To Follow the Lead or Retrocede to Followers? An Auction Model of the Reinsurance Market*

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Abstract

This study models competing reinsurance syndicates vying for underwriting risk in a common-value setting. In the market-wide follow-the-lead practice, the lead reinsurer makes an offer directly to clients based on their risk assessment, while followers, who typically provide capital and capacity, are locked into a single unit price determined at the tender stage. Whether this premium alignment feature benefits the client remains underexplored. Inspired by the design of spectrum auctions, we restructure the allocation process into multi-stages, with information revelation occurring between these stages, referred to as the retrocession case. In this scenario, the lead reinsurer makes an offer for the entire business and cedes partial risk to the followers. Similar risk allocation and capital-saving outcomes are achieved. We compare this situation with the follow-the-lead practice and find that, under the follow-the-lead scenario, lead reinsurers shade their offers to avoid the winner's curse, allowing followers to benefit. Conversely, in the case of a retrocession, lead reinsurers make more aggressive offers to signal information. This design benefits the initial client insurer, which is achieved through an information transmission channel.

JEL Classification: C72; D44; D47; G22

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1 Introduction

Rising threats, such as catastrophic climate change, and pandemics fuel a surge in demand for reinsurance.[1] In response, reinsurers often form syndicates to diversify risks.[2] A syndicate has a lead reinsurer and followers. The role of the lead is to determine the terms and conditions based on expertise and experience. The role of followers is to provide capital and the capacity to hold risk. Terms and conditions are typically uniform for all reinsurers.[3] This is referred to as the *follow-the-lead* practice, or premiums alignment. Does this practice truly benefit clients, who are insurers in this market? Are there other ways to organize the market to generate better terms for client insurers?

To address these questions, this study develops a theoretical model to analyze the follow-the-lead practice and propose a new design, hereafter referred to as the *retrocession case*.[4] We show that if the allocation process is restructured into multi-stages, in which the lead reinsurer makes an offer for the business first and cedes risks to followers, with information disclosed in between, this may not only favor the lead reinsurer but, more importantly, benefit the initial client insurer.

Our contributions to the literature are as follows. First, our modeling framework is novel in reinsurance economics. We introduce asymmetric information between reinsurers and model their competition as a first-price common-value auction, which is a natural choice given their information structure. This contrasts with existing studies that model price formation through Cournot-type interactions (e.g., Powers and Shubik (2001); Boulatov and Dieckmann (2013)), Stackelberg-type interactions (e.g., Bäuerle and Glauner (2018); Chen et al. (2020)), or within a general equilibrium framework (e.g., Borch (1992); Bernis (2002); Chi and Tan (2013); Boonen et al. (2021)). Second, to the best of our knowledge, this is the first study to analyze this market practice and design a reinsurance market. Third, our model is based on auctions with resale, where the object being sold is divisible in our setting. Our results may also be applicable to other markets with divisible goods like energy markets (Anatolitis et al., 2022). Fourth, in addition to other financial markets featuring common values, such as IPO auctions where

[1] Deloitte reports that for US non-life insurance in 2022, it was the eighth consecutive year featuring at least 10 US catastrophes, causing over US$1 billion in losses. Property-catastrophe reinsurance costs for primary non-life carriers were driven up by 30.1% in 2023, which was double the prior year's hike of 14.8%. The US demand for catastrophe reinsurance alone is expected to grow as much as 15% by 2024, putting further pressure on prices.

[2] As of 31 December 2022, there were 77 syndicates, 8 special purpose arrangements, and 7 syndicates in a box (SIAB) at Lloyd's, a leading insurance and reinsurance marketplace located in London, United Kingdom. Source: https://www.lloyds.com/about-lloyds/our-market/lloyds-market

[3] See https://www.investopedia.com/terms/l/lead-reinsurer.asp. See also https://www.investopedia.com/terms/c/coreinsurance.asp

[4] Retrocession is a transaction in which a reinsurer transfers risks it has already insured to other reinsurers. After signing a treaty with the client insurer, the retrocedent (the original reinsurer) cedes part of the risks it has assumed to retrocessionaires. Source: https://blog.ccr-re.com/en/what-is-retrocession

investors have to estimate the future cash flows of firms (Sherman, 2005), or securities markets (Yuan, 2022), we emphasize the common value feature in the insurance and reinsurance market, given that the risk can be estimated ex-ante and realized and fixed ex-post.

Below, we briefly outline the model, key results, and underlying intuitions. The model is depicted in Figure 1. The lead reinsurer makes an offer on behalf of the syndicate to the client. Other syndicates also make offers, and the client selects the most favorable one to underwrite the risk. Followers then subscribe to the pre-contractual shares at the same price as their lead reinsurer. In the follow-the-lead case, lead reinsurers shade their offers in the tender stage to protect themselves against the *winner's curse*,[5] earning a positive surplus. Since followers subscribe to a portion of the business at the same unit price as the lead, they benefit from the leader's expertise in pricing and also earn positive payoffs in equilibrium.

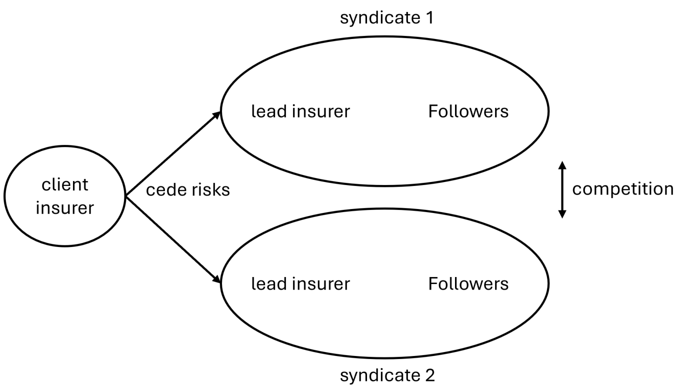

Figure 1: An illustration of the model

In contrast, in the retrocession design, the allocation process is divided into multiple stages, with offers disclosure occurring in between. The lead reinsurer makes an offer for the entire business and then sells part of the risk to followers to save capital. The client insurer, as the initial auctioneer, collects offers and reveals them to the follow market to promote transparency, allowing followers to have some information for evaluating the risk. In the follow market, followers with unit demand purchase the remaining risk in a first-price auction. Ultimately, the lead reinsurer retains a share while the followers hold the remaining risk. This approach achieves similar objectives in terms of risk allocation and capital savings as the follow-the-lead case.

The main result is that clients benefit more from the retrocession case than from the follow-the-lead case. In the retrocession case, a lead reinsurer balances the potential to exploit private

[5]The "winner's curse" is a phenomenon often observed in auctions and competitive bidding situations. It occurs when the winning bidder ends up overpaying for an asset due to overly optimistic estimates. This behavior is considered irrational in auctions. One behavioral concept to model this behavior is the "cursed equilibrium", proposed by Eyster and Rabin (2005).

information by retaining more shares and increasing capital savings by retaining less. Given the informational linkage across two stages, a lead reinsurer makes a more aggressive offer. The additional signaling component arises from the disclosure of bids, reducing uncertainty and limiting reinsurers' ability to leverage private information for profit. In the second stage, with offers from the first stage made public, followers' payoffs are reduced and drop to zero as there is no private information to exploit. Their surplus is extracted and transferred to the client through the lead reinsurer's aggressive bidding in the first stage.

Justification of Our Design. The optimal selling mechanism for an object when bidders' values are correlated is well-established in theory. Crémer and McLean (1985) (hereafter CM) show that when bidders' values are slightly correlated, the entire rent can be extracted. McAfee et al. (1989) extends this result to cases where agents' types are continuously distributed. The details of this mechanism are provided in Appendix C.

Though the full surplus extraction result is insightful, it has been criticized as unrealistic and has not been observed in practice. Several theoretical explanations include bidders' risk aversion and limited liability (See Robert (1991)), information acquisition about competitors' types (See Bikhchandani (2010)), and non-robustness to bidders' beliefs (See Pham and Yamashita (2024)). Börgers (2015) argues that "one should view the Cremer-McLean result as a paradox rather than guidance for constructing practical mechanisms."

Our design is motivated by Milgrom's spectrum auction design for the Federal Communications Commission, specifically the Simultaneous Multi-Round Auction (SMRA) (Milgrom, 2000). In SMRA, the original single-round bidding process is restructured into multiple rounds, with some bidding information revealed between stages to encourage more aggressive bidding in subsequent rounds. Although our model is not as sophisticated as the SMRA, which involves repeated bidding and multiunit allocation, it borrows some of its key features. For example, we restructure the allocation process from static, one-time allocation to a two-round dynamic allocation, accounting for the information disparity between lead reinsurers and followers. Additionally, bidding information from the first round is disclosed before the opening of the second round, establishing informational linkages across markets to promote aggressive bidding. Moreover, this phased disclosure enhances transparency by allowing followers to better understand the risk environment, leading to more informed decision-making and improved price discovery.

Another merit of the retrocession design is that the lead reinsurer in the syndicate is more willing to participate, as their equilibrium payoffs are higher under this structure. Although the lead reinsurer bids more aggressively, the signaling component pertains only to the cession shares allocated to followers, which is compensated by followers in secondary markets. Our design may shed light on the organization of this market and other markets with common-value features for policymakers.

Related Literature. Our study relates to the literature on economics of reinsurance. The pioneering work by Borch (1992) studies optimal risk-sharing among reinsurers to rationalize the syndication structure in the general equilibrium framework. Plantin (2006) provides a rationale for reinsurers arising endogenously from risk managers by focusing on their ability to mitigate the moral hazard problem faced by the cedant. Studies on the strategic interaction between insurers and reinsurers, typically in actuarial pricing. Zhu et al. (2023) study how competing reinsurers strategically set prices using the Stackelberg model. However, little attention has been given to follow-the-lead practices from an economic perspective. This study is the first to explore market-wide practices and design this market.

We are related to the literature on common-value auctions with a resale market. Bukhchandani and Huang (1989) model speculators bidding to resell investors to study the debatable question of which payment rule, uniform pricing or discrimination, is advantageous in treasury bill auctions. Haile (2003) studies the first-stage winner reselling a single-unit good to first-stage losers instead of a third party in our setting. In our model, the object is divisible and we allow the bidder to resell arbitrary shares. This is relevant not only to the insurance market, where risks are inherently divisible, but also to other markets, such as the energy market (e.g., Anatolitis et al. (2022)). In the auctions with resale literature, there are different motives for resale, such as cooperation on collusion through resale (Garratt et al., 2009), misperception of the resale market (Georganas, 2011), asymmetry leading to inefficiency (Hafalir and Krishna, 2008), and delaying the resale to achieve an expected gain (Khurana, 2024). In our model, the resale motive is the lead reinsurer's need to save capital. We endogenize the retention ratio by introducing capital constraints. The lead reinsurer balances ceding more to conserve capital with ceding less to retain greater uncertainty, which preserves the potential to exploit private information.

Our paper relates to auctions with signaling concerns. Perry et al. (2000) propose a two-round selling procedure in which only the two highest buyers are allowed to participate in the second round and bid above their first-round bid. This is equivalent to an English auction in an interdependent value setting. The revenue comparison between our model and an English auction was ambiguous. In independent private value settings, several studies have examine how different post-auction competitions, such as Cournot or Bertrand competition or disclosure policies, shape first-stage bidding behavior (e.g., Jehiel and Moldovanu (2000); Rhodes-Kropf and Katzman (2001); Varma (2003); Goeree (2003)). Calzolari and Pavan (2006) study optimal mechanisms under resale to third parties and inter-bidder resale. Dworczak (2020) characterizes the optimal mechanism with an aftermarket in a private value setting within the set of cutoff mechanisms. Bos and Pollrich (2022) analyze the optimal disclosure policy when bidders have concave or convex signaling concerns.

Plan for the Paper. The rest of the paper is organized as follows: Section 2 outlines the model and justifies the key assumptions in the follow-the-lead and retrocession cases. Section 3 analyzes the equilibrium strategies and compares the payoffs of the client insurer and reinsurers in the two settings. Section 4 is an extension section, which shows that the main results are robust against the disclosure of private information by the client insurer and the reserve price. Section 5 discusses the design. Section 5 concludes. Proofs and background information are provided in the appendix.

2 The Model

Competing Syndicates. Consider a setting where $n \geq 2$ reinsurance syndicates compete to underwrite reinsurance risks denoted by a random variable V for a client insurer in a first-price auction. Each lead reinsurer i has a private signal X_i, where $i = 1, 2, ..., n$. A signal can be understood as their assessment of risk. V and all signals $X_1, X_2, \ldots, X_n$ are assumed to be affiliated, as defined in Definition 1.

Definition 1. *(Affiliation Condition) For all $\boldsymbol{x}_1, \boldsymbol{x}_2 \in [\underline{x}, \bar{x}]^n$, and $v_1, v_2 \in [\underline{v}, \bar{v}]^n$, the random variables $X_1, X_2, \ldots, X_n$ and V are said to be strictly affiliated if the following condition holds:*

$$f\left((v_1, \boldsymbol{x_1}) \vee (v_2, \boldsymbol{x}_2)\right) \cdot f\left((v_1, \boldsymbol{x_1}) \wedge (v_2, \boldsymbol{x_2})\right) > f\left(v_1, \boldsymbol{x_1}\right) \cdot f\left(v_2, \boldsymbol{x}_2\right),$$

where "$\vee$" denotes component-wise maximum and "$\wedge$" denotes component-wise minimum.

Let $f(v, x)$ denote the joint density function of V and the vector of signals $X = (X_1, X_2, ..., X_n)$. It is assumed that f, strictly positive, with full support and twice continuously differentiable on $[\underline{v}, \overline{v}] \times [\underline{x}, \overline{x}]^n$, is symmetric in the last n arguments. A bidding strategy is a measurable function denoted as $b(X_i) : [\underline{x}, \overline{x}] \to \mathbb{R}$, for $i = 1, 2, ..., n$.

Small followers are assumed to be uninformed.[6] The role of followers is to provide capital and capacity to hold the risks. They do not approach clients directly to compete with the lead because they are either less experienced or are smaller firms that cannot take on a large share

[6]The assumption of an exogenous information structure will be discussed later in the discussion section. We maintain a minimal assumption in this section. In the extension section, we allow followers to have information, provided certain technical conditions hold. The main results remain qualitatively unaffected.

of the risks.[7] Their roles can vary depending on the types of associated risk; here, we focus on one specific business. All reinsurers are assumed to be risk-neutral.[8] Each syndicate, consisting of a lead reinsurer and multiple followers, is assumed to be able to provide full coverage for the client insurer.

Affiliation is a strong form of positive correlation. Intuitively, it means if a subset of $X_i's$ are all large, then this implies an increased likelihood that the remaining $X_j's$ are also large. Suppose the insurance companies Amlin, Beazley and Catlin believe that a flood is more likely to occur in southern Germany, then it is likely that their competitor, Hiscox, would also tend to hold a similar belief. The lemmas implied bylied the affiliation condition used in the analysis of equilibrium are presented below.

Lemma 1 (Milgrom and Weber, 1982, Theorem 3)**.** *If random variables $Z_1, \ldots, Z_k$ are affiliated, and $g_1, \ldots, g_k$ are all increasing functions, then $g_1(Z_1), \ldots, g_k(Z_k)$ are also affiliated.*

Lemma 2 (Milgrom and Weber, 1982, Theorem 4)**.** *If random variables $Z_1, \ldots, Z_k$ are affiliated, and $g_1, \ldots, g_k$ are all increasing functions, then $g_1(Z_1), \ldots, g_k(Z_k)$ are also affiliated.*

Lemma 3 (Milgrom and Weber, 1982, Theorem 5)**.** *Let $Z_1, \ldots, Z_k$ be affiliated, and let H be any increasing function. Then the function h defined by*

$$h(a_1, b_1; \ldots; a_k, b_k) = E[H(Z_1, \ldots, Z_k) \mid a_1 \leq Z_1 \leq b_1, \ldots, a_k \leq Z_k \leq b_k]$$

is increasing in all of its arguments. In particular, for $l = 1, \ldots, k$, the functions $h_l(z_1, \ldots, z_l) = E[H(Z_1, \ldots, Z_k) \mid z_1, \ldots, z_l]$ are all increasing.

Throughout the analysis, we keep the following technical assumption.

Assumption 1. *$E[V|X_1, \ldots, X_n] := g(x_1, \ldots, x_n)$ is supermodular in signals; i.e., $\frac{\partial^2 g}{\partial x_i \partial x_j} \geq 0$.*

The assumption roughly means signals are information complements to value. This is standard in common value models with resale and ensures the monotonicity of bidding strategies. This assumption does not contradict strict affiliation; to illustrate, we construct a numerical example based on the normal distribution.

A Numerical Example. Let V, X_1, and X_2 be jointly normally distributed random variables

[7]Source: https://blog.ccr-re.com/en/what-is-a-follower

[8]This assumption makes the analysis tractable. Introducing risk aversion or non-linearity of utility in monetary transfer would render the differential equation in the leader's decision problem non-linear and hard to solve analytically.

with means μ_V, μ_{X_1}, and μ_{X_2} and a covariance matrix

$$\Sigma = \begin{pmatrix} \sigma_V^2 & \sigma_{VX_1} & \sigma_{VX_2} \\ \sigma_{VX_1} & \sigma_{X_1}^2 & \sigma_{X_1X_2} \\ \sigma_{VX_2} & \sigma_{X_1X_2} & \sigma_{X_2}^2 \end{pmatrix}.$$

By the projection theorem,[9] the conditional expectation $E[V|X_1, X_2]$ is a linear function of X_1 and X_2: $E[V|X_1, X_2] = \beta_1 X_1 + \beta_2 X_2$. The vector $\beta = (\beta_1, \beta_2)^T$ is given by $\beta = \Sigma_{VX}\Sigma_{XX}^{-1}$, where $\Sigma_{VX} = (\sigma_{VX_1}, \sigma_{VX_2})^T$ and Σ_{XX} is the covariance matrix of X_1 and X_2. The joint density function is $f(v, x_1, x_2) = \frac{1}{(2\pi)^{3/2}|\Sigma|^{1/2}} \exp\left(-\frac{1}{2}\mathbf{Z}^T\Sigma^{-1}\mathbf{Z}\right)$, where $\mathbf{Z} = (v, x_1, x_2)^T$. Taking the natural logarithm of f, we obtain $\ln f(v, x_1, x_2) = -\frac{3}{2}\log(2\pi) - \frac{1}{2}\log|\Sigma| - \frac{1}{2}\mathbf{Z}^T\Sigma^{-1}\mathbf{Z}$. Expanding the last term gives $\mathbf{Z}^T\Sigma^{-1}\mathbf{Z} = a_{11}v^2 + a_{22}x_1^2 + a_{33}x_2^2 + 2a_{12}vx_1 + 2a_{13}vx_2 + 2a_{23}x_1x_2$, where a_{ij} are elements of Σ^{-1}, the inverse of the covariance matrix. Hence, the cross-partial derivative is $\frac{\partial^2}{\partial x_1 \partial x_2}(\ln f) = -a_{23}$, as the first two terms in $\ln f$ are constant.

We assume each variable has zero mean (i.e., $\mu_V = \mu_{X_1} = \mu_{X_2} = 0$) and specific values for the covariance matrix:

$$\Sigma = \begin{pmatrix} 1 & 0.8 & 0.6 \\ 0.8 & 1 & 0.5 \\ 0.6 & 0.5 & 1 \end{pmatrix}.$$

The conditional expectation is $E[V|X_1, X_2] = \frac{2}{3}X_1 + \frac{1}{3}X_2$, so the cross-partial derivative is zero and Assumption 1 is satisfied. The inverse of Σ can be computed as

$$\Sigma^{-1} = \begin{pmatrix} 2.439 & -1.707 & -0.805 \\ -1.707 & 2.927 & -0.878 \\ -0.805 & -0.878 & 1.707 \end{pmatrix},$$

where the off-diagonal elements are negative. The cross-partial derivative is $\frac{\partial^2 \ln f}{\partial x_1 \partial x_2} = -(-0.878) = 0.878$, which shows that strict affiliation holds. Hence, the constructed example satisfies both assumptions.

Capital-constrained Reinsurers. Reinsurers are capital-constrained. The amount of capital needed to underwrite the entire risk is denoted by I.[10] The leader i's retention ratio is denoted by α_i. We assume that the lead reinsurer i has an initial net capital $K < I$ for all i. We consider a symmetric case for simplicity. If this assumption is violated, one reinsurer would

[9] See, e.g., DeGroot (2005)

[10] For current capital requirements in the insurance industry in Europe, see Solvency II: https://www.eiopa.europa.eu/browse/regulation-and-policy/solvency-ii_en

suffice to provide the coverage, which is rare in the reinsurance market. Lead reinsurers can tap followers who jointly have sufficient financial capacity to provide coverage. Such logic resembles that in syndicated loans.[11]

Structure of the Game in the Follow-the-Lead Case. The sequence of moves under the follow-the-lead practice is as follows.

(1) Pre-contractual Stage. The lead reinsurer i decides on the retention ratio α_i^*.

(2) Tender Stage. The lead reinsurer i makes an offer $b(X_i)$ to compete for underwriting the risk in a first-price auction. The winner is the one with the best offer, and the payment is the winner's offer.[12]

(3) Subscription Stage. Suppose syndicate i wins. Then, the leader i subscribes to an α_i share of the risk, and its followers jointly subscribe to the remaining share, $1-\alpha_i$, at the same unit price determined during the tender stage.

(4) All payoffs are realized.

Payoffs in the Follow-the-lead Case. In the first-price auction, the price $\tilde{p}$ is the winner's offer: $\tilde{p} = b(X_i)$.[13] Assuming that the winning offer is made by $i = 1$. Given the price $\tilde{p}$ and retained share α_1, the wining lead reinsurer 1's payoff is $\tilde{\pi}_L = \alpha_1(V - \tilde{p})$. Since the unit price of risk is mandated uniformly, the followers' payoff is $\tilde{\pi}_F = (1-\alpha_1)(V-\tilde{p})$. The reinsurance syndicate 1's payoff is $\tilde{\pi}_1 = V - \tilde{p}$. The expected payoff of lead reinsurer 1, conditional on winning, is $E\tilde{\pi}_L = \alpha_1 E[V - b(X_1)|X_1 = x, Y_1 < x]$, where Y_1 is highest order statistics of $(X_2, \ldots, X_n)$, i.e., the highest competing signal.[14] The client insurer, as the initial auctioneer, with payoff $\tilde{\pi}_C = \tilde{p}$ is better off with a higher $\tilde{p}$.

Structure of the Game in the Retrocession Case. In the retrocession case, the follow market is no longer locked into the same unit price as determined in the tender stage. Suppose there are m_i followers in the winning syndicate i. As in the follow-the-lead case, the leader i determines the retention ratio α_i prior to the contract. The remaining share is split into $k < m_i$

[11]The details of loan syndication are provided in Appendix B.

[12]Ties (if any) are assumed to be broken at random.

[13]One can consider a negative $\tilde{p}$ to mean the premium cost paid by the client insurer to get coverage from reinsurers, so a higher negative $\tilde{p}$ means a lower premium cost in absolute value to get full coverage. We use a standard forward auction here to simplify comparison with the literature; the results do not change qualitatively in a reverse auction, but the analysis introduces additional complexity.

[14]The implicit assumption is that bids are monotonic in signals, which can be verified after explicitly deriving it. This is the logic in Milgrom and Weber (1982) and applies similarly in our retrocession game. Moreover, they establish the existence of equilibrium by explicitly identifying a candidate using the first-order differential equation derived from the incentive compatibility condition, and showing that the equilibrium payoff is positive so that individual rationality holds. We follow the same approach.

units and sold to followers in a first-price auction, with each follower assumed to have unit demand for simplicity.[15] The retrocession price for the shares ceded to its followers is denoted by p_{re}. Between the two stages, it is assumed that all bids in the tender stage are revealed publicly.[16] The sequence of moves is as follows. The difference is that the lead reinsurer secures the entire business and then cedes portions to the followers, with bidding information disclosed between the stages.

(1) Pre-contractual Stage. The lead reinsurer i decides the retention ratio α_i^*.

(2) Tender Stage. The lead reinsurer i makes an offer $b(X_i)$ to compete for underwriting the risk in a first-price auction. The winner is the one with the best offer, and the payment is the winner's offer.

(3) All offers are disclosed after the tender stage.

(4) Subscription Stage. The winning leader i sells the remaining k units of risk to m_i followers in a first-price auction.

(5) All payoffs are realized.

Payoffs in the Retrocession Case. In the retrocession case, followers hold $1 - \alpha_i$ share of risk, their ex-post payoff is $\hat{\pi}_F = (1 - \alpha_i)V - p_{re}$. The lead reinsurer i keeps α_i share of the business, receives p_{re} for reselling $1 - \alpha_i$ share of the business, and pays $\hat{p}$ for the entire business initially, so the lead reinsurer's payoff is $\hat{\pi}_L = \alpha_i V + p_{re} - \hat{p}$. Assume that the winning bid is made by $i = 1$. Since the price $\hat{p}$ is the winner's offer $b(X_1)$, the leader's interim payoff conditional on winning is $E\hat{\pi}_L = E[\alpha_1 V + p_{re} - b(X_1)|X_1 = x, Y_1 < x]$. The followers' joint expected payoff is given by $E\hat{\pi}_F = E[(1-\alpha_1)V - p_{re}|X_1, \ldots, X_n]$. Similar to the follow-the-lead case, the client insurer, as the initial auctioneer, benefits from a high price and his payoff is $\hat{\pi}_C = \hat{p}$.

Solution Concept. We focus on separating equilibrium, in which bidders of different types submit distinct bids, and followers' beliefs are Bayesian updated wherever possible.

[15] Suppose two or more units are sold to two followers. In this case, both followers bidding zero is an equilibrium point, as they would still acquire the share at the lowest cost. However, if only one unit is sold to two followers, bidding zero is no longer an equilibrium since one would have an incentive to deviate. Thus, we require the number of bidders to exceed the number of units to prevent this situation.

[16] In government auctions, it is mandated that all bids are revealed afterward. Relaxing this assumption would significantly complicate the derivation of continuation payoffs. We do not address the optimal disclosure policy of bids in this paper. In the independent value setting within a class of cutoff mechanisms, Dworczak (2020) characterizes the optimal disclosure policy to the aftermarket. Full disclosure of bids is also an assumption made in the literature, such as Bukhchandani and Huang (1989) and Haile (2003).

3 Equilibrium Analysis

In this section, we analyze the equilibria in the follow-the-lead and retrocession case. We then compare the price for the client insurer and the payoffs of reinsurers in the two cases.

3.1 Equilibrium Price in the Follow-the-lead Case

Tender Stage. The lead reinsurer 1 chooses $b(X_1)$ to maximize his expected payoff conditional on winning, $E\tilde{\pi}_L = \alpha_1 E[V - b(X_1)|X_1 = x, Y_1 < x]$. This is the standard first-price common value auction with affiliated signals (see, e.g., Krishna (2009)).

Pre-contractual Stage. The expected joint equilibrium payoff of reinsurers in the winning is nonnegative, otherwise bidding zero would be preferable. For the lead reinsurer, retaining more shares leads to a weakly higher payoff, so the leader 1 chooses to pool his capital to retain a share $\alpha_1^* = K/I < 1$. The equilibrium result is summarized in the following proposition.

Proposition 1. *In the follow-the-lead case, the equilibrium price $\tilde{p}$ is equal to $\tilde{b}(x)$. The winning lead retains $\alpha^* = K/I$ share of business, and $\tilde{b}(x)$ is the symmetric equilibrium bidding strategy of the lead with signal x, defined as:*

$$\tilde{b}(x) = q(x, x) - \int_{\underline{x}}^{x} L(s \mid x) dq(s, s),$$

with

$$q(x, y) = E[V|X_1 = x, Y_1 = y];$$
$$L(s \mid x) = \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\}.$$

Proof. See Krishna (2009). □

The second term, $\int_{\underline{x}}^{x} L(s \mid x) dq(s, s)$, represents bid shading. Due to this term, the equilibrium bid is less than the expected value conditional on winning, expressed as $\tilde{b}(X_1) < E\left[V \mid X_1 = x, Y_1 < x\right]$.[17] A rational lead reinsurer shade his offer to protect himself from the winner's curse. In the follow-the-lead case, followers benefit from the lead reinsurer's expertise and also gain positive payoffs in equilibrium. In the next section, we study the retrocession case.

3.2 Equilibrium Price in the Retrocession Case

We analyze the game using backward induction. We first analyze the retrocession price in the subscription stage, then derive the equilibrium bidding strategy of the lead reinsurers in the

[17] See p.100 in Krishna (2009) for details.

tender stage. Finally, we determine the pre-contractual retention ratio.

Retrocession Price in the Subscription Stage. In the subscription stage, followers can learn information from bid revelation if the bidding strategy is separating. Given their information, the retrocession price of the risk in the secondary market is the $1-\alpha$ share of the expected value V, conditional on all publicly available information from the first-round bids, i.e., $p_{\text{re}} = (1-\alpha)E[V \mid X_1, Y_1, \ldots, Y_{n-1}]$, where $Y_1, \ldots, Y_{n-1}$ are the order statistics from the highest to the lowest of signals $(X_2, \ldots, X_n)$.

Bidding Strategy in the Tender Stage. Suppose that lead reinsurers $i = 2, \ldots, n$ adopt strategy $\hat{b}$ and lead reinsurer 1 receives information $X_1 = x$ and submits a bid equal to b. Then if the lead reinsurer 1 wins and followers believe that he is following $\hat{b}$, the retrocession price will be $(1-\alpha)E\left[V \mid X_1 = \hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right] := (1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right)$, where $\hat{b}^{-1}$ denotes the inverse of $\hat{b}$. Define $p(x', x, y) = E\left[q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) \mid X_1 = x, Y_{n-1} = y\right]$, which is the expected retrocession price conditional on lead 1's true signal x, highest competing signal y, and the followers' perception of lead 1's signal x' when the lead 1 wins. By the assumption of strict affiliation, both p and q are increasing in each of their arguments. The payoff of lead reinsurer 1, denoted as $\pi(b|x)$, is the resale price of the $1-\alpha$ share, plus the retained value of the α share, minus the initial price paid, which can be written in the integral form:

$$
\begin{aligned}
\pi(b \mid x) &\equiv E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}(Y_1)\right) \mid X_1 = x\right] \\
&= E\left[E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}(Y_1)\right) \mid X_1, Y_1\right] \mid X_1 = x\right] \\
&= E\left[\left((1-\alpha)p\left(\hat{b}^{-1}(b), X_1, Y_1\right) + \alpha q(x,y) - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}(Y_1)\right) \mid X_1 = x\right] \\
&= \int_{\underline{x}}^{\hat{b}^{-1}(b)} \left[(1-\alpha)p\left(\hat{b}^{-1}(b), x, y\right) + \alpha q(x,y) - b\right] \cdot f_{Y_1}(y \mid x)dy.
\end{aligned}
$$

The second equality applies the law of iterated expectation. The third follows from the definition of $q(x,y)$, and the last equation is in integral form, where $f_{Y_1}(y \mid x)$ is the conditional density of the order statistic Y_1. The equilibrium bidding strategy $\hat{b}(x)$ for solving the optimization problem is presented below, where $\beta = 1-\alpha$ is the retrocession share to followers.

$$
\begin{aligned}
\hat{b}(x) &= (1-\alpha)p(x,x,x) + \alpha q(x,x) - \int_{\underline{x}}^{x} L(s \mid x)dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x) \\
&= p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x)dp(s,s,s) + \beta\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x),
\end{aligned}
$$

where

$$L(s \mid x) = \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\},$$
$$J(s) = \int_{\underline{x}}^s p_1(s, s, y) \cdot f_{Y_1}(y \mid s) dy.$$

Retention Ratio in the Pre-contractual stage. The optimal retention ratio is $\alpha^* = K/I$. This follows from the observation that the signaling component in $\hat{p} = \hat{b}(x)$ price increases proportionally with the shares β ceded to the followers, or equivalently, decreases with the retention ratio α. The payoffs for the client insurer and reinsurers are $\hat{p}$ and $V - \hat{p}$, respectively. Since the followers' payoffs are zero, the lead reinsurer's payoff is $V - \hat{p}$, which decreases as the ceding share β increases. On the one hand, ceding more leads to more aggressive bidding, reduces the potential to leverage private information for profit, and results in lower profits. On the other hand, ceding more shares results in capital savings. Hence, given the capital constraint, the lead reinsurer at most retains a share $\alpha^* = K/I$, i.e., they pool their capital into the business.

Proposition 2. *In the retrocession case, the price of the entire risk, $\hat{p}$ is equal to $\hat{b}(x)$, where $\hat{b}(x)$ is the symmetric equilibrium bidding strategy of lead reinsurer with signal x, defined as:*

$$\begin{aligned}\hat{b}(x) &= (1-\alpha^*)p(x,x,x) + \alpha^* q(x,x) - \int_{\underline{x}}^x L(s \mid x) dp(s,s,s) + (1-\alpha^*)\int_{\underline{x}}^x \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x) \\ &= p(x,x,x) - \int_{\underline{x}}^x L(s \mid x) dp(s,s,s) + \beta^* \int_{\underline{x}}^x \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x),\end{aligned}$$

where

$$\alpha^* = K/I,$$
$$L(s \mid x) = \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\},$$
$$J(s) = \int_{\underline{x}}^s p_1(s, s, y) \cdot f_{Y_1}(y \mid s) dy.$$

Proof. See the Appendix. □

In this equilibrium, $J(s)$ measures the average of the responsiveness of the retrocession price to the perceived signal. The third term $S(\beta^*, x) := \beta^* \int_{\underline{x}}^x \frac{J(s)}{f_{Y_1}(s|s)} dL(s \mid x)$ captures the informational linkage between the primary and secondary markets, where β^* is the retrocesion shares to followers. The retrocession price factors into the lead reinsurer's decision problem initially and causes them to bid more aggressively. It is the disclosure of bids across markets that gives the lead reinsurer an incentive to signal.

Technically, his result can be viewed as a weighted average of two extreme settings. In the no-resale setting ($\alpha = 1$ or $\beta = 0$), the bidding strategy in Proposition 2 corresponds to $\tilde{b}(x) = q(x,x) - \int_{\underline{x}}^{x} L(s \mid x)\, dq(s,s)$, as in the follow-the-lead case. In this setting, since the followers' payoff is zero, the lead reinsurer's payoff is the syndicate's payoff $E\tilde{\pi}_1$ without resale.

