Causal Effects in Matching Mechanisms with Strategically Reported Preferences*

Marinho Bertanha† Margaux Luflade‡ Ismael Mourifié§

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Abstract

A growing number of central authorities use assignment mechanisms to allocate students to schools in a way that reflects student preferences and school priorities. However, most real-world mechanisms incentivize students to strategically misreport their preferences. In this paper, we provide an approach for identifying the causal effects of school assignment on future outcomes that accounts for strategic misreporting. Misreporting may invalidate existing point-identification approaches, and we derive sharp bounds for causal effects that are robust to strategic behavior. Our approach applies to any mechanism as long as there exist placement scores and cutoffs that characterize that mechanism's allocation rule. We use data from a deferred acceptance mechanism that assigns students to more than 1,000 university–major combinations in Chile. Matching theory predicts that students' behavior in Chile should be strategic because they can list only up to eight options, and we find empirical evidence consistent with such behavior. Our bounds are informative enough to reveal significant heterogeneity in graduation success with respect to preferences and school assignment.

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†Department of Economics, University of Notre Dame. Address: 3060 Jenkins Nanovic Halls, Notre Dame, IN 46556. Email: mbertanha@nd.edu.

‡Department of Economics, University of Pennsylvania. Address: 133 South 36th Street, Philadelphia, PA 19104. Email: mluflade@sas.upenn.edu.

§Department of Economics, Washington University in St. Louis & NBER. Address: One Brookings Drive St. Louis, MO 63130-4899, USA. Email: ismaelm@wustl.edu.

1 Introduction

One of the most important decisions students make is their choice of field and institution of education. Identification of the impact of such choices on future outcomes is a critical step in the study of this decision process and of public education policies. The causal effects of schooling may vary widely across economic agents because of heterogeneous skills and preferences. Moreover, individuals' expectations about potential returns may prompt them to choose schools strategically. Heterogeneity and selection make identification of causal effects very challenging, especially when students face a large number of unordered options. On the positive side, a growing number of schools use centralized assignment mechanisms, which produce credible instruments based on the discontinuities generated by the assignment of comparable students to different schools (Kirkeboen et al. (2016) and Abdulkadiroglu et al. (2022)). Such discontinuities arise from unpredictable admission cutoffs that characterize the matching of students to schools.

Many centralized school assignment mechanisms effectively amount to a quasi-experimental design where two groups of individuals who share similar scores are assigned to different schools based on how their scores relate to admission cutoffs. The assignment in a matching characterized by such cutoffs depends on the student's preferences over feasible schools. Unlike in a typical regression discontinuity (RD) design, in this setting, students on the same side of the cutoff do not necessarily receive the same assignment. For example, individuals with similar scores just above a certain cutoff may all prefer to go to the same "school a" but could have very different second-best options if they fall on the other side of the cutoff. In such a context, Kirkeboen et al. (2016) construct comparable groups of individuals near a cutoff by conditioning on local preferences—that is, by selecting students whose preferences yield identical first- and second-best options if they fall, respectively, above and below that cutoff. Controlling for local preferences that equal a pair of schools, e.g., (a, b), allows the RD to identify the causal effect of a change in the school assignment from b to a, averaged over individuals who prefer a over b. A natural question to ask, then, is how often students submit their true preferences and what the consequences are of controlling for misreported preferences in the RD strategy. In fact, most real-world school assignment mechanisms create incentives for students to misreport their preferences. Agarwal and Somaini (2018) and Fack et al. (2019) provide thorough discussions with several real-world examples of this. Although a powerful identification strategy, this paper shows that the RD strategy that controls for reported preferences raises two important concerns, unless we assume everyone is a truth-teller. First, we argue that an average effect over students that truly prefer a over b is more externally valid than an average effect over students that only report to prefer a over b because preference reports could easily change in counterfactuals with different incentives. Second, even if interest lies on the second type of average effect, we show that controlling for reported preferences may invalidate the fundamental continuity assumption of the RD identification strategy.

This paper derives sharp bounds for causal effects of school assignment on future outcomes in mechanisms with cutoff characterization and strategic student behavior. We propose a two-step identification approach that is robust to strategic reporting of preferences. In the first step, the researcher partially identifies local preferences and constructs local preference sets for each student. We provide several tools for constructing these sets in the context of a student-proposing deferred acceptance (DA) mechanism where constraints on the submitted preferences lead to strategic behavior. Outside such a context, researchers may employ alternative tools to partially identify local preferences, as the second step of our procedure does not require a particular method to be used in the first step. For example, the identification method of Agarwal and Somaini (2018) applies to a general class of mechanisms with cutoff representation that includes variants of the DA mechanism, Boston, First Preferences First, Chinese Parallel, etc. Finally, in the second step, the researcher employs an RD identification strategy that controls for local preference sets and partially identifies the causal effects of school assignment.

Strategic behavior among students depends on the characteristics of the assignment mechanism. A mechanism is said to be strategy proof if submitting true preferences is a weakly dominant strategy for all students. For example, Dubins and Freedman (1981) demonstrate that the DA mechanism is strategy proof. However, this result breaks down when the mechanism imposes constraints on the preferences that students can submit. In many real-world school assignment mechanisms, the number of schools is too large for students to feasibly rank all schools. The central authorities running these systems may either limit the number of schools that students may rank or impose costs on the basis of the number of schools submitted. See Table 1 Panel B by Fack et al. (2019) for examples.

Our first-step tools for partial identification of local preferences naturally require assumptions on students' strategic behavior. We motivate our assumptions following the important contributions of Haeringer and Klijn (2009). They study a game where students submit constrained preference rankings and a central mechanism allocates the students to schools. One of their important findings is that it is rational for students to submit partial orders of their true preferences in some mechanisms. Specifically, suppose that a mechanism is strategy proof when students are free to rank any number of schools, as under, e.g., the unconstrained DA or Top Trading Cycles (TTC) mechanisms. Then, if the preference rankings are constrained to having at most K schools, a student can do no better than selecting K schools among her acceptable schools and ranking them according to her true preferences.

The key assumption for our first-step tools is that students submit only partial orders of their preferences. This feature is in addition to the cutoff characterization of the matching, which we assume throughout the paper. Cutoff characterization means that a student is matched to her best feasible school, where "best" is defined according to her true preferences and a school is feasible if the student's placement score clears that school's admission cutoff. Our assumption about cutoff characterization is satisfied when the matching outcome is stable (Azevedo and Leshno, 2016). Stability means that each student is matched to an acceptable school and all slots in preferred schools have been filled with people who have better placement scores. In constrained DA and TTC mechanisms, stability occurs in Nash equilibrium of the preference revelation game under appropriate conditions on the placement scores (Theorems 6.3 and 6.4, Haeringer and Klijn

(2009)). Stability occurs in Nash equilibrium without restrictions on the scores in the constrained serial dictatorship (SD) mechanism, which is a particular case of DA. We characterize sharp local preference sets for every individual that are compatible with the observed data and these model assumptions. Our random local preference sets contain the true local preference random variable with probability one. We also show a way to shrink these sets by imposing assumptions on students' expectations regarding the outcome of the match and by assuming students maximize expected utility, in line with Agarwal and Somaini (2018).

The form of our local preference sets has two important implications for applied work. First, if we ignore strategic behavior and condition only on reported local preferences, the RD strategy may yield inconsistent estimates. The reason is that, when we select individuals with the same (strategically) reported local preferences near a cutoff, our local preference sets change discontinuously at that cutoff. The problem is akin to the manipulation problem in RD: we are controlling for a variable that is manipulable, that is, reported preferences. Second, suppose that we select individuals with reported local preferences (a, b) and see that the local preference sets are all singletons $\{(a, b)\}$; in other words, our selection contains only individuals who do not manipulate. Even in this case, an RD strategy that controls for reported local preferences may still be inconsistent because the local preference sets may well contain the pair (a, b) for someone whose reported local preferences differ from (a, b). Thus, selecting individuals who *report* (a, b) does not necessarily imply that we select all individuals whose *true* local preferences equal (a, b).

The second step of our approach relies on the local preference sets constructed in the first step with either our method or an alternative method. Given interest in a pair of schools (a, b), we select all individuals whose local preference sets contain (a, b) and whose placement scores are close to the cutoff for admission at school a. This subpopulation of individuals contains all individuals whose true local preferences equal (a, b), but also other individuals. The average outcome in the subpopulation equals a weighted average of two averages: first, the average outcome for individuals with true local preferences (a, b), which is interesting for the identification of causal effects, and second, the average outcome for individuals with true local preferences that differ from (a, b). We do not know which individuals have preferences (a, b), but we do characterize sharp bounds on the proportion of such individuals in the subpopulation using the random sets constructed in the first step. Thus, our setting fits the identification problem with corrupted data studied by Horowitz and Manski (1995). This method allows us to derive closed-form bounds on the first out of the two average outcomes above, which then leads to bounds on the average causal effects. This closed-form approach offers some intuition on when we can expect the bounds to be informative about or equal to the actual average causal effect (i.e., point identification). Although practical and intuitive, these closed-form bounds may not be sharp. Thus, building on Molinari (2020) and using random set theory, we characterize sharp bounds that are numerically computable when outcomes take finitely many values.

Methods combining RD identification with school matching data have been popular among applied and theoretical researchers in economics (Jackson, 2010; Bertanha, 2020). To the best of our knowledge, our paper is the first to prove RD identification of returns of school assignment in matching mechanisms with strategically reported preferences. We note that our two-step approach differs from the usual control function approach because our first step partially identifies the control variable instead of point-identifying it as in the usual approach. This paper unifies and complements two branches of the literature. One branch features the methods proposed by Agarwal and Somaini (2018) and Fack et al. (2019) that take into account strategic reporting and identify students' true preferences, but they do not focus on causal effects of school matches on future outcomes. The other branch features the methods of Kirkeboen et al. (2016) and Abdulkadiroglu et al. (2022) that identify the causal effects of different assignments but control for reported instead of true preferences. Chen (2023) discusses methods applicable in the case when assignment is based both on lottery- and RD-driven variation.

We apply our two-step identification strategy to matching data from Chile. Chile has a centralized DA mechanism that assigns students to university–major pairs. In 2010, 88,000 students ranked at most eight university–major pairs out of a total of 1,092 options available. Thus, the mechanism constrains students' preference rankings, and students have incentives to behave strategically. The methods proposed by Kirkeboen et al. (2016) and Abdulkadiroglu et al. (2022) are hence not directly applicable in the Chilean case, even if all students report their preferences truthfully.

The methods of Kirkeboen et al. (2016) apply to the SD mechanism—a particular case of the DA mechanism in which all schools utilize the same placement score. In the SD case, the counterfactual set of schools for students just above or just below a cutoff does not vary across students. For example, for students just above a cutoff, their counterfactual set includes all schools whose cutoffs are lower than the cutoff in question. The same does not apply in DA, the mechanism used in Chile. Students have multiple placement scores, and the set of feasible schools may vary widely across students. Defining the counterfactual set is an important step in the RD identification strategy because local preferences are defined over these sets, which then become the control variable in the RD strategy.

Abdulkadiroglu et al. (2022) propose a solution to the DA counterfactual problem by constructing a propensity score control variable. They study New York City public high schools, where placement scores are functions of integer priority scores plus a continuously distributed variable with full support. This particular structure of school priorities does not correspond to the Chilean case, where program-specific placement scores are computed as functions of five primitive scores: math, language, history, science, and an average score from high school. These functions are different across programs and sometimes nonlinear. This setting requires the counterfactual set of schools to be carefully defined such that controlling for local preferences does not violate the continuity assumptions required by RD. We therefore propose a general method that applies to such empirical contexts and leads to point identification if students are truthful but partial identification otherwise.

We present several pieces of evidence that students in Chile behave strategically. In settings where behavior is strategic, existing methods that control for reported preferences are invalid, and identification requires the tools that we introduce in this paper. Our data contain more than three hundred university–major pairs for which the localized RD samples are large enough for our procedure to be implemented. For each pair, we construct bounds on the average effect of a change in the initial assignment between the second- and first-best options in that pair. We examine the effects on two binary outcomes: graduation from the first-best university–major and graduation from a top university. The bounds that we compute are informative about the sign of average treatment effects in 33–67% of the university–major pairs, depending on the outcome. The bounds are also informative in that they reveal substantial heterogeneity in the treatment effects across local preferences and second-best options. Finally, we compare our bounds to point estimates that we obtain by naively ignoring strategic behavior and simply controlling for reported preferences. The naive estimates fall outside our bounds for 25–57% of the university–major pairs, consistent with the fact that naive estimates are generally inconsistent for an average treatment effect parameter when students behave strategically.

The rest of this paper proceeds as follows. Section 2 lays out the matching model for a continuum population of students and a finite number of schools. Section 3 examines point identification of average treatment effects when students are truth-tellers. Section 4 examines partial identification when students strategically report their preferences, with two subsections: Section 4.1 provides tools for construction of local preference sets that apply to constrained DA mechanisms, and Section 4.2 discusses how to use local preference sets constructed in this or other ways to derive bounds on the average treatment effects. We illustrate our identification approach with the Chilean data in Section 5. The appendix presents all proofs for the paper.

2 Model

We consider a continuum population of students and a set of J schools, $\mathcal{J} := \{1, \ldots, J\}$, that have capacities $\{q_1, \ldots, q_J\}$ defined in terms of shares of the student population (Azevedo and Leshno, 2016). Denote by Ω the set of all students in the universe of interest and use ω to index an individual student type. The student type consists of three objects. First, $Q(\omega)$ denotes the true (strict) preference relation of student ω over the set of options $\mathcal{J}^0 := \mathcal{J} \cup \{0\}$, which includes schools $\mathcal{J}$ and an outside option 0. For example, if $J = 2$ and $Q(\omega) = \{1, 2, 0\}$, then 1 is preferred to 2 (i.e., $1Q(\omega)2$), 1 is preferred to 0 (i.e., $1Q(\omega)0$), and 2 is preferred to 0 (i.e., $2Q(\omega)0$). Let $\mathcal{Q}$ be the set of all strict preference relations over $\mathcal{J}^0$ that admit at least one school that is acceptable. A school $j \in \mathcal{J}$ is "acceptable" for student ω if it is preferred to that student's outside option, i.e., $jQ(\omega)0$. We define $\bar{Q}$ as the weak preference relation induced by Q, i.e., $j\bar{Q}k \Leftrightarrow jQk$ or $j = k$. The second object of the student type is a vector of scores $\mathbf{R}(\omega) := (R_1(\omega), \ldots, R_J(\omega)) \in \mathcal{R} \subseteq \mathbb{R}^J$, where each school j utilizes R_j to rank students for admission. The third and last object, $Y(\omega, d)$, is the potential outcome of student ω if the student is assigned to option $d \in \mathcal{J}^0$. Each student has a potential outcome function $Y(\omega, \cdot)$ that maps from $\mathcal{J}^0$ to $\mathcal{Y} \subseteq \mathbb{R}$. We call $\mathbf{\Gamma}$ the set of all possible potential outcome functions. The set of all student types is $\Omega := \mathcal{Q} \times \mathcal{R} \times \mathbf{\Gamma}$. In a continuum economy, there is a probability measure $\mathbb{P}$ over Ω and the Borel σ-algebra of the product space Ω. We suppress the argument ω whenever it is unnecessary for ease of notation, e.g., $Y(d)$ vs. $Y(\omega, d)$ and Q vs. $Q(\omega)$.

A "matching" is described by a measurable function $\mu : \Omega \to \mathcal{J}^0$ that satisfies two conditions: for every $j \in \mathcal{J}$, (i) the mass of students matched to j is less than or equal to the capacity of school j, i.e., $\mathbb{P}\{\omega : \mu(\omega) = j\} \leq q_j$; and (ii) the set of students who weakly prefer option $j \in \mathcal{J}^0$ over their matching, i.e., $\{\omega : j\bar{Q}(\omega)\mu(\omega)\}$, is an open set.[1] For every student type ω, $\mu(\omega)$ is either the school j to which the student is matched or zero. When $\mu(\omega) = 0$, the student is unmatched and takes an outside option. An important definition for this paper is that of stability.

Definition 1 (Stability)**.** *The matching $\mu : \Omega \to \mathcal{J}^0$ is a stable matching if three conditions are satisfied for every $\omega \in \Omega$: (i) $\mu(\omega)\bar{Q}(\omega)0$ (individual rationality); (ii) for any $j \in \mathcal{J}$, if $jQ(\omega)\mu(\omega)$, then j is full (no waste); and (iii) for any $j \in \mathcal{J}$ that is full, if $\mu(\omega') = j$ and $jQ(\omega)\mu(\omega)$, then $R_j(\omega') > R_j(\omega)$ (no justified envy).*

A mechanism φ matches students to schools by mapping the students' scores and submitted preference lists to schools. Student ω submits a preference list $P(\omega) \subseteq \mathcal{J}$, which is an ordered list of her acceptable schools. For example, for $J = 3$, if $Q(\omega) = \{1, 2, 0, 3\}$, then $P(\omega) = \{1, 2\}$, as long as the student submits her true list of acceptable schools. The number of schools in P, denoted $|P|$, is at least one because everyone participating in the match has at least one acceptable school. As with $\bar{Q}$, we also define $\bar{P}$ as the weak preference relation induced by P. A mechanism takes as inputs everyone's submitted preferences (i.e., a correspondence $P : \Omega \rightrightarrows \mathcal{J}$) and everyone's scores (i.e., a vector-valued function $\mathbf{R} : \Omega \to \mathcal{R}$) and gives rise to a matching function. Formally, $\varphi(P, \mathbf{R}) : \Omega \to \mathcal{J}^0$. We say a mechanism φ is strategy proof if submitting her true ranking of acceptable schools is weakly dominant for every student—in other words, if any student ω's misreporting of P never leads to a better option and sometimes leads to a worse option, depending on what other students submit. We say a student is a truth-teller if her P equals her true ranking of acceptable schools. Otherwise, we say she is strategic or not a truth-teller.

The ability to characterize a matching allocation on the basis of cutoffs is fundamental for this paper.

Definition 2 (Cutoff Characterization)**.** *For placement scores $\mathbf{S} : \Omega \to \mathcal{S} \subseteq \mathbb{R}^J$, $\mathbf{S}(\omega) := (S_1(\omega), \ldots, S_J(\omega))$ and admission cutoffs $\mathbf{c} \in \mathcal{S}$, $\mathbf{c} := (c_1, \ldots, c_J)$, the set of feasible options of a student ω equals all schools for which her placement scores clear the admission cutoffs plus the outside option: $\{0\} \cup \{j \in \mathcal{J} : S_j(\omega) \geq c_j\}$; student ω's best feasible option is the option that ranks first according to $Q(\omega)$ among her feasible options. We say the matching $\mu : \Omega \to \mathcal{J}^0$ has cutoff characterization if there exist placement scores $\mathbf{S} : \Omega \to \mathcal{S}$ and admission cutoffs $\mathbf{c} \in \mathcal{S}$ such that, for every $\omega \in \Omega$, the matching $\mu(\omega)$ equals student ω's best feasible option according to $Q(\omega)$.*

[1] Azevedo and Leshno (2016) impose the same condition to rule out multiplicity of stable matchings that differ in a set of types with measure zero.

This paper considers mechanisms that produce matching functions with a cutoff characterization according to Definition 2. Placement scores $\mathbf{S}$ may or may not equal school priority scores $\mathbf{R}$. The definition gives researchers the freedom to construct special placement scores $\mathbf{S}$ if the mechanism that they consider does not admit cutoff characterization by means of priority scores $\mathbf{R}$. The idea behind this definition stems from the logic of the general class of mechanisms of Agarwal and Somaini (2018). Moreover, we assume throughout the paper that, for any $j \in \mathcal{J}$, S_j has a distribution that is absolutely continuous with respect to (wrt) the Lebesgue measure and support $\mathcal{S}_j$ that contains a closed interval around c_j. The support of $\mathbf{S}$ is $\mathcal{S}$. For any two scores S_j and S_l, we assume that either $\mathbb{P}[S_j = S_l] = 1$ or $\mathbb{P}[f(S_j) = S_l] < 1$ for any measurable function f. This says that the only deterministic function relating any two scores may be the identity function. Azevedo and Leshno (2016) demonstrate that, if the matching function is stable, the matching has cutoff characterization with $\mathbf{S} = \mathbf{R}$ and admission cutoffs constructed as follows. For each $j \in \mathcal{J}$, $c_j := \inf\{S_j(\omega) :\ \text{for } \omega \text{ with } \mu(\omega) = j\}$ if some individuals are matched to j or $c_j := \inf \mathcal{S}_j$ if nobody is matched to j. Many mechanisms produce stable matchings. For example, SD and DA are strategy proof and lead to stable matchings if agents are truth-tellers. Regarding settings where agents are not truth-tellers, e.g., because they face constraints in the submission of P, Haeringer and Klijn (2009) study how the SD, DA, Boston, and TTC mechanisms lead to stable matchings.

This paper is about identification of moments of treatment effects $Y(d') - Y(d)$. The econometrician has access to an infinite amount of data and observes the joint distribution of the following random objects: $P(\omega)$, $\mathbf{S}(\omega)$, $\mu(\omega)$, and $Y(\omega) := Y(\omega, \mu(\omega))$, where we abuse the notation and employ the letter Y for both the observed outcome, $Y(\omega)$, and the potential outcome of being assigned to school d, $Y(\omega, d)$.

3 Identification with Truthful Reports

In this section, we consider identification of causal effects when all students are truth-tellers, that is, when they submit their true ranking of acceptable schools. This behavior is rational in strategy-proof mechanisms. For instance, Dubins and Freedman (1981) and Roth (1982) show that the DA mechanism without constraints on preference submission is strategy proof in the finite economy; Abdulkadiroglu et al. (2015) show the same for the continuum economy.[2] The SD is also strategy proof, as it is a special case of DA. On the other hand, experimental evidence show that some individuals deviate from the rational behavior of truth-telling (Chen and Sönmez (2006), Pais and Pintér (2008)). Thus, we explicitly assume every one is a truth-teller.

Assumption 1 (Truth-telling)**.** *Students submit their true list of acceptable schools.*

The identification strategy of this paper resembles a sharp RD design. Our goal is to identify the effects of the school of assignment on future outcomes. Another interesting question is the effect of the school of *graduation* on future outcomes—to answer it we would require a strategy resembling

[2] See also the work by Azevedo and Budish (2019), who advocate a robust notion of strategyproofness in a large economy.

a fuzzy RD because some students do not graduate from the same school they are assigned to. We defer this identification problem to future work as several issues beyond the scope of this paper (e.g., multiple compliance types with unordered treatments) arise in that case.

Unlike in a standard sharp RD, that $S_j(\omega)$ clears the cutoff c_j does not automatically determine that student ω is allocated to school j. This is the case only when j is the most preferred school among the schools that are feasible to the student, that is, when j is the favorite school in the set of schools for which the student clears the cutoff.

The first step in the RD is to correctly identify the marginal individuals for a given cutoff and a given change in schools. For example, for any individual with score S_j just to the right of c_j, we need to determine two things: that the individual is matched to school j, and that the individual would have been matched to school k had her score been just to the left of c_j. The cutoff representation implies that these two things depend on the counterfactual sets of available schools on either side of the cutoff and on the individual's preferences over these sets.

It is straightforward to obtain counterfactual sets of available schools in the case where all schools rely on the same placement score, that is, $S_j = S_1$ for every j. For example, this is the case under the SD mechanism. In this case, the set of feasible schools is all schools with a cutoff below or equal to score S_1. Note that everyone just above (or just below) cutoff c_j has exactly the same set of feasible schools. For someone with $S_1 \geq c_j$, the counterfactual scenario has the score crossing to the left of cutoff c_j, and school j is dropped from the set of feasible schools. In turn, for someone with $S_1 < c_j$, the counterfactual scenario adds school j to the set of feasible schools. Unlike under SD, agents near and on the same side of a cutoff in DA differ in their sets of feasible schools. It is not immediately obvious which schools appear in their counterfactual sets. In DA, schools use different scores, and these scores may be functions (e.g., weighted averages) of a small set of primitive scores. That is the case in our application with the Chilean data. This makes the joint support of the distribution of scores highly dependent and complicates the counterfactual analysis. Dealing with this complexity is empirically relevant since many real-world higher education assignment mechanisms use DA. See Table 1 Panel B in Fack et al. (2019) for a list of examples.[3]

Our framework allows for a variety of joint distributions on the vector of scores $\mathbf{S}$ and works with the definition of counterfactual budget sets below.

Definition 3 (Counterfactual Budget Sets)**.** *Consider a student with a vector of scores* $\mathbf{S}$. *The*

[3] A common practice in applied work consists of "cleaning" irrelevant schools from the submitted preference lists in cases where $S_j = S_1$ for every j. For instance, say an individual submits $P = \{1, 2, 3\}$ and $c_2 > S_1 > c_1 > c_3$. Given the cutoff characterization and truth-telling, the matching assignment of this individual is school 1; the counterfactual assignment is school 3 even though $2P3$; this is the case because school 2 has a cutoff higher than the cutoff of school 1. In this case, the irrelevant school to be cleaned from P is school 2. The general idea is to remove all schools ranked below 1 that have cutoffs higher than c_1. See the description of this practice by Estrada and Gignoux (2017). The practice cannot be used to identify counterfactual assignments in cases where different schools use different placement scores, as under, e.g., the DA mechanism.

budget set for this student is her set of feasible options,

$$B(\mathbf{S}) := \{0\} \cup \{m \in \mathcal{J} \ : \ S_m \geq c_m\}.$$

Fix a school $j \in \mathcal{J}$ with cutoff c_j. The right-counterfactual budget set for this student at cutoff c_j is $B_j^+(\mathbf{S}) := B(\mathbf{S}) \cup \{m : S_m = S_j \text{ and } c_m = c_j\}$; the left-counterfactual is $B_j^-(\mathbf{S}) := B(\mathbf{S}) \setminus \{m : S_m = S_j \text{ and } c_m = c_j\}$, where $C \setminus D$ equals the set C minus the elements of set D.

To fix ideas, we consider a simple example throughout the paper in the context of the SD mechanism.

Example (SD Example, Part I)**.** *In SD, $S_j = S_1$ for every j, and the definition of the budget set above equals $B(\mathbf{S}) = \{m : S_1 \geq c_m\}$. Suppose we have four schools with cutoffs $c_1 < c_2 < c_3 < c_4$. Individuals in this economy have five possible budget sets: $\{0\}$, $\{0,1\}$, $\{0,1,2\}$, $\{0,1,2,3\}$, and $\{0,1,2,3,4\}$. For individuals near cutoff c_4, the counterfactual budget sets are $B_4^-(\mathbf{S}) = \{0,1,2,3\}$ and $B_4^+(\mathbf{S}) = \{0,1,2,3,4\}$.*[4]

Next, we define the concept of local preferences, that is, the first- and second-best choices for a marginal individual at any given cutoff.

Definition 4 (Local Preferences)**.** *Fix a school $j \in \mathcal{J}$ with cutoff c_j. Consider a student ω with preference $Q(\omega)$ and scores $\mathbf{S}(\omega)$. For any pair of options $(k,l) \in \mathcal{J}^0 \times \mathcal{J}^0$, we say that (k,l) is the local preference of student ω at cutoff c_j if the favorite feasible option of student ω shifts from l to k as we exogenously increase $S_j(\omega)$ from being smaller than c_j to being larger than c_j. We define the true local preference of this student as the pair $Q_j(\omega) := (k,l)$. Formally, for a set of options $B \subseteq \mathcal{J}^0$, define the best option in B according to Q as $Q(B)$. We have that $Q(B) = m \Leftrightarrow m \in B$ and $m\bar{Q}(\omega)n \ \forall n \in B$. Finally, $Q_j(\omega) = (k,l)$ if, and only if, $Q(B_j^+(\mathbf{S})) = k$ and $Q(B_j^-(\mathbf{S})) = l$. The reported local preference $P_j(\omega)$ is defined in a similar fashion. If $B \cap P \neq \emptyset$, $P(B) = m \Leftrightarrow m \in B$ and $m\bar{P}(\omega)n \ \forall n \in B$; otherwise, if $B \cap P = \emptyset$, $P(B) = 0$. We have that $P_j(\omega) = (k,l)$ if, and only if, $P(B_j^+(\mathbf{S})) = k$ and $P(B_j^-(\mathbf{S})) = l$.*

Example (SD Example, Part II)**.** *Consider four individuals whose submitted preferences are $P^{(1)} = \{3,4,2,1\}$, $P^{(2)} = \{4,1,2,3\}$, $P^{(3)} = \{4,2,3,1\}$, and $P^{(4)} = \{4,3,1,2\}$. Their corresponding local preferences at cutoff c_4 are $P_4^{(1)} = (3,3)$, $P_4^{(2)} = (4,1)$, $P_4^{(3)} = (4,2)$, and $P_4^{(4)} = (4,3)$.*

There is no distinction between P_j and Q_j at this stage because of Assumption 1. Students are truthful when they submit their list of acceptable schools, so P equals the schools listed higher than 0 in Q and $P_j = Q_j$. Section 4 considers the case of strategic misreporting. In that case, P_j is observed but Q_j is not. Cutoff characterization implies that an individual with $Q_j = (k,l)$ is matched to school k if S_j is just above c_j or to school l if S_j is just below c_j. The same applies for individuals with $P_j = (k,l)$ under Assumption 1.

[4] In SD or in DA with independent placement scores, the definitions of the counterfactual equal $B_j^+(\mathbf{S}) = B(\mathbf{S}) \cup \{m : c_m = c_j\}$ and $B_j^-(\mathbf{S}) = B(\mathbf{S}) \setminus \{m : c_m = c_j\}$; moreover, if the cutoffs are unique, $B_j^+(\mathbf{S}) = B(\mathbf{S}) \cup \{j\}$ and $B_j^-(\mathbf{S}) = B(\mathbf{S}) \setminus \{j\}$.

The local preference pair (j, k) at a certain cutoff c_j is useful for identification only if there exists a positive fraction of individuals in the data near cutoff c_j with those local preferences. We collect such useful pairs in the set $\mathcal{P}$.

Definition 5 (Comparable Pairs)**.** *We say $(j, k) \in \mathcal{J} \times \mathcal{J}$, $j \neq k$, is a comparable pair of alternatives if (i) c_j is an interior point of the support $\mathcal{S}_j$ and (ii) $\mathbb{P}[Q_j = (j, k)|S_j = s]$ is bounded away from zero for s in an open neighborhood of c_j. Finally, we define $\mathcal{P} \subseteq \mathcal{J} \times \mathcal{J}$ as the set of all comparable pairs.*

We adopt the convention that comparable pairs do not involve the outside option 0. We do this because it may be hard to interpret the treatment effects of a change from the outside option to a school when the outside option varies across individuals. We do not consider pairs with $j = k$ because the initial school assignment does not change for these individuals. We also exclude pairs $Q_j = (j', k)$ with $j' \neq j$ from $\mathcal{P}$ to avoid redundancy. We may find individuals with $Q_j = (j', k)$ whenever schools j and j' use the same score and have the same cutoff. As the score $S_j = S_{j'}$ crosses the cutoff $c_j = c_{j'}$, access is granted to both schools j and j', and individuals may differ in their preferences for these schools. Individuals who prefer j' will not appear in $\mathcal{P}$ as having $Q_j = (j', k)$, but they may appear in $\mathcal{P}$ with $Q_{j'} = (j', k)$.

The purpose of defining counterfactual sets and local preferences is to construct a local preference variable for every student and use it as a control variable in the RD. In this section, this variable is P_j, which equals to Q_j because of Assumption 1. When we focus on students with $P_j = (j, k)$, $k \neq j$, the marginal switch in the allocation around cutoff c_j becomes a function of S_j. Controlling for P_j ensures that we apply the RD strategy to all individuals whose assignment switches from k to j at the cutoff; this identifies the effect of the change in the assignment as long as the typical RD continuity assumptions are satisfied.

RD identification requires continuity assumptions on the distribution of individual types conditional on the relevant placement score. In our case, we also need to verify continuity after we condition on $P_j = Q_j$. After all, we do not want to condition on a variable that breaks the central argument for identification in RD: that individuals to the right and the left of the cutoff are "similar on average". Below, we state an assumption on the continuity of types and prove that it implies the kind of smoothness required by RD. Before we do so, we define the following set of events. For every school $j \in \mathcal{J}$, partition the placement scores $\mathbf{S}$ as the score of school j and all other scores: $\mathbf{S} \equiv (S_j, \mathbf{S}_{-j})$. Define $\mathbf{A}_{-j}$ to be the collection of events on $\mathbf{S}_{-j}$ that determine the availability of all non-j schools. There are 2^{J-1} such events in $\mathbf{A}_{-j}$. For example, if $J = 2$, $\mathbf{A}_{-1} = \{\{S_2 \geq c_2\}, \{S_2 < c_2\}\}$; if $J = 3$, $\mathbf{A}_{-1} = \{\{S_2 \geq c_2, S_3 \geq c_3\}, \{S_2 \geq c_2, S_3 < c_3\}, \{S_2 < c_2, S_3 \geq c_3\}, \{S_2 < c_2, S_3 < c_3\}\}$; and so forth.

Assumption 2. *(Continuity of Types) Consider a school j with cutoff c_j in the interior of the support $\mathcal{S}_j$. Assume the following functions of s are all continuous at $s = c_j$: (i) $\mathbb{P}[\mathbf{S}_{-j} \in A_0, Q = Q_0|S_j = s]$ for any $A_0 \in \mathbf{A}_{-j}$ and $Q_0 \in \mathcal{Q}$ and (ii) $\mathbb{E}[\ g(Y(d))\ \mathbb{I}\{\mathbf{S}_{-j} \in A_0, Q = Q_0\} \mid S_j = s]$ for any $A_0 \in \mathbf{A}_{-j}$, $Q_0 \in \mathcal{Q}$, and $g \in \mathcal{G}$, where $\mathcal{G}$ is a set of measurable functions $g : \mathbb{R} \to \mathbb{R}$ that includes the constant function $g(y) = 1$ and the identity function $g(y) = y$.*

Lemma 1. *Suppose Assumption 2 holds. Consider a school j with cutoff c_j in the interior of the support $\mathcal{S}_j$, and choose two schools $k, l \in \mathcal{J}^0$ such that $\mathbb{P}[Q_j = (k,l)|S_j = c_j] > 0$. Then, for any function $g \in \mathcal{G}$ and any $d \in \mathcal{J}^0$, we have that $\mathbb{E}[g(Y(d))|Q_j = (k,l), S_j = s]$ and $\mathbb{P}[Q_j = (k, l)|S_j = s]$ are continuous functions of s at $s = c_j$.*

The proof of this lemma and all other proofs appear in the appendix. Finally, Assumptions 1 and 2 give sufficient conditions for identification for comparable pairs of school changes.

Proposition 1. *Suppose Assumptions 1–2 hold. For any pair $(j,k) \in \mathcal{P}$,*

$$\mathbb{E}[g(Y(j))|Q_j = (j,k), S_j = c_j] = \mathbb{E}[g(Y)|P_j = (j,k), S_j = c_j^+]$$

$$\mathbb{E}[g(Y(k))|Q_j = (j,k), S_j = c_j] = \mathbb{E}[g(Y)|P_j = (j,k), S_j = c_j^-]$$

$$\begin{aligned}&\mathbb{E}[g(Y(j)) - g(Y(k))|Q_j = (j,k), S_j = c_j] \\ &\qquad = \mathbb{E}[g(Y)|P_j = (j,k), S_j = c_j^+] - \mathbb{E}[g(Y)|P_j = (j,k), S_j = c_j^-],\end{aligned}$$

where the condition $S_j = c_j^+$ denotes the limit as $S_j \downarrow c_j$ and the condition $S_j = c_j^-$ denotes the limit as $S_j \uparrow c_j$.

Proposition 1 shows that a standard RD is valid in the truth-telling case as long as we control for P_j. The parameter of interest is the average treatment effect on $g(Y)$ from a change in the school of assignment from j to k, averaged over individuals at the cutoff c_j and with true local preferences (j,k). Indeed, these are the parameters of interest for the rest of this paper:

$$\mathbb{E}\left[g(Y(j)) - g(Y(k))\,|Q_j = (j,k), S_j = c_j\right],\ (j,k) \in \mathcal{P},\ g \in \mathcal{G}. \tag{1}$$

Parameters like these are of economic interest not only for evaluating current mechanisms in place but also for studying the effects of counterfactual assignments (e.g., of students near c_j currently at school k being offered admission at school j.) The fact that the parameter is conditional on Q_j instead of P_j is important for the external validity of (1) when P is manipulable by agents.

4 Identification with Strategic Reports

This section studies identification of causal effects when students are strategic in reporting their rankings of acceptable schools. Strategic reports make P_j generally different from Q_j, and Q_j is not observed. Identification strategies that condition on the observed P_j are potentially invalid for two reasons. First, in contrast to Proposition 1, controlling for P_j does not identify the parameters of interest in (1), which condition on Q_j. Second, controlling for P_j breaks the internal validity of the RD in some cases because P_j is a variable subject to manipulation by agents.

We propose a two-step identification approach that is robust to strategic reporting. In the first step, the researcher characterizes the set of true local preferences Q_j that is compatible with the data and appropriate behavioral assumptions. In the second step, the researcher controls for the constructed local preference sets and partially identifies the parameters in (1). We discuss the first and second steps in Sections 4.1 and 4.2, respectively. Note that our two-step approach differs from the usual two-step control function approach in econometrics. The usual approach is to point-identify the control variable in the first step, while our approach involves partially identifying the control variable.

Section 4.1 presents several tools for the identification of local preference sets. These tools rely on assumptions known to be appropriate in SD and DA contexts, although we do not rule out their applicability in contexts with other mechanisms; e.g., the TTC mechanism satisfies one of our assumptions, such that some of the tools from Section 4.1 are still useful. More generally, researchers may utilize preference identification tools that work under alternative assumptions, for example, the methods of Agarwal and Somaini (2018) and Fack et al. (2019). Either way, the researcher must construct a set of local preferences for each individual in the first step.

Section 4.2 describes the second step of our procedure. This step features high-level assumptions imposed on the local preference sets such that the researcher is not restricted to the methods in Section 4.1.

4.1 Partial Identification of Local Preferences

This section provides tools for set identification of local preferences using assumptions on agents' behavior and the mechanism. These assumptions are specific to this subsection, and we motivate them with reference to the context of the constrained DA mechanism studied by Haeringer and Klijn (2009). Haeringer and Klijn (2009) study a game where students submit constrained preference rankings and a mechanism matches students to schools as a function of P, $\mathbf{R}$, and schools' capacities. Although the unconstrained DA mechanism is strategy proof, many real-world implementations of DA restrict the number of schools that students can submit in their rankings. In this case, there is a cap $K < J$ such that $1 \leq |P| \leq K$, and the submitted ranking P is generally different from the list of acceptable schools in Q. When implemented in this way, the DA mechanism is not strategy proof, and there are no dominant strategies. Strategyproofness also breaks down if, instead of facing a cap, students incur an application cost as a function of the number of schools submitted (Fack et al., 2019).

Lemma 4.2 by Haeringer and Klijn (2008) shows that, if a mechanism is strategy proof when $K = J$, then, in the game with $K < J$, any constrained ranking of schools is weakly dominated by the same set of schools ranked according to true preferences. This result implies that a student cannot lose and may possibly gain by taking any arbitrary list with less than or equal to K schools, dropping the unacceptable schools, and ranking the acceptable schools according to her true preferences. A further implication is that if a student's number of acceptable schools is less than or equal to K, then her dominant strategy is to submit her true list of acceptable schools (Proposition

4.2 of Haeringer and Klijn (2009)). These implications give rise to a class of undominated strategies according to the following definitions of partial order.

Definition 6. *(Weak and Strong Partial Order) We say P is a weak partial order of Q if P is any selection of up to K schools among the acceptable schools in Q and that selection of schools is ranked according to Q. Formally, (i) $1 \leq |P| \leq K$, $P \subseteq \{d \in Q : dQ0\}$; and (ii) for every $d, d' \in P$, $d'Pd \Leftrightarrow d'Qd$. We say P is a strong partial order of Q when a third condition holds in addition to (i) and (ii). Namely, (iii) $|P| = \min\{K, |\{d \in Q : dQ0\}|\}$. In other terms, if the number of acceptable schools in Q is less than or equal to K and P is a strong partial order of Q, then P equals the list of acceptable schools in Q; otherwise, if the number of acceptable schools in Q is greater than K, P is a subset of K schools among the acceptable schools in Q.*

Lemma 2 in Section A.3 of the appendix summarizes the implications of the result on partial orders from Haeringer and Klijn (2008, 2009) in terms of our Definition 6. In short, for a student with true preferences Q, any P is weakly dominated by a weak partial order P^* of Q that has the same acceptable schools as P; in turn, P^* is weakly dominated by a strong partial order P^{**} of Q that contains the same set of acceptable schools as P^*. Every strong partial order is a weak partial order, but the converse is not true. Our definition of a weak partial order strategy is similar to the definition of the dropping strategy from Kojima and Pathak (2009).

Assuming that agents always submit a strong partial order implies they reveal their true ordered list of acceptable schools whenever they submit P with fewer schools than the cap K. This could be a strong behavioral assumption in some contexts where agents have more than K acceptable schools but have a strong expectation that they will gain admission to a smaller-than-K set of schools. In this case, they may submit $|P| < K$ not because it reflects their full list of acceptable schools but simply because they may not want to incur the mental costs of ranking all schools up to K. In the rest of this subsection, we consider mechanisms that impose a cap K on P and assume that students submit a weak partial order of their true preferences.

Assumption 3 (Submission of Weak Partial Order)**.** *Students submit a weak partial order of their true preferences.*

Assumption 3 replaces Assumption 1 to accommodate mechanisms that are not strategy proof. Submitting a weak partial order is rational in DA mechanisms with cap constraints. That is true for any mechanism that becomes strategy proof once we remove the cap constraint, for example, the TTC mechanism.

An assumption maintained throughout this paper is that the cutoff characterization from Definition 2 applies to the mechanism. This assumption says that $\mu(\omega) = Q(B(\mathbf{S}(\omega))$ for every $\omega \in \Omega$. Azevedo and Leshno (2016) show that stability is equivalent to cutoff characterization with $\mathbf{S} = \mathbf{R}$ and cutoffs that equal the minimum score of the admitted students in each school. Thus, it is worth discussing the stability of the constrained DA mechanism. Theorem 6.3 from Haeringer and Klijn (2009) demonstrates that any Nash equilibrium in constrained DA where $\mathbf{R}$ satisfies Ergin acyclicity leads to a stable matching in the finite economy.[5] Even without Ergin acyclicity, some Nash equilibria still produce stability. SD always satisfies Ergin acyclicity, and thus every Nash equilibrium in SD produces a stable matching. Fack et al. (2019) also study the constrained DA mechanism. They extend Theorem 6.3 from Haeringer and Klijn (2009) to (pure-strategy) Bayesian Nash equilibria in the continuum economy (Proposition A3, Online Appendix A.2.5, Fack et al. (2019)). Fack et al. (2019) also provide primitive conditions for finite economies where students play partial orders to converge to a continuum economy with a stable equilibrium (Proposition 5, Fack et al. (2019)). They further provide a test for implications of stability and find no empirical or simulation evidence against it. In the context of the constrained TTC mechanism, any Nash equilibrium leads to a stable matching as long as $\mathbf{R}$ satisfies Kesten acyclicity (Theorem 6.4 from Haeringer and Klijn (2009)). Therefore, constrained SD, DA, and TTC all satisfy the assumption of cutoff characterization as in Definition 2 with $\mathbf{S} = \mathbf{R}$ under the appropriate conditions.

There is another interesting feature of the cutoff characterization of DA mechanisms. We know that DA produces a stable matching if agents are truth-tellers. In case agents are not truth-tellers, the matching outcome continues to be "stable" if we replace Q with P in the definition of stability.

Definition 7 (Stability wrt P)**.** *We say the matching $\mu : \Omega \to \mathcal{J}^0$ is a stable matching wrt P if three conditions are satisfied for every $\omega \in \Omega$: (i) $\mu(\omega)\bar{P}(\omega)0$ (individual rationality); (ii) for any $j \in \mathcal{J}$, if $jP(\omega)\mu(\omega)$, then j is full (no waste); and (iii) for any $j \in \mathcal{J}$ that is full, if $\mu(\omega') = j$ and $jP(\omega)\mu(\omega)$, then $R_j(\omega') > R_j(\omega)$ (no justified envy), where we adopt the convention that $mP0$ for every $m \in P$. This is the same as Definition 1 except that P appears in the place of Q.*

The DA mechanism, constrained or unconstrained, produces a matching that is stable wrt reported preferences P. Stability wrt P leads to a cutoff characterization wrt P according to the work of Azevedo and Leshno (2016). This cutoff characterization has scores $\mathbf{S} = \mathbf{R}$ and admission cutoffs that equal the smallest scores of admitted students in each school. In other words, this is the same cutoff characterization from Definition 2 except that Q is replaced with P. Cutoff characterization wrt P is natural in DA but not necessarily in other mechanisms, so we state it in the following assumption.

Assumption 4 (Cutoff Characterization wrt P)**.** *In addition to the maintained assumption of cutoff characterization as in Definition 2, the matching function μ satisfies $\mu(\omega) = P(B(\mathbf{S}(\omega))$ for every $\omega \in \Omega$.*

Assumption 4 essentially says that agents are matched to their best feasible options, where best is now defined according to P. Assumption 4 is convenient because it allows us to write a simple expression for the identified set of local preferences in Proposition 2 below; however, it is not a necessary assumption for the identification of those sets. The convenience comes from the fact that Assumption 4 implies $\mu = P(B(\mathbf{S})) = Q(B(\mathbf{S}))$, where both μ and P are observed and P and Q are related via the weak partial order assumption. If we drop Assumption 4, we have only one equality

[5] Ergin acyclicity ensures that no student can block a potential improvement for any two other students without affecting her own assignment. See Ergin (2002) for the formal definition.

$\mu = Q(B(\mathbf{S}))$, which leads to larger sets of local preferences in Proposition 2 below. This is useful to know for settings such as those with the TTC mechanism, which is not stable wrt P.

Next, we characterize all possible pairs of local preferences at a cutoff that are compatible with the data and Assumptions 3 and 4.

Proposition 2 (Identification of Local Preference Sets)**.** *Suppose Assumptions 3 and 4 hold. Select a school j with cutoff c_j. Consider a student with scores $\mathbf{S} \equiv (S_j, \mathbf{S}_{-j})$ and submitted preferences P. Call $(a,b) = P_j$. For this student, define $N_j^+ = B_j^+(\mathbf{S}) \setminus \{P \cup \{0\}\}$ and $N_j^- = B_j^-(\mathbf{S}) \setminus \{P \cup \{0\}\}$, respectively, the sets of unlisted feasible schools in the counterfactual budget sets to the right and the left of the cutoff. Then, the Q_j of this student belongs to $\mathbf{Q}_j$, where the set $\mathbf{Q}_j$ is defined as follows:*

$$\mathbf{Q}_j = \begin{cases} \{(a,b)\}, & \text{if } S_j \geq c_j \text{ and } a = b, \\ \{(a,b)\} \cup \Big(\{a\} \times N_j^-\Big), & \text{if } S_j \geq c_j \text{ and } a \neq b, \\ \{(a,b)\} \cup \Big((N_j^+ \setminus N_j^-) \times \{b\}\Big) & \text{if } S_j < c_j, \end{cases} \tag{2}$$

where $\Big(\{a\} \times N_j^-\Big)$ denotes the set formed by the Cartesian product of a and elements in N_j^- and $\Big(\{a\} \times N_j^-\Big) = \emptyset$ if $N_j^- = \emptyset$.

Moreover, assume P is a strong partial order of Q. Then, $\mathbf{Q}_j$ becomes:

$$\mathbf{Q}_j = \begin{cases} \{(a,b)\}, & \text{if } |P| < K, \text{ or if } |P| = K,\ S_j \geq c_j, \text{ and } a = b, \\ \{(a,b)\} \cup \Big(\{a\} \times N_j^-\Big), & \text{if } |P| = K,\ S_j \geq c_j, \text{ and } a \neq b, \\ \{(a,b)\} \cup \Big((N_j^+ \setminus N_j^-) \times \{b\}\Big) & \text{if } |P| = K \text{ and } S_j < c_j. \end{cases} \tag{3}$$

Finally, the characterization in (2) *is sharp if the distribution of Q conditional on P and $\mathbf{S}$ has full support, that is, if every $Q \in \mathcal{Q}$ that satisfies Assumptions 3 and 4 is in that support. Likewise,* (3) *is sharp if the distribution of Q conditional on P and $\mathbf{S}$ has full support under Assumptions 3 and 4 and P being a strong partial order.*

Example (SD Example, Part III)**.** *Suppose the cap constraint is $K = 3$ and the four schools are acceptable for everyone. We consider all agents whose $P_4 = (4,2)$. For example, if agents submit strong partial orders, they submit either $P = \{4,2,1\}$ or $P = \{4,2,3\}$. The assumption of cutoff characterization wrt P (Assumption 4) says that these agents are matched to school* 4 *if $S_1 \geq c_4$ and to school* 2 *otherwise. To keep things simple, consider five different types of true preferences: $Q^{(1)} = \{4,2,3,1,0\}$, $Q^{(2)} = \{4,3,2,1,0\}$, $Q^{(3)} = \{4,1,3,2,0\}$, $Q^{(4)} = \{3,4,2,1,0\}$, and $Q^{(5)} = \{1,3,2,4,0\}$. The weak partial order assumption rules out $Q^{(5)}$ because $2Q^{(5)}4$ contradicts* 4 *being reported preferred to* 2*. The maintained assumption of cutoff characterization (Definition 2) further rules out more types of Q, depending on whether $S_1 \geq c_4$ or $S_1 < c_4$:*

1. *if $S_1 \geq c_4$, $Q^{(4)}$ is not possible because the matching assignment is* 4 *but the best feasible option according to $Q^{(4)}$ is* 3*; in this case, the possible true local preferences are: $Q_4^{(1)} = (4,2)$,*

$Q_4^{(2)} = (4,3)$, and $Q_4^{(3)} = (4,1)$; for a student who submits $P = \{4,2,1\}$, $\mathbf{Q}_4 = \{(4,2),(4,3)\}$; otherwise, for someone who submits $P = \{4,2,3\}$, $\mathbf{Q}_4 = \{(4,2),(4,1)\}$;

2. *if $S_1 < c_4$, none of $Q^{(2)}$, $Q^{(3)}$, or $Q^{(4)}$ is possible because the matching assignment is 2 but the best feasible options according to these Qs differ from 2; in this case, the only possible true local preference is $Q_4^{(1)} = (4,2)$, so that $\mathbf{Q}_4 = \{(4,2)\}$.*

This example illustrates why an RD that controls only for $P_4 = (4,2)$ may fail to identify the treatment effect of a change in assignment from school 2 to school 4. As the score S_1 crosses the cutoff c_4, the support of the distribution of true local preference types generally changes discontinuously from having only $(4,2)$ to having $(4,1)$, $(4,2)$, and $(4,3)$. This possibility invalidates RD identification because true preferences and potential outcomes are plausibly dependent on each other; in this case, the distribution of potential outcomes conditional on $S_1 = s$ and $P_4 = (4,2)$ changes discontinuously at $S_1 = c_4$. For the possibility to arise, a positive mass of agents must change their submission P as their score S_1 crosses c_4. This does not necessarily require that these agents are certain ex ante about the cutoff value c_4 but simply that they have expectations about that cutoff that change as S_1 crosses c_4.

Proposition 2 identifies all possible values of Q_j for students near a cutoff c_j as a function of their scores and submitted preferences. In some settings, many unlisted programs may be feasible for students, which translates into sets $\mathbf{Q}_j$ with many values. For example, the Chilean data have $K = 8$ but more than 1,000 options; a student may have numerous feasible options but not consider many of them. It is possible to obtain smaller sets $\mathbf{Q}_j$ by imposing further assumptions on the expectations that students have when they submit P.

Agarwal and Somaini (2018) propose a general framework to rationalize strategic reporting as the optimal solution to an expected utility maximization problem. In this framework, agents have private information about their preferences and scores and form beliefs about the distribution of other people's preferences and scores. These beliefs plus knowledge of the mechanism lead the rational agent to derive probabilities of admission to the various schools as a function of the agent's private information and expectations about other agents. The agent then chooses the submission P that maximizes her expected utility.

We assume that agents are expected utility maximizers and note that it is sufficient for each individual to consider beliefs on admission cutoffs. This is the case in our continuum economy with cutoff characterization because cutoffs and scores fully characterize the agent's budget set. Given the student's scores, a distribution of possible cutoffs translates into a distribution of possible budget sets. Under Assumption 4, the student is admitted to the best school according to the submission P among the available schools in the budget set. Therefore, beliefs on cutoffs translate into probabilities of admission to various schools for any given P. We make an assumption on the distribution of cutoffs expected by agents that has to do with the concept of uniformly more accessible schools.

Definition 8 (Uniformly More Accessible Schools)**.** *For a pair of distinct schools (d, e), we say e is uniformly more accessible than d if two conditions are satisfied: first, if access to school d implies access to school e,*

$$\{\omega : S_d(\omega) \geq c_d\} \subseteq \{\omega : S_e(\omega) \geq c_e\},$$

and second, if replacing option d with option e in any submission P alters the likelihood of admission for at least one school listed in P; formally, for any two fixed (i.e., nonrandom) submissions P and $\widetilde{P}$ such that P has d but does not have e and $\widetilde{P}$ equals P except for e in the place of d, there exists $u \in \{0, 1, \ldots, |P|\}$ for which

$$\mathbb{P}\left[P(B(\mathbf{S})) = P^u\right] \neq \mathbb{P}\left[\widetilde{P}(B(\mathbf{S})) = \widetilde{P}^u\right],$$

where $P(B)$ denotes the best choice in set B according to P (Definition 4) and P^u denotes the school ranked in the u-th position in P. In short, we say $(d, e) \in UMAS$, where $UMAS \subseteq \mathcal{J} \times \mathcal{J}$ is the set of all such pairs.

Definition 8 says that e is uniformly more accessible than d if everyone who qualifies for school d also qualifies for school e. Schools d and e must also be relevant in the sense of the second condition: there is always a strictly positive fraction of individuals for whom listing e in the place of d changes their best feasible options. In the SD case, a sufficient condition for Assumption 5 is that $c_d > c_e$ and the cutoffs are distinct interior points in the support of the placement score. Uniformly more accessible schools do not always exist. Whether they do depends on the mechanism in place and the joint distribution of the placement scores. We use this definition to impose a mild restriction on the expectations of agents regarding cutoffs.

Assumption 5. *Consider a student with scores $\mathbf{s} \in \mathcal{S}$ who views uncertain cutoffs as random variables $C_1, \ldots, C_J$ before the matching assignment. Let $\widetilde{B} = \{0\} \cup \{j \in \mathcal{J} \ : \ s_j \geq C_j\}$ be the student's corresponding random budget set. For every pair $(d, e) \in UMAS$, the distribution of cutoffs for this student is such that two conditions are satisfied: first,*

$$\{s_d \geq C_d\} \subseteq \{s_e \geq C_e\},$$

and second, for any two fixed (i.e., nonrandom) submissions P and $\widetilde{P}$ such that P has d but does not have e and $\widetilde{P}$ equals P except for e in the place of d, there exists $u \in \{0, 1, \ldots, |P|\}$ for which

$$\mathbb{P}\left[P(\widetilde{B}) = P^u\right] \neq \mathbb{P}\left[\widetilde{P}(\widetilde{B}) = \widetilde{P}^u\right].$$

This is true for every student in the economy.

Assumption 5 says that students correctly anticipate which schools will be uniformly more accessible after the matching assignment. For example, agents may learn this information by observing past realizations of the matching in the economy. If a school e is well known to be accessible to everyone who has access to school d, then it is natural for a student to expect to have access to e if she ever has access to school d. Note that the assumption does not pin down the expected probability of admission or the set of schools to which the student will have access in the ex post economy. It restricts only the expected hierarchy of school access according to $UMAS$. This assumption has implications for the joint distribution of (P, Q).

Proposition 3. *Suppose Assumptions 3–5 hold. Consider a student with reported preference ranking P. Let $(P \times P^c)$ be the Cartesian product of listed and unlisted schools, respectively, P and P^c. If $(d, e) \in UMAS \cap (P \times P^c)$, then dQe.*

Proposition 3 says that if an agent lists school d but does not list the uniformly more accessible school e, it must be that this agent prefers d over e. This result offers a refinement of Proposition 2 above.

Corollary 1. *Consider the setup of Proposition 2, where $P_j = (a, b)$, and suppose Assumption 5 holds. Define $A_j^- = N_j^- \setminus \{e : \exists d \in P \text{ with which } (d, e) \in UMAS \text{ and } b\bar{P}d\}$ and $A_j^+ = \left(N_j^+ \setminus N_j^-\right) \setminus \{e : \exists d \in P \text{ with which } (d, e) \in UMAS \text{ and } a\bar{P}d\}$. Then, under weak partial order,*

$$\mathbf{Q}_j = \begin{cases} \{(a, b)\}, & \text{if } S_j \geq c_j \text{ and } a = b, \\ \{(a, b)\} \cup \left(\{a\} \times A_j^-\right), & \text{if } S_j \geq c_j \text{ and } a \neq b, \\ \{(a, b)\} \cup \left(A_j^+ \times \{b\}\right) & \text{if } S_j < c_j. \end{cases} \tag{4}$$

Under strong partial order,

$$\mathbf{Q}_j = \begin{cases} \{(a, b)\}, & \text{if } |P| < K, \text{ or if } |P| = K, \ S_j \geq c_j, \text{ and } a = b, \\ \{(a, b)\} \cup \left(\{a\} \times A_j^-\right), & \text{if } |P| = K, \ S_j \geq c_j, \text{ and } a \neq b, \\ \{(a, b)\} \cup \left(A_j^+ \times \{b\}\right) & \text{if } |P| = K \text{ and } S_j < c_j. \end{cases} \tag{5}$$

These characterizations are sharp as long as (2)–(3) are sharp in their respective contexts in Proposition 2 and imposing Assumption 5 sets to zero only the following probabilities: $\mathbb{P}[eQd|P, \mathbf{S}]$ for every $(d, e) \in UMAS \cap (P \times P^c)$.

Corollary 1 describes how to use Proposition 3 to potentially reduce the number of elements in the $\mathbf{Q}_j$ constructed in Proposition 2. The intuition runs as follows. Suppose that a student submits P and $P_j = (a, b)$. If school e is uniformly more accessible than school d and d is listed in P but e is not listed in P, then we know the student truly prefers d over e. This excludes some possibilities of Q_j in the $\mathbf{Q}_j$ defined by Proposition 2. For instance, this person cannot have $Q_j = (a, e)$ if $b\bar{P}d$ because that presupposes $eQb\bar{Q}d$, which contradicts dQe. Likewise, this person cannot have $Q_j = (e, b)$ if $a\bar{P}d$.

Example (SD Example, Part IV)**.** *The set of uniformly more accessible schools is $UMAS = \{(2, 1), (3, 2), (3, 1), (4, 3), (4, 2), (4, 1)\}$. Assumption 5 shrinks the set $\mathbf{Q}_4$ of those agents with $S_1 \geq c_4$*

and $P = \{4, 2, 3\}$. *Applying the assumption changes* $\mathbf{Q}_4 = \{(4, 2), (4, 1)\}$ *to* $\mathbf{Q}_4 = \{(4, 2)\}$ *because* 1 *is uniformly more accessible than* 2, 2 *is listed, and* 1 *is not listed, so Proposition 3 implies* $2Q1$.

4.2 Partial Identification of Causal Effects

In this section, we lay out conditions and derive bounds on average treatment effects. We assume that the researcher has already identified the set of local preferences at a cutoff of interest. This means that the researcher has a set-valued variable $\mathbf{Q}_j$ for all students in the vicinity of a cutoff j corresponding to a comparable pair (j, k) in $\mathcal{P}$. Researchers may construct $\mathbf{Q}_j$ using the methods in Section 4.1 if they find it reasonable to rely on at least some of the specific assumptions in that subsection; otherwise, they may use any other method to construct $\mathbf{Q}_j$. There is no restriction on the choice of the method for constructing $\mathbf{Q}_j$ except a couple of high-level conditions that we assume to hold in this section. We start by defining the conditional support of partially identified true local preferences.

Definition 9 (Support of Local Preference Sets)**.** *Consider a pair* $(j, k) \in \mathcal{P}$ *and corresponding cutoff* c_j. *The support of partially identified true local preferences conditional on* $S_j = s$ *is defined as*

$$\mathbf{\Lambda}_j(s) = \left\{B \subseteq \mathcal{J}^0 \times \mathcal{J}^0 : \mathbb{P}\left[\mathbf{Q}_j = B | S_j = s\right] > 0\right\}.$$

The union set of this support is defined as the collection of all unions of sets in $\mathbf{\Lambda}_j(s)$, *namely,*

$$\mathbf{\Lambda}_j^{\cup}(s) = \left\{B^{\cup} \subseteq \mathcal{J}^0 \times \mathcal{J}^0 : \exists B_1, B_2, \ldots \in \mathbf{\Lambda}_j(s) \text{ with } B^{\cup} = \cup_i B_i\right\}.$$

The set $\mathbf{\Lambda}_j(s)$ collects all values of $\mathbf{Q}_j$ that occur with positive probability conditional on $S_j = s$. In the specific context of Section 4.1, $\mathbf{Q}_j$ is constructed from the mapping of observables $(P, \mathbf{S})$ to a subset of $\mathcal{J}^0 \times \mathcal{J}^0$, i.e., $\mathbf{Q}_j = \psi_j(P, \mathbf{S})$. For example, Proposition 2 and Corollary 1 give examples of such mapping ψ_j. A set B of pairs $(a, b) \in \mathcal{J}^0 \times \mathcal{J}^0$ belongs to the support set $\mathbf{\Lambda}_j(s)$ if there is a set of values in the support of the conditional distribution of $(P, \mathbf{S})$ given $S_j = s$ such that ψ_j maps those values to the set B. The union set $\mathbf{\Lambda}_j^{\cup}(s)$ collects all possible unions of support points of $\mathbf{Q}_j$ conditional on $S_j = s$. These definitions are instrumental in the computation of the partially identified distribution of Q_j, as explained in Proposition 4 below.

Sharpness of identification of the distribution of Q_j requires sharpness in the construction of the sets $\mathbf{Q}_j$. Proposition 2 and Corollary 1 gave the conditions for sharpness of $\mathbf{Q}_j$ in the context of Section 4.1. Outside that context, researchers may construct $\mathbf{Q}_j$ in a different way, so we impose sharpness of $\mathbf{Q}_j$ in the general form of the assumption below.

Assumption 6 (Sharp Local Preference Sets)**.** *Consider a pair* $(j, k) \in \mathcal{P}$ *and corresponding cutoff* c_j. *Assume that:*

(i) the random variable Q_j *and the random set* $\mathbf{Q}_j$ *are both measurable maps on the same probability space and* $\mathbb{P}\left[Q_j \in \mathbf{Q}_j \mid S_j\right] = 1$ *with probability 1; and*

(ii) $\text{supp}\left[Q_j \mid \mathbf{Q}_j, S_j\right] = \mathbf{Q}_j$ *with probability* 1*, where* $\text{supp}\left[Y \mid X\right]$ *denotes the support set of the distribution of* Y *conditional on* X*.*

Assumption 6(i) says that $\mathbf{Q}_j(\omega)$ of individual ω contains the true pair of local preferences $Q_j(\omega)$ of that individual (for almost all individuals), which is a minimum requirement for the construction of $\mathbf{Q}_j(\omega)$. This does not say anything about the sharpness of $\mathbf{Q}_j$. For example, $\mathbf{Q}_j = \mathcal{J}^0 \times \mathcal{J}^0$ is completely uninformative and trivially satisfies Assumption 6(i). The sharpness requirement is stated in Assumption 6(ii). It says that all possibilities of local preferences listed in $\mathbf{Q}_j$ actually occur in the data with positive probability. This rules out unnecessarily large sