sell the asset
- B: Measure of non-owners finding dealers to buy the asset
- The value functions, partitions and the price p have to satisfy this equation

The distribution of assets over time

- Φ^o , Φ^n : distributions of owners and non-owners

$$\begin{aligned} \dot{\Phi}^o(\nu) = & \eta\Phi^n(\nu) - \mu\Phi^o(\nu) - \underbrace{\lambda_C \int_{-\infty}^{\nu} \int_{\nu}^{\infty} \mathbb{1}_{\{\tilde{\nu}, \hat{\nu} \in \Omega_C, \Delta(\hat{\nu}) > \Delta(\tilde{\nu})\}} d\Phi^n(\hat{\nu}) d\Phi^o(\tilde{\nu})}_{\text{sell to non-owners}} \\ & - \underbrace{\lambda_D \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu} \in \Omega_D, \Delta(\tilde{\nu}) < p - \tau\}} d\Phi^o(\tilde{\nu})}_{\text{sell to dealers}} + \underbrace{\lambda_D \int_{-\infty}^{\nu} \mathbb{1}_{\{\tilde{\nu} \in \Omega_D, \Delta(\tilde{\nu}) > p + \tau\}} d\Phi^n(\tilde{\nu})}_{\text{buy from dealers}} \end{aligned}$$

- $\dot{\Phi}^o(\nu) = 0$ in the steady state
- The distributions of owners and non-owners satisfy $\Phi^o(\nu) + \Phi^n(\nu) = F(\nu)$

Equilibrium

Equilibrium

- An equilibrium is a set $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \mathcal{P}\}$ of
 - value functions V^o, V^n ,
 - reservation values Δ ,
 - price p ,
 - distributions Φ^o, Φ^n , and
 - partitions $\mathcal{P} = \{\Omega_D^o, \Omega_C^o, \Omega_D^n, \Omega_C^n\}$

satisfying the equations above, given the distribution F of utility types and the search and bargaining parameters.

How should the equilibrium look like?

- In principle, the sets of customers that search for other customers could be different for owners and non-owners
- We might have $\Omega_C^o \neq \Omega_C^n$
 - In this case, owners of a certain type ν would search for customers to sell
 - But non-owners of the same type would not search for other customers to buy
- We anticipate that customers that search for customers share certain characteristics
- The same holds for those that search for dealers
- We then define what we call a regular equilibrium

Regular equilibrium

- In this equilibrium, the sets of owners and non-owners that search for other customers are equal

$$\Omega_C^o = \Omega_C^n = \Omega_C \quad (\text{customer-customer})$$

- The same holds for owners and non-owners that search for dealers

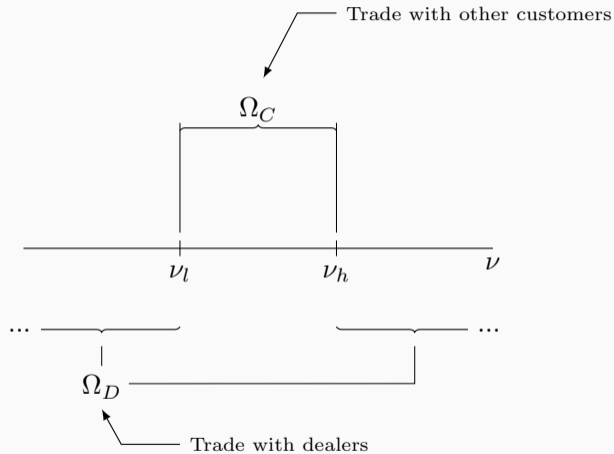
$$\Omega_D^o = \Omega_D^n = \Omega_D \quad (\text{customer-dealer})$$

- Customers that share similar characteristics ν make similar decisions on their counterparty

Regular equilibrium – partition

- Moreover, Ω_C is an interval and Ω_D is the union of two intervals
- Partition of \mathbb{R}
 - $\Omega_C = (\nu_l, \nu_h)$ (customer-customer)
 - $\Omega_D = \mathbb{R} \setminus \Omega_C = (-\infty, \nu_l] \cup [\nu_h, +\infty)$ (customer-dealer)
- Types $\nu \in \Omega_C$ search for customers
- Types $\nu \in \Omega_D$ search for dealers
- We concentrate on regular equilibria
- Conjecture: all equilibria are regular equilibria

Partitions – choice of counterparty



Definition

An equilibrium $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \{\Omega_C, \Omega_D\}\}$ is a regular equilibrium if

$$\Omega_C^o = \Omega_C^n = \Omega_C = (\nu_l, \nu_h),$$

$$\Omega_D^o = \Omega_D^n = \Omega_D = (-\infty, \nu_l] \cup [\nu_h, +\infty),$$

given two types $\nu_l, \nu_h \in \mathbb{R}$, and the reservation value Δ is continuous and strictly increasing

- An equilibrium always exists—but may not be unique

Results

Two-blocks

- Given ν_l and ν_h , we can divide the economy into two blocks
- Customers searching for dealers $\implies \Omega_D$
- Customers searching for customers $\implies \Omega_C$
- Solve each block separately
 - Customer-dealer: Lagos and Rocheteau (2009)
 - Customer-customer: Hugonnier, Lester, and Weill (2022)
- Find ν_l and ν_h

Customer-dealer block — reservation value

- Owners and non-owners searching for dealers

$$rV_D^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_D\theta_D \max\{(p - \tau) - \Delta(\nu), 0\}$$

$$rV_D^n(\nu) = \eta\Delta(\nu) + \lambda_D\theta_D \max\{\Delta(\nu) - (p + \tau), 0\}$$

- Taking the difference $rV_D^o(\nu) - rV_D^n(\nu)$ implies

$$\Delta(\nu) = \frac{\nu + \lambda_D\theta_D [p + \tau + \max\{\Delta(\nu), p - \tau\} - \max\{\Delta(\nu), p + \tau\}]}{r + \nu + \mu + \lambda_D\theta_D}$$

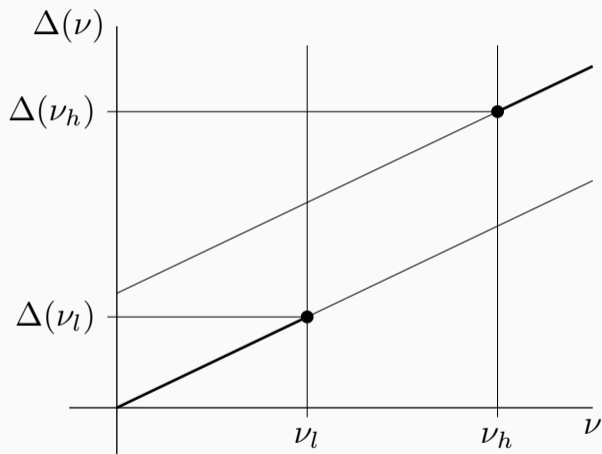
Lemma

The reservation value $\Delta : \mathbb{R} \rightarrow \mathbb{R}$ is strictly increasing, continuous and satisfies

$$\Delta(\nu) = \begin{cases} \frac{\nu + \lambda_D \theta_D (p - \tau)}{r + \eta + \mu + \lambda_D \theta_D}, & \nu \leq \nu_l, \\ \frac{\nu + \lambda_D \theta_D (p + \tau)}{r + \eta + \mu + \lambda_D \theta_D}, & \nu \geq \nu_h, \end{cases}$$

on the sub-domain $\Omega_D = (-\infty, \nu_l] \cup [\nu_h, \infty)$

Reservation value for customer-dealer block



$$\Delta(v) = \begin{cases} \frac{v + \lambda_D \theta_D (p - \tau)}{r + \eta + \mu + \lambda_D \theta_D}, & v \leq v_l, \\ \frac{v + \lambda_D \theta_D (p + \tau)}{r + \eta + \mu + \lambda_D \theta_D}, & v \geq v_h, \end{cases}$$

$$\Delta(v_l) \leq p - \tau$$

→ owners $v < v_l$ sell to dealers

$$\Delta(v_h) \geq p + \tau$$

→ non-owners $v > v_h$ buy from dealers

Measure of frictions

- Define a measure of frictions by

$$\sigma_D = \frac{\partial \Delta(\nu)}{\partial \nu}$$

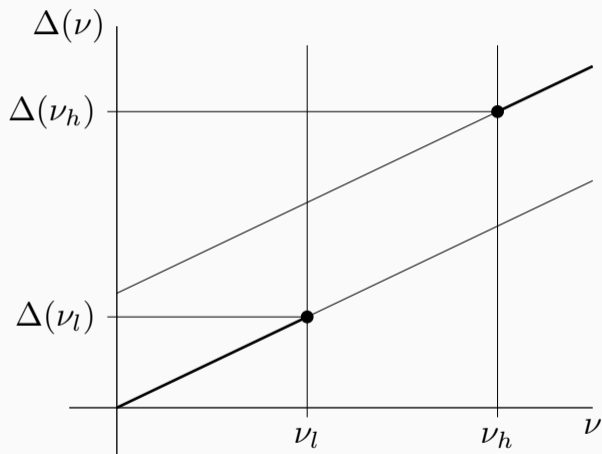
- We have

$$\sigma_D = \frac{1}{r + \eta + \mu + \lambda_D \theta_D},$$

constant for the Customer-Dealer market

- In particular, as $\lambda_D \rightarrow \infty$, $\sigma_D \rightarrow 0$

Reservation value for customer-dealer block



$$\Delta(\nu) = \begin{cases} \sigma_D[\nu + \lambda_D\theta_D(\rho - \tau)], & \nu \leq \nu_l, \\ \sigma_D[\nu + \lambda_D\theta_D(\rho + \tau)], & \nu \geq \nu_h, \end{cases}$$

Distributions of owners and non-owners

- $\dot{\Phi}^o(\nu)$ for $\nu = \nu_l$, $\dot{\Phi}^o(\nu_l) = 0$, $\Phi^o(\nu_l) + \Phi^n(\nu_l) = F(\nu_l)$

$$\dot{\Phi}^o(\nu_l) = \eta\Phi^n(\nu_l) - \mu\Phi^o(\nu_l) - \lambda_D\Phi^o(\nu_l) \implies \Phi^n(\nu_l) = \frac{\eta F(\nu_l)}{\eta + \mu + \lambda_D}$$

- From the market clearing condition, $\Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h)$
- With $\Phi^o(\infty) + \Phi^n(\infty) = 1$ and

$$\Phi^o(\infty) = \frac{\eta}{\mu + \eta} \implies \Phi^n(\infty) = 1 - \frac{\eta}{\mu + \eta}$$

Lemma

A regular equilibrium $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \{\Omega_C, \Omega_D\}\}$ satisfies

$$\begin{cases} \Phi^o(\nu) = \frac{\eta F(\nu)}{\eta + \mu + \lambda_D}, & \text{for all } \nu \leq \nu_l, \\ \Phi^n(\nu) = \frac{\mu}{\eta + \mu} - \frac{\mu[1 - F(\nu)]}{\eta + \mu + \lambda_D}, & \text{for all } \nu \geq \nu_h \end{cases}$$

Moreover, the measures of sellers and buyers in the inter-dealer market are equal,

$$\Phi^o(\nu_l) = \Phi^n(\infty) - \Phi^n(\nu_h) \implies \eta F(\nu_l) = \mu[1 - F(\nu_h)]$$

Customer-customer block — reservation value

- Owners and non-owners searching for other customers

$$rV_C^o(\nu) = \nu - \mu\Delta(\nu) + \lambda_C\theta_C^o \int_{\nu_l}^{\nu_h} [\Delta(\tilde{\nu}) - \Delta(\nu)] \mathbb{1}_{\{\Delta(\tilde{\nu}) > \Delta(\nu)\}} d\Phi^n(\tilde{\nu})$$

$$rV_C^n(\nu) = \eta\Delta(\nu) + \lambda_C\theta_C^n \int_{\nu_l}^{\nu_h} [\Delta(\nu) - \Delta(\tilde{\nu})] \mathbb{1}_{\{\Delta(\nu) > \Delta(\tilde{\nu})\}} d\Phi^o(\tilde{\nu})$$

- Taking the difference $rV_C^o(\nu) - rV_C^n(\nu)$ implies $\Delta(n)$ for the customer-customer block

