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The weakness of common job contacts☆

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ABSTRACT

Many people obtain job information from friends and acquaintances. However, one factor influencing labor-market outcomes that is ignored in the literature is the presence of overlapping friendship circles in social networks. We find that overlapping friendship networks produce correlated information flows, resulting in an increased probability of two events: either receiving redundant job offers or receiving no job offers at all. Consequently, people with common contact networks exhibit worse employment prospects even if they have the same number of information providers and compete with the same number of people for vacancies. In quantitative terms, the impact of overlapping friendship circles rivals that of the number of direct contacts and contacts' contacts. This implies that the results in Calvó-Armengol (2004) only apply for networks where people's friends are neither connected nor have common contacts. Because overlapping friendship circles are a crucial aspect of strong relationships, our findings uncover an alternative mechanism behind "The Strength of Weak Ties" (Granovetter, 1973): their ability to maintain independence in job information flows. We further show that people with common job contacts earn lower incomes on average. However, conditional on being employed, their expected wage is higher because they can take advantage of the multiple job offers received by selecting the one with the highest pay.

1. Introduction

The old saying "it's not what you know but who you know" suggests that social connections bring labor advantages. Empirical evidence has corroborated this popular saying, documenting that between 30% and 50% of jobs are obtained through friends and acquaintances (Myers and Shultz, 1951; Granovetter, 1973; Pellizari, 2010; Munshi and Rosenzweig, 2016; Bewley, 2021). In a survey conducted in Kolkata, India, Beaman and Magruder (2012) reported that around 40% of respondents had helped a friend or relative to acquire a job with their current employer. In a study of British households, Cappellari and Tatsiramos (2015) found that an additional employed friend increases the probability of finding a job by 3.7%. Many studies yield similar results, regardless

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of the socioeconomic status and profession of the population under study (for reviews, see Ioannides and Loury, 2004, Beaman, 2016, or Granovetter, 2018).

The economic literature has echoed the relevance of networks in the labor market. Following early contributions by Boorman (1975) and Diamond (1981), the path-breaking model of Calvó-Armengol (2004) showed that the employment probability of people depends on their number of friends and the number of friends of their friends. In his model, employed people who hear about jobs pass the information on to their unemployed acquaintances. Direct contacts are beneficial because they are a source of job information, whereas contacts of contacts are detrimental because they compete with agents for their friends' information. Further research has built on these contributions to explain how people, anticipating the risk of becoming unemployed, decide to create new links (Calvó-Armengol, 2004) or invest in relationships from which they expect to obtain job-related information (Galeotti and Merlino, 2014). The model has also been used to characterize job matching (Calvó-Armengol and Zenou, 2005) as well as to explain drop-out decisions, duration dependence, and long-run correlations in employment and wages (Calvó-Armengol and Jackson, 2004, 2007).[1]

Our paper uncovers a driver of employment outcomes that is ignored in this literature: the presence of overlapping friendship circles in social networks. Overlapping friendship networks or transitive relationships embody a number of stylized facts in real-world social networks: people's friends often know each other, co-authors of co-authors are often also co-authors, competing firms usually share suppliers and customers, students have teachers and classmates in common, etc. (Jackson et al., 2017). In other words, social networks contain many triangles (people whose connections are also connected) and squares (people whose friends have other common friends). Empirical evidence shows that high levels of such network clustering or close-knittedness help us to understand phenomena such as favor exchange (Jackson et al., 2012), trust building (Karlan et al., 2009), and the spread of certain behaviors (Centola, 2010).

In this paper, we show that the results in Calvó-Armengol (2004) only apply if people's friends are not friends with each other nor have other friends in common. In other networks, the employment probability of people depends on three aspects: their number of friends, the number of friends of their friends, and the set of triangles and squares they form with their contacts. Overlapping friendship circles generate correlation in the job information flows that people receive, increasing their probability of unemployment. Thus, holding constant the number of friends and friends of friends, their employment probability decreases monotonically with the number of triangles and squares that they form with their contacts. At the macro level, the unemployment rate in a society increases monotonically with its level of close-knittedness, all else being equal. In fact, people with friends belonging to different social circles may have better employment prospects than those with a tightly clustered circle of friends, even if the former have fewer friends or a greater number of job competitors. That is, the impact of the local network geometry on labor-market outcomes can outweigh that of the number of connections characterized in Calvó-Armengol (2004).

Intuitively, the existence of overlapping friendship circles prevents information from flowing efficiently. When a friend of i, such as j, has information about a job, the probability that this information ends up in i's hands is $1/u_j$, where u_j is the number of unemployed friends of j. However, if two friends of i, say j and k, have another mutual friend l (so that i, j, k, and l form a square), the probabilities that j and k pass information to i are correlated because both probabilities depend on a common event: whether or not l is unemployed and needs a job. Such a correlation increases the probability that i receives two redundant offers (if l is employed) or no offers from the two friends (if l is unemployed). Because people can take only one job, this decreases the probability that i receives job information through i's connections and thus lowers i's employment probability. A similar mechanism operates if j and k are connected (i.e., i, j, and k form a triangle).

We then propose an extension of the model, where agents may hear about jobs with different wages. People with overlapping friendship networks earn lower incomes because of their higher unemployment probability. However, conditional on being employed, their expected wage is higher because they can make use of the multiple offers received by selecting the one with the highest pay.

We uncover an alternative mechanism behind what arguably is one of the most influential theories on networks of the past century: "The Strength of Weak Ties" (Granovetter, 1973).[2] This theory states that people are more likely to find a job through weak ties—acquaintances with whom they have little contact—than through strong ones—family members or close friends with whom they interact frequently. The reason is the different content of the information that the two links provide. Strong ties tend to connect people with common friendship circles, often providing information that is identical to what individuals already have. Weak ties, in contrast, act as bridges between people with different friendship circles, enabling access to novel information from distant parts of the social system. According to Granovetter, "the stronger the tie between two people, the greater the extent of overlap in their friendship circles" (Granovetter, 1983). We provide an alternative explanation for the strength of weak ties: their ability to maintain independence in the information flows that reach people. We show that weak ties—those connecting people with few or no mutual friends—are more advantageous than strong ties even when the content of the information provided by both links is the same and people connected by strong ties learn about vacant jobs with independent probabilities. Hence, the strength of weak ties lies not only in their informational content but also in their capacity to diffuse information efficiently.

[1] In Calvó-Armengol (2004), all agents are initially employed and lose their jobs randomly before information transmission takes place. In Calvó-Armengol and Jackson (2004), some agents are initially employed while others are not, and individuals lose their jobs randomly after information transmission. Both assumptions lead to identical results on job information diffusion.

[2] Granovetter's thesis has underpinned several follow-on theories in a variety of contexts (see Aral, 2016 for a review).

Empirical studies have examined the effects of weak ties on the probability of finding a job.[3] Granovetter, 1973 discovered that 16.7% of interviewed workers learned about job openings through strong ties (i.e., people they saw "often"), while 83.4% found their jobs through weak ties (i.e., people they saw "occasionally" or "rarely"). Subsequent studies have produced results consistent with Granovetter's hypothesis (see, e.g., Yakubovich, 2005 and Bian et al., 2015), but some have found opposite results (see Murray et al., 1981 and Bian, 1997). However, none of the aforementioned studies provide information about the extent of overlap in the friendship circles of the analyzed population. Among the studies providing this information are Gee et al. (2017a,b), and Rajkumar et al. (2022). These studies consider the number of friends that two linked individuals have in common (i.e., the number of triangles they form) to measure the strength of the tie connecting them.[4] The greater the number of friends they have in common, the stronger is their connection. Gee et al. (2017a,b) analyzed data from six million Facebook users and found that people are more likely to find jobs through weak ties because they are more numerous, but a single strong tie can be more valuable at the margin. These results suggest a positive effect of overlapped friendship circles on employment. However, the authors acknowledge that their work "may not represent the true causal effect of tie strength on the probability of a sequential job" due to its non-experimental nature. Rajkumar et al. (2022) used data from several large-scale randomized experiments on LinkedIn and found causal evidence supporting the advantage of weak ties in job transmission. They also show that the strength of weak ties is not linear and varies across industries: weak ties are more beneficial in more digital industries, while strong ties are more beneficial in industries that rely less on software and automation. Rajkumar et al. (2022) provide partial support for our theory by demonstrating that a lower degree of overlap in the friendship circles of two individuals (i.e., a weaker connection between them) can increase job transmission. This implies that future empirical research on weak ties may encounter an omitted variable bias if it does not adequately consider the existence of common friendship circles within individuals' social networks.

We contribute to the literature on the economics of social networks by establishing a clear relationship between a common feature of social networks (a high level of clustering) and labor-market outcomes. We demonstrate that the extent to which people's friends are also friends among themselves or share common acquaintances may be as important for their employment outcomes as the number of connections they have. Some papers have found a positive relationship between individuals' number of connections and their employment prospects and wages (Calvó-Armengol, 2004; Beaman, 2012; Cappellari and Tatsiramos, 2015), between their number of friends and their expected payoffs in network games (Galeotti et al., 2010), between their number of weak ties and their propensity to engage in criminal activities (Patacchini and Zenou, 2008), and between the centrality of their network position and their strategic behavior (Ballester et al., 2006; Calvó-Armengol et al., 2009). Regarding network clustering, although the literature has emphasized its relevance in explaining cooperative behavior (Bloch et al., 2008; Lippert and Spagnolo, 2011; Jackson et al., 2012; Ali and Miller, 2016), its role in other contexts remains relatively unknown (see Jackson et al., 2017). Nevertheless, because the goal of incorporating a network into economic models is precisely to understand the effects of network topology on economic phenomena, overlooking important features of the structure of real-life social networks may lead to biased predictions, as demonstrated in this paper.