In the full resale setting ($\alpha = 0$ or $\beta = 1$), the strategy in Proposition 2 becomes $\hat{b}^{Full}(x) = p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x)\, dp(s,s,s) + \int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s|s)}\, dL(s|x)$. In this setting, the lead reinsurer's payoff is the syndicates's payoff $E\hat{\pi}_1^{Full}$ since the followers's payoff is zero. Taken together, when partial risk α is held by lead reinsurer and the rest β is ceded, the equilibrium bidding strategy $\hat{b}(x)$ is a weighted average of the two settings, i.e., $\hat{b}(x) = \alpha\tilde{b}(x) + \beta\hat{b}^{Full}(x)$ as reflected in Proposition 2. Also, the payoff of the lead reinsurer is a weighted average of their payoffs in the two settings, i.e., $\hat{\pi}_L = \alpha\tilde{\pi}_1 + \beta\hat{\pi}_1^{Full}$, where $\alpha + \beta = 1$.

Corollary 1. *Under Assumption 1, the equilibrium bidding function $\hat{b}(x)$ is monotone increasing in signal x.*

Proof. See the Appendix. □

The separating bidding strategy is monotonic, enabling reinsurers in the follow market to invert lead reinsurers' bids from the tender stage to glean information.

3.3 Comparison of Payoffs in the Follow-the-lead Case and Retrocession Case

Given the equilibrium results in the follow-the-lead case (Proposition 1) and retrocession case (Proposition 2), we can compare the payoffs of client and reinsurers in the two cases.

Proposition 3. *The client insurer is better off in the retrocession case than in the follow-the-lead case, i.e., $\tilde{\pi}_C > \hat{\pi}_C$. Reinsurers are better off in the follow-the-lead case, i.e., $\hat{\pi}_1 > \tilde{\pi}_1$. Specifically, the lead reinsurer is better off in the retrocession case, i.e., $\tilde{\pi}_L > \hat{\pi}_L$, while the followers are better off in the follow-the-lead case, i.e., $\hat{\pi}_F > \tilde{\pi}_F$.*

Proof. See the Appendix. □

The main results of the paper can be summarized in Figure 2. α^* is the equilibrium retention ratio. The left panel show the follow-the-lead case. The upper rectangle represents the client's payoff varying with α, the middle triangle corresponds to the followers' payoff, and the lower triangle denotes the lead reinsurer's payoff.

The right panel shows the retrocession case. The purple dotted line corresponds to the price function $\hat{p}$ derived in Proposition 2, which decreases with the pre-contractual retention

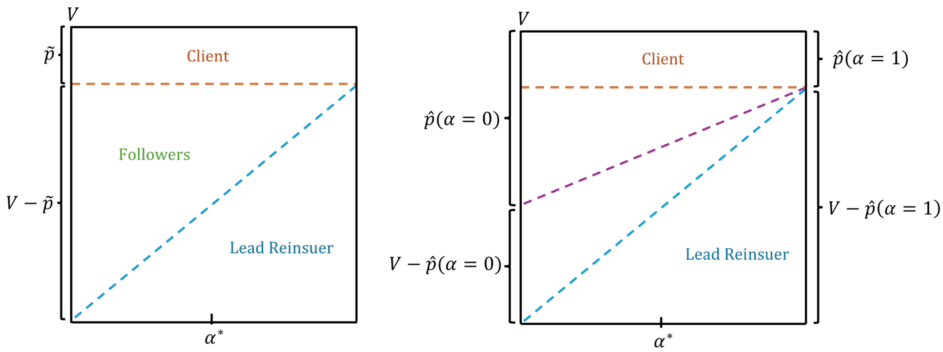

Figure 2: Payoffs Comparison in the Two Organizational Structures

ratio α. At the right boundary ($\alpha = 1$), the price converges to $\tilde{p}$, as in the follow-the-lead case. At the left boundary ($\alpha = 0$), it is full resale. The followers' payoffs are divided by this pricing function into two small triangles, which are then allocated to the client and the lead reinsurer, respectively. The upper trapezoid above $\hat{p}$ represents the client's payoff, while the lower trapezoid below $\hat{p}$ corresponds to the lead reinsurer's payoff. In the retrocession case, the followers' payoffs are zero.

Intuitively, for followers, their rent is zero in the retrocession case, whereas in the follow-the-lead case, they benefit from the lead reinsurer's expertise, shielding themselves from the winner's curse. Their rent comes from sharing the lead reinsurer's private knowledge of the risk. For the lead reinsurer, although they bid aggressively, this is only for the portion ceded to the followers, while their retention share remains unaffected. The partial surplus transferred from the followers compensates for their aggressive bidding on the ceded share, leaving them better off. For the client insurer, the additional signaling component in the first-stage offers improves their position. The extra surplus for the client insurer is extracted from the reinsurance syndicates, achieved through reselling risk and signaling to the follower market.

4 Several Extensions

This section provides a robustness check when some assumptions of the model are altered.

4.1 Announcement of Client Insurer's Information

In the main section, we assume that the client insurer does not have information because it is the reinsurers who assess, understand, and price the risk. We then relax this assumption and introduce a disclosure stage at the beginning of the games, allowing for pre-trading communication between the client insurer and reinsurers. We assume that the client has private information X_0 that satisfies the affiliation condition defined in Definition 1 and supermodularity condition defined in Assumption 1, i,e., $E\left[V|X_1,\ldots,X_n\right] := g\left(x_1,\ldots,x_n;x_0\right)$ is supermodular in signals; i.e., $\frac{\partial^2 g}{\partial x_i \partial x_j} \geq 0$. We show that the disclosure of the client insurer's private information does not qualitatively change the main results from the previous section.

4.1.1 Equilibrium Price in the Follow-the-Lead Case

To study the client insurer's disclosure policy, we first derive the equilibrium price when X_0 is revealed. Assume w.l.o.g. that syndicate 1 wins the business. In a similar way, the joint interim expected payoff conditional on winning, is given by $E\left[V - b(X_1) \mid X_1 = x, Y_1 < X_1; X_0\right]$, where Y_1 is the highest signal among the competitors, i.e., $Y_1 = max(X_2,\ldots,X_n)$. When the client insurer reveals publicly X_0, the equilibrium price of the reinsurance contract is summarized in the following proposition.

Proposition 4. *Under the follow-the-lead case, when X_0 is revealed, the equilibrium price of the entire contract $\tilde{p}$ is equal to $\tilde{b}_I(X_1;X_0)$. $\alpha^* = K/I$, and $\tilde{b}_I(X_1;X_0)$ is the symmetric equilibrium bidding strategy of leader 1 with signal x, defined as:*

$$\tilde{b}_I(X_1;X_0) = q(x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x;x_0)dq(s,s;x_0),$$

with

$$q(x,y;x_0) = E[V|X_1 = x, Y_1 = y; X_0 = x_0];$$
$$L(s \mid x;x_0) = \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t;x_0)}{F_{Y_1}(t \mid t;x_0)}dt\right\}.$$

Proof. See the Appendix. □

To show that publicly revealing X_0 before trading benefits the client, we need the revenue ranking result or the linkage principle as shown in Lemma 4. Let $W(z,x)$ be the expected price paid by bidder 1 if he is the winning bidder when he receives a signal x but bids as if his signal were z, i.e., $W(z,x) = b(z)$. Let $W_2(z,x)$ denote the partial derivative of the function $W(z,x)$ with respect to the second argument. Then the following result holds.

Lemma 4. *Let A and B be two auctions in which the highest bidder wins. Suppose that each auction has a symmetric and increasing equilibrium such that (1) for all x, $W_2^A(x,x) \geq W_2^B(x,x)$; (2) $W^A(\underline{x},\underline{x}) = E\left[V \mid X_1 = \underline{x}, Y_1 = \underline{x}\right] = W^B(\underline{x},\underline{x})$. Then the expected revenue in A is at least as large as that in B.*

Proof. See Proposition 7.1 in Krishna (2009). □

In our setting, auctions A and B correspond to the scenarios with and without revealing X_0, respectively. Based on Lemma 4, we obtain the following result.

Proposition 5. *Under the follow-the-lead case, publicly revealing X_0 benefits the client.*

Proof. See the Appendix. □

Based on the above results, we know that the client insurer would disclose X_0 to reinsurers in the disclosure stage. One might wonder would the client insurer censor information, say, she discloses information if it is above some threshold and withhold it otherwise. A simple unraveling argument under passive beliefs of reinsurers rules out such a possibility.[18]

4.1.2 Equilibrium Price in the Retrocession Case

Given that the client insurer has private information X_0, we consider the impact of revealing X_0 on the equilibrium price of the reinsurance business. Let $\hat{b}(x; x_0)$ be the symmetric bidding strategy in the tender stage conditional on $X_0 = x_0$. Then if the lead reinsurer 1 wins and followers believe that he is following $\hat{b}(X_1; X_0)$, the retrocession price will be $(1-\alpha)q\left(\hat{b}^{-1}(b; x_0), Y_1, \ldots, Y_{n-1}; X_0\right) = E\left[(1-\alpha)V \mid X_1 = \hat{b}^{-1}(b), Y_1, \ldots, Y_{n-1}; X_0\right]$, where $\hat{b}^{-1}(\cdot; x_0)$ denotes the inverse of $\hat{b}(\cdot; x_0)$.

Define $p\left(x', x, y; x_0\right) = E\left[q\left(x', Y_1, \ldots, Y_{n-1}; X_0\right) \mid X_1 = x, Y_1 = y\right]$, which is the expected retrocession price conditional on X_1 and Y_1 when the lead 1 wins and followers believe that bidder 1's private signal is equal to x'. The payoff of lead reinsurer 1, denoted as $\pi(b \mid x; x_0)$, by a similar way, can be written in the following integral form:

$$\begin{aligned}\pi(b \mid x; x_0) &\equiv E\left[\left((1-\alpha)q\left(\hat{b}^{-1}(b), Y_1; X_0\right) + \alpha V - b\right) \cdot \mathbf{1}\left(b \geq \hat{b}\left(Y_1\right)\right) \mid X_1 = x; X_0 = x_0\right] \\ &= \int_{\underline{x}}^{\hat{b}^{-1}(b)} \left[(1-\alpha)p\left(\hat{b}^{-1}(b), x, y; x_0\right) + \alpha q(x, y; x_0) - b\right] \cdot f_{Y_1}(y \mid x; x_0) dy.\end{aligned}$$

In the Appendix, we derive the first-order condition using the Leibniz integral rule, solve the differential equation, and demonstrate that the bidding strategy leads to a maximum payoff, not a minimum. From this, we obtain the following equilibrium result.

Proposition 6. *In the retrocession case, when X_0 is revealed, the winning leader keeps a share $\alpha^* = K/I$ of risks and cedes the rest $\beta^* = 1-\alpha^*$ to followers. The price of the entire business $\hat{p}$ is equal to $\hat{b}(x_1; x_0)$, where $\hat{b}(x; x_0)$ is the symmetric equilibrium bidding strategy of lead reinsurer*

[18] Suppose X_0 is uniformly distributed on $[0,1]$ and such a cutoff $x^* \in [0,1]$ exists. Upon not receiving information, reinsurers believe that x is uniformly in $[0, x^*]$ with an average $x^*/2$. Thus, a client insurer with value $x \in (x^*/2, 1]$ would disclose to avoid being perceived as a low type. After n rounds of reasoning, a client insurer with value $x \in (x^*/2^n, 1]$ would disclose. Hence, full disclosure is optimal when there are no other frictions, such as disclosure costs (Dye (1985)), uncertainty of information endowment(Wagenhofer (1990)), etc.

with signal x, defined as:

$$\begin{aligned}\hat{b}(x;x_0) &= (1-\alpha^*)p(x,x,x;x_0) + \alpha^* q(x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) \\ &\qquad + \beta^* \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0) \\ &= p(x,x,x;x_0) - \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) + \beta^* \int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0)\end{aligned}$$

where

$$\begin{aligned}L(s \mid x;x_0) &= \exp\left\{-\int_s^x \frac{f_{Y_1}(t \mid t;x_0)}{F_{Y_1}(t \mid t;x_0)} dt\right\}, \\ J(s;x_0) &= \int_{\underline{x}}^{s} p_1(s,s,y;x_0) \cdot f_{Y_1}(y \mid s;x_0) dy.\end{aligned}$$

Proof. See the Appendix. □

Based on the linkage principle, we have the following revenue ranking result.