Lemma

The reservation value $\Delta : \mathbb{R} \rightarrow \mathbb{R}$ satisfies

$$\Delta(\nu) = \Delta(\nu_l) + \int_{\nu_l}^{\nu} \sigma_C(\tilde{\nu}) d\tilde{\nu},$$

where

$$\sigma_C(\nu) = \frac{1}{r + \mu + \eta + \lambda_C \left\{ \theta_C^o [\Phi^n(\nu_h) - \Phi^n(\nu)] + \theta_C^n [\Phi^o(\nu) - \Phi^o(\nu_l)] \right\}},$$

on the sub-domain $\Omega_C = (\nu_l, \nu_h)$

Lemma

The asset distribution $\Phi^o(\nu)$ satisfies

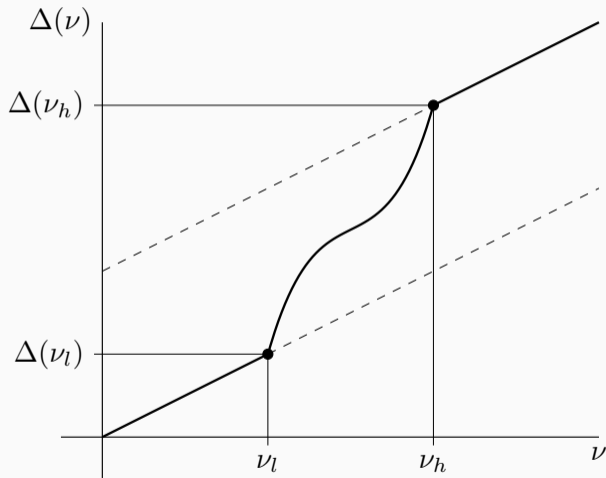
$$\Phi^o(\nu) - \Phi^o(\nu_l) = -\frac{\mu + \eta + \lambda_C [F(\nu_h) - F(\nu) - s_C]}{2\lambda_C} + \frac{\sqrt{\{\mu + \eta + \lambda_C [F(\nu_h) - F(\nu) - s_C]\}^2 + 4\lambda_C \eta [F(\nu) - F(\nu_l)]}}{2\lambda_C}$$

for $\nu \in (\nu_l, \nu_h)$, where

$$s_C = \Phi^o(\nu_h) - \Phi^o(\nu_l) = \frac{\eta}{\mu + \eta} [F(\nu_h) - F(\nu_l)]$$

s_C : measure of assets traded customer-customer

Complete reservation value



$$\Delta(v) = \begin{cases} \sigma_D[v + \lambda_D \theta_D(p - \tau)], & v \leq v_l, \\ \Delta(v_l) + \int_{v_l}^v \sigma_C(\tilde{v}) d\tilde{v}, & v_l < v < v_h, \\ \sigma_D[v + \lambda_D \theta_D(p + \tau)], & v \geq v_h, \end{cases}$$

Equilibrium — Solving for ν_l and ν_h

Lemma

A regular equilibrium $\{V^o, V^n, \Delta, p, \Phi^o, \Phi^n, \nu_l, \nu_h\}$ satisfies

$$\frac{1}{2\lambda_D\theta_D} \int_{\nu_l}^{\nu_h} \frac{\sigma_C(\nu; \nu_l, \nu_h) - \sigma_D}{\sigma_D} d\nu = \tau$$

- $\mu[1 - F(\nu_h)] = \eta F(\nu_l)$ defines ν_h as a function of ν_l
- We have that

$$G(\nu_l) = \tau$$

implies equilibrium value for ν_l

- Existence and uniqueness around $\tau = 0$

Lemma

A regular equilibrium exists for any $\tau \geq 0$, and it is unique in a neighborhood of $\tau = 0$

- Unique equilibrium
 - For low or high intermediation costs \Rightarrow stability
- Multiplicity of equilibria
 - For intermediary intermediation costs \Rightarrow possibly unstable
- New result: multiplicity and instability

