Inflation Uncertainty from Firms' Perspective, Overconfidence and Credibility of Monetary Policy*

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Abstract

This paper uses survey data to gauge firms' inflation uncertainty. First, it shows how commonly used proxies of uncertainty, such as *ex post* squared forecast errors or forecast dispersion differ from measures of actual *ex ante* inflation uncertainty. Second, this paper documents novel stylized facts: firms' uncertainty and overconfidence – low *ex ante* variances compared to *ex post* (squared) forecast errors – are shown to be relevant for how firms' form their beliefs about inflation and their inflation forecasts accuracy (firms know what they do not know) and to impact firms' beliefs about credibility of monetary policy.

Keywords: inflation expectations, inflation uncertainty, overconfidence, subjective probability distribution, monetary policy, central bank's credibility

JEL classification codes: D84, E31, E58

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1 Introduction

The conventional wisdom has it that monetary policy is, to a large extent, the management of expectations, see for example Svensson (2004), Woodford (2005). However, still little is known about the precise channels through which a central bank can affect the expectation formation process by different groups of private agents: households, firms, and financial market participants.

In the context of the New Keynesian model – a workhorse model used at the central banks around the world to guide monetary policy design – of particular importance are firms' inflation expectations, which facilitate real effects of monetary policy through firms' pricing decisions. Naturally, the inflation expectations of individuals who lead firms can influence the prices that their firms charge customers, impacting overall inflation. A recent burgeoning literature on firms' inflation expectations based on surveys of expectations, summarized in Candia et al. (2022), provides valuable insights. Some of the most notable contributions document a causal link between firms' inflation expectations, employment, and investment decisions (Coibion et al. (2020)) as well as macroeconomic uncertainty and firms decisions (Kumar et al. (2023)). Interestingly, firms' attention to inflation and firms' inflation expectations in high- and low-inflation countries seem to differ substantially (Candia et al. (2022), Borraz et al. (2023)), and so may the ability of monetary policy to affect these.

In this paper, we attempt to shed light on the relationship between firms' beliefs about inflation and the design of monetary policy in a (relatively) high-inflation environment. In particular, rather than focusing only on expected paths of inflation, we study inflation uncertainty from firms' perspective, defined as the conditional variance of firms' beliefs about inflation. The idea of focusing on the probabilistic survey questions rather than point predictions for understanding the expectation formation mechanism has been gaining ground in recent years (Engelberg et al. (2009), Clements (2014), Manski (2018)). Based on survey questions that ask firms to assign likelihood to alternative inflation outcomes (bins), we approximate firms' subjective probability distributions and study the evolution of uncertainty around inflation for firms in Uruguay in 2014-2021. Our paper concludes that accounting for firms' inflation uncertainty is essential for understanding firms' forecast accuracy – we show that firms know what they do not know (using the terminology of Bryan et al. (2015)) in that measures of subjective inflation uncertainty help explain firms' forecast errors.

Endowed with measures of firms' subjective uncertainty, we also study the extent to which firms may be overconfident in their forecasts, that is, the extent to which a firm's *ex-ante* uncertainty differs from *ex-post* realized forecast errors. The literature has documented the effects of overconfidence. Less studied are the reasons for firms becoming overconfident. We analyze and quantify how the degree of firms' overconfidence may be affected by three characteristics of inflation forecasts: i) rounding (as in Glas & Hartmann (2022)), ii) internal inconsistency (when point predictions differ from the subjective expectation inferred from subjective distributions), iii) nature of information available to firms at the time of constructing their forecasts (public vs. private). We find that both rounding and internal inconsistency matter in explaining variation in firms' uncertainty and overconfidence. This result teaches us important lessons about using survey data to guide policy, contributing to Coibion et al. (2020). Similarly, the nature of the information firms use to condition their forecast is essential. What drives variation in firms' forecast errors is the accuracy of the average forecast more so than disagreement amongst firms. Consequently, forecast dispersion – a commonly used proxy of uncertainty – would not provide the full picture of firm's inflation uncertainty.

Next, we ask how firms' inflation uncertainty and overconfidence may matter for monetary policy. In particular, we investigate whether and how a firm's uncertainty affects firm's *perceived credibility* of the central bank – the likelihood a given firm assigns to future inflation outcomes falling within the central bank's inflation target range. The importance of central banks' credibility and the public's perceptions (mainly rational) about credibility have long been understood, starting with the seminal works of Kydland & Prescott (1977) and Barro & Gordon (1983). As noted in Blinder (2000), "Credibility matters in theory, and it is certainly believed to matter in practice—although empirical evidence on this point is hard to come by because credibility is not easy to measure". The early papers focused on the idea of proxying credibility and the related stability of inflation expectations with the extent to which private agents' expectations are "anchored", and focused on inflation expectations of professionals, households, or financial market participants.[1] Concerning monetary policy influencing firms' inflation expectations, an early study Afrouzi et al. (2015) concludes that inflation targeting does not anchor firms' inflation expectations based on a survey of data by firms in New Zealand. Indeed, firms in New Zealand seem to have minimal knowledge about monetary policy or prevailing (realized) inflation.[2] Our paper contributes to this growing body of literature in that: 1) we study firms' beliefs about central bank's credibility in a high-inflation economy with an inflation-targeting central bank, and 2) we extend the definition of anchored expectations (a notion of firm's perceived credibility) to account for the im-

[1]There are several measures of anchoring which have been proposed in the literature including: the closeness of average beliefs to the central bank's inflation target (Afrouzi et al. (2015), Łyziak & Paloviita (2016)), response or correlation of long-term inflation expectations to/with short-term inflation expectations (Afrouzi et al. (2015), Gerlach et al. (2017)), precision around estimates of the level of inflation (Mehrotra & Yetman (2018)), the volatility of shocks to trend inflation (Mertens (2016)), the response of inflation measures or interest rates to macroeconomic news (Gürkaynak et al. (2007), Goldberg & Klein (2011)).

[2]By contrast, firms in Uruguay are very attentive to inflation, a finding which our earlier work, Borraz et al. (2023), attributes to the high inflation experience of that country.

portance of firm's inflation uncertainty as well as the distance to central bank's inflation target. Specifically, while the measures cited above relate to stability in the conditional mean of inflation (by definition or construction), we show that much of the variation in the firms' perceived credibility can be explained by uncertainty (conditional variance of inflation) and overconfidence. To our knowledge, ours is the first paper that looks at the survey data of firms' inflation expectations to define a firm's perceived credibility and to link it to survey-based measures of firms' uncertainty. Two papers in the literature that are perhaps the most closely related to our work in the way anchored expectations or perceived credibility are defined are Grishchenko et al. (2019) and Ehrmann et al. (2024). This paper differs from the first one in that we study firms' expectations and do not rely on econometric models to compute the measures of uncertainty (which, in turn, requires imposing relations between inflation factors and conditional mean and variance of inflation). Concerning the latter paper, our paper studies firms' perceived credibility (as opposed to households') in a high-inflation economy.

Our paper also contributes to recent literature on measuring and explaining sources of uncertainty fluctuations.[3] Importantly, we clarify – theoretically and through our survey data of firms' inflation expectations – the differences between commonly used proxies of uncertainty: mean squared forecast errors and forecast dispersion. Our results are consistent with recent studies that highlight the advantage of using probabilistic expectations to measure uncertainty over using forecaster disagreement (Glas (2020), Rich & Tracy (2021)). Also, it is closely related to the work of Broer & Kohlhas (2022), who define absolute and relative overconfidence, but these notions have to do with the interpretation of variances of private signals only. Private signals are one of the pieces of information that enter the measure of uncertainty, which we show formally in a simple motivating conceptual example in Section 2, so we consider many potential explanations for overconfidence. Importantly, we show how firms' overconfidence is related to their beliefs about the central bank's credibility. Finally, our paper builds on work on imperfect credibility, see, for example, Bodenstein et al. (2012), Nunes et al. (2021), which remains largely theoretical.

The remainder of the paper is organized as follows: Section 2 lays out the definitions of basic objects, including those of proxies of inflation uncertainty and perceived confidence, and sets up simple forecasting models as conceptual frameworks. Section 3 describes the survey of firms' inflation expectations in Uruguay and presents empirical characterization of firms' inflation uncertainty and overconfidence. Section 4 studies potential sources of firms' uncertainty and overconfidence. Section 5 relates firms' uncertainty and overconfidence to firms' perceived credibility. Section 6 concludes.

[3]See, for example, the seminal work by Jurado et al. (2015) and Baker et al. (2016).