Our findings do not invalidate the results of Calvó-Armengol and Jackson (2004) on long-run employment correlations and duration dependence.[5] They demonstrate that the employment status of path-connected individuals is correlated in the long run. This is because people receive job offers from their friends only if their friends do not need them (i.e., if they are already employed). Consequently, individuals with employed friends are more likely to receive job information, which results in long-term employment correlations. Calvó-Armengol and Jackson (2004) also illustrate how network geometry leads to variations in steady-state employment probabilities, even in cases where individuals have a low likelihood of hearing about job opportunities or experiencing unemployment. They attribute these differences to the different average path lengths separating network members, which affect the magnitude of the employment correlations observed in the long run. Our contribution is to demonstrate that beyond the distribution of the number of friends and the number of friends of friends, individuals' employment probabilities depend on the triangles and squares they form with their contacts, even within a single period.[6] This finding may provide us with a better understanding of the patterns of employment inequality observed among agents.

Lastly, because the model can be easily adapted to analyze the diffusion of other behaviors or resources that spread through networks, we discuss the compatibility of our findings with the theories positing that clustered networks may be beneficial and foster diffusion.

The paper is organized as follows. Section 2 presents the model. Section 3 presents the results concerning the effect of overlapping networks on employment (Section 3.1), along with a discussion of them (Section 3.2). Section 4 analyzes the impact of overlapping social networks on wages, and Section 5 provides the conclusions.

2. The model

Contact networks. There is a set of agents $N = \{i, j, \dots, n\}$ distributed on a fixed network g. The number of agents in the network is n. Each pair of agents either knows each other or not. Let $g_{ij} = 1$ if i and j know each other (i.e., they are linked in network g)

[3] There are also several studies that examine the effects of weak ties on income and status attainment with mixed results (see, for example, Lin et al., 1981a,b, Bridges and Villemez, 1986, Lin, 1999 or Smith, 2000).

[4] However, these studies do not consider the number of squares.

[5] None of their proofs require independence in information flows.

[6] We analyze the static version of the model; however, since our results are not contingent on the initial employment distribution, the observed impact of overlapping friendship networks on diffusion would persist in each period of the dynamic model.

and $g_{ij} = 0$ if they do not. Links are reciprocal, $g_{ij} = g_{ji}$. The set of neighbors or contacts of person i is $N_i(g) = \{j \in N : g_{ij} = 1\}$. The degree of person i, denoted $n_i(g)$, is the number of contacts of i (i.e., the cardinality of $N_i(g)$). A three-cycle or triangle is a set of three distinct individuals $\{i, j, k\}$ such that $g_{ij} = g_{ik} = g_{jk} = 1$. Analogously, a four-cycle or square is a set of four different people $\{i, j, k, l\}$ such that $g_{ij} = g_{ik} = g_{jl} = g_{kl} = 1$. Thus, if i forms a triangle, two friends of i know each other, whereas if i forms a square, two friends of i have another common friend l, where $l \neq i$. The set of all triangles and squares in network g is $O(g)$ and reflects the degree of overlap in i's neighborhood.

Job transmission. People live one period. At the beginning of the period, each person is employed.[7] Then, every agent becomes unemployed with probability $b \in (0, 1)$. Afterward, information randomly arrives: Each person exogenously hears about a vacant job with probability $a \in (0, 1)$. All jobs are identical. Losing the job and hearing about a vacancy are independently distributed and independent across individuals. At this point, people can be in one of four possible states:

1. With probability $\alpha = (1 - b)a$ they keep their job and hear about a job offer (status 1)
2. With probability $b(1 - a)$ they lose their job and do not hear about any job offer (status 2)
3. With probability ba they lose their job and hear about a job offer (status 3)
4. With probability $(1 - b)(1 - a)$ they keep their job and do not hear about any job offer (status 4).

Unemployed individuals who hear about a job (those in status 3) immediately become employed. Workers who are employed and hear about a job offer (those in status 1) randomly pick a friend who has lost their job (a contact in status 2 or 3) to pass the information onto.[8]

Observe that only those individuals in status 1 can pass information onto their contacts. Therefore, individuals in status 1 are called *providers*. Likewise, only those agents in status 2 and 3 can receive information from contacts. Individuals in status 2 and 3 are called *competitors*. By construction, competitors may receive several offers from different providers simultaneously. In this case, they accept one, while the other offers are lost.

Information flows. Suppose person i is unemployed and has a friend j. If j has another friend besides i, say, z, the probability that i does not receive an offer from j is

$$\underbrace{\alpha}_{j \text{ is a provider}} \left[\frac{1}{2} \underbrace{b}_{z \text{ is a competitor}}\right] + \underbrace{1-\alpha}_{j \text{ is not a provider}} = 1 - \alpha \frac{1-(1-b)^2}{2b}. \tag{1}$$

Generally, if j has $n_j(g)$ friends, the probability that i does not receive information from j is

$$q_j\left(n_j(g)\right) = 1 - \alpha \frac{1-(1-b)^{n_j(g)}}{n_j(g)b}, \tag{2}$$

as proved in Calvó-Armengol (2004).[9] Note that $q_j(n_j(g))$ increases with $n_j(g)$: the greater $n_j(g)$, the greater the number of potential recipients of j's information, and the less likely i is to be the one that j will pass information onto.

Suppose that none of i's friends knows each other nor has any common contact $l \neq i$. As established by Calvó-Armengol (2004), the probability that i receives at least one offer from contacts in network g is

$$P_i(g) = 1 - \prod_{j \in N_i(g)} q_j(n_j(g)). \tag{3}$$

Thus, $P_i(g)$ depends on two aspects: (i) the number of friends of i and (ii) the number of friends of i's friends. A greater number of contacts broadens the information channels available to i, increasing the probability that i receives a job offer. Conversely, a higher number of friends of friends increases the expected number of job competitors of i, decreasing i's employment prospects. The employment probability of i in network g is

$$E_i(g) = \underbrace{(1-b)}_{i \text{ keeps job}} + \underbrace{\underbrace{b}_{i \text{ fired}} \Big[\underbrace{a}_{\text{hears about a job}} + (1-a) \underbrace{P_i(g)}_{\text{receives info. from contacts}} \Big]}_{i \text{ re-employed}},$$

while $E(g) = \frac{1}{n} \sum_{i \in N} E_i(g)$ and $U(g) = 1 - E(g)$ are the employment and unemployment rates in network g, respectively.

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[7] This assumption is inconsequential. All of our results hold as long as the probability of being employed is the same for all individuals.

[8] In Calvó-Armengol (2004), employed individuals with a job offer pass the vacancy onto one of their contacts who has lost their job. Unemployed individuals may have later received an offer, but this is not observed by other agents. Although this is not explicitly stated, it can be seen in his Proposition 1, where the decision to pass information depends on b instead of $\beta = b(1 - a)$. To facilitate comparisons, we maintain this assumption. However, all of our results hold if agents pass information onto contacts in status 2 only.

[9] For details, see Proposition 1 in his paper.

3. Results

3.1. The effects of common job contacts on employment

Our objective is to characterize the impact of overlapping friendship networks on employment prospects by controlling for the number of friends of people (information channels) and for the number of friends of their friends (job competitors). With that aim, we define some relationships that allow us to compare the network position of an individual i in two different networks g and g'. Assume i's friends are $N_i(g) = \{1, 2, \dots, n_i(g)\}$ and $N_i(g') = \{1, 2, \dots, n_i(g')\}$ in g and in g', respectively. We say that i is *at least as well connected* in g as in g' if i has at least the same number of friends in g as in g' and no friend of i has fewer friends in g' than in g. Likewise, i is said to be *comparable* in g and in g' if i and all i's friends have the same number of contacts in both networks. Formally,

- agent i is at least as *at least as well connected* in g as in g' if and only if $n_i(g) \geq n_i(g')$ and $n_x(g) \leq n_x(g')$ for each $x = 1, 2, \dots, n_i(g')$, while
- agent i is *comparable* in g and in g' if and only if $n_i(g) = n_i(g')$ and $n_x(g) = n_x(g')$ for each $x = 1, 2, \dots, n_i(g) = n_i(g')$.

Analogously, we say that two networks g and g' are *comparable* if they comprise the same people and each person is comparable in the two networks. In network terminology, g and g' are comparable if they have the same joint degree distribution. Proposition 1 shows that increasing local network close-knittedness has negative consequences for employment, at both the individual and macro level.[10]

Proposition 1. *Let g, g^t, and g^s be three networks. Assume g^t has an additional triangle to g, $O(g^t) = O(g) \cup \{i, j, k\}$, and g^s has an additional square to g, $O(g^s) = O(g) \cup \{i, j, k, l\}$:*

(i) If person i is at least as well connected in g as in g^x, $P_i(g) > P_i(g^x)$, $x \in \{t, s\}$.
(ii) If i is at least as well connected in g^s as in g^t, $P_i(g^s) > P_i(g^t)$.
(iii) If the three networks are comparable, $U(g) < U(g^s) < U(g^t)$.

Part (i) of Proposition 1 encompasses multiple changes in i's neighborhood, allowing for a general comparison. It establishes that, if the number of i's friends (the expected number of providers of information to i) and the number of friends of i's friends (the expected number of job competitors of i) are kept fixed in the three networks, then $P_i(g) > P_i(g^t)$ and $P_i(g) > P_i(g^s)$.

Part (i) further includes less restrictive changes in networks. Consider, for example, a network g_0. In g_0, j and k are two contacts of i and l is a friend of k. Suppose g^t is the resulting network from adding a link between j and k in g_0, g^s is the resulting network from adding a link between j and l in g_0, and g the resulting network from adding a link in g_0 between two nodes s and t outside i's neighborhood. Proposition 1(i) implies that the link jk (jl) harms i's employment prospects more than the link st. This is due to two reasons. On the one hand, the link jk (jl) increases the expected number of job competitors of i (the number of friends of i's friends), whereas the link st does not. On the other hand, the link jk (jl) generates dependence in the information flows arriving to i, whereas the link st does not have such an effect.