Measure of liquidity

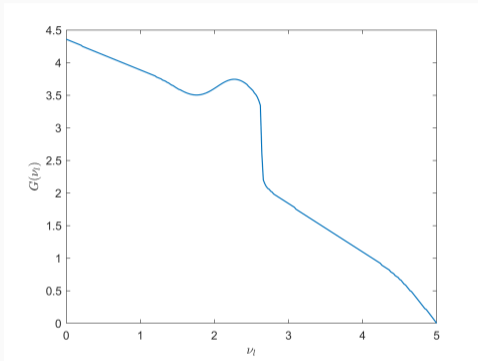
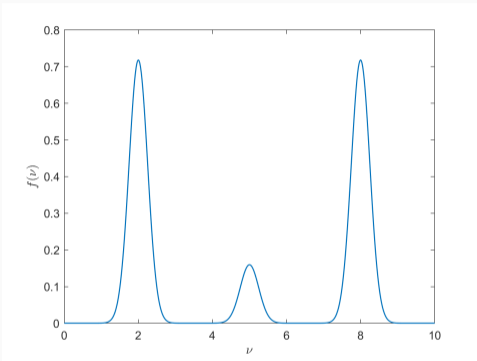
- We can define the following measure of illiquidity

$$\sigma = \int \sigma_i(\nu) dF(\nu)$$

where $\sigma_i(\nu) = \sigma_D(\nu)$ for $\nu \in \Omega_D$ and $\sigma_i(\nu) = \sigma_C(\nu)$ for $\nu \in \Omega_C$

- $\sigma \rightarrow 0$ as $\lambda_D \rightarrow \infty$ and $\lambda_C \rightarrow \infty$
- σ increases if it is more difficult to make a trade

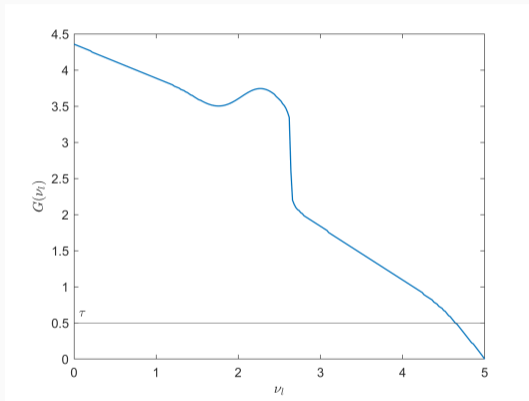
Equilibrium—Numerical simulation



Density of types f and function G

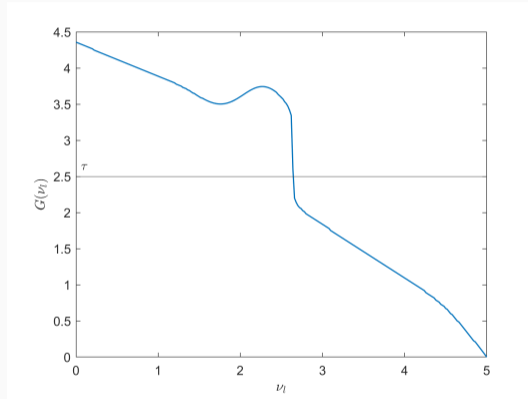
$$r = 0.05, \mu = \eta = 0.3, \theta_D = \theta_C^n = \theta_C^o = 0.5, \lambda_D = 3 \text{ and } \lambda_C = 1$$

Regions of stability and instability

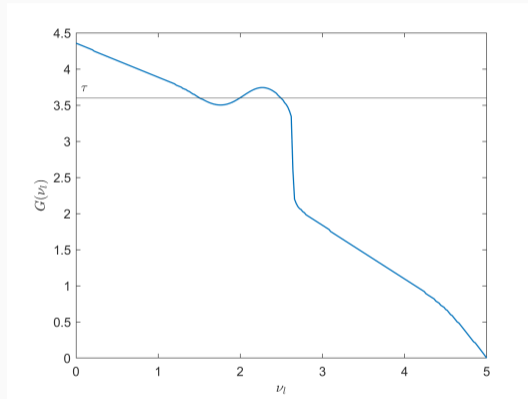


Stability

Stability

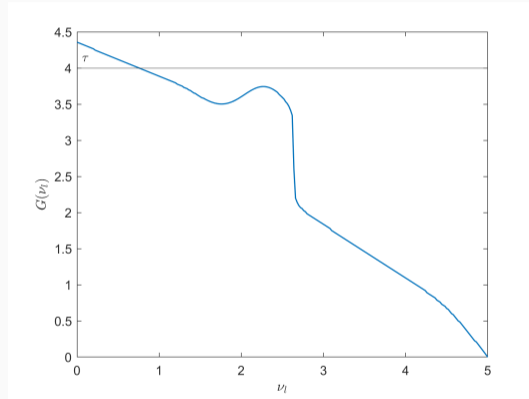


Stability

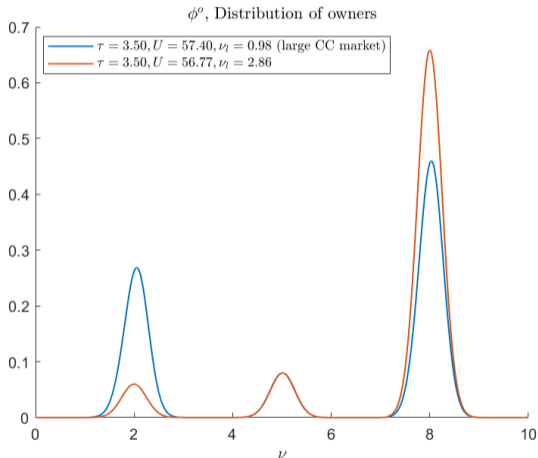


Multiplicity of equilibrium and liquidity crises

Instability



Stability



$\tau = 3.5$, two equilibria:

1. $\nu_l = 0.98$: more matchmaking
2. $\nu_l = 2.86$: more inventory trade

More inventory trade: more efficient trades

- More trade from low ν to high ν investors
- Investors with higher ν are more likely to buy the asset and become owners

Turnover (trade volume)

$$T \equiv \lambda_D \Phi^o(\nu_l) + \lambda_C \int_{\nu_l}^{\nu_h} \int_{\nu}^{\nu_h} d\Phi^n(\tilde{\nu}) d\Phi^o(\nu)$$

Lemma

Turnover is always decreasing in τ in a neighborhood of $\tau = 0$

Moreover, turnover is decreasing in τ in any region with a unique equilibrium

$$\text{Turnover} \equiv \frac{\text{Volume}_t}{\text{Amount outstanding}_t}$$

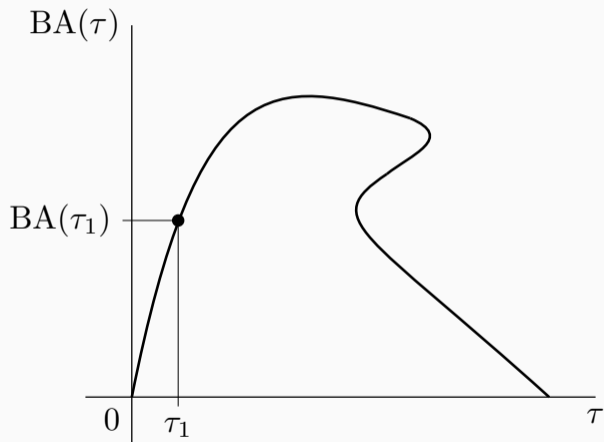
Bid-ask spread

- $BA = \omega_D BA_D + \omega_C BA_C$, bid-ask spread given by $|p^{sell} - p^{buy}|$

Lemma

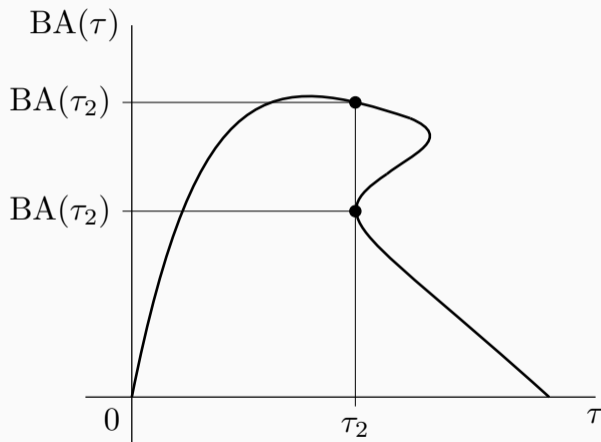
1. *The bid-ask spread is increasing in τ in a neighborhood of $\tau = 0$*
 2. *There exists $\tau_1, \tau_0 > 0$, with $\tau_1 > \tau_0$, and two respective regular equilibria \mathcal{E}_0 and \mathcal{E}_1 such that the bid-ask spread in \mathcal{E}_0 is strictly bigger than the bid-ask spread in \mathcal{E}_1*
- Intensive margin: BA_D increases with τ (dominates for low τ)
 - Extensive margin: ω_D decreases with τ

Bid-ask spread



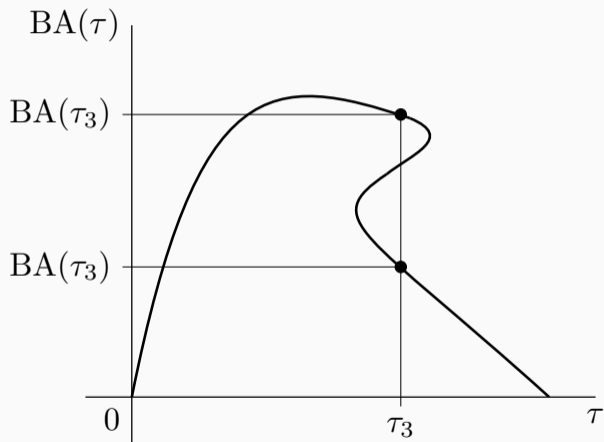
Increasing bid-ask spread for low values of τ

Bid-ask spread



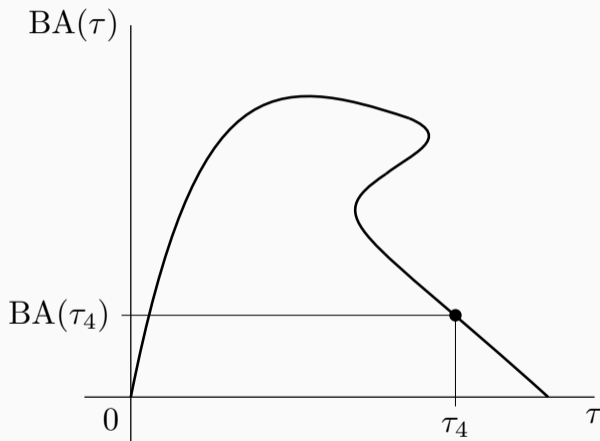
Multiplicity of equilibria for higher values of τ

Bid-ask spread



Multiplicity of equilibria for higher values of τ

Bid-ask spread



Return to low bid-ask spread for high values of τ

More frequent direct trades
customer-to-customer

Customer liquidity provision

Discussion

- Interpret the Dodd-Frank regulation as an increase in dealer's intermediation costs
- Our main results are then consistent with several documented facts in corporate bond markets
- Decreased in bid-ask spreads generally
- Increase in bid-ask spreads of dealers (Choi, Huh, and Shin 2024)
- Reduction in turnover (Adrian, Fleming, Shachar, and Vogt 2017)
- Change in composition from DC trades to CC trades (Choi, Huh, and Shin 2024)

Conclusions

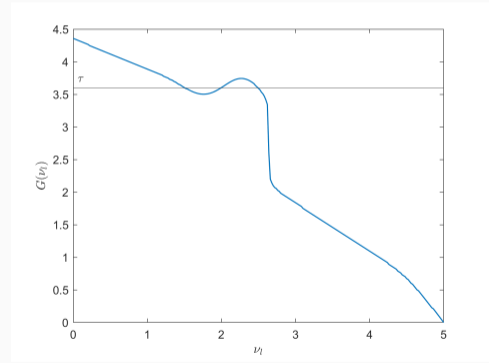
Conclusions

- Model of dealer-customer and customer-customer trade composition in OTC markets
- It captures empirical trends in the US corporate bond market
 - Change in composition, decline in BA, decline in turnover, impact of BA on yields
- Policy implications
 - Measures of liquidity such as bid-ask spread might be misleading when dealer's balance-sheet costs are high
 - High balance sheet costs might be associated multiple equilibria and financial instability

Conclusions

- We define a new measure of illiquidity
- The Volcker rule might imply multiple equilibria and instability

$$\sigma = \int \sigma_i(\nu) dF(\nu)$$



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