2 Preliminaries: Definitions and a Conceptual Framework

The agents, who we call forecasters, are indexed by $i = 1, 2, ..., N$. At each date, t, a forecaster conditions on the information set, $\mathcal{I}_{it}$, and forms beliefs about the distribution of inflation for different time horizons, $h = 0, 1, 2, ...$ We call the expected value, $E(\pi_{t+h}|\mathcal{I}_{it})$, a forecaster i's *forecast* and the conditional variance, $Var(\pi_{t+h}|\mathcal{I}_{it})$, is what we call *uncertainty*.[4] Let $\pi^t \equiv \{\pi_\tau\}_{\tau=1}^{t}$ denote a series of inflation data available to the forecaster at time t. In forming their beliefs about future inflation, each forecaster may also have access to private, idiosyncratic signals and/or public signals, which we describe in detail below. The forecasts may differ from the realized inflation rate. This difference is what we call a *forecast error*.

Definition 1 ***Forecast error***
An agent i's forecast error is the distance, in absolute value, between the forecast and the realized inflation, for a given horizon h: $FE_{i,t+h} = |\pi_{t+h} - E[\pi_{t+h}|\mathcal{I}_{it}]|$.

We date the forecast error $t+h$ because it depends on a variable π_{t+h} that is not observed at time t. If there are N_t forecasters at date t, an average forecast error is $\bar{FE}_{t+h} = \frac{1}{N_t}\sum_{i=1}^{N_t} FE_{i,t+h}$.

Definition 2 ***Uncertainty***
Inflation uncertainty is the variance of the time-$(t+h)$ inflation, conditional on forecaster's time-t information: $U_{i,t}^h = E\left[(\pi_{t+h} - E[\pi_{t+h}|\mathcal{I}_{it}])^2 \middle| \mathcal{I}_{it}\right]$.

Two remarks concerning this definition are in order. First, the conditional variance of the forecaster's beliefs about future inflation is an *ex-ante* measure of uncertainty. Second, it is forecaster-specific in that, by definition, each forecaster conditions on their own information set, which – other than different sources of information or signals – may contain a particular model a forecaster uses to determine the subjective conditional distribution of inflation. By contrast, some authors use *forecast dispersion* to measure uncertainty because it is regarded as "model free" and measurable from observations of forecaster's forecasts, i.e., point forecasts only.[5]

[4] When applied to the data, we will think of firms' forecasts as representing the conditional mean of firms' beliefs. This assumption is valid as long as firms (rationally) minimize a squared loss function. However, forecasts may also represent the belief distribution's conditional median, mode, or any quantile under an appropriate loss function. We discuss this in Section 4.2.

[5] See e.g. Diether et al. (2002), and Johnson (2004).

Definition 3 ***Dispersion***

Dispersion of forecasters' beliefs about inflation, D_{t+h}, is the average squared difference of each forecast from the average forecast, $D_{t+h} \equiv \frac{1}{N_t}\sum_{i=1}^{N_t}(E[\pi_{t+h}|\mathcal{I}_{it}] - \bar{E}[\pi_{t+h}])^2$ where $\bar{E}\left(\pi_{t+h}\right) = \frac{1}{N_t}\sum_{i=1}^{N_t} E\left[\pi_{t+h}|\mathcal{I}_{it}\right]$ is the average forecast.

To fix ideas, we start with a simple conceptual forecasting framework.[6]

To form beliefs about inflation, each firm i is assumed to be endowed with a prior belief, $\pi_{t+1} \sim \mathcal{N}(\mu_{it}, \tau_\pi^{-1})$ and to observe two types of signals: a private signal and a public signal defined below, respectively

$$x_{it} = \pi_{t+1} + \epsilon_{it}^x \tag{1}$$

$$s_t = \pi_{t+1} + \epsilon_t^s \tag{2}$$

where $\epsilon_{it}^x \sim \mathcal{N}(0, \tau_x^{-1})$, $\epsilon_t^s \sim \mathcal{N}(0, \tau_s^{-1})$. In addition, the noise terms ϵ_{it}^x are assumed to be independent across time and forecasters while ϵ_t^s is assumed to be independent across time, and of ϵ_{it}^x, for all t and i.

Let $\mathcal{I}_{it} = \{x_{it}, s_t, \mu_{it}, \tau_\pi, \tau_x, \tau_s\}$ denote information set of forecaster i at time t. Then, each firm forming its beliefs in a Bayesian way would construct its inflation forecast as follows

$$E[\pi_{t+1}|\mathcal{I}_{it}] = (1 - \kappa_x - \kappa_s)\mu_{it} + \kappa_x x_{it} + \kappa_s s_t \tag{3}$$

where coefficients κ_x and κ_s are determined using Bayes law as $\kappa_x = \frac{\tau_x}{(\tau_\pi+\tau_x+\tau_s)}$ and $\kappa_s = \frac{\tau_s}{(\tau_\pi+\tau_x+\tau_s)}$.[7]

We will use eq. (3) and Definitions 1-3 to investigate differences between various proxies used to measure uncertainty. First, notice that eq. (3) can be rewritten in terms of primitive shocks as

$$E[\pi_{t+1}|\mathcal{I}_{it}] = \pi_{t+1} + (1 - \kappa_x - \kappa_s)\epsilon_{it}^\pi + \kappa_x \epsilon_{it}^x + \kappa_s \epsilon_t^s \tag{4}$$

where we used eq. (1) and eq. (2) as well as an isomorphic specification of the prior belief, $\mu_{it} = \pi_{t+1} + \epsilon_{it}^\pi$ with $\epsilon_{it}^\pi \sim \mathcal{N}(0, \tau_\pi^{-1})$. As a result, the firm-specific forecast error is then given by

$$FE_{i,t+1} = |(1 - \kappa_x - \kappa_s)\epsilon_{it}^\pi + \kappa_x \epsilon_{it}^x + \kappa_s \epsilon_t^s| \tag{5}$$

Second, with a large number of forecasters, invoking the law of large numbers, we can

[6] From now on, unless we define new objects in general form, we work with $h = 1$, without loss of generality. With a slight abuse of notation, we use $U_{i,t}$ to mean $U_{i,t}^{h=1}$. Also, in the empirical part of the paper, we use data on 12-month ahead inflation forecasts.

[7] See Veldkamp (2011), p. 12.

write the average inflation forecast as

$$\bar{E}\left(\pi_{t+1}\right)=\pi_{t+1}+\kappa_s\epsilon_t^s \tag{6}$$

Thus, by Definition 3, forecast dispersion is given by

$$D_{t+1}=\frac{1}{N_t}\sum_{i=1}^{N_t}[(1-\kappa_x-\kappa_s)\epsilon_{it}^{\pi}+\kappa_x\epsilon_{it}^{x}]^2 \tag{7}$$

Notice that forecast dispersion reflects only private noise and heterogeneous prior information. By contrast, uncertainty is the conditional standard deviation, which depends on all the sources of information

$$\begin{aligned} U_{i,t} &= E[(1-\kappa_x-\kappa_s)\epsilon_{it}^{\pi}+\kappa_x\epsilon_{it}^{x}+\kappa_s\epsilon_t^s]^2 && (8)\\ &= (1-\kappa_x-\kappa_s)^2\tau_{\pi}^{-2}+\kappa_x^2\tau_x^{-2}+\kappa_s^2\tau_s^{-2} && (9)\end{aligned}$$

Therefore, whether or not dispersion accurately reflects uncertainty will depend on the nature of the information and its relative importance.

A measure that does capture both private and public sources of information is the forecast mean-squared error, $MSE_{t+1}=\frac{1}{N_t}\sum_{i=1}^{N_t}FE_{i,t+1}^2$. Also, mean squared error and forecast dispersion are *ex-post* measures. If forecast errors were completely idiosyncratic, with no common component, then dispersion in forecasts and mean-squared forecasting errors would be equal. Note that we can express the (squared) mean forecast error in terms of accuracy of average forecast and forecast dispersion in the following way

$$MSE_{t+1}=\frac{1}{N_t}\sum_{i=1}^{N_t}[(E[\pi_{t+1}|\mathcal{I}_{it}]-\bar{E}\left(\pi_{t+1}\right))+(\bar{E}\left(\pi_{t+1}\right)-\pi_{t+1})]^2 \tag{10}$$

If the first term of the bracketed sum is orthogonal to the second (which is, indeed, the case in the current framework), then the mean-squared error is simply a sum of forecast dispersion and the squared error of the average forecast. We will use this insight in the empirical part of the paper to evaluate how much variation in mean-squared errors in inflation forecasts of firms in Uruguay comes from the accuracy of average (across firms) inflation forecasts and how much comes from forecast dispersion. This will teach us what possible sources of firms' overconfidence are.