Part (ii) compares the effect of increasing the number of connections among i's contacts with that of increasing the number of common friends of i's friends, showing that the former change harms i's employment probability more. Observe that the link jk increases the number of job competitors of i more than the link jl. Again, and more importantly, the effect holds even if we control for the number of friends and friends of friends of i. For instance, $P_i(g^s) > P_i(g^t)$, where g and g^s are the networks in Fig. 1.

Part (iii) establishes that increasing network clustering, keeping fixed the number of friends and friends of friends of each person (i.e., the joint distribution of the number of friends and friends of friends), increases unemployment rates. Fig. 1 shows an example of three comparable networks.

The economic intuition behind these results is as follows. Transitive relationships in people's friendship circles break the independence in information flows, preventing information from diffusing efficiently. If a person's friends are connected or have common friends, it is more likely that the person will receive several job offers but also that none will be received. Because all jobs are identical, a higher probability of receiving multiple job offers does not entail any advantage, because a person can accept only one job. However, a higher likelihood of not receiving information makes a person more vulnerable. Thus, the existence of triangles and squares in people's friendship circles decreases the probability that they receive at least one job offer by putting more weight on the extreme outcomes (receiving redundant offers or none).

The above findings imply that the employment probability *cannot* be exclusively ranked according to the number of friends of people and the number of friends of their friends. In arbitrary networks, an individual i, with a non-overlapping network neighborhood may have better employment prospects than another individual j, who is embedded in a tightly clustered friendship circle, even if i has fewer friends than j. The same can occur if i's friends have more friends than j's: $P_i(g)$ may be greater than $P_j(g)$ if i's friendship circle is less tightly clustered. Example A in the online Appendix illustrates this point.

Remark 1. Having more friends (or fewer friends of friends) does not necessarily imply a higher probability of employment.

[10] The proof can be found in Appendix A.

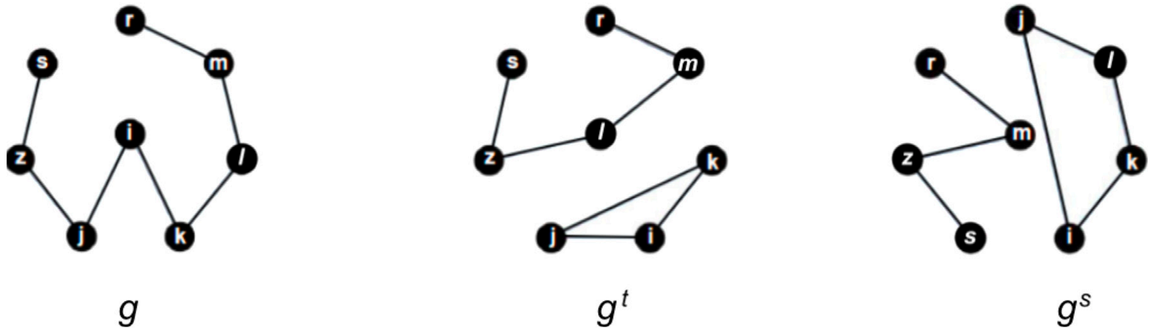

Fig. 1. Example of g, g^t and g^s in Proposition 1.

Table 1
Probability that i does *not* receive any offer from contacts in networks g, g^t, and g^s in Fig. 1.

case $x \in \{$A,B, …,F$\}$	prob. case x	$[Q_i(g) \mid \text{case } x]$	$[Q_i(g^t) \mid \text{case } x]$	$[Q_i(g^s) \mid \text{case } x]$
A: j, k providers	α^2	$\left(\frac{b}{2}\right)^2$	0	$\frac{b}{4}$
B: j provider, k competitor	αb	$\frac{b}{2}$	$\frac{1}{2}$	$\frac{b}{2}$
C: j competitor, k provider	$b\alpha$	$\frac{b}{2}$	$\frac{1}{2}$	$\frac{b}{2}$
D: j provider, k in status 4	$\alpha(1-\alpha-b)$	$\frac{b}{2}$	0	$\frac{b}{2}$
E: j in status 4, k provider	$(1-\alpha-b)\alpha$	$\frac{b}{2}$	0	$\frac{b}{2}$
F: j, k not providers	$(1-\alpha)^2$	1	1	1

We next provide an illustration of Proposition 1. Consider the networks in Fig. 1. Suppose i (with two friends j and k) is unemployed. The first two columns of Table 1 provide the possible combinations of status of i's friends (column 1) and their probabilities (column 2). Let $[Q_i(g) \mid \text{case } x]$ be the probability that i receives no offer in network g, conditional on case x in column 1. Columns 3, 4, and 5 contain the probability that i does not receive any offer from contacts in g, g^t, and g^s, respectively, conditional on each case in column 1. For example, conditional on j being a provider and k being a competitor, i receives no offer in network g if agent z is a competitor and j passes the information onto z. That is, with probability $[Q_i(g) \mid \text{case B}] = \frac{b}{2}$.

In network g, the probability that i does not get information from j is characterized in Eq. (1). Observe that this probability depends only on the status of j and z. Analogously, $q_k\left(n_k(g)\right)$ depends only on k's and l's status. Because $q_j\left(n_j(g)\right)$ and $q_k\left(n_k(g)\right)$ depend on the status of different people, flows of information from j and k to i are independent,

$$P_i(g) = 1 - \sum_{x \in \{\text{A,B, …,F}\}} \left(\text{prob. case } x\right) * [Q_i(g) \mid \text{case } x] = 1 - \underbrace{\left(1 - \alpha \frac{1-(1-b)^2}{2b}\right)^2}_{q_j\left(n_j(g)\right) = q_k\left(n_k(g)\right)}. \tag{4}$$

In network g^t, in contrast, flows of information from j and k to i are not independent. As shown in the corresponding column in Table 1, the status of j affects the probability that k does not transmit information to i and vice versa: provider j (k) does not pass a job offer onto i with probability $\frac{1}{2}$ if k (j) is a competitor and with probability 0 otherwise. Then,

$$P_i(g^t) = 1 - \sum_{x \in \{\text{A,B, …,F}\}} \left(\text{prob. case } x\right) * [Q_i(g^t) \mid \text{case } x] = \alpha(2-\alpha-b) < P_i(g). \tag{5}$$

Thus, although i has the same number of friends and friends of friends in both g and g^t, the link between j and k in g^t decreases i's employment probability compared with g.

Similar results hold when we compare $P_i(g)$ and $P_i(g^s)$. As can be seen in Table 1, the lack of independence in information flows to agent i in network g^s stems from their mutual dependence on l's status when j and k are providers. If l is a competitor (event that occurs with probability b), i does not receive information from j (k) with probability $\frac{1}{2}$, while if l is not a competitor, i does not receive an offer from j (k) with probability 0. Note, however, that the probability that i does not receive information in cases B-F is the same in g as in g^s. In all of these situations, the probability that i gets no information from i's non-provider contact is 1, independently of l's status. Then,

$$P_i(g^s) = 1 - \left[\left(1 - \alpha \frac{1-(1-b)^2}{2b}\right)^2 + \underbrace{\alpha^2 \frac{b(1-b)}{4}}_{P_i(g) - P_i(g^s)}\right] < P_i(g). \tag{6}$$

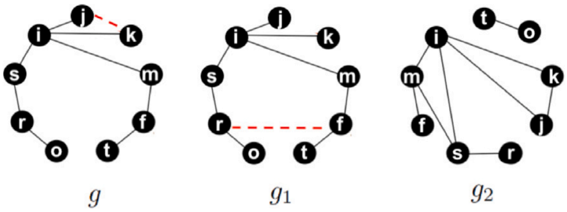

Fig. 2. Networks in Example 1.

where $P_i(g) - P_i(g^s) = \alpha^2\Big([Q_i(g^s) \mid \text{case A}] - [Q_i(g) \mid \text{case A}]\Big) > 0$. Thus, the fact that i's friends have a common contact $l \neq i$ in g^s but not in g entails a greater employment probability for i in the second network, even though i, j, and k have the same number of friends in both networks.

Finally, let us compare $P_i(g^t)$ and $P_i(g^s)$. Given that

$$P_i(g^s) - P_i(g^t) = \alpha^2 b\frac{3}{4} > 0,$$

the presence of a link between i's neighbors in g^t reduces i's employment prospects more than the presence of an additional common contact between them in g^s.

Let us now compare the unemployment rates in the three (comparable) networks in Fig. 1. Proposition 1(iii) establishes that $U(g) < U(g^s) < U(g^t)$. Note that $P_i(g) = P_o(g)$ for $o \in \{j,k,l\}$, $P_i(g^t) = P_o(g^t)$ for $o \in \{j,k\}$ and $P_i(g^s) = P_o(g^s)$ for $o \in \{j,k,l\}$. Since

$$P_o(g) = P_o(g^x) = 1 - \underbrace{\left(1 - \alpha\frac{1-(1-b)^2}{2b}\right)}_{q_m(n_m(g)) = q_z(n_z(g))}$$

for $o \in \{r,s\}$ and $x \in \{t,s\}$,

$$P_o(g) = P_o(g^x) = 1 - \underbrace{(1-\alpha)}_{q_r(n_r(g)) = q_s(n_s(g))}\left(1 - \alpha\frac{1-(1-b)^2}{2b}\right)$$

for $o \in \{m,z\}$ and $x \in \{t,s\}$, and $P_i(g) = P_l(g^t)$, then

$$U(g) = (1-a)b\Big(\frac{1}{2} - (3-b)\frac{\alpha}{8}\Big)\Big(2 - (2-b)\alpha\Big),$$

$$U(g^s) = (1-a)b\left[1 - \frac{\alpha}{4}\Big(7 - 3\alpha - b(3-2\alpha)\Big)\right],$$

and

$$U(g^t) = (1-a)b\left[1 - \frac{\alpha}{4}\left(7 - 3b - \alpha\Big(3 - (1 - \frac{b}{8})b\Big)\right)\right].$$

For all parameter values, $U(g) < U(g^s) < U(g^t)$.

One natural question is whether our results can be strengthened by relaxing the assumptions in Proposition 1. The following example shows that this is not possible. Adding a link that closes a triangle does not generally induce higher unemployment rates than adding a link between two distant agents if the conditions in Proposition 1 do not hold. The same applies if we increase the number of triangles keeping constant the number of friends of network members (the degree distribution) but not the number of friends and friends of friends of each person (the joint degree distribution). The conditions in Proposition 1 are, therefore, necessary.

Example 1. Consider the networks in Fig. 2. At an individual level, agent j forms an additional triangle in g compared with g_1. However, because j is not at least as well connected in g_1 as in g, $E_j(g) > E_j(g_1)$ for all parameter values. At a global level, because the degree distribution of g and g_1 is different, $E(g) > E(g_1)$, even though g has an additional triangle to g_1. Likewise, although g_2 and g_1 have the same degree distribution, $E(g_2) > E(g_1)$, because the joint degree distribution of the two networks differs (the networks are not comparable). To see the values of these probabilities, see the online Appendix. ■

Proposition 1 characterizes the effect of varying the number of triangles and squares on employment. Yet, there are different statistics that capture the degree of overlap in friendship circles. The most popular one is the clustering coefficient, which counts the fraction of triangles in i's neighborhood out of the number of all possible triangles among all i's contacts. Formally, the clustering coefficient is $C_i^3(g) = \frac{2t_i}{n_i(g)(n_i(g)-1)}$, where t_i is the number of connections among i's friends. Analogously, the clustering coefficient proposed by Lind et al. (2005) measures the proportion of squares formed by i over the number of all possible squares that i could form with them, $C_i^4(g)$. Due to the popularity of these statistics, we could try to relate unemployment with these coefficients in Proposition 1. However, the relationship between these coefficients and $P_i(g)$ is not one-to-one. Consider for instance the networks

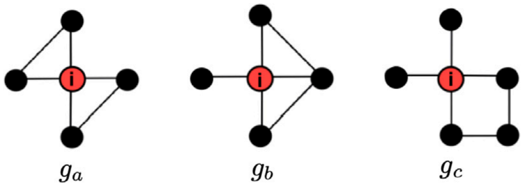

Fig. 3. Triangles, squares, and clustering.

in Fig. 3. Although $C_i^3(g_a) = C_i^3(g_b)$, $P_i(g_a) \neq P_i(g_b)$, because the two triangles in g_b additionally form a four-cycle that affects the information flows reaching i. Likewise, although $C_i^4(g_b) = C_i^4(g_c)$, $P_i(g_b) \neq P_i(g_c)$, because i forms an additional triangle in g_b compared to g_c.

3.2. Discussion

The above results provide support for “The Strength of Weak Ties” (Granovetter, 1973). Granovetter’s hypothesis is that weak ties might be more relevant than strong ones in job information transmission. According to his hypothesis, the strength of the tie between two individuals is closely related to the overlap in their friendship circles such that people linked through strong ties are expected to have a higher proportion of common contacts than people connected through weak ties. Therefore, if individuals involved in weak connections are indeed less likely to have overlapping friendship circles,[11] the advantage of weak ties in job transmission is proved here. Note, however, that the explanation is different from that of Granovetter. His emphasis was on the content of the information provided by these links: people embedded in weak relationships would have access to novel information coming from distant network areas, while individuals with many strong ties are “confined to the provincial news and views of their close friends” (Granovetter, 1983). Proposition 1 uncovers a distinct mechanism: the ability of the weak connections to preserve independence in the information flows that reach the agents. This reveals that weak ties might be more relevant than strong ties even if the content of the information they provide is the same.

Observe that weak ties—those connecting people with no overlapping friendship circles—are more beneficial than strong ones, even if people receive information about job vacancies with probabilities that are independent of their position in the network. However, it could be argued that the probability that i and j receive information about a job vacancy is not a^2, but a function $a(O_{ij}(g))$ that depends positively on the set of triangles and squares they form together, $O_{ij}(g)$. This makes sense: if two agents have many common friends, they are more likely to frequent the same environments and hear about jobs at similar times. Conversely, if they share few contacts, they are more likely to learn about job opportunities independently. If we assume that people with overlapping neighborhoods hear about vacancies with correlated probabilities, Proposition 1 reinforces.

Other important aspects of the model are worth stressing here. First, we have assumed that agents pass information exclusively onto one friend. Our findings are robust to different diffusion protocols. For example, we may assume that information can be passed on (at no cost) to all friends who have lost their job. If all workers are identical (and therefore equally likely to be hired) the probability that i gets the job from j’s information is $1/u_j$, where u_j is the number of friends of j who have been informed by j about the vacancy. Clearly, the probability that j gets a job is the same under both protocols.

Second, the model suggests that the ideal friend is someone who does not know anyone else.[12] This is not very realistic, as people generally want to create links with well-connected people in professional environments. In a world where knowledge flows through chains, friends of friends are not competitors but sources of information. A way of addressing this issue is to assume that the job arrival rate a depends positively on agents’ degree (their number of contacts, $a(n_i(g))$. This assumption is reasonable because the popularity of people results from a variety of factors that might be correlated with the probability of receiving information about vacancies, such as their professional status or the variety of places that they frequent. Under this assumption, the negative impact of a higher number of competitors would rival the positive effect of being connected to someone with a greater knowledge of job opportunities. The main statement of Proposition 1, however, would be unaffected: holding constant the number of friends, and friends of friends, people’s employment prospects decrease with the number of triangles and squares they form with their contacts.

Positive effects of overlapping networks. There may be situations in which being embedded in transitive relationships may pose labor advantages. If jobs are not homogeneous, the higher probability of receiving multiple job offers by individuals with overlapping networks may allow them to discriminate between different offers. This can entail pay increases for people with overlapping friendship networks (as we show in Section 4) or the possibility of accepting jobs more aligned with their preferences. Conversely, people without overlapping networks may be forced to accept a less preferred job, given their lower probability of receiving different offers from which to choose.

In contrast, overlapped contact networks—as compared to networks without triangles and squares—provide a natural ground for norm enforcement (see, e.g., Granovetter, 1985), generating a social obligation to share information within dense circles. This would

[11] This statement is consistent with empirical evidence (see, e.g., Onnela et al., 2007).

[12] However, this is an artifact of the static model. Once the model is repeated, friends of friends keep their friends employed. Hence, having many friends of friends is beneficial in the long run (see Calvó-Armengol and Jackson, 2004).

increase (decrease) the likelihood of receiving information from strong (weak) ties. There would thus be a trade-off between this force and the mechanism detected in the previous section. Characterizing under which condition the social-enforcement mechanism would dominate is an interesting direction for future research.

Setting aside the labor market, there may be other contexts where overlapping friendship networks can be beneficial. One of the most influential theories on this matter, "The Weakness of Long Ties" (Centola and Macy, 2007), argues that clustered networks foster the spread of behaviors that require social reinforcement to be adopted, such as the adoption of technological innovations or participation in high-risk social movements. In this type of contagion processes, where exposure to a sufficiently high number of "adopting contacts" is required for a node to adopt a behavior or technology, overlapping friendship networks would favor the emergence of common sources of contagion and, consequently, propagation.

Our results are compatible with this theory. As an example, consider the networks g and g^t in Fig. 1. Imagine a scenario where what is spread is a behavior requiring multiple sources of reinforcement to be adopted. Each individual can be in one of three states: a promoter of the behavior, with probability α; susceptible to adopting the behavior, with probability b; or not in either of the previous states, with probability $(1-\alpha-b)$. Suppose each promoter can convince only one responsive neighbor per unit of time to adopt the behavior, and this neighbor is randomly picked by the promoter among all of the promoter's responsive contacts. If agent i is responsive, the probability that i is prompted by two promoters is $\alpha^2\left(1-\frac{b}{2}\right)^2$ in network g and α^2 in network g^t. Because social reinforcement is required for adoption, the probability that i adopts the behavior is higher in g^t than in g. Note that the advantage of overlapping neighborhoods in this example does not lie in their ability to generate common sources of contagion (people do not need to observe several adopters to adopt the technology themselves) but in their capacity to create redundancies in the signals received from contacts. Thus, the statements in Proposition 1 uncover an alternative mechanism for clustered networks to spread behaviors that propagate as complex contagions.

4. Wages

We next analyze the effects of overlapping friendship networks on wages. With that aim, we propose an extension of the model to allow agents to receive information about jobs that differ in their wages.

Let $W_i(g)$ be a random variable denoting the wage of agent i in network g. Assume that there are two wage levels in the economy: low-paying positions with wage w_0 and high-paying positions with wage $w_1 > w_0$. Initially, all people are employed in a high-paying job.[13] At stage one, each worker may become unemployed with probability $b \in (0,1)$. Then, each worker hears about a low- or a high-paying job with probabilities a_0 and a_1, respectively, with $a_0 + a_1 = a \in (0,1)$.

Unemployed workers who learn about a job opportunity immediately take the job, regardless of whether the job is high or low paying.[14] Employed workers who learn about a low- or a high-paying job (i.e., individuals who are providers) pass the offer uniformly at random onto one of their unemployed contacts, who immediately accepts the offer. Thus, the probability of being a provider is $\alpha = \alpha_0 + \alpha_1$, where $\alpha_0 = a_0(1-b)$ is the probability of being a provider of a low paying job and $\alpha_1 = a_1(1-b)$ the probability of being a provider of a high paying one. If unemployed workers learn of several vacancies, either directly or through word-of-mouth, they select the one with the highest wage, and the other job positions remain unfilled.