Proposition 7. *In the retrocession case, publicly revealing X_0 benefits the clients.*

Proof. See the Appendix. □

Based on the above result, we know that under the retrocession case, the client insurer reveals X_0 publicly to reinsurers, and the price of the reinsurance business is $\hat{p}$ as defined in Proposition 6. The clients are better off since the equilibrium price in Proposition 6 is higher than $\tilde{p}$ in Proposition 4 by the signaling component.

4.2 Followers' Information

We relax the assumption that followers are uninformed. Our main results remain unaffected if the followers have some information, provided that their information is garbled by that of the lead, as defined in Definition 2.

Definition 2. *(Garbling Condition) A random variable Z_{n+k} is a garbling of $(Z_1, Z_2, \ldots, Z_n)$ if the joint density of $V, Z_1, Z_2, \ldots, Z_n, Z_{n+k}$ can be written as $g\left(V, Z_1, \ldots, Z_n\right) \cdot h\left(Z_{n+k} \mid Z_1, \ldots, Z_n\right)$, where g and h are joint density and conditional density of respective variables.*

It is a strong sufficient statistic condition.[19] Should a following reinsurer denoted as $n+k$ ground her estimate Z_{n+k} on the information accessible to lead reinsurers, the garbling condition

[19] Milgrom and Weber (1982) use this to show that when a less-informed bidder competes with better-informed bidders in a common value auction, she receives zero payoffs in an equilibrium.

is satisfied. In an extreme case where followers are uninformed and leaders are informed, this condition holds as well, because empty information sets are subsets of any non-empty set. One implication from the garbling condition is Lemma 5.

Lemma 5. *If a random variable Z_{n+k} is a garbling of random variables $Z_1, Z_2, \ldots, Z_n$, then for $k = 1, 2, \ldots, m$, it holds that*

$$E\left[V \mid Z_1, Z_2, \ldots, Z_n, Z_{n+k}\right] = E\left[V \mid Z_1, Z_2, \ldots, Z_n\right].$$

Proof. See the Appendix. □

Lemma 5 says that the evaluation of reinsurance risk V at a more precise information set $(Z_1, Z_2, \ldots, Z_n)$ remains unchanged when an additional piece of coarse information Z_{n+k} is introduced. To see that the retrocession price is not affected, note that the signals of followers are garbled by those of lead reinsurers. For an individual follower, her evaluation of the risk conditional on her private signal X_{n+k} and the lead reinsurers' signals, is equivalent to evaluating it based on the lead reinsurers' signals by Lemma 5, i.e., $E\left[V \mid X_1, Y_1, \ldots, Y_{n-1}, X_{n+k}\right] = E\left[V \mid X_1, Y_1, \ldots, Y_{n-1}\right]$, where $k = 1, 2, \ldots, m$. Given that their value remains unaffected, the analysis in the main section remains unchanged.

4.3 Reserve Price

This section discusses the introduction of a reserve price or participation fee. These two are equivalent to excluding some bidders with lower values from participation. We exclude this in the main section for two reasons. First, from a technical perspective, it could lead to the nonexistence of monotone equilibria when signals are affiliated (See Landsberger (2007) for counterexamples). Second, in cases where equilibrium exists with a reserve price $r \in [\underline{v}, \bar{v}]$, the main result remains qualitatively unaffected. In the follow-the-lead case, given a reserve price r, any lead reinsurer with a signal x below r would not participate, as their payoff would be negative if they win. For lead reinsurers participating in the business, the boundary condition for the lowest type changes to $\hat{b}(r) = q(r, r)$. The rest of the derivation remains unchanged. Hence, the symmetric equilibrium bidding strategy for the lead reinsurer with signal x is given by:

$$\tilde{b}(r, x) = q(x, x) - \int_r^x L(s \mid x) dq(s, s).$$

In the retrocession case, the same set of lead reinsurers with value $X < r$ are excluded. The boundary changes to $\hat{b}(r) = p(r, r, r)$, and the symmetric bidding strategy is given by:

$$\hat{b}(r, x) = p(x, x, x) - \int_r^x L(s \mid x) dp(s, s, s) + \beta^* \int_r^x \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s \mid x),$$

and the comparison of prices remains unaffected.

5 Discussion of the Model

Below, we discuss several assumptions in the model.

Collusive Bidders. Our model assumes no collusive behavior in either scenario, as reinsurers typically interact across various business lines and prioritize long-term reputations, discouraging deceptive or illegal actions. We acknowledge that preventing bidder collusion is a key aspect of auction design. In the retrocession scenario, the lead reinsurer profits more than in the follow-the-lead case. This increased profit incentive encourages the lead reinsurer's participation but may also heighten collusion risks, such as forming tacit alliances to demand higher premiums and share surplus after winning. Exploring a collusion-proof design is an intriguing avenue for future research.

Exogenous Information Structure. The information structure in our model is exogenously given. We assume that the leader has information while followers are uninformed, modeling followers' reliance on the leader's expertise to price risk. The leader's role often depends on specialized expertise; for example, Tokio Marine excels in assessing earthquake risks, while Taiping Re specializes in typhoons. Their roles as leaders and followers may interchange depending on the business context. Our model focuses on one specific business line, and the information assumption is a simplification of reality. A potential direction for future research could be to allow reinsurers to acquire information and study their information acquisition behavior under various organizational structures.

6 Concluding Remarks

This paper presents a model for studying the organization of the reinsurance market by comparing the current follow-the-lead practice with a proposed retrocession design that offers greater benefits to clients. In follow-the-lead practice, a capital-constrained lead reinsurer sets the terms of the offer, and followers subscribe to the remaining shares at a uniform price. The lead reinsurer benefits from private information about the underlying risk, while the followers earn rent by leveraging the lead's expertise in risk assessment.

In the proposed retrocession design, the allocation process is divided into two stages with information revelations occurring between them. In the first stage, the lead reinsurer balances leveraging their potential to exploit private information by retaining more shares and increasing capital savings by retaining fewer shares. In the second stage, with all information public, followers' payoffs drop to zero as there is no private information to exploit. Their surplus is extracted and transferred to the client through aggressive bidding by the lead reinsurer.

This retrocession design is not widely observed in practice, possibly for reasons similar to the initial introduction of the SMRA in spectrum auctions. There may be a lack of understanding regarding its mechanism, as restructuring a simple allocation process into a dynamic one introduces additional complexity and requires more effort to organize the subsequent interactions.

A Appendix A

A.1 Proof of Proposition 2

Proof. Given the lead reinsurer's payoff, the first-order condition, derived using the Leibniz rule, is expressed as follows:

$$\begin{aligned}
\hat{b}'(x) &= \Big[(1-\alpha)p(x,x,x) + \alpha q(x,x) - \hat{b}(x)\Big] \cdot \frac{f_{Y_1}(x \mid x)}{F_{Y_1}(x \mid x)} + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x \mid x)}\, dy \\
&= [p(x,x,x) - \hat{b}(x)] \cdot \frac{f_{Y_1}(x \mid x)}{F_{Y_1}(x \mid x)} + (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x \mid x)}\, dy
\end{aligned}$$

With the boundary condition of the lowest type $\hat{b}(\underline{x}) = p(\underline{x},\underline{x},\underline{x})$, the solution to the above differential equation is

$$\begin{aligned}
\hat{b}(x) &= (1-\alpha)p(x,x,x) + \alpha q(x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x) \\
&= p(x,x,x) - \int_{\underline{x}}^{x} L(s \mid x) dp(s,s,s) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)} dL(s,x).
\end{aligned}$$

where

$$\begin{aligned}
L(s \mid x) &= \exp\left\{-\int_{s}^{x} \frac{f_{Y_1}(t \mid t)}{F_{Y_1}(t \mid t)} dt\right\}, \\
J(s) &= \int_{\underline{x}}^{s} p_1(s,s,y) \cdot f_{Y_1}(y \mid s) dy.
\end{aligned}$$

To demonstrate that the lead reinsurer indeed achieves the maximum profit, not the minimum, consider that if $x' < x$, her payoff when bidding $\hat{b}(x')$ is then $\pi\left(\hat{b}(x') \mid x\right)$. Note that $\frac{\partial \pi\left(\hat{b}(x') \mid x'\right)}{\partial \hat{b}} = 0$, and the following inequality holds.

$$\begin{aligned}
\frac{\partial \pi\left(\hat{b}(x') \mid x'\right)}{\partial b} =& \frac{\Big[p(x',x',x') - \hat{b}(x)\Big] \cdot f_{Y_1}(x' \mid x') + \int_{\underline{x}}^{x'} (1-\alpha) p_1(x',x',y) \cdot f_{Y_1}(y \mid x')\, dy}{\hat{b}'(x')} \\
& - F_{Y_1}\left(x' \mid x'\right) \\
=& \Big[\hat{b}'(x')\Big]^{-1} F_{Y_1}\left(x' \mid x'\right) \left\{ \begin{array}{c} \left(p(x',x',x') - \hat{b}(x')\right) \frac{f_{Y_1}(x' \mid x')}{F_{Y_1}(x' \mid x')} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1(x',x',y) \cdot \frac{f_{Y_1}(y \mid x')}{F_{Y_1}(x' \mid x')} dy - \hat{b}'(x') \end{array} \right\} \\
\leq& \Big[\hat{b}'(x')\Big]^{-1} F_{Y_1}\left(x' \mid x'\right) \left\{ \begin{array}{c} \left(p(x',x,x') - \hat{b}(x')\right) \frac{f_{Y_1}(x' \mid x)}{F_{Y_1}(x' \mid x)} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1(x',x,y) \cdot \frac{f_{Y_1}(y \mid x)}{F_{Y_1}(x' \mid x)} dy - \hat{b}'(x') \end{array} \right\} \\
=& \left[\frac{F_{Y_1}(x' \mid x')}{F_{Y_1}(x' \mid x)}\right] \left[\frac{\partial \pi(b(x') \mid x)}{\partial \hat{b}}\right].
\end{aligned}$$

The above inequality holds because $F_{Y_1}(\cdot \mid x)$ dominates $F_{Y_1}(\cdot \mid x')$ in terms of reverse hazard rate for all $x' < x$ (see p. 287 in the Appendix of Krishna, 2009). Moreover, p and its derivative p_1 are nondecreasing in their arguments due to affiliation and Assumption 1, and $\frac{F_{Y_1}(\cdot|x')}{F_{Y_1}(x'|x')}$ first-order dominates $\frac{F_{Y_1}(\cdot|x)}{F_{Y_1}(x'|x)}$, also by affiliation. Note that $\frac{\partial\pi(\hat{b}(x'))}{\partial\hat{b}} \geq 0$ implies that when the true signal of a lead reinsurer is x and he bids $b(x')$ where $x' \leq x$, he would increase the bid to maximize his payoff. Symmetrically, it holds that $\frac{\partial\pi(\hat{b}(x'))}{\partial\hat{b}} \leq 0$ if $x' \geq x$. The lead would lower his bid $\hat{b}(x')$ to increase his payoff if $x' \leq x$. We must also have that for all x, $\alpha p(x,x,x) + (1-\alpha)q(x,x) - \hat{b}(x) = p(x,x,x) - \hat{b}(x) > 0$ holds for all x in $[\underline{x}, \bar{x}]$, otherwise a bid of zero would be better. Note the following results hold.

$$\begin{aligned}
p(x,x,x) - \hat{b}(x) &= \int_{\underline{x}}^{x} L(s \mid x)\, dp(s,s,s) - (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s)}{f_{Y_1}(s \mid s)}\, dL(s,x) \\
&= \int_{\underline{x}}^{x} L(s \mid x)\,[p_1(s,s,s) + p_2(s,s,s) + p_3(s,s,s)]\, ds - (1-\alpha)\int_{\underline{x}}^{x} L(s,x)\frac{J(s)}{F_{Y_1}(s \mid s)}\, ds \\
&= \int_{\underline{x}}^{x} L(s \mid x)\left[p_1(s,s,s) + p_2(s,s,s) + p_3(s,s,s) - (1-\alpha)\frac{J(s)}{F_{Y_1}(s \mid s)}\right] ds \\
&\geq \int_{\underline{x}}^{x} L(s \mid x)\,[p_2(s,s,s) + p_3(s,s,s)]\, ds > 0.
\end{aligned}$$

The first equality uses total differentiation of dp and the fact that $\frac{dL(s,x)}{L(s,x)} = \frac{f_{Y_1}(s|s)}{F_{Y_1}(s|s)}ds$. The inequality uses $(1-\alpha)\frac{J(s)}{F_{Y_1}(s|s)} := \beta\frac{J(s)}{F_{Y_1}(s|s)} = \beta\int_{\underline{x}}^{s} p_1(s,s,y)\cdot\frac{f_{Y_1}(y|s)}{F_{Y_1}(s|s)}dy \leq p_1(s,s,s)$ for $y \leq s$ and $0 \leq \beta \leq 1$. The last inequality holds since $p_2 > 0$ and $p_3 > 0$ by affiliation. Hence, bidding $b(x)$ is indeed an equilibrium when the lead's signal is x. This completes the proof. □

A.2 Proof of Corollary 1

Proof. In the proof of Proposition 2, we establish that $p(x,x,x) - b(x) > 0$. Substituting this result into the expression for $\hat{b}'(x)$ and noting that $p_1(x,x,y) \geq 0$ by Assumption 1, we conclude that the derivative $\hat{b}'(x)$ is positive. This completes the proof. □

A.3 Proof of Propostion 3

Proof. First, for the client, in the follow-the-lead case, the client's payoff is $\hat{\pi}_C = \hat{p}$, while in the retrocession case, the client's payoff is $\tilde{\pi}_C = \tilde{p}$. Since $\tilde{p} - \hat{p} = S(\beta^*, x) > 0$, the client is better off in the retrocession case. This is illustrated in Figure 2, where the distance between the purple dotted line and the upper bound of the box is greater than that between the orange dotted line and the upper bound of the box.

Second, for the followers, in the follow-the-lead case, the followers' payoff is $\tilde{\pi}_F = (1 - \alpha^*)(V - \tilde{p}) > 0$, whereas in the retrocession case, their payoff is zero, i.e., $\hat{\pi}_F = 0$. Therefore, followers are better off in the follow-the-lead case. This is illustrated in Figure 2, where the middle triangle is divided by the purple line into two parts in the right-hand panel, which are transferred to the client and the lead reinsurers.

Third, for the lead reinsurer, in the follow-the-lead case, the leader's payoff is $\tilde{\pi}_L = \alpha^* \tilde{\pi}_1 > 0$. In the retrocession case, the lead's payoff is $\hat{\pi}_L = \alpha^* \tilde{\pi}_1 + \beta^* \hat{\pi}_1^{\text{Full}} > \tilde{\pi}_L$, where $\alpha^* + \beta^* = 1$. The fact that $\hat{\pi}_1^{\text{Full}} > 0$ can be seen in the proof of Proposition 2 that the equilibrium payoff is positive if $\alpha = 0$ or $\beta = 1$. This is illustrated in Figure 2, where the lower trapezoid in the right-hand panel covers the lower triangle in the left-hand panel.

Fourth, for the reinsurers, in the common-value setting, their joint payoff, $V - p$, and the client's payoff, p, sum to a constant V in both cases. If the client is better off in the retrocession case than in the follow-the-lead case, the reinsurers are correspondingly better off in the follow-the-lead case. This can be seen in Figure 2, where the lower rectangle in the left panel covers the trapezoid below the purple dotted line in the right panel. □

A.4 Proof of Proposition 4

Proof. Taking the FOC of equilibrium payoff to get the equilibrium bidding strategy candidate

$$\tilde{b}(x) = q(x, x; x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0)\, dq(s, s; x_0).$$

We need to show that the bidding strategy candidate $\tilde{b}(x; x_0)$ leads to a maximum in payoff. If the leader bids $\tilde{b}(x'; x_0)$ instead when his signal is x, the payoff is $\pi(\tilde{b}(x'; x_0) \mid x)$. Taking the derivative:

$$\begin{aligned} \frac{\partial \pi(\tilde{b}(x'; x_0) \mid x)}{\partial x'} &= (q(x, x'; x_0) - \tilde{b}(x'; x_0)) f_{Y_1}(x' \mid x; x_0) - \tilde{b}'(x; x_0) F_{Y_1}(x' \mid x; x_0) \\ &= F_{Y_1}(x' \mid x; x_0) \left[(q(x, x'; x_0) - \tilde{b}(x'; x_0)) \frac{f_{Y_1}(x' \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} - \tilde{b}'(x; x_0) \right]. \end{aligned}$$

If $x' < x$, then since $q(x, x'; x_0) > q(x', x'; x_0)$ and $\frac{f_{Y_1}(x'|x;x_0)}{F_{Y_1}(x'|x;x_0)} > \frac{f_{Y_1}(x'|x';x_0)}{F_{Y_1}(x'|x';x_0)}$, it holds that

$$\begin{aligned} \frac{\partial \pi(\tilde{b}(x'; x_0) \mid x)}{\partial x'} &> F_{Y_1}(x' \mid x; x_0) \left[(q(x', x'; x_0) - \tilde{b}(x'; x_0)) \frac{f_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} - \tilde{b}'(x; x_0) \right] \\ &= \frac{\partial \pi(\tilde{b}(x'; x_0) \mid x')}{\partial x'} = 0. \end{aligned}$$

In words, the payoff increases with a higher bid when $x' < x$. Similarly, we have that $\frac{\partial\pi(\tilde{b}(x';x_0)|x)}{\partial x'} < 0$ when $x' > x$. The payoff increases with a lower bid when $x' < x$. Hence, choosing $\tilde{b}(x;x_0)$ when the signal is x indeed leads to a maximum payoff. This completes the proof. ☐

A.5 Proof of Proposition 5

Proof. When X_0 is revealed, the expected payment of a winning leader when he receives a signal x but bids as if his signal were z (i.e., for all $X_0 = x_0$, he bids $\tilde{b}(z;x_0)$) is

$$W^A(z,x) = E\left[\tilde{b}_I(z;x_0) \mid X_1 = x\right],$$

so $W_2^A(z,x) \geq 0$, because X_0 and X_1 are affiliated.

When X_0 is not revealed, similarly we have that

$$W^B(z,x) = \tilde{b}(z),$$

so $W_2^B(z,x) = 0$.

Hence, $W_2^A(z,x) \geq W_2^B(z,x)$. Publicly revealing X_0 raises $\tilde{p}$. ☐

A.6 Proof of Proposition 6

Proof. The proof is similar to that in Proposition 2. The FOC is derived using the Lebnitz's integral rule, and $\hat{b}(x;x_0)$ satisfies a first-order differential equation:

$$\begin{aligned}
\widehat{b}'(x) = &\left[(1-\alpha)p(x,x,x;x_0) + \alpha q(x,x;x_0) - \hat{b}(x;x_0)\right] \cdot \frac{f_{Y_1}(x \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)} \\
&+ (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y;x_0) \cdot \frac{f_{Y_1}(y \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)}\, dy \\
= &\left[p(x,x,x;x_0) - \hat{b}(x;x_0)\right] \cdot \frac{f_{Y_1}(x \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)} \\
&+ (1-\alpha)\int_{\underline{x}}^{x} p_1(x,x,y;x_0) \cdot \frac{f_{Y_1}(y \mid x;x_0)}{F_{Y_1}(x \mid x;x_0)}\, dy.
\end{aligned}$$

With the boundary condition of the lowest type $\hat{b}(\underline{x}; x_0) = p(\underline{x}, \underline{x}, \underline{x}; x_0)$, the solution to the above differential equation is

$$\begin{aligned}\hat{b}(x; x_0) &= (1-\alpha)p(x, x, x; x_0) + \alpha q(x, x; x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0) dp(s, s, s; x_0) \\ &\qquad + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s; x_0)}{f_{Y_1}(s \mid s; x_0)} dL(s, x; x_0) \\ &= p(x, x, x; x_0) - \int_{\underline{x}}^{x} L(s \mid x; x_0) dp(s, s, s; x_0) + (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s; x_0)}{f_{Y_1}(s \mid s; x_0)} dL(s, x; x_0),\end{aligned}$$

where

$$\begin{aligned}L(s \mid x; x_0) &= \exp\left\{-\int_{s}^{x} \frac{f_{Y_1}(t \mid t; x_0)}{F_{Y_1}(t \mid t; x_0)} dt\right\}, \\ J(s; x_0) &= \int_{\underline{x}}^{s} p_1(s, s, y; x_0) \cdot f_{Y_1}(y \mid s; x_0) dy.\end{aligned}$$

Given the equilibrium bidding strategy candidate, to show that the lead reinsurer indeed achieves the maximum profit, not the minimum, consider that if $x' < x$, their payoff when bidding $\hat{b}(x'; x_0)$ is then $\pi\left(\hat{b}(x'; x_0) \mid x\right)$. Note that $\frac{\partial \pi\left(\hat{b}(x'; x_0) \mid x'\right)}{\partial \hat{b}} = 0$, and the following inequality holds.

$$\begin{aligned}\frac{\partial \pi\left(\hat{b}(x'; x_0) \mid x'\right)}{\partial \hat{b}} &= \frac{\left[p(x', x', x'; x_0) - \hat{b}(x)\right] \cdot f_{Y_1}(x' \mid x'; x_0) + \int_{\underline{x}}^{x'} (1-\alpha) p_1(x', x', y; x_0) \cdot f_{Y_1}(y \mid x'; x_0)\, dy}{\hat{b}'(x'; x_0)} \\ &\quad - F_{Y_1}(x' \mid x'; x_0) \\ &= \left[\hat{b}'(x'; x_0)\right]^{-1} F_{Y_1}(x' \mid x'; x_0) \left\{\begin{array}{c} \left(p(x', x', x'; x_0) - \hat{b}(x'; x_0)\right) \frac{f_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1(x', x', y; x_0) \cdot \frac{f_{Y_1}(y \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)} dy - \hat{b}'(x'; x_0) \end{array}\right\} \\ &\leq \left[\hat{b}'(x'; x_0)\right]^{-1} F_{Y_1}(x' \mid x'; x_0) \left\{\begin{array}{c} \left(p(x', x, x'; x_0) - \hat{b}(x'; x_0)\right) \frac{f_{Y_1}(x' \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} \\ + \int_{\underline{x}}^{x} (1-\alpha) p_1(x', x, y; x_0) \cdot \frac{f_{Y_1}(y \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)} dy - \hat{b}'(x'; x_0) \end{array}\right\} \\ &= \left[\frac{F_{Y_1}(x' \mid x'; x_0)}{F_{Y_1}(x' \mid x; x_0)}\right] \left[\frac{\partial \pi\left(\hat{b}(x'; x_0) \mid x\right)}{\partial \hat{b}}\right].\end{aligned}$$

The above inequality holds because $F_{Y_1}(\cdot \mid x; x_0)$ dominates $F_{Y_1}(\cdot \mid x'; x_0)$ in terms of reverse hazard rate for all $x' < x$ (see p.287 in the Appendix of Krishna, 2009). Moreover, p and its derivative p_1 are nondecreasing in their arguments due to affiliation and Assumption 1, and $\frac{F_{Y_1}(\cdot \mid x'; x_0)}{F_{Y_1}(x' \mid x'; x_0)}$ first-order dominates $\frac{F_{Y_1}(\cdot \mid x; x_0)}{F_{Y_1}(x' \mid x; x_0)}$, also by affiliation. Note that $\frac{\partial \pi(\hat{b}(x'))}{\partial \hat{b}} \geq 0$ implies that when the true signal of a lead reinsurer is x and he bids $\hat{b}(x'; x_0)$ where $x' \leq x$, he would increase the bid to maximize his payoff. Symmetrically, it holds that $\frac{\partial \pi(\hat{b}(x'; x_0))}{\partial \hat{b}} \leq 0$ if $x' \geq x$. The lead would lower his bid $\hat{b}(x')$ to increase his payoff if $x' \leq x$. We must also have that for all x, $\alpha p(x,x,x;x_0) + (1-\alpha)q(x,x;x_0) - \hat{b}(x;x_0) = p(x,x,x;x_0) - \hat{b}(x;x_0) > 0$ holds for all x in $[\underline{x}, \bar{x}]$, otherwise a bid of zero would be better.

$$
\begin{aligned}
p(x,x,x;x_0) - \hat{b}(x;x_0) &= \int_{\underline{x}}^{x} L(s \mid x;x_0)\, dp(s,s,s;x_0) - (1-\alpha)\int_{\underline{x}}^{x} \frac{J(s;x_0)}{f_{Y_1}(s \mid s;x_0)}\, dL(s,x;x_0) \\
= \int_{\underline{x}}^{x} & L(s \mid x;x_0)\left[p_1(s,s,s;x_0) + p_2(s,s,s;x_0) + p_3(s,s,s;x_0)\right] ds - (1-\alpha)\int_{\underline{x}}^{x} L(s,x;x_0) \frac{J(s;x_0)}{F_{Y_1}(s \mid s;x_0)}\, ds \\
= \int_{\underline{x}}^{x} & L(s \mid x;x_0)\left[p_1(s,s,s;x_0) + p_2(s,s,s;x_0) + p_3(s,s,s;x_0) - (1-\alpha)\frac{J(s;x_0)}{F_{Y_1}(s \mid s;x_0)}\right] ds \\
\geq \int_{\underline{x}}^{x} & L(s \mid x;x_0)\left[p_2(s,s,s;x_0) + p_3(s,s,s;x_0)\right] ds \geq 0.
\end{aligned}
$$

The first equality uses total differentiation of dp and the fact that $\frac{dL(s,x;x_0)}{L(s,x;x_0)} = \frac{f_{Y_1}(s|s;x_0)}{F_{Y_1}(s|s;x_0)}ds$. The inequality uses $\frac{J(s;x_0)}{F_{Y_1}(s|s;x_0)} = \int_{\underline{x}}^{s} p_1(s,s,y;x_0) \cdot \frac{f_{Y_1}(y|s;x_0)}{F_{Y_1}(s|s;x_0)}dy \leq p_1(s,s,s;x_0)$ for $y \leq s$. The last inequality holds since $p_2 \geq 0$ and $p_3 \geq 0$ by affiliation. Hence, bidding $\hat{b}(x;x_0)$ is indeed an equilibrium when the lead's signal is x. This completes the proof. □

A.7 Proof of Corollary 2

Proof. The proof of corollary 2 is similar to that in the proof of corollary 1. In the proof of proposition 6, we establish that $p(x,x,x;x_0) - \hat{b}(x;x_0) > 0$. Substituting this result into the derivative of $\tilde{b}(x)$ and noting that $p_1(x,x,y;x_0) \geq 0$ by Assumption 1, we then conclude the derivative $\hat{b}'(x;x_0)$ is positive. This completes the proof. □

A.8 Proof of Proposition 7

Proof. When X_0 is revealed, the expected payment of a winning leader when he receives a signal x but bids as if his signal were z (i.e., for all $X_0 = x_0$, he bids $\hat{b}(z;x_0)$) is

$$
W^A(z,x) = E\left[\hat{b}(z;x_0) \mid X_1 = x\right],
$$

so $W_2^A(z,x) \geq 0$, because X_0 and X_1 are affiliated.

When X_0 is not revealed, similarly we have that

$$
W^B(z,x) = \hat{b}(z),
$$

so $W_2^B(z,x) = 0$. Hence, $W_2^A(z,x) \geq W_2^B(z,x)$.

Thus, publicly revealing X_0 raises $\hat{p}$ by Lemma 4. □

A.9 Proof of Lemma 5

Proof.

$$
\begin{aligned}
&E\left[V \mid Z_1, Z_2, \ldots, Z_n, Z_{n+k}\right] = \int_{\mathbb{R}} v \cdot f_{V|Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v \mid z_1, z_2, \ldots, z_n, z_{n+k}\right) dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,z_2,\ldots,Z_n,Z_{n+k}}\left(v, z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right) \cdot f_{Z_{n+k}|Z_1,Z_2\ldots,Z_n}\left(z_{n+k} \mid z_1, z_2, \ldots, z_n\right)}{f_{Z_1,Z_2\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right) \cdot \frac{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}(z_1,z_2,\ldots,z_n,z_{n+k})}{f_{Z_1,Z_2,\ldots,Z_n}(z_1,z_2,\ldots,z_n)}}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)} dv \\
&= \int_{\mathbb{R}} v \cdot \frac{f_{V,Z_1,Z_2,\ldots,Z_n}\left(v, z_1, z_2, \ldots, z_n\right)}{f_{Z_1,Z_2,\ldots,Z_n}\left(z_1, z_2, \ldots, z_n\right)} dv \\
&= E\left[V \mid Z_1, Z_2, \ldots, Z_n\right].
\end{aligned}
$$

The first and last equalities correspond to the definitions of conditional expectation. The second equality uses

$$
f_{V|Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v \mid z_1, z_2, \ldots, z_n, z_{n+k}\right) = \frac{f_{V,Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(v, z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)}.
$$

The numerator in the third equality follows the definition of the garbling condition. The fourth equality uses

$$
f_{Z_{n+k}|Z_1,Z_2,\ldots,Z_n}\left(z_{n+k} \mid z_1, z_2, \ldots, z_n\right) = \frac{f_{Z_1,Z_2,\ldots,Z_n,Z_{n+k}}\left(z_1, z_2, \ldots, z_n, z_{n+k}\right)}{f_{Z_1,Z_2,\ldots,Z_n}\left(z_1, z_2, \ldots, z_n\right)}.
$$

This completes the proof. □

B Appendix B

This section contrasts syndicated loans with reinsurance syndicates, as these two are similar in organization, and the former is extensively studied in banking.

B.1 Comparison of Syndicated Loan and Reinsurance Syndicate

Below we briefly describe the basic background of syndicated loans and reinsurance syndicates. A syndicated loan is provided by a group of lenders to a single borrower, often for large-scale loans exceeding a single lender's capacity. It is formed by banks or financial institutions, with a lead bank arranging and managing the loan. The loan amount is divided among participants, limiting each lender's risk to their portion.

In a co-reinsurance syndicate, multiple reinsurers pool resources to underwrite large or complex risks. A lead reinsurer is responsible for determining the terms and conditions, which are binding for the follow market. Reinsurers each assume a portion of the risk according to their participation percentages. Large risks are spread to reduce the financial impact on any single insurer.

Both organizations share similarities, requiring multiple banks or reinsurers to jointly raise capital and share risks. They adopt a leader-follower structure to reduce transaction costs in business. Though the organizational structures have some commonalities between a syndicated loan and a co-insurance syndicate, there are crucial differences.

First, the industry applications are different. Syndicated loans are used in banking and finance, while reinsurance syndicates serve the broader insurance and reinsurance industry.

Second, the underlying risk types involved in these two structures differ. A syndicated loan is subject to the credit risk of the borrower, while the default risk has been historically rare in insurance; one example we know is American International Group (AIG)'s near-failure in 2008 due to its exposure to credit default swaps (CDS) and the housing market collapse.[20] In contrast, the main risk related to pricing in reinsurance is the underwriting risk covered by the contracts, such as potential damages caused by a hurricane in Florida or an earthquake in Japan.

Third, the pricing practices induced by covering different risks in these two industries differ. The pricing in syndicated loans is typically standardized, often based on benchmark interest rates such as London Inter-bank Offered Rate (LIBOR) or Secured Overnight Financing Rate

[20] AIG, a global company with about 1 trillion US Dollars in assets before the crisis, lost 99.2 billion in 2008. On September 16 of that year, the Federal Reserve Bank of New York stepped in with an 85 billion loan to keep the failing company from going under. See https://insight.kellogg.northwestern.edu/article/what-went-wrong-at-aig

(SOFR), plus an applicable margin.[21] Conversely, there is no such benchmark in reinsurance treaties. The price of a reinsurance policy varies case by case, depending on the specific business and the individual underwriter's assessment of the underwriting risk.

Lastly, the regulatory focuses differ between the insurance and banking sectors. In insurance, regulations aim to prevent unfair pricing and exclusionary practices, driven by the sector's reliance on competitive risk assessment and pricing. In contrast, banking regulations prioritize financial stability, risk management, and the prevention of systemic failures, viewing syndicated loans as a means to spread risk and increase lending capacity for large projects. While competition concerns are monitored, the overarching goal in banking is to ensure that collaborative lending practices do not compromise the resilience of the financial system.

B.2 Reinsurance Business and Follow-the-lead Practice

This section offers background information on the reinsurance business and the follow-the-lead practice.

B.2.1 Reinsurance Business and Syndicates

The reinsurance can be classified into external reinsurance and internal reinsurance. The follow-the-lead practice discussed in this paper is used in external reinsurance, wherein cedants rely on professional reinsurers for risk cession. Internal reinsurance, or captive reinsurance, is when a parent company forms its own reinsurance entity to manage risks from its insurance operations internally, acting as a form of self-insurance instead of outsourcing to external reinsurance firms. External reinsurance typically constitutes the primary business line. As Hsiao and Shiu (2019) shows, in the UK life insurance industry, 80.24% of the insurers used at least one type of reinsurance. The participation rate for external reinsurance usage is 76.33%. This means non-affiliated professional external reinsurers play an important role in diversifying the risks.

In addition to reinsurance, there are various Alternative Risk Transfer (ART) mechanisms like Insurance-Linked Securities (ILS), which offer additional options for managing risk. These financial instruments allow insurers to transfer risk to investors in the financial markets, similar to how banks distribute loan risks through securitization. For example, by issuing $1 million in catastrophe bonds, an insurer can raise the same amount from investors. If a catastrophe occurs, the principal is used to cover losses, and investors receive only coupon payments. Otherwise, the insurer repays the principal plus coupons. Despite their presence in diversifying risks, these instruments fall outside the scope of this paper.

[21]Source: https://www.srsacquiom.com/our-insights/syndicated-loan-market/

Client insurers depend on professional reinsurers for both risk management and pricing, as reinsurers typically have a deeper understanding of underlying risks than their clients, drawing on historical data, experience, and new technology. For instance, newly emerging risks such as cybercrime require specialized knowledge and tools. Lloyd's syndicates use the Axio360 platform to develop solutions for cyber-physical damage coverage, leveraging it as a decision-making engine for comprehensive cyber risk management. This includes cybersecurity assessments, cyber risk quantification (CRQ), risk transfer, and cyber insurance analysis.[22] Individual clients may lack access to such advanced technology when evaluating the potential risks.

In external reinsurance, reinsurers often collaborate to spread risk more effectively. Notable examples include insurance and reinsurance syndicates like Hiscox ESG 3033, which, brokered by Aon, provided coverage for a new wind farm in Spain and a solar farm risk in the USA in 2023.[23] Furthermore, Beazley launched Syndicate 4321 in 2022, offering exclusive capacity for clients with high ESG ratings.[24]

B.2.2 Follow-the-lead Practice

Follow-the-lead practice locks the aftermarket on a single set of terms and conditions determined in the tender phase. Regarding the details, the Commission outlines some market elements contained in the subscription procedure:

(a) Alignment on the contractual terms offered by the lead (re)insurer.

(b) Revealing the price offered by the lead (re)insurer to the follow market.

(c) Potentially, guaranteeing to the lead that the price and conditions, and the share of the risk, that were agreed with it at the end of the first round, will not be changed to its detriment if participants in the follow market were to offer a lower price;

(d) Alignment on the premium.

Up to today, follow-the-lead remains a prevalent market practice, evidenced by recent articles. One notes,"Follower reinsurers accept to participate in a reinsurance treaty in which the final terms and conditions have already been agreed, but they don't necessarily influence the terms and conditions involved."[25] Another states,"Despite not being in the lead, they (followers) enjoy the same level of compensation as the lead reinsurer."[26]

[22] See https://www.reinsurancene.ws/lloyds-of-london-investment-in-axio-to-support-company-growth-and-benefit-the-market/
[23] See https://www.hiscoxgroup.com/news/press-releases/2023/02-08-23
[24] See https://www.beazley.com/en-us/news-and-events/esg-syndicate-4321/
[25] See an article written by CCR Re in 2021 https://blog.ccr-re.com/en/what-is-a-follower
[26] See an article by insurance professionals in 2022: https://www.investopedia.com/terms/l/lead-reinsurer.asp

C Appendix C

Below, we discuss the implementation of the optimal selling mechanism, sufficient conditions for full surplus extraction, and its distinction from a second-price auction, based on McAfee et al. (1989).

C.1 Implementation

1. The insurer (seller) *randomly* selects two syndicates labeled i and j among n competing syndicates.

2. The insurer then asks leader j to report his signal, the realization is denoted as x_j, but offers the business to syndicate i at a price $z(x_j)$, where $z(\cdot)$ is a price function that depends only on j's report.

C.2 Incentive Compatibility and Participation Constraint

The first observation is that the mechasim above is weakly incentive-compatible (IC). This is because a reinsurer's payoff does not depend on its own actions: syndicate i's payoff depends on syndicate j's report, while the payoffs for all other participants are zero.

Second, we need to show the existence of a price function $z(\cdot)$ such that the participation constraints are satisfied, i.e., the buyer i's expected payoff is nonnegative. Specifically, if his payoff is zero, the mechanism is optimal from the seller's perspective.

Denote the conditional distribution of each lead reinsurer i's signal x_i as $F(x_i|v)$, with density $f(x_i|v)$ continuous and strictly positive on $[0,1] \times [0,1]$.[27] Let the distribution of v be $G(v)$. The payoff of leader i is given by:

$$\pi_i = \int_0^1 \left[v - \int_0^1 z(x) f(x \mid v) dx \right] \frac{f(x_i \mid v)}{\int_0^1 f(x_i \mid u)\, dG(u)} dG(v).$$

Thus, characterizing the optimal mechanism reduces to finding $z(\cdot)$ such that $\pi_i = 0$.

C.3 Surplus Extraction as a Minimum Norm Problem

The full surplus extraction problem can be transformed into a minimum norm problem. Consider a Hilbert space of square-integrable functions $L^2([0,1], G)$ and define the norm as $\|x\| =$

[27] We consider $[0,1]$ w.l.o.g. since it is isomorphic to interval $[\underline{v}, \overline{v}]$.

$(x,x)^{\frac{1}{2}}$, where $(x,y) = \int_0^1 x(v)y(v)dG(v)$. Define a set Y as:

$$Y = \left\{ y \in L^2([0,1],G) \mid \exists z \in L^2([0,1],G), y(v) = \int_0^1 z(x)f(x \mid v)dx \right\}.$$

Then we have the following results.

Lemma 6 (McAfee et al., 1989, Lemma)**.** $\exists z(\cdot) \in L^2([0,1],G)$ *satisfying* $\pi_i = 0$ *if and only if* $\min_{y \in Y} \|y - v\|$ *has a solution.*

Theorem 1 (McAfee et al., 1989, Main Theorem)**.** $\forall \varepsilon > 0, \exists z \in L^2([0,1],G)\,\text{s. t.}\,\pi_i \in [0,\varepsilon]$.

The intuition behind leaving small rents ε is that the mechanism is only weakly IC, meaning the buyer is not worse off if they lie. To ensure strict incentives, a cost shall be imposed on the seller to leave strict rents for the buyer to report truthfully.

C.4 Sufficient Conditions for Full Surplus Extraction

Balakrishnan (2012) states that every closed convex set in a Hilbert space has a unique element of minimum norm. Two examples provided by McAfee et al. (1989) satisfy the condition that the set Y defined above is closed.

1. The conditional density satisfies a separability condition, i.e., $f(s \mid v) = \sum_{i=1}^n a_i(s)b_i(v)$.
2. The value v has a finite support, i.e., $G = \{v_1, v_2, \ldots, v_m\}$.

C.5 Distinction from Second-Price Auction

The mechanism discussed here resembles but is essentially different from a second-price auction (SPA) in key aspects:

1. A SPA is not optimal as it is dominated by a English auction in a common value setting.
2. In this mechanism, the two buyers are selected arbitrarily. In contrast, in a SPA, the highest and second-highest bidders are chosen by the buyers themselves based on their signals or bids. The buyers' knowledge of this prevents full surplus extraction.

References

Anatolitis, V., Azanbayev, A., & Fleck, A.-K. (2022). How to design efficient renewable energy auctions? empirical insights from europe. *Energy Policy*, *166*, 112982.

Balakrishnan, A. V. (2012). *Applied functional analysis: A* (Vol. 3). Springer Science & Business Media.

Bäuerle, N., & Glauner, A. (2018). Optimal risk allocation in reinsurance networks. *Insurance: Mathematics and Economics*, *82*, 37–47.

Bernis, G. (2002). Equilibrium in a reinsurance market with short sale constraints. *Economic Theory*, *20*(2), 295–320.

Bikhchandani, S. (2010). Information acquisition and full surplus extraction. *Journal of Economic Theory*, *145*(6), 2282–2308.

Boonen, T. J., Tan, K. S., & Zhuang, S. C. (2021). Optimal reinsurance with multiple reinsurers: Competitive pricing and coalition stability. *Insurance: Mathematics and Economics*, *101*, 302–319.

Borch, K. (1992). Equilibrium in a reinsurance market. In *Foundations of insurance economics: Readings in economics and finance* (pp. 230–250). Springer.

Börgers, T. (2015). *An introduction to the theory of mechanism design*. Oxford University Press, USA.

Bos, O., & Pollrich, M. (2022). Auctions with signaling bidders: Optimal design and information disclosure. *Available at SSRN 4252493*.

Boulatov, A., & Dieckmann, S. (2013). The risk-sharing implications of disaster insurance funds. *Journal of Risk and Insurance*, *80*(1), 37–64.

Bukhchandani, S., & Huang, C.-f. (1989). Auctions with resale markets: An exploratory model of treasury bill markets. *The Review of Financial Studies*, *2*(3), 311–339.

Calzolari, G., & Pavan, A. (2006). Monopoly with resale. *The RAND Journal of Economics*, *37*(2), 362–375.

Chen, L., Shen, Y., & Su, J. (2020). A continuous-time theory of reinsurance chains. *Insurance: Mathematics and Economics*, *95*, 129–146.

Chi, Y., & Tan, K. S. (2013). Optimal reinsurance with general premium principles. *Insurance: Mathematics and Economics*, *52*(2), 180–189.

Crémer, J., & McLean, R. P. (1985). Optimal selling strategies under uncertainty for a discriminating monopolist when demands are interdependent. *Econometrica*, *53*(2), 345–361.

DeGroot, M. H. (2005). *Optimal statistical decisions*. John Wiley & Sons.

Dworczak, P. (2020). Mechanism design with aftermarkets: Cutoff mechanisms. *Econometrica*, *88*(6), 2629–2661.

Dye, R. A. (1985). Disclosure of nonproprietary information. *Journal of Accounting Research*, 123–145.

Eyster, E., & Rabin, M. (2005). Cursed equilibrium. *Econometrica*, *73*(5), 1623–1672.

Garratt, R. J., Tröger, T., & Zheng, C. Z. (2009). Collusion via resale. *Econometrica*, *77*(4), 1095–1136.

Georganas, S. (2011). English auctions with resale: An experimental study. *Games and Economic Behavior*, *73*(1), 147–166.

Goeree, J. K. (2003). Bidding for the future: Signaling in auctions with an aftermarket. *Journal of Economic Theory*, *108*(2), 345–364.

Hafalir, I., & Krishna, V. (2008). Asymmetric auctions with resale. *American Economic Review*, *98*(1), 87–112.

Haile, P. A. (2003). Auctions with private uncertainty and resale opportunities. *Journal of Economic theory*, *108*(1), 72–110.

Hsiao, C.-Y., & Shiu, Y.-M. (2019). The effects of business mix on internal and external reinsurance usage. *The Geneva Papers on Risk and Insurance-Issues and Practice*, *44*, 624–652.

Jehiel, P., & Moldovanu, B. (2000). Auctions with downstream interaction among buyers. *Rand Journal of Economics*, 768–791.

Khurana, S. (2024). Auctions with resale at a later date. *Economic Theory*, 1–33.

Krishna, V. (2009). *Auction theory*. Academic press.

Landsberger, M. (2007). Non-existence of monotone equilibria in games with correlated signals. *Journal of Economic Theory*, *132*(1), 119–136.

McAfee, R. P., McMillan, J., & Reny, P. J. (1989). Extracting the surplus in the common-value auction. *Econometrica: Journal of the Econometric Society*, 1451–1459.

Milgrom, P. R. (2000). Putting auction theory to work: The simultaneous ascending auction. *Journal of Political Economy*, *108*(2), 245–272.

Milgrom, P. R., & Weber, R. J. (1982). A theory of auctions and competitive bidding. *Econometrica: Journal of the Econometric Society*, 1089–1122.

Perry, M., Wolfstetter, E., & Zamir, S. (2000). A sealed-bid auction that matches the english auction. *Games and Economic Behavior*, *33*(2), 265–273.

Pham, H., & Yamashita, T. (2024). Auction design with heterogeneous priors. *Games and Economic Behavior*, *145*, 413–425.

Plantin, G. (2006). Does reinsurance need reinsurers? *Journal of Risk and Insurance*, *73*(1), 153–168.

Powers, M. R., & Shubik, M. (2001). Toward a theory of reinsurance and retrocession. *Insurance: Mathematics and Economics*, *29*(2), 271–290.

Rhodes-Kropf, M., & Katzman, B. (2001). The consequences of information revealed in auctions. *Columbia Business School, Economics and Finance Working Paper*.

Robert, J. (1991). Continuity in auction design. *Journal of Economic theory*, *55*(1), 169–179.

Sherman, A. E. (2005). Global trends in ipo methods: Book building versus auctions with endogenous entry. *Journal of Financial Economics*, *78*(3), 615–649.

Varma, G. D. (2003). Bidding for a process innovation under alternative modes of competition. *International Journal of Industrial Organization*, *21*(1), 15–37.

Wagenhofer, A. (1990). Voluntary disclosure with a strategic opponent. *Journal of Accounting and Economics*, *12*(4), 341–363.

Yuan, Y. (2022). Security design under common-value competition. *Available at SSRN 4271432*.

Zhu, M. B., Ghossoub, M., & Boonen, T. J. (2023). Equilibria and efficiency in a reinsurance market. *Insurance: Mathematics and Economics*, *113*, 24–49.