Definition 4 ***Overconfidence***
A forecaster is considered to be overconfident, ex-post, if $FE_{i,t+1}>\sqrt{U_{i,t}}$.

In practice, to the extent uncertainty is measured ex-ante, it will be useful to define the ex-ante probability of a forecaster being overconfident, that is $\eta_{it}\equiv Prob(FE_{i,t+1}>\sqrt{U_{i,t}})$.

To determine that ex-ante firm- and time-specific probabilities it will be useful to define the forecast error, in levels, as $FF_{i,t+1} = \pi_{t+1} - E[\pi_{t+1}|\mathcal{I}_{it}]$, with $FE_{i,t+1} = |FF_{i,t+1}|$, for all i and all t. With that, the ex-ante probability of a firm being overconfident can be written as

$$\eta_{it} = Prob(|FF_{i,t+1}| > \sqrt{U_{i,t}}) = 1 - Prob(-\sqrt{U_{i,t}} \leq FF_{i,t+1} \leq \sqrt{U_{i,t}}) \tag{11}$$

Note that, since $E[FF_{i,t+1}|\mathcal{I}_{it}] = 0$, and, by definition of uncertainty, $U_{i,t} = E[FF^2_{i,t+1}|\mathcal{I}_{it}]$, then the variance of the forecast errors is given by $Var[FF_{i,t+1}|\mathcal{I}_{it}] = U_{i,t}$ and the expression in eq. (11) becomes (one minus) the expression for a normal variable to be within one standard deviation from its mean of zero, which equals 0.68. In other words, in this simple forecasting model, the ex-ante probability of a forecaster being overconfident is $\eta_{it} = 0.32$, constant for all i and all t.

Instead, what we measure in the data is an *ex-post* measure of overconfidence of a given forecaster – variance misalignment ratio – $VMR_{i,t+h} \equiv \frac{FE_{i,t+h}}{U^h_{i,t}}$. Notice that in the simple forecasting model outlined here $VMR_{i,t+h}$ will fluctuate with the shocks to firm's prior beliefs, idiosyncratic and public signals, ϵ^{π}_{it}, ϵ^{x}_{it} and ϵ^{s}_{t} to the extent governed by the properties of these latter fundamental shocks, by eqs. (5) and (9). Interestingly and perhaps somewhat unrealistically, to the extent uncertainty in this simple model is constant across firms and over time, it serves merely as a scaling factor in computing overconfidence: Because uncertainty is constant (and, in particular, it does not depend on the level of the realized inflation), the higher the realized inflation turns out to be (and, hence, the higher the forecast error), the higher the $VMR_{i,t+h}$. In addition, for any given firm i, and on average over time t, the model cannot talk about overconfidence in a meaningful way as the firm's beliefs are unbiased on average, and average forecast errors are zero. In what follows, we present a straightforward extension of our simple model, allowing for time variation in uncertainty and a meaningful discussion of a notion of the firm's (heterogeneous) beliefs about the central bank's credibility.

In the extended model, we assume that the forecasters think of inflation as following the hidden Markov switching process

$$\pi_t = \theta_{z_t} + \sigma_{z_t}\epsilon_{\theta} \tag{12}$$

where a discrete state $z_t = \{C, D\}$ follows a Markov chain with transition matrix (λ_{mn}) with $\sum_n \lambda_{mn} = 1$ and $\epsilon_{\theta} \sim \mathcal{N}(0, 1)$. We may identify state $z_t = C$ (commitment) with a situation where inflation is on target (in the target range), and we will assume that $\theta_C < \theta_D$ and that $\sigma_C < \sigma_D$. If the central bank can credibly bring inflation down to its target, then inflation will be lower, and so will its (unconditional) variance in this regime.

In that sense, we can think of λ_{CC} as a proxy for the central bank's actual credibility, consistently with the interpretation in Blinder (2000) and Debortoli & Lakdawala (2016). Let ω_{it} =Prob$(z_t = C|\pi^t, \omega_{i,t-1})$ with $\omega_{i,0}$ given for all i. At the beginning of the period, every firm observes realized inflation and updates the state belief using Bayes' law as follows

$$\omega_{it} = \frac{\text{Prob}(z_t = C, \pi_t)}{\text{Prob}\,(\pi_t)} = \frac{\lambda_{CC} f\,(\pi_t, 1)\,\omega_{i,t-1} + \lambda_{DC} f\,(\pi_t, 2)\,(1 - \omega_{i,t-1})}{f\,(\pi_t, 1)\,\omega_{i,t-1} + f\,(\pi_t, 2)\,(1 - \omega_{i,t-1})} \tag{13}$$

where $f\,(x, j) = \frac{1}{\sqrt{2\pi\sigma_j}} \exp\left[-\,(x - \theta_j)^2 \,/\, \left(2\sigma_j^2\right)\right]$ is the density function of the normal distribution with mean θ_j and variance σ_j^2 for $j = \{C, D\}$. The higher the actual credibility of the central bank, λ_{CC}, all else equal, the higher the firm's state belief. The recursive updating of the firm's state belief builds in history-dependence and persistence to expectations formation of a given firm, and the central bank's credibility governs the force of that effect.

Notice that, conditional on firms' state beliefs, inflation is normally distributed, and this regime-switching inflation model can be considered a simple extension of the forecasting model we wrote down above, where all we have done in this extended model was to change the firms' prior beliefs.[8] To see this, first note that firms' state beliefs are recursive so that today's belief about inflation being at its target next period (without the benefit of observing tomorrow's inflation) is given as

$$\text{Prob}(z_{t+1} = C|\pi^t, \omega_{i,t}) = \omega_{i,t}\lambda_{CC} + (1 - \omega_{i,t})\lambda_{DC} \tag{14}$$

Then, firms' beliefs about inflation (conditional on the state) are $\pi_{t+1}|\omega_{it} \sim \mathcal{N}(\mu_{it}^C, \tau_{\pi,it}^{-1})$ with $\mu_{it}^C = \text{Prob}(z_{t+1} = C|\pi^t, \omega_{i,t})\theta_C + (1 - \text{Prob}(z_{t+1} = C|\pi^t, \omega_{i,t}))\theta_D$ and $\tau_{\pi,it}^{-1} = \text{Prob}(z_{t+1} = C|\pi^t, \omega_{i,t})\sigma_C^2 + (1 - \text{Prob}(z_{t+1} = C|\pi^t, \omega_{i,t}))\sigma_D^2$. The rest of the forecasting model from the section above with private and public signals can be layered on top using eq. (3-5) and (8-9).[9] Specifically, let $\mathcal{I}_{it}^C = \{x_{it}, s_t, \mu_{it}^C, \tau_{\pi,it}, \tau_x, \tau_s, \omega_{it}\}$. Then, we can write down the firm's forecasts and uncertainty, respectively, as

$$\begin{aligned} E[\pi_{t+1}|\mathcal{I}_{it}^C] &= (1 - \kappa_{x,it} - \kappa_{s,it})\mu_{it}^C + \kappa_{x,it}x_{it} + \kappa_{s,it}s_t && (15) \\ U_{i,t}^C &= (1 - \kappa_{x,it} - \kappa_{s,it})^2\tau_{\pi,it}^{-2} + \kappa_{x,it}^2\tau_x^{-2} + \kappa_{s,it}^2\tau_s^{-2} && (16) \end{aligned}$$

where coefficients $\kappa_{x,it}$ and $\kappa_{s,it}$ are determined using Bayes law as $\kappa_{x,it} = \frac{\tau_x}{(\tau_{\pi,it}+\tau_x+\tau_s)}$ and $\kappa_{s,it} = \frac{\tau_s}{(\tau_{\pi,it}+\tau_x+\tau_s)}$.

[8] Also, note that this forecasting model can be extended further to three hidden states (with the inflation target range becoming an intermediate state) if – as it may be the case for some economies – inflation occasionally falls below the target range.

[9] Notice that for simplicity of the exposition, we assumed that the private and public signals are used only to forecast future realizations of inflation but not to update state beliefs.

Contrary to the simple model presented above, both firm-specific forecasts and uncertainty fluctuate over time. Second, the relative importance of firm's prior beliefs as well as idiosyncratic and public signal changes over time as well. We will use the conditional mean and variance of the firm-specific predictive distribution for inflation to construct firm's beliefs about central bank's credibility as follows:

Definition 5 ***Perceived Credibility***
A firm i's beliefs about central bank's credibility – perceived credibility – are given by $\phi_{it}^{h} = Prob(\underline{\pi} \leq \pi_{t+h} \leq \overline{\pi} | \mathcal{I}_{it}^{C})$ *for a given time horizon h and central bank's inflation target range* $[\underline{\pi}, \overline{\pi}]$.

3 Uncertainty and Overconfidence: From Theory to Data

3.1 Data Description

The survey we use – Inflation Expectations Survey (IES) – has been conducted monthly by the National Statistical Office of Uruguay (Instituto Nacional de Estadística, INE) in agreement with the Central Bank of Uruguay (Banco Central del Uruguay, BCU) since October 2009 based on a sample covering all the sectors of the economy except for the agricultural, financial, and public sectors. We will use waves of survey when firms were asked probabilistic questions regarding potential inflation outcomes on 12 occasions between 2014 and 2021. Regarding the representativeness of the survey, firms are selected using stratified random sampling. The stratification is according to the number of employees (50 to 99; 100 to 199; 200 or more) and the economic sector. Therefore, only firms with 50 or more employees are participating. The survey is currently sent out monthly to around 500 firms by email. Eventually, approximately 335 questionnaires are received (with an average response rate of 65%) each month. If a firm does not respond, the weights are recalculated to guarantee the representativeness of the results.
The firms are asked each month about their inflation expectations for the calendar year, the 12-month horizon, and the long-run expectations (i.e., 18-month-ahead inflation expectation before July 2013 and 24-month-ahead inflation expectation since July 2013).[10]
Table 1 shows summary statistics for our database. The total number of observations decreased from 381 in April 2014 to 204 in September 2020 but seems to be coming back somewhat in the most recent survey wave of September 2021.[11]

[10] The change in the definition of the long-run inflation expectation corresponded to the shift in the monetary policy horizon planning.

[11] To remedy the issue, we use the population weight in all analyses below.

Table 1: **Summary statistics of 12-month-ahead expected inflation and realized inflation**

	Aug 14	Sep 15	Mar 16	Mar 17	Sep 17	Mar 18	Jun 18	Sep 18	Jun 19	Mar 20	Sep 20	Sep 21
Panel A. Expected inflation												
# firms	381	383	378	334	314	306	300	288	259	218	204	234
Mean	9.70	10.28	10.83	9.54	8.57	8.42	8.78	9.21	9.00	9.77	9.56	8.20
Median	9.5	10	10	9	8	8	8	8.9	8.5	9	9.5	8
Min	7	7	7.5	6.15	3.25	5.30	6	5	5	5	6.42	3
Max	30	29	36	16.8	17	20	25	25	20	25	21	16
Std dev.	1.70	2.05	2.27	1.82	2.10	1.85	2.18	2.01	1.18	2.26	1.82	1.69
Panel B. Realized inflation												
12-month inflation$_t$	8.75	9.14	10.60	6.71	5.75	6.65	8.11	8.26	7.36	9.16	9.92	7.41
12-month inflation$_{t+12}$	9.02	9.38	7.09	7.07	8.31	7.49	7.73	7.76	11.05	9.12	7.59	9.53

Source: IES, INE, authors' calculations.

Realized inflation during the period considered varies between 5.75% in September 2017 to 10.60% in March 2016. Firms' expected inflation forecast for the 12-month horizon differs significantly, with a minimum value of 8% for the median inflation forecast and 8.20% for the mean inflation forecast and a maximum of 10% and 10.83% for the median and mean inflation forecasts, respectively. An important characteristic of this inflation forecast data set is the high and relatively stable level of disagreement in the cross-section of firms' inflation expectations, as given by a standard deviation of close to 2%.

We work with selected waves from this survey, carried out between August 2014 and September 2021, focusing our attention on firms' answers to questions that are meant to help gauge firms' subjective probability distributions over expected inflation outcomes. The table below, Table 2, lists intervals the firms were presented with in each of the 12 survey waves. The firms answered subjective probability questions correctly (in the sense that the probabilities summed up to one), in all the survey waves except for the first one.

Table 2: **Density intervals**

Aug 14, Sep 15	Mar 16	Mar 17	Sep 17, Mar 18, Jun 18, Sep 18	Jun 19, Mar 20, Sep 20, Sep 21
<3	<3	<3	<0	<3
			[0-3]	[3-4]
[3-7]	[3-7]	[3-7]	[3-5]	[4-5]
			[5-7]	[5-6]
				[6-7]
[7-10]	[7-10]	[7-10]	[7-10]	[7-8]
				[8-9]
				[9-10]
>10	[10-12]	[10-15]	[10-12]	[10-12]
	[12-15]		[12-15]	[12-15]
	>15	>15	[15-20]	[15-20]
			>20	>20

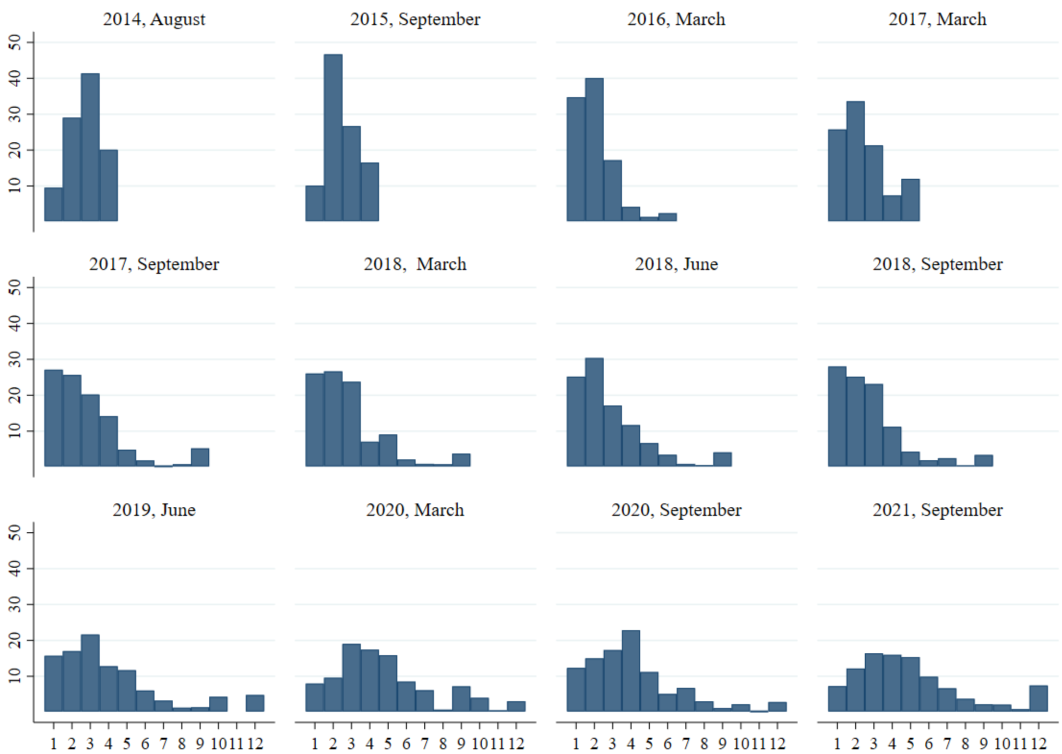
Source: IES, author's calculations.

For example, consider the question asked in the most recent survey wave of September 2021: "*What do you think are the chances that the following will happen with inflation in September 2021-August 2022? (Please note that numbers must add up to 100%)*:

- The inflation will be less than 3%: ... ... chances per cent
- The inflation will be between 3 and 4%: ... ... chances per cent
- The inflation will be between 4 and 5%: ... ... chances per cent
- The inflation will be between 5 and 6%: ... ... chances per cent
- The inflation will be between 6 and 7%: ... ... chances per cent
- The inflation will be between 7 and 8%: ... ... chances per cent
- The inflation will be between 8 and 9%: ... ... chances per cent
- The inflation will be between 9 and 10%: ... ... chances per cent
- The inflation will be between 10 and 12%: ... ... chances per cent
- The inflation will be between 12 and 15%: ... ... chances per cent
- The inflation will be between 15 and 20%: ... ... chances per cent
- Inflation will be higher than 20% ... ... chances per cent
- Total (column must add 100%): 100 chances per cent“

Figure 1 shows the distribution of answers according to the number of intervals firms use assigning positive probability.

Figure 1: **Distribution of number of intervals used by forecasters (%)**



Source: IES, authors' calculations.

First, we observe that at the beginning of our sample in August 2014 and September 2015, about 10 percent of the firms placed the whole probability in one interval only. In

March 2016, this figure exceeded 30 percent and stabilized at around 25 percent until the September 2018 survey. Second, in the last four waves, in which the number of intervals increased, the number of firms using one interval only decreased, reaching a low of about 8 percent in the most recent survey of September 2021. Having firms assign all probability mass to a single interval can be problematic when measuring and interpreting firms subjective beliefs and views on inflation uncertainty. Therefore, in addition to baseline results presented throughout the paper, we restrict our sample to firms that assign positive probability to three or more bins as a robustness check for all results. Another aspect worth highlighting is that – except for a few firms only – firms assign positive probabilities to adjacent intervals. These types of responses facilitate the non-parametric estimation of the measures of central tendency as the mean, the median, or the mode.[12]

3.2 Uncertainty and Overconfidence

Following the approach of Engelberg et al. (2009) and Armantier et al. (2017), we assume uniform distribution when the respondent assigns all probability mass to one interval, and we fit an isosceles triangular distribution when a respondent puts all probability mass in two adjacent intervals. For the remaining cases, we use the "mass-at-midpoint approach" (Glas (2020)). In what follows, variables with 'tilde' denote empirical counterparts – obtained through the fitting of distribution to firms' answers – to previously defined theoretical objects. Specifically, the inflation forecast of firm i at time t is given by

$$E\left(\pi_t|\mathcal{I}_{i,t-1}\right) \cong \tilde{E}_{i,t} \equiv \sum_{k=1}^{K} p_{i,k,t} * m_k \tag{17}$$

where $p_{i,k,t}$ is the probability assigned by firm i to inflation forecast in period t -performed 11 months ago- falling within the k-th interval and m_k is the midpoint of the k-th interval. Correspondingly, the firm's inflation uncertainty is computed as the variance of the individual probability distribution as

$$U\left(\pi_t|\mathcal{I}_{i,t-1}\right) \cong \tilde{U}_{i,t-1} \equiv \sum_{k=1}^{K} p_{i,k,t} * \left(m_k - \tilde{E}_{i,t}\right)^2 \tag{18}$$

Figure 2 shows, for each survey wave, the mean forecast inflation rate on the horizontal axis and standard deviation on the vertical axis at the individual level. (The lines repre-

[12]Starting with a total number of observations of 3,533, we eliminate observations for which: a) the sum of the probabilities does not add up to one, or b) forecaster's responses fall only into two non-adjacent intervals. As a result, we lost 2 percent of the observations. In addition, we discard observations that are associated with extreme values of mean squared error or uncertainty (1 percent highest or 1 percent lowest). We ended up with 3,326 observations.

sent means of forecasts and uncertainty). The data show a positive correlation between the mean and the standard deviation.[13] Second, the mean of the firms' subjective inflation distributions shifts to the left as inflation decreases – from March 2016 to September 2017, the inflation rate falls from 10.6% to 5.75%, and then again between September 2020 and 2021 when inflation fell by two percentage points. At the same time, the corresponding inflation uncertainty did not change much.[14]

Figure 2: **Firms' inflation forecasts and inflation uncertainty**

Source: IES, authors' calculations.

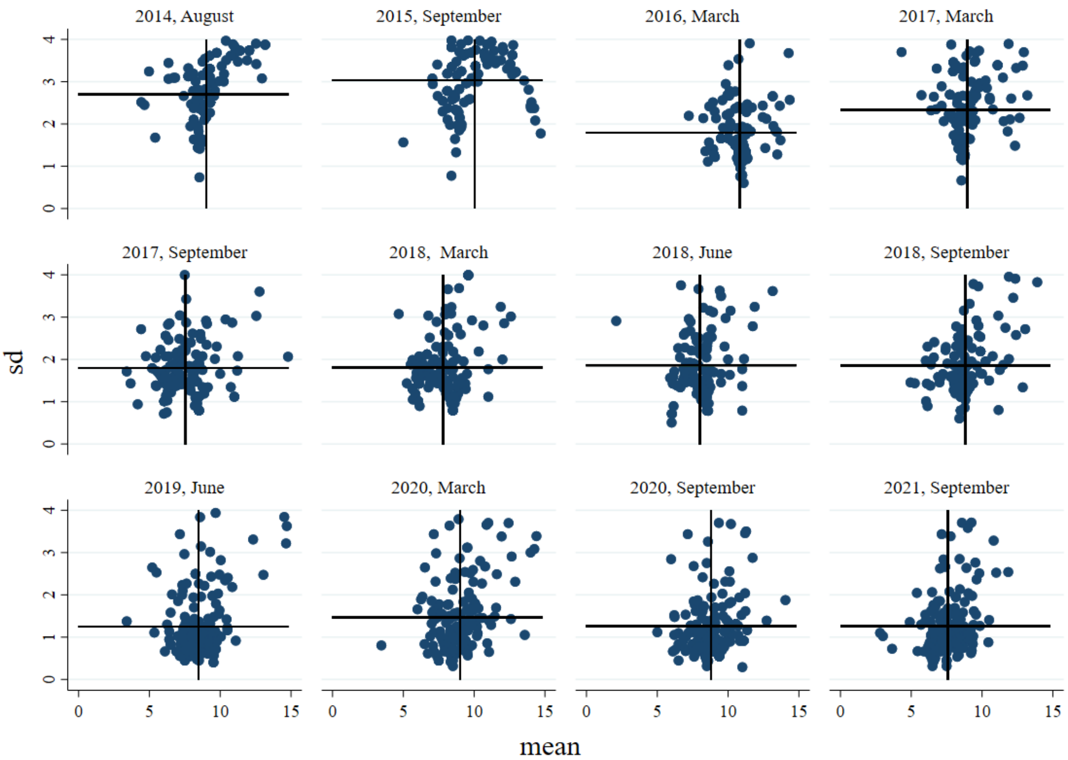
Finally, the figure below, Figure 4, depicts - for each survey wave, the evolution of average mean squared errors and uncertainty (variance) over time. Whenever the mean-squared error is higher than the corresponding uncertainty, we would conclude that – in the aggregate – firms are overconfident. Interestingly, uncertainty seems to be less volatile than mean-squared errors. Also, changes in the mean squared errors over time are of a higher magnitude than changes in uncertainty.[15]

[13] This result is confirmed with a formal regression with time effects, resulting in a coefficient of 0.12, significant at a 1-percent level.

[14] As a robustness check, Figure 3 shows the mean and uncertainty for a restricted sample which excludes firms that assign positive probabilities to only one or two bins.

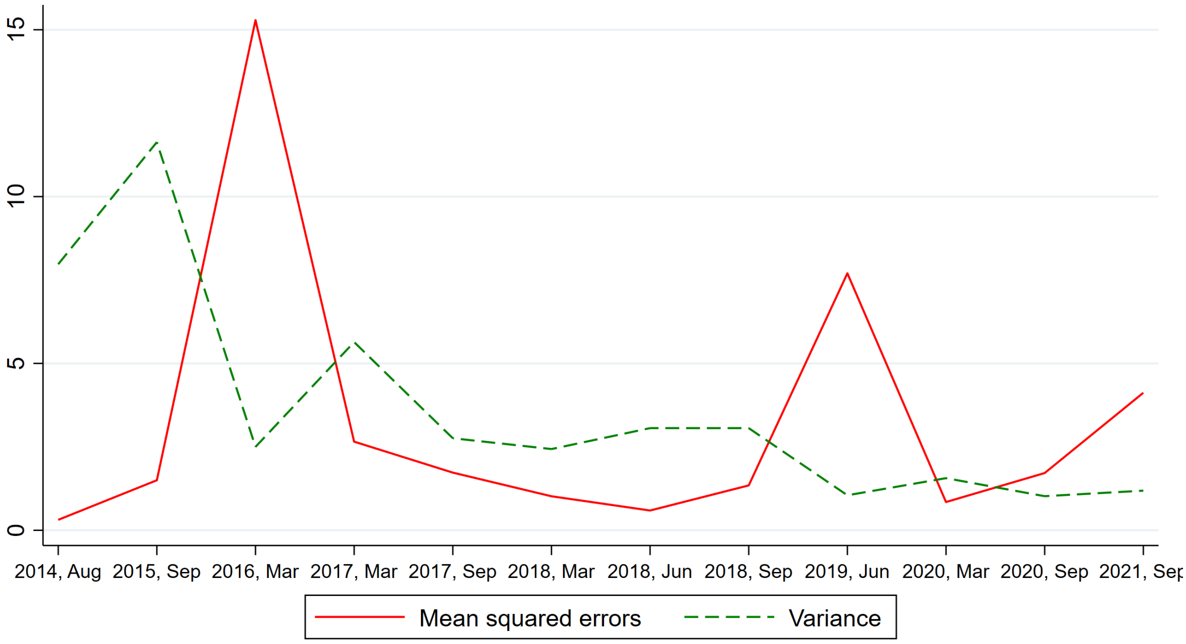
[15] When we exclude from our sample firms that report positive probabilities in one or two bins only, the qualitative pattern of differences between mean-squared errors and uncertainty remains (see Figure 5).

Figure 3: **Firms' inflation forecasts and inflation uncertainty, subsample**



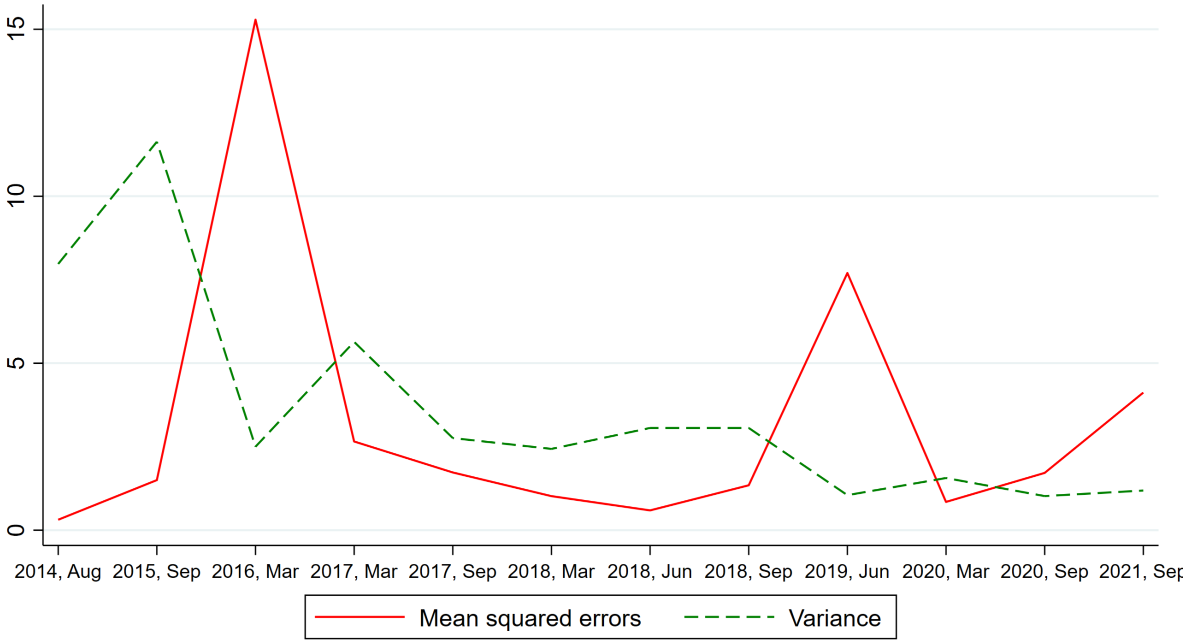
Source: IES, authors' calculations.

Figure 4: **Mean squared errors and uncertainty**



Source: IES, INE, authors' calculations.

Figure 5: **Mean squared errors and uncertainty, subsample**



Source: IES, authors' calculations.

While the proceeding statements refer to aggregate/ average quantities, a very tight and economically meaningful connection exists between mean-squared errors, uncertainty, and overconfidence at a firm level. Specifically, Table 3 presents the results of 4 regressions of firm-level absolute forecast error on conditional standard deviation ("sd") and, potentially, a measure of overconfidence ("vmr"). The forecast errors are computed in one of two ways – using the point forecasts reported directly by the firms (PF) or mean forecasts from the fitted subjective probability distributions (PDF), as in eq. (17). In either case, it is shown that firms know what they do not know in that inflation uncertainty (as measured by the "sd" variable) helps predict firms' inflation forecast errors, even when controlling for industry- and time-fixed effects.[16] When we control for the measure of firm's confidence ("vmr"), in addition to firms' inflation uncertainty (still controlling for industry and time-fixed effects), the latter is also highly statistically significant and helps explain forecast errors further (adjusted R^2 increases from 0.502 to 0.621, in the case of PDF-based forecast errors, and from 0.33 to 0.379, for PF-based forecast errors, respectively). That is, uncertainty and the extent to which firms are overconfident in their predictions materially affect their inflation forecast accuracy.[17] The next section investigates the forces behind

[16] To our knowledge, this terminology was first used by Bryan et al. (2015) in the context of the discussion of unit cost forecasts of US firms.

[17] As a robustness check, when we exclude firms which report probabilities in less than three bins, the results are even stronger, both in the sense of the magnitude of the statistically significant coefficients and variation explained, as measured by adjusted R^2. Therefore, the results are not driven by the potential mis-measurement of uncertainty for firms that use one or two bins only.

this result. In particular, we ask about the sources of fluctuations in firms' inflation uncertainty and overconfidence and how these may translate into firms' beliefs about inflation and the resulting forecast accuracy.

Table 3: **Forecast errors, uncertainty, and overconfidence**

	Dependent variable: absolute forecast error Probability Distribution Forecast (PDF) and Point Forecast (PF)			
Variables	PDF	PF	PDF	PF
sd	0.880***	0.148***	0.987***	0.203***
	(0.160)	(0.053)	(0.137)	(0.067)
vmr			0.061***	0.031***
			(0.004)	(0.003)
Constant	-2.199***	-0.331***	-1.353	0.102
	(0.601)	(0.325)	(0.499)	(0.317)
Observations	3,381	3,381	3,381	3,381
Industry FE	Y	Y	Y	Y
Time FE	Y	Y	Y	Y
Adjusted R-squared	0.502	0.3300	0.621	0.379
Robust standard errors in parentheses. *** p<0.01.				
Source: IES, INE, authors' calculations.				

4 Sources of Firms' Inflation Uncertainty and Overconfidence

We start our discussion of sources of firms' inflation uncertainty and overconfidence by investigating potential measurement and/or behavioral issues, such as inconsistency and rounding, typically studied in the literature which uses survey data.

4.1 Consistency of forecasts

The objective of the analysis performed in this subsection is to compare point forecasts reported by the firms with forecasts derived based on fitted subjective probability distributions to analyze what is known in the literature as internal consistency of forecasters. Based on firms' probability distribution, we non-parametrically estimate bounds on the inflation forecast. Following Engelberg et al. (2009), we compute the lower bound assigning all of the probability to the lower value of the interval, and the upper bound allocating all of the probability to the higher value of the interval. Below, we present the equation for the lower bound. The upper bound for the mean is built analogously.

$$\text{mean\_ lower\_ bound}_{i,t} = \sum_{k=1}^{K} \left(\mathrm{p}_{i,k,t} * \text{lower\_ bound}_{k,t}\right) \tag{19}$$

where k is the total number of intervals for each respective survey, $p_{i,k,t}$ is the probability that the firm i assigns to the k-th interval, and t corresponds to the date of the survey.

Because firms are asked for their point forecast for inflation, we need to know which measure of central tendency is reported. Some firms may answer with the mean and others with the median or yet some other measure.[18] As an example of an inconsistent forecaster, consider a specific case from the March 2017 survey where one firm assigned a 0.05 probability to inflation being between 7% and 10%, 0.15 to inflation between 10% and 15% and expected inflation greater or equal to 15% with a 0.80 probability. At the same time, its point prediction for the next 12 months' inflation rate was 12%. In this case, the point forecast is not the subjective probability distribution's mean, median, or mode. Therefore, we classify this forecaster as inconsistent.

Table 4 shows the percentage of point predictions of firms within the mean from the non-parametric bounds of the subjective probability distribution. For a non-negligible share of firms, the point prediction is outside of the non-parametric bounds. On average, 32% of the forecasts are inconsistent, and the highest figure corresponds to March 2020, where six in ten inflation forecasts are inconsistent. This result of inconsistency of inflation forecasts and magnitude is similar to Engelberg et al. (2009) and Clements (2010) for the US Survey of Professional Forecasters. Note that the share of consistent responses is much higher for the first few survey waves which included fewer bins that are also much wider because a higher bin width inflates the bounds, which, in turn, increases the likelihood of classifying forecasts as consistent.

Table 4: **Point predictions within non-parametric bounds of subjective probability distribution (%)**

Aug-14	91
Sep-15	90
Mar-16	81
Mar-17	82
Sep-17	66
Mar-18	74
Jun-18	75
Sep-18	77
Jun-19	48
Mar-20	37
Sep-20	43
Sep-21	56
Total sample	68

IES, authors' calculations.

[18] Borraz & Zacheo (2018) compare the point prediction with various measures of central tendency from the probability distribution as the mean, the median, and the mode.

As mentioned above, Clements (2010) finds that the point forecasts are more accurate than the predictions based on probability distribution. Therefore, because not all forecasters update their histogram as new information arrives, we sometimes observe inconsistencies between the point forecast and the probability distribution prediction. However, in our case, Table 5 indicates that the point forecasts are not always more precise predictors than the probability distribution forecasts.

Table 5: **Mean squared error: point forecast (PF) vs. probability distribution forecast (PDF) by consistency**

	Consistent	Non-consistent	Total
Aug-14 PF	0.7	2.3	1.0
Aug-14 PDF	0.3	0.8	0.3
Sep-15 PF	0.4	0.4	0.4
Sep-15 PDF	0.8	3.8	0.8
Mar-16 PF	8.5	7.9	8.5
Mar-16 PDF	11.6	15.3	15.3
Mar-17 PF	3.7	15.4	3.7
Mar-17 PDF	2.2	7.6	2.8
Sep-17 PF	1.7	1.7	1.7
Sep-17 PDF	0.7	4.0	1.7
Mar-18 PF	0.3	6.3	1.0
Mar-18 PDF	1.0	1.5	1.0
Jun-18 PF	0.5	5.2	1.5
Jun-18 PDF	0.6	0.4	0.6
Sep-18 PF	0.1	10.5	1.5
Sep-18 PDF	0.5	4.3	0.6
Jun-19 PF	6.5	4.2	6.5
Jun-19 PDF	6.5	7.6	6.5
Mar-20 PF	1.0	1.3	1.2
Mar-20 PDF	0.5	1.2	0.7
Sep-20 PF	3.6	1.7	2.0
Sep-20 PDF	3.6	1.9	2.2
Sep-21 PF	6.4	2.3	4.1
Sep-21 PDF	5.1	3.7	4.1

Source: IES and INE, authors' calculations.

4.2 Rounding

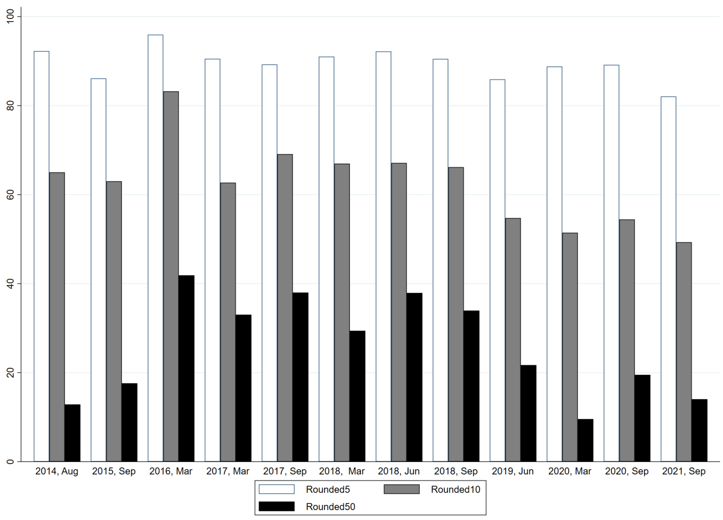
Following Manski & Molinari (2010) and Clements (2011) rounding is defined as the common practice of reporting one value whenever a real number lies in an interval. The forecaster could round their prediction to a particular value in this case. Moreover, the forecasters usually do not use the full 0-100 percent range and round to multiples of 5 or 10. For example, a forecaster reporting a probability of 0.5 reports any value between 0 and 1.

Dominitz & Manski (1997) not only were among the first proponents of using probabilistic questions from surveys to measure uncertainty, but they also discussed the problem of rounding. They state that while it is only sometimes possible to determine why agents round, it is feasible to recognize the presence of rounding despite the survey not requiring rounding to any specific level. The rounding behavior of forecasters can reflect uncertainty about the inflation process. It may also be one of the reasons for the inconsistency of forecasts which we discussed above (Clements (2011), Engelberg et al. (2009)).

Because there is no consensus regarding rounding, we define a forecaster as a rounder when all the probabilities in the subjective probability distribution are multiples of 5 or 10%. We analyze the sensitivity of our results to these two definitions of rounding. We define a forecaster as a rounder in each of the data waves.

Figure 6 shows that rounding is widespread among Uruguayan firms, with rounding at a level of 5% being the most common form (89% on average across all sample individuals). The behavior of rounding at 10% comes in second place (63%), followed by rounders who round with 0, 50, or 100% (26%).

Figure 6: **Rounders (%)**

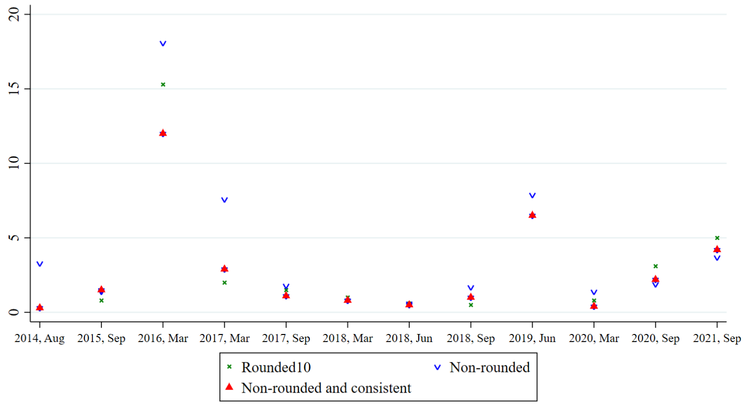


Source: IES, authors' calculations.

As we focus on the implication of rounding on prediction errors, we must consider the possibility of misclassifying a rounder as an inconsistent forecaster. Figure 7 shows that the mean squared error (MSE) is lower for the non-rounders than for rounders. When we exclude inconsistent forecasts, the results remain. This fact is verified by the t-test results, considering the categorization of rounders at 10%. Therefore, the rounding forecasters differ from those that do not round in terms of mean squared error.

Next, we analyze the behavior of mean-squared errors and uncertainty in the subsamples of rounders and non-rounders. The prior finding of higher volatility of the mean squared

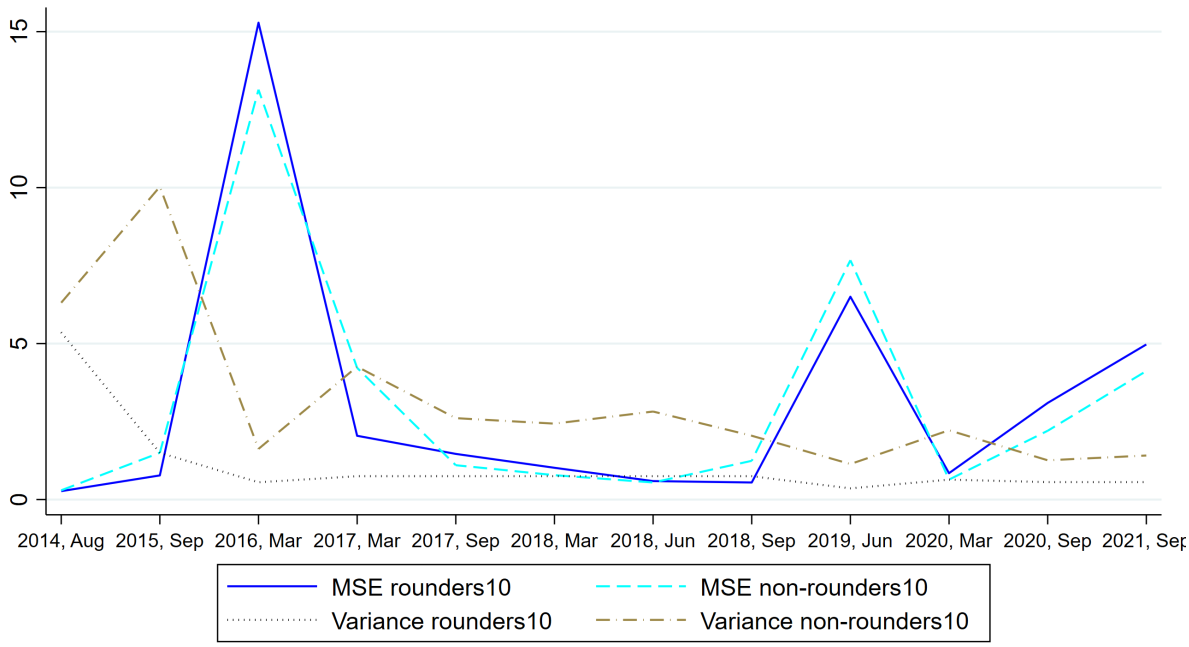
Figure 7: **Mean squared error: rounders vs. non-rounders (10%)**



Source: IES and INE, authors' calculations.

forecast errors than that of uncertainty remains when we consider agents that do not round (see Figure 8).

Figure 8: **Mean squared errors and variance over time by rounding**



Source: IES, authors' calculations.

Finally, we put the two measures together and analyze the variance misalignment ratio. The results are shown in Table 6 for the whole sample, while Table 7 excludes the inconsistent forecasters. For all firms, the variance misalignment ratio is above 1, implying that the firms are overconfident. Therefore, they have less uncertainty at the time they report their probability distribution for the inflation forecast than what the forecast errors turn out to be, on average. This result is in line with the empirical findings (Glas & Hartmann (2022)). Rounders at 10% and rounders at 5% have higher ratios than the non-rounders. Excluding firms with at least one inconsistent forecast also leads to the diminishing of the VMR (Table 7) (except for the 5% criteria). In this case, we find overconfidence only for rounded and consistent forecasters. The tables also include information on the share of overconfident firms which is higher than the theoretically predicted (constant across firms and time) *ex-ante* probability of overconfident forecast.

Table 6: **Variance misalignment ratio: rounders vs. non-rounders**

	Share of vmr>1	Median - Ratio	N
All	0.55	1.23	620
Rounders10	0.66	2.46	212
Non-Rounders10	0.51	1.05	408
Rounders5	0.62	1.39	431
Non-Rounders5	0.43	0.86	189

Source: IES and INE, authors' calculations.

Table 7: **Variance misalignment ratio - excl. inconsistent forecasts**

	Share of vmr>1	Median Ratio	N
Rounders10	0.52	1.10	102
Non-Rounders10	0.46	0.98	116
Rounders5	0.50	1.11	171
Non-Rounders5	0.43	0.78	47

Source: IES and INE, authors' calculations.

4.3 Information: Private vs. Public

Next, we ask whether the finding of firms' overconfidence can be explained by the different types of information they condition on when forming their inflation forecasts, and if so, what kind of information.

We use the insights from eq. (10) to measure how much variation in mean-squared errors comes from changes in the accuracy of average forecasts and how much comes from changes in dispersion. When we regress mean-squared errors on the second term of summation on the right-hand side of eq. (10) we find that the R^2 of this regression is 98%. The remaining variation is due to changes in forecast dispersion. This teaches us that most variation in mean squared errors comes from changes in the accuracy of forecast errors.[19] In turn, the average forecast's accuracy variation would come from variations

[19]This result also implies that firms' inflation expectations in Uruguay are an example of a dataset for which using forecast dispersion as a proxy for uncertainty may not be a good idea as one would miss an important source of variation by doing so.

in the level of realized inflation, the variation of the public signal, and/or the interaction between the two. This finding is potentially of particular importance for central bank's communication policies – the issue we discuss further below.

5 Firms' Uncertainty, Overconfidence, and Perceived Credibility

Underlying our concept of perceived credibility is a simple idea that a credible central bank should be able to deliver on its mandate.[20] Contrary to the prior studies, however, which attempt to evaluate the extent to which the central bank is considered credible through expectations anchoring based on conditional mean of inflation only, our measure of perceived credibility also accounts for inflation uncertainty.[21] Intuitively, consider a situation where the announcement of monetary policy action (or a macroeconomic or industry-specific data release) results in a substantial increase in uncertainty (conditional variance) but leaves the inflation forecast (conditional mean) unchanged. In other words, while inflation expectation remains stable, a forecaster judges that both upside and downside risks to inflation have increased, which is at odds with the idea of stability (anchoring) of expectations associated with the central bank's credibility. Per our definition, the possible scenarios entailing a firm's expectations becoming potentially unstable (unanchored) include a situation in which a given firm's inflation expectation displays low uncertainty but is far away from the target or a situation in which another firm's inflation expectation is close to the target but displays high uncertainty. Then, depending on the actual realized inflation – and the resulting forecast error – we would be able to categorize these firms into over- or under-(less) confident. Ultimately, the extent to which a firm's inflation uncertainty matters for its perceived credibility is an empirical question. The survey of firms' inflation expectations in Uruguay allows us to shed light on this relationship.

Table 8 contains results of the following regressions: we regress the probability with which a given firm at a given point in time believes that inflation will fall within inflation target bounds on a measure of uncertainty ("sd") and/or a measure of over- ("vmr>1") or under-confidence ("vmr"<1). Overall, across all firms, and perhaps somewhat surprisingly at

[20]In her speech, Lagarde (2023), ECB's President Lagarde stated: "My definition of credibility is that we deliver on our mandate." This notion is related to – but different from – trust in the central bank (Christelis et al. (2020)).

[21]Borraz & Mello (2020) is an earlier study on the credibility of the Uruguayan central bank as seen by the firms. However, it uses a different definition of credibility: a central bank is assumed to be regarded as credible by a given firm if its inflation forecast falls within the target range. Another difference is the horizon of the forecasts considered. While the authors look at 24-month ahead forecasts, which they can afford given their notion of credibility, the nature of probabilistic questions forces us to work with 12-month ahead inflation forecasts.

Table 8: **Credibility**

	Dependent variable: Probability of 12-month inflation inside the target				
Variables	All	All	All	VMR>1	VMR<1
sd	1.140***	1.135***		-0.737	4.881***
	(0.442)	(0.487)		(0.702)	(0.645)
vmr		0.122**	0.095	0.056	-6.577**
		(0.059)	(0.059)	(0.069)	(2.575)
Constant	-8.221***	-6.512***	-3.387	26.20*	13.77***
	(7.724)	(7.383)	(7.333)	(14.510)	(3.454)
Observations	3,381	3,381	3,381	1,781	1,600
Industry FE	Y	Y	Y	Y	Y
Time FE	Y	Y	Y	Y	Y
Adjusted R-squared	0.231	0.234	0.229	0.292	0.369

Robust standard errors in parentheses. *** p<0.01, ** p<0.05, * p<0.1.

Source: IES, authors' calculations.

first, the higher the inflation uncertainty, the higher the reported probability that inflation will, indeed, reach its target range. When we split the sample into two subsamples depending on firms' degree of overconfidence, it turns out that the result is mainly driven by firms that are less confident in their forecasts, *ex-ante*. In a country like Uruguay, where inflation has been historically high and running outside of the target of the central bank, it seems to be the case that a firm has little confidence, *ex-ante* in its inflation prediction (that is, relatively large confidence bands) for it to assign a meaningful probability to inflation falling within the target range of the central bank. In fact, conditional on being classified as under-confident ($VMR < 1$), the less confident the firm is, the higher the probability that the firm assigns to inflation reaching its target range within the next 12 months.

So what is the central banker to do? Ideally, a credible central banker would not only be able to bring the public's inflation forecast to its target but also create high confidence in this belief (resulting in low inflation uncertainty). Endowed with typically only one policy tool at a time, like interest rate policy, the policymakers turn to communication strategies, and our results bring to the forefront the importance of such communications. In a country like Uruguay, where realized inflation has been coming in above the central bank's target for a while (which – through the lenses of the model – would make the firms, all else equal, increase both the probability and confidence in the high-inflation state), the central bank could try to tailor its communication policies (which in the model would mean adjusting the precision and/or the size of the public signal) so as to lower inflation uncertainty while lowering firm's forecast towards the target. Of course, the central bank must be able to – sooner rather than later – actually deliver on the promises it makes in its communications, or else its communications themselves would not be perceived as credible, eventually. How exactly to tailor the central bank's communication policies towards firms would certainly be a worthy subject for future research.

6 Conclusions

This paper uses a rich dataset with the subjective probability distribution of inflation expectations of firms to find that the *ex-ante* conditional variance of inflation is often lower than the *ex-post* forecast errors - a phenomenon we define as firms' overconfidence. We document that firms in Uruguay know what they do not know in that measures of their subjective inflation uncertainty and overconfidence help explain firms' inflation forecast accuracy. We also find that firms' inflation expectations in Uruguay are an example of a dataset for which using forecast dispersion as a proxy for uncertainty may not be a good idea as one would miss an important source of variation by doing so.

We also show how firms' beliefs about central banks' credibility, understood as the likelihood firms assign to inflation falling within the target range within the next 12 months, correlate with firms' inflation uncertainty and overconfidence.

Future research should explore that connection further. Within general equilibrium, firms' beliefs would be a fixed point: firms' inflation uncertainty affects firms' beliefs about the credibility of the central bank, but these very expectations greatly influence the central bank's credibility and actual ability to achieve the target.

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