The following proposition clarifies the effect of overlapping friendship networks by showing that the expected wage of people decreases with the number of triangles and squares that they form with their contacts. The expected wage of i in network g is denoted by $E[W_i(g)]$.

Proposition 2. *Consider the networks in Proposition 1. If agent i is at least as well connected in g as in g^x, $E[W_i(g)] > E[W_i(g^x)]$, $x \in \{t,s\}$.*

Proposition 2 complements our previous findings by showing that people's expected wage decreases with the number of connections among their contacts and with the number of contacts their contacts have in common, *ceteris paribus*. Note that forming triangles and/or squares decreases the probability of being employed. Thus, it may not be surprising that the expected wage decreases as people's networks are more tightly clustered, because the wage in unemployment is zero and unemployment has a higher probability in clustered networks. However, we show in the next proposition that, conditional on being employed, the expected wage increases with the number of connections and common contacts among an individual's friends.

Proposition 3. *Consider the networks in Proposition 1. If agent i is comparable in g and g^x, $x \in \{t,s\}$, the expected wage of i conditional on being employed is greater in g^x than in g.*

In other words, employed people are more likely to have a higher wage if their contacts are connected or have common friends. The explanation for this again relies on the lack of independence in information flows from different providers: when i is immersed in a triangle or a square, i is more likely to end up with no offer, but also more likely to receive several offers at once. Receiving multiple information from different sources does not constitute an advantage for obtaining employment, because i can accept only one job. However, concerning wage, receiving multiple offers is beneficial because it allows agents to choose among different job options.

[13] Once again, this assumption is inconsequential.

[14] We assume that unemployed agents who directly hear about a low-paying job cannot afford to wait to see whether some of their contacts pass them a high-paying offer. This assumption is inconsequential: Propositions 2 and 3 hold if unemployed agents who directly hear about a job accept it only if it corresponds to a high-paying offer.

5. Concluding remarks

The model of Calvó-Armengol (2004) provides much of the foundations of the literature on social networks in the labor market. Our paper identifies a driver of employment prospects ignored in this model: the existence of overlapping friendship circles in social networks. When two friends, j and k, of i are connected or have another common contact z, the information that arrives to i is correlated, increasing i's unemployment probability. However, if jobs differ in the level of wages, employed people with common friendship circles earn a wage premium, because they can make use of the multiple received offers by selecting the one that is better paid.

We provide a theoretical proof of Granovetter's core proposition. An extension of the model may also be compatible with "The Weakness of Long Ties", the main competing theory to Granovetter's thesis. It should be noted, however, that we have focused on a particular aspect of weak ties. Our understanding of the strength of weak ties would benefit from exploring the incidence of other factors and mechanisms that might explain their strength or weakness. In the same vein, although the positive relationship between the strength of ties and the degree of overlap in the friendship circles of people connected by them has been generally accepted, there is a scarcity of studies analyzing the validity of this relationship in specific contexts. Exploring this issue might help to resolve the mixed empirical findings concerning the role of weak ties in job information transmission.

We have assumed that people pass information mechanically. While this assumption enables us to isolate the effect of overlapping contact networks on diffusion of job information, the decision to pass on job information might also be strategic. If agents live several periods (as in the dynamic version of the model considered in Calvó-Armengol and Jackson, 2004), the employment status of i in period t depends on the patterns of information exchange in period $t-1$. Anticipating this, people may decide strategically who to pass information onto. Suppose for instance that i, j, and k form a triangle. If i passes information onto j in $t-1$, then j is more likely to be employed in period t and to provide job information to i in that period. At the same time, if i passes an offer onto j, the probability that j competes with i in period t for any job offer of k decreases. Agent i might therefore prefer to pass information onto j rather than onto a friend with whom she does not form any triangle. Thus, one avenue for future research is to endogenize to whom agents pass job information onto. As illustrated in this example, some of our results might reverse under such an approach.

We can extract some policy lessons from our results. First, because employment prospects increase with the number of professional contacts, any policy aimed at increasing the number of connections (such as networking activities or social events) should improve employment prospects. However, these policies can be more effective if they bring people together from distant parts of the social system. In the same vein, given the diffusion inefficiencies induced by overlapping friendship networks, any initiative seeking to spread information related to job openings, training programs, or educational opportunities can be more effective if implemented in loosely knit network areas rather than in more close-knit neighborhoods.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A

Proofs

First we introduce Lemma 1, which is used in the proofs of the propositions. Let $S_i^x(g)$ be a random variable such that $s_i^x(g)=1$ if i is in state $x \in \{1,2,3,4\}$ in network g and $s_i^x(g)=0$ otherwise. Let $C_i(g)=S_i^2(g)+S_i^3(g)$. The realization of $C_i(g)$ is $c_i(g)$: $c_i(g)=1$ if i is a competitor and $c_i(g)=0$ otherwise. We define $N_{jk}^{-i}(g)=\{N_j(g)\cap N_k(g)\}\setminus\{i\}$ as the set of common contacts of j and k in network g excluding i, and $n_{jk}^{-i}(g)$ as the number of common contacts of j and k in g excluding i. If j and k are two friends of i, $n_{jk}^{-i}(g)$ is the number of squares that i forms with j and k in g. The number of agents in $N_{jk}^{-i}(g)$ who are competitors is a random variable $C_{N_{jk}^{-i}}(g)=\sum_{m\in N_{jk}^{-i}(g)} C_m(g)$. The realization of $C_{N_{jk}^{-i}}(g)$ is $c_{N_{jk}^{-i}}(g)$.

Recall that g_{jk} denotes the link between j and k in network g: $g_{jk}=1$ if j and k are linked in g and $g_{jk}=0$ otherwise. Similarly, g_{jk}^x is the link of j and k in network g^x.

Lemma 1. *Assume j and k are two neighbors of i, $z\in N\setminus\{i,j\}$, and i is unemployed. The probability that j does not pass an offer to i conditional on $c_z(g)$ and on $c_{N_{jk}^{-i}}(g)$ is*

$$q_j\left(n_j(g)-n_{jk}^{-i}(g)-g_{jz}\mid c_{N_{jk}^{-i}}(g)+g_{jz}c_z(g),\,s_j^1(g)=1\right)$$

$$=\sum_{x=0}^{n_j(g)-n_{jk}^{-i}(g)-g_{jz}-1}\binom{n_j(g)-n_{jk}^{-i}(g)-g_{jz}-1}{x}b^x(1-b)^{n_j(g)-n_{jk}^{-i}(g)-g_{jz}-1-x}\;\frac{x+c_{N_{jk}^{-i}}(g)+g_{jz}c_z(g)}{x+c_{N_{jk}^{-i}}(g)+g_{jz}c_z(g)+1}.\quad\blacksquare$$

To simplify the notation,

$$q_j\left(n_j(g)-n_{jk}^{-i}(g)-g_{jz}\mid c_{N_{jk}^{-i}}(g)+g_{jz}c_z(g), s_j^1(g)=1\right)=q_j^1\left(n_j(g)-n_{jk}^{-i}(g)-g_{jz}\mid c_{N_{jk}^{-i}}(g)+g_{jz}c_z(g)\right)$$

in what follows.

Example of Lemma 1. Suppose $N_j(g)=\{i,m,l,k\}$ and $N_k(g)=\{i,m,j\}$. Then, $N_{jk}^{-i}(g)=\{m\}$, $n_{jk}^{-i}(g)=1$ and $g_{jk}=1$. The probability that provider j does not pass on an offer to i conditional on $c_{N_{jk}^{-i}}(g)=c_m(g)=1$ and $c_k(g)=0$ is

$$q_j^1\left(n_j(g)-n_{jk}^{-i}(g)-g_{jk}\mid c_{N_{jk}^{-i}}(g)+g_{jk}c_k\right)=q_j^1(2\mid 1)=\frac{2}{3}\underbrace{b}_{l\text{ competitor}}+\frac{1}{2}\underbrace{(1-b).}_{l\text{ not competitor}}$$

If rather $c_{N_{jk}^{-i}}(g)=1$ and $c_k(g)=1$,

$$q_j^1\left(n_j(g)-n_{jk}^{-i}(g)-g_{jk}\mid c_{N_{jk}^{-i}}(g)+g_{jk}c_k\right)=q_j^1(2\mid 2)=\frac{3}{4}\underbrace{b}_{l\text{ competitor}}+\frac{2}{3}\underbrace{(1-b).}_{l\text{ not competitor}}$$

Assume now that $N_j(g)=\{i,m,l,r\}$, $N_k(g)=\{i,m\}$ (so that $n_{jk}^{-i}(g)=1$ and $g_{jk}=0$). The probability that provider j does not pass on an offer to i conditional on $c_{N_{jk}^{-i}}(g)=c_m=1$ and $c_k(g)$ is:

$$q_j^1\left(n_j(g)-n_{jk}^{-i}(g)-g_{jk}\mid c_{N_{jk}^{-i}}(g)+g_{jk}c_k\right)=q_j^1(3\mid 1)=\frac{3}{4}\underbrace{b^2}_{l,\ r\text{ competitors}}+\frac{2}{3}\underbrace{b(1-b)}_{l\text{ competitor, }r\text{ not}}+\frac{2}{3}\underbrace{(1-b)b}_{r\text{ competitor, }l\text{ not}}+\frac{1}{2}\underbrace{(1-b)^2.}_{l,\ r\text{ not competitors}}\quad\blacksquare$$

Observe in this example that $q_j^1(2\mid 2)>q_j^1(2\mid 1)$, $q_j^1(3\mid 1)>q_j^1(2\mid 1)$ and $q_j^1(3\mid 1)=bq_j^1(2\mid 2)+(1-b)q_j^1(2\mid 1)$, as highlighted below.

Remark A. $q_j^1(x\mid y+1)>q_j^1(x\mid y)$ and $q_j^1(x+1\mid y)>q_j^1(x\mid y)$, $x,y\in\mathbb{N}$.

Remark B. $q_j^1(x\mid y)=bq_j^1(x-1\mid y+1)+(1-b)q_j^1(x-1\mid y)$, $x,y\in\mathbb{N}$.

Proof of Proposition 1. *Part (i).* Let us first assume that i is comparable in g, g^t and g^s. Let $Q_i^{jk}(g^x)$ be the probability that neither j nor k pass an offer to i in network $g^x\in\{g,g^t,g^s\}$. The probability that i receives at least one offer from contacts in g^x is

$$P_i(g^x)=1-Q_i^{jk}(g^x)*[Q_i^{-jk}(g^x)\mid \text{no offer from } j \text{ and } k],\tag{7}$$

where $[Q_i^{-jk}(g^x)\mid$ no offer from j and $k]$ is the probability that i receives no information from contacts other than j and k in g^x conditional on having received no offer from j or k in that network. Since all neighbors of i except j and k form the same triangles and squares with i in the three networks, $[Q_i^{-jk}(g)\mid$ no offer from j and $k]=[Q_i^{-jk}(g^x)\mid$ no offer from j and $k]$ for $x\in\{t,s\}$. Then, $P_i(g)>P_i(g^x)$ if and only if $Q_i^{jk}(g^x)>Q_i^{jk}(g)$ for $x\in\{t,s\}$.

From Lemma 1, the probability that provider j does not pass an offer to i conditional on the status of j and k in g can be expressed as

$$\sum_{h=0}^{n_{jk}^{-i}(g)}\binom{n_{jk}^{-i}(g)}{h}b^h(1-b)^{n_{jk}^{-i}(g)-h}q_j^1\Big(n_j(g)-n_{jk}^{-i}(g)-g_{jk}\mid \underbrace{h}_{c_{N_{jk}}^{-i}(g)}+g_{jk}c_k(g)\Big),\tag{8}$$

and similarly for this probability in g^t.

To simplify notation, let $n_x(g)=n_x(g^y)=n_x$ for $x\in\{j,k\}$ and $y\in\{t,s\}$, $n_{jk}^{-i}(g)=n_{jk}^{-i}(g^t)=\eta$, and

$$\sum_\eta=\sum_{h=0}^{n_{jk}^{-i}(g)}\binom{n_{jk}^{-i}(g)}{h}b^h(1-b)^{n_{jk}^{-i}(g)-h}.$$

Then, (8) is

$$\sum_\eta q_j^1\Big(n_j-\eta\mid h\Big).\tag{9}$$

Table 2
Probability that neither j nor k will pass on an offer to i in g and g^t (Proposition 1).

case $x \in \{\text{A,B}, \dots, \text{F}\}$	prob. case x	$[Q_i^{jk}(g) \mid \text{case } x]$	$[Q_i^{jk}(g^t) \mid \text{case } x]$
A: j,k providers	α^2	$\sum_\eta q_j^1(n_j-\eta \mid h) q_k^1(n_k-\eta \mid h)$	$\sum_\eta q_j^1(n_j-\eta-1 \mid h) q_k^1(n_k-\eta-1 \mid h)$
B: j provider, k competitor	αb	$\sum_\eta q_j^1(n_j-\eta \mid h)$	$\sum_\eta q_j^1(n_j-\eta-\underbrace{1}_{g^t_{jk}} \mid h+\underbrace{1}_{g^t_{jk}c_k(g^t)})$
C: j competitor, k provider	$b\alpha$	$\sum_\eta q_k^1(n_k-\eta \mid h)$	$\sum_\eta q_k^1(n_k-\eta-\underbrace{1}_{g^t_{jk}} \mid h+\underbrace{1}_{g^t_{jk}c_j(g^t)})$
D: j provider, k in status 4	$\alpha(1-\alpha-b)$	$\sum_\eta q_j^1(n_j-\eta \mid h)$	$\sum_\eta q_j^1(n_j-\eta-1 \mid h)$
E: j in status 4, k provider	$(1-\alpha-b)\alpha$	$\sum_\eta q_k^1(n_k-\eta \mid h)$	$\sum_\eta q_k^1(n_k-\eta-1 \mid h)$
F: j,k not providers	$(1-\alpha)^2$	1	1

Table 2 contains the probability that neither j nor k pass an offer to i in g and in g^t conditional on each possible state of j and k. Considering these probabilities and Remark B,

$$Q_i^{jk}(g^t) - Q_i^{jk}(g) = \sum_{x \in \{\text{A,B}, \dots, \text{F}\}} \Big(\text{prob. case } x\Big) * \Big([Q_i^{jk}(g^t) \mid \text{case } x] - [Q_i^{jk}(g) \mid \text{case } x]\Big)$$

$$= \alpha^2 \sum_\eta \Bigg[\Big(q_j^1(n_j-\eta \mid h) - q_j^1(n_j-\eta-1 \mid h)\Big)\Big(1 - q_k^1(n_k-\eta-1 \mid h)\Big) + \Big(q_k^1(n_k-\eta \mid h) - q_k^1(n_k-\eta-1 \mid h)\Big) \Big(1 - q_j^1(n_j-\eta \mid h)\Big)\Bigg], \tag{10}$$

which is positive by Remark A. Then, $P_i(g) > P_i(g^t)$.

In g^s, i forms $n_{jk}^{-i}(g^s) = \eta + 1$ squares with j and k. Then, the probability that provider j does not pass an offer to i conditional on the status of j and k is the corresponding expression to (9) in g^s, that is,

$$\sum_{\eta+1} q_j^1\Big(n_j - \eta - 1 \mid h\Big) = \sum_\eta q_j^1\Big(n_j - \eta \mid h\Big). \tag{11}$$

Table 3 contains the probability that neither j nor k pass an offer to i in g^s conditional on each possible state of j and k. Considering these probabilities and Remark B,

$$Q_i^{jk}(g^s) - Q_i^{jk}(g) = \sum_{x \in \{\text{A,B}, \dots, \text{F}\}} \Big(\text{prob. case } x\Big) * \Big([Q_i^{jk}(g^s) \mid \text{case } x] - [Q_i^{jk}(g) \mid \text{case } x]\Big)$$

$$= \sum_\eta b(1-b)\Bigg[\Big(q_k^1\Big(n_k-\eta-1 \mid h+1\Big)\Big) - \Big(q_k^1\Big(n_k-\eta-1 \mid h\Big)\Big)\Bigg]\Bigg[\Big(q_j^1\Big(n_j-\eta-1 \mid h+1\Big)\Big) - \Big(q_j^1\Big(n_j-\eta-1 \mid h\Big)\Big)\Bigg], \tag{12}$$

which is positive by Remark A. Hence, $P_i(g) - P_i(g^s) > 0$.

The expression (10) is based on the assumption that i is comparable in g and in g^t. If the degree of any contact of i in g is lower than in g^t, then the expression (10) increases. Suppose for instance that $n_j(g) = n_j(g^t) - x$. Then, by Remark A, all the probabilities in the third column of Table 2 are lower as in the previous case (where $n_j(g) = n_j(g^t)$). The same is true for (12): it increases if the degree of any contact of i in g is lower than in g^s. Similarly, if the degree of i is greater in g than in g^t (g^s), e.g., because m is a friend of i in g but not in g^t (g^s), then the probability that i receives no information from m is lower than 1 in network g, while it is 1 in g^t (g^s). Then $P_i(g) > P_i(g^x)$, $g^x \in \{g^s, g^t\}$.

Part (ii). If i is comparable in g^t and in g^s, applying Remark B and operating,

$$Q_i^{jk}(g^t) - Q_i^{jk}(g^s) = \sum_{x \in \{\text{A,B}, \dots, \text{F}\}} \Big(\text{prob. case } x\Big) * \Big([Q_i^{jk}(g^t) \mid \text{case } x] - [Q_i^{jk}(g^s) \mid \text{case } x]\Big) =$$

$$\sum_\eta \Bigg[\Big(q_j^1(n_j-\eta \mid h) - q_j^1(n_j-\eta-1 \mid h)\Big)\Big(1 - q_k^1(n_k-\eta-1 \mid h+1)\Big)$$

$$+ \Big(q_k^1(n_k-\eta-1 \mid h+1) - q_k^1(n_k-\eta-1 \mid h)\Big)\Big(1 - q_j^1(n_j-\eta-1 \mid h)\Big)\Bigg] > 0.$$

Thereby, $P_i(g^s) > P_i(g^t)$. By the arguments above, the difference $P_i(g^s) - P_i(g^t)$ increases if i's degree is greater in g^s than in g^t or the degree of some contact of i is lower in g^s than in g^t.

Part (iii). As shown above, $P_i(g) > P_i(g^s) > P_i(g^t)$, meaning that $E_i(g) > E_i(g^s) > E_i(g^t)$. By the same arguments, $E_x(g) > E_x(g^s) > E_x(g^t)$ for $x \in \{j,k,l\}$. Since agents not in $\{i,j,k,l\}$ form identical triangles and squares in the three networks and have the same number of friends and friends of friends, $E_x(g) = E_x(g^t) = E_x(g^s)$ $\forall x \notin \{i,j,k,l\}$. This completes the proof. ■

Table 3
Probability that neither j nor k will pass on an offer to i in g^s (Proposition 1).

case $x \in \{A,B,\dots,F\}$	prob. case x	$[Q_i^{jk}(g^s)\ \lvert \text{case } x]$
A: j,k providers	α^2	$\sum_\eta \left[\underbrace{b}_{l \text{ competitor}} \left(q_j^1(n_j-\eta-\underbrace{1}_{g^s_{jl}} \mid h+\underbrace{1}_{g^s_{jl}c_l(g^s)}) q_k^1(n_k-\eta-1 \mid h+1) \right) \right.$ $+ \sum_\eta \left[\underbrace{(1-b)}_{l \text{ non competitor}} \left(q_j^1(n_j-\eta-\underbrace{1}_{g^s_{jl}} \mid h+\underbrace{0}_{g^s_{jl}c_l(g^s)}) q_k^1(n_k-\eta-1 \mid h) \right) \right.$
B: j provider, k competitor	αb	$\sum_\eta q_j^1(n_j-\eta \mid h)$
C: j competitor, k provider	$b\alpha$	$\sum_\eta q_k^1(n_k-\eta \mid h)$
D: j provider, k in status 4	$\alpha(1-\alpha-b)$	$\sum_\eta q_j^1(n_j-\eta \mid h)$
E: j in status 4, k provider	$(1-\alpha-b)\alpha$	$\sum_\eta q_k^1(n_k-\eta \mid h)$
F: j,k not providers	$(1-\alpha)^2$	1

Proof of Proposition 2. Suppose first that i is comparable in g and in g^t, $n_x(g) = n_x(g^y) = n_x$ for $x \in \{j,k\}$ and $y \in \{t,s\}$. The number of squares that i forms with j and k in g is $n_{jk}^{-i}(g) = n_{jk}^{-i}(g^t) = \eta$.

(i) *We first proof that* $E[W_i(g)] > E[W_i(g^t)]$.

Scenario (a) Suppose that i did not receive an offer from an agent other than j and k. In such a case, the expected wage of i in g^t (g^s) depends exclusively on the information flows from j and k in g^t (g^s). Let $E[W_i^{jk}(g)]$ be the expected wage of agent i in g if we consider only the information coming from j and k. Table 4 lists all possible states of j and k (with the exception of the one in which neither j nor k are providers), as well as $E[W_i^{jk}(g)]$ and $E[W_i^{jk}(g^t)]$ conditional on each case. For a better understanding of the probabilities in this table, we recommend that the reader look at the proof of Proposition 1 (expressions (8) and (9)). Based on the probabilities in Table 4 and applying Remark B,

$$
\begin{aligned}
&E[W_i^{jk}(g^t)] - E[W_i^{jk}(g)] = \sum_{x\in\{A,B,\dots,L\}} \Big(\text{prob. case } x\Big) * \Big(E[W_i^{jk}(g^t) \mid \text{case } x] - E[W_i^{jk}(g) \mid \text{case } x]\Big) \\
&= \sum_\eta -(\alpha_0^2 w_0 + \alpha_1^2 w_1) \Big[\Big(q_k^1(n_k-\eta \mid h) - q_k^1(n_k-1-\eta \mid h)\Big)\Big(1-q_j^1(n_j-\eta \mid h)\Big) + \\
&\Big(q_j^1(n_j-\eta \mid h) - q_j^1(n_j-1-\eta \mid h)\Big) * \Big(1-q_k^1(n_k-1-\eta \mid h)\Big)\Big] - \alpha_0\alpha_1 * w_0 \Big[\Big(q_j^1(n_j-\eta \mid h) - q_j^1(n_j-1-\eta \mid h)\Big) \\
&\Big(2-q_k^1(n_k-\eta \mid h) - q_k^1(n_k-1-\eta \mid h)\Big) + \Big(q_k^1(n_k-\eta \mid h) - q_k^1(n_k-1-\eta \mid h)\Big) \\
&\Big(2-q_j^1(n_j-\eta \mid h) - q_j^1(n_j-1-\eta \mid h)\Big)\Big],
\end{aligned}
\tag{13}
$$

which is negative by Remark A.

Scenario (b). Suppose that individual i receives a job offer for a low-paying job from an agent other than j and k and does not receive any offer for a high-paying job. In this scenario, the expected wage of individual i is higher in network g compared to network g^t if she has a greater probability of receiving a high-paying job from agents j and k in the former network. The difference in the probability of receiving a high-paying job from agents j and k in networks g and g^t is equal to the sum of the terms multiplying w_1 in Eq. (13),

$$
-\alpha_1^2 \sum_\eta \Big[\Big(q_k^1(n_k-\eta \mid h) - q_k^1(n_k-1-\eta \mid h)\Big)\Big(1-q_j^1(n_j-\eta \mid h)\Big) + \Big(q_j^1(n_j-\eta \mid h) - q_j^1(n_j-1-\eta \mid h)\Big)\Big(1-q_k^1(n_k-1-\eta \mid h)\Big)\Big].
$$

Since this difference is negative, the expected wage conditional on this scenario is greater in g than in g^t.

Scenario (c) If i receives at least one high-paying job offer from an agent other than j and k, i's expected wage is w_1 in both networks, regardless of the information flows from j and k.

(ii) We show now that $E[W_i(g)] > E[W_i(g^s)]$.

Scenario (a). If i does not receive an offer from any s other than j and k, the expected wage of i depends only on the information flows from j and from k. Let $P_i^{jk}(g) = 1 - Q_i^{jk}(g)$ be the probability that i receives at least one offer from j and k in network g. Since (12) is positive, $P_i^{jk}(g^s) < P_i^{jk}(g)$, and

$$
E[W_i^{jk}(g^s) \mid \text{case A}] = P_i^{jk}(g^s) w_1 < E[W_i^{jk}(g) \mid \text{case A}] = P_i^{jk}(g) w_1,
$$

where case A is defined in Table 4. Analogously, $E[W_i^{jk}(g^s) \mid \text{case B}] < E[W_i^{jk}(g) \mid \text{case } B]$.

Note that

$$
E[W_i^{jk}(g) \mid \text{case C}] = \sum_\eta \Big(1-q_j^1(n_j-\eta \mid h)\Big) w_1 + q_j^1(n_j-\eta \mid h)\Big(1-q_k^1(n_k-\eta \mid h)\Big) w_0,
\tag{14}
$$

and

$$E[W_i^{jk}(g^s) \mid \text{case C}] = \sum_\eta \Big(1 - q_j^1(n_j - \eta \mid h)\Big) w_1 + \Big[\underbrace{b}_{l \text{ competitor}} q_j^1(n_j - \eta - \underbrace{1}_{g^s_{jl}=1} \mid h + \underbrace{1}_{g^s_{jl} c_l(g^s)}) \Big(1 - q_k^1(n_k - \eta - 1 \mid h+1)\Big) + (1-b) q_j^1(n_j - \eta - 1 \mid h)\Big(1 - q_k^1(n_k - \eta - 1 \mid h)\Big) \Big] w_0. \tag{15}$$

Subtracting (14)–(15), operating and applying Remark B,

$$E[W_i^{jk}(g) \mid \text{case C}] - E[W_i^{jk}(g^s) \mid \text{case C}] = \sum_\eta (1-b)\Big(q_k^1(n_k - \eta - 1 \mid h+1) - q_k^1(n_k - \eta - 1 \mid h)\Big) \Big(q_j^1(n_j - \eta \mid h) - q_j^1(n_j - \eta - 1 \mid h)\Big) w_0 + q_j^1(n_j - \eta \mid h)\Big(1 - q_k^1(n_k - \eta \mid h)\Big) w_1 > 0. \tag{16}$$

Analogously, $E[W_i^{jk}(g) \mid \text{case D}] - E[W_i^{jk}(g^s) \mid \text{case D}] > 0$. Since the only difference in the information flows from j and k to i in g and in g^s corresponds to the cases in which j and k are providers (i.e., cases A-D), then $E[W_i^{jk}(g)] > E[W_i^{jk}(g^s)] > 0$.

Scenario (b). If individual i receives a low-paying job offer from an agent other than j and k but does not receive any offer for a high-paying job, then $E[W_i(g)] > E[W_i(g^s)] > 0$ if and only if the probability that i receives a high-paying job offer from j and k is greater in network g than in network g^s.

The probability that i receives information about a high-paying job when j has information about a high-paying job and k does not is the same in both networks (i.e., $\sum_\eta q_j^1(n_k - \eta \mid h)$), and likewise when k has information about a high-paying job and j does not. If both j and k have information about a high-paying job, the probability that at least one of them passes the job offer to i is greater in network g than in network g^s (see (12)). Therefore, the expected wage of i under this scenario is higher in network g than in network g^s.

Scenario (c). If individual i receives information about a high-paying job from an agent other than j and k, the expected wage is the same in both network g and network g^s.

(iii) The above analysis compares the expected wage of individual i in networks g and g^t (g^s), assuming that i is comparable in both networks. If i is at least as well connected in g as in g^t (g^s), the above results are further strengthened. It is important to note that when i is at least as well connected in network g as in g^t (g^s), the probability of her receiving information from each neighbor is at least as high in network g as in g^t (g^s). On the other hand, the probability that i receives information from *at least one* neighbor is greater in network g than in g^t (g^s) due to the correlation of information flows. Therefore, if i is at least as well connected in network g as in g^t (g^s), then $E[W_i(g)] > E[W_i(g^x)]$, for $x \in \{t, s\}$. ■

Proof of Proposition 3. To simplify the notation, let $n_x(g) = n_x(g^y) = n_x$ for $x \in \{j,k\}$ and $y \in \{t,s\}$, and $n_{jk}^{-i}(g) = n_{jk}^{-i}(g^t) = \eta$.

Since the difference in i's expected wage between g and g^t arises solely from network transmission by j and k, we will focus on i's expected wage conditioned on being employed through j or k. To calculate i's expected wage conditioned on having received an offer from either j or k, we divide i's expected wage in each case in Table 4 by the probability that i receives at least one offer from these neighbors in each respective case. For a more detailed explanation of the probabilities presented in Table 4, we recommend referring to the proof of Proposition 1.

Define $E[W_i^{jk}(g) \mid \text{case } x, \text{offer from } j,k]$ as:

$$E[W_i^{jk}(g) \mid \text{case } x, \text{offer from } j,k] = \frac{E[W_i^{jk}(g) \mid \text{case } x]}{[P_i^{jk}(g) \mid \text{case } x]}.$$

To simplify notation, $E[W_i^{jk}(g) \mid \text{case } x, \text{offer from } j,k] = \bar{E}[W_i^{jk}(g) \mid \text{case } x]$. It is straightforward to note that $\bar{E}[W_i^{jk}(g) \mid \text{case } x] = \bar{E}[W_i^{jk}(g^t) \mid \text{case } x] = \bar{E}[W_i^{jk}(g^s) \mid \text{case } x]$ for all cases but cases C and D.

(i). We show that the expected wage of i conditioned on being employed is greater in g^t than in g.

Note that

$$\bar{E}[W_i^{jk}(g) \mid \text{case C}] = \frac{\Big[\Big(1 - q_j^1(n_j - \eta \mid h)\Big) w_1 + q_j^1(n_j - \eta \mid h)\Big(1 - q_k^1(n_k - \eta \mid h)\Big) w_o\Big]}{1 - q_j^1(n_j - \eta \mid h) q_k^1(n_k - \eta \mid h)} = w_0 + \sum_\eta \frac{1 - q_j^1(n_j - \eta \mid h)}{1 - q_j^1(n_j - \eta \mid h) q_k^1(n_k - \eta \mid h)} (w_1 - w_0). \tag{17}$$

and analogously for $\bar{E}[W_i^{jk}(g) \mid \text{case D}]$. Then

$$\bar{E}[W_i^{jk}(g) \mid \text{case C}] + \bar{E}[W_i^{jk}(g) \mid \text{case D}] = 2w_0 + \sum_\eta \frac{2 - q_j^1(n_j - \eta \mid h) - q_k^1(n_k - \eta \mid h)}{1 - q_j^1(n_j - \eta \mid h) q_k^1(n_k - \eta \mid h)} (w_1 - w_0). \tag{18}$$

The corresponding expression to (18) in network g^t is

$$\bar{E}[W_i^{jk}(g^t) \mid \text{case C}] + \bar{E}[W_i^{jk}(g^t) \mid \text{case D}] = 2w_0 + \sum_\eta \frac{2 - q_j^1(n_j - \eta - 1 \mid h) - q_k^1(n_k - \eta - 1 \mid h)}{1 - q_j^1(n_j - \eta - 1 \mid h) q_k^1(n_k - \eta - 1 \mid h)} (w_1 - w_0). \tag{19}$$

Table 4
Expected wage of i in function of the information received by j and k.

case $x \in \{\text{A,B}, \dots, \text{L}\}$	prob. case x	$E[W_i^{jk}(g) \mid \text{case} \quad x]$	$E[W_i^{jk}(g^t) \mid \textit{case} \quad x]$
A: j,k low providers	α_0^2	$\sum_\eta \left[1-q_j^1(n_j-\eta \mid \underbrace{h}_{c^{-i}_{N_{jk}}(g)})q_k^1(n_k-\eta \mid h)\right]w_0$	$\sum_\eta \left[1-q_j^1(n_j-\eta-\underbrace{1}_{g^t_{jk}} \mid h)q_k^1(n_k-\eta-1 \mid h)\right]w_0$
B: j,k high providers	α_1^2	$\sum_\eta \left[1-q_j^1(n_j-\eta \mid h)q_k^1(n_k-\eta \mid h)\right]w_1$	$\sum_\eta \left[1-q_j^1(n_j-\eta-1 \mid h)q_k^1(n_k-\eta-1 \mid h)\right]w_1$
C: j high provider, k low provider	$\alpha_0\alpha_1$	$\sum_\eta \left[\left(1-q_j^1(n_j-\eta \mid h)\right)w_1 + q_j^1(n_j-\eta \mid h) * \left(1-q_k^1(n_k-\eta \mid h)\right)w_0\right]$	$\sum_\eta \left[\left(1-q_j^1(n_j-\eta-1 \mid h)\right)w_1 + q_j^1(n_j-\eta-1 \mid h)\left(1-q_k^1(n_k-\eta-1 \mid h)\right)w_0\right]$
D: k high provider, j low provider	$\alpha_1\alpha_0$	$\sum_\eta \left[\left(1-q_k^1(n_k-\eta \mid h)\right)w_1 + q_k^1(n_k-\eta \mid h)\left(1-q_j^1(n_j-\eta \mid h)\right)w_0\right]$	$\sum_\eta \left[\left(1-q_k^1(n_k-\eta-1 \mid h)\right)w_1 + q_k^1(n_k-\eta-1 \mid h)\left(1-q_j^1(n_j-\eta-1 \mid h)\right)w_0\right]$
E: k high provider, j unemployed	$\alpha_1 b$	$\sum_\eta \left(1-q_k^1(n_k-\eta \mid h)\right)w_1$	$\sum_\eta \left(1-q_k^1(n_k-\eta-\underbrace{1}_{g^t_{jk}} \mid h+\underbrace{1}_{g^t_{jk}c_j(g^t)})\right)w_1$
F: k high provider, j neither provider nor unemployed	$\alpha_1(1-\alpha-b)$	$\sum_\eta \left(1-q_k^1(n_k-\eta \mid h)\right)w_1$	$\sum_\eta \left(1-q_k^1(n_k-\eta-1 \mid h)\right)w_1$
G: j high provider, k unemployed	$b\alpha_1$	$\sum_\eta \left(1-q_j^1(n_j-\eta \mid h)\right)w_1$	$\sum_\eta \left(1-q_j^1(n_j-\eta-1 \mid h+1)\right)w_1$
H: j high provider, k neither provider nor unemployed	$(1-\alpha-b)\alpha_1$	$\sum_\eta \left(1-q_j^1(n_j-\eta \mid h)\right)w_1$	$\sum_\eta \left(1-q_j^1(n_j-\eta-1 \mid h)\right)w_1$
I: j low provider, k unemployed	$b\alpha_0$	$\sum_\eta \left(1-q_j^1(n_j-\eta \mid h)\right)w_0$	$\sum_\eta \left(1-q_j^1(n_j-\eta-1 \mid h+1)\right)w_0$
J: j low provider, k neither provider nor unemployed	$(1-\alpha-b)\alpha_0$	$\sum_\eta \left(1-q_j^1(n_j-\eta \mid h)\right)w_0$	$\sum_\eta \left(1-q_j^1(n_j-\eta-1 \mid h)\right)w_0$
K: k low provider, j unemployed	$\alpha_0 b$	$\sum_\eta \left(1-q_k^1(n_k-\eta \mid h)\right)w_0$	$\sum_\eta \left(1-q_k^1(n_k-\eta-1 \mid h+1)\right)w_0$
L: k low provider, j neither provider nor unemployed	$\alpha_0(1-\alpha-b)$	$\sum_\eta w_0\left(1-q_k^1(n_k-\eta \mid h)\right)\ w_0$	$\sum_\eta \left(1-q_k^1(n_k-\eta-1 \mid h)\right)w_0$

Note that (19) is greater than (18) if

$$\begin{aligned}&\sum_\eta \left[2-q_j^1(n_j-\eta \mid h)-q_k^1(n_k-\eta \mid h)\right]\left[1-q_j^1(n_j-\eta-1 \mid h)q_k^1(n_k-\eta-1 \mid h)\right]\\ &-\left[2-q_j^1(n_j-\eta-1 \mid h)-q_k^1(n_k-\eta-1 \mid h)\right]\left[1-q_j^1(n_j-\eta \mid h)q_k^1(n_k-\eta \mid h)\right] \le 0.\end{aligned} \tag{20}$$

Operating in (20)

$$\begin{aligned}&\sum_\eta \left[q_j^1(n_j-\eta-1 \mid h)-q_j^1(n_j-\eta \mid h)\right]\left[1-q_k^1(n_k-\eta-1 \mid h)\left(2-q_k^1(n_k-\eta \mid h\right)\right]\\ &+\left[q_k^1(n_k-\eta-1 \mid h)-q_k^1(n_k-\eta \mid h)\right]\left[1-q_j^1(n_j-\eta \mid h)\left(2-q_j^1(n_j-\eta-1 \mid h\right)\right] \le 0,\end{aligned} \tag{21}$$

and the result follows.

(ii) We prove that the expected wage conditional on being employed is greater in g^s than in g.

Let $P_i^j(g)$ probability that i receives information from j in g. Recall that $P_i^{jk}(g)$ is the probability that i receives at least one offer from contacts j and k. Note that

$$\begin{aligned}\bar{E}[W_i^{jk}(g^x) \mid \text{case C}] &= \frac{E[W_i^{jk}(g^x) \mid \text{case C}]}{[P_i^{jk}(g^x) \mid \text{case C}]} = \frac{[P_i^j(g^x) \mid \text{case C}]w_1 + \overbrace{\left[[P_i^k(g^x) \mid \text{case C}]-[P_i^{jk}(g^x) \mid \text{case C}]\right]}^{\text{prob. only } k \text{ passes an offer to } i} w_0}{[P_i^{jk}(g^x) \mid \text{case C}]}\\ &= \frac{[P_i^j(g^x) \mid \text{case C}]w_1 + [P_i^k(g^x) \mid \text{case C}]w_0}{[P_i^{jk}(g^x) \mid \text{case C}]} - w_0.\end{aligned} \tag{22}$$

As explained above, $P_i^y(g) = P_i^y(g^s)$ for $y \in \{j,k\}$. Since (12) is positive, $[P_i^{jk}(g) \mid \text{case C}] > [P_i^{jk}(g^s) \mid \text{case C}]$, meaning that $\bar{E}[W_i^{jk}(g^s) \mid \text{case C}] > \bar{E}[W_i^{jk}(g) \mid \text{case C}]$. Following the same reasoning, $\bar{E}[W_i^{jk}(g^s) \mid \text{case D}] > \bar{E}[W_i^{jk}(g^s) \mid \text{case D}]$. ■

Appendix B. Supplementary data

Supplementary material related to this article can be found online at https://doi.org/10.1016/j.euroecorev.2023.104594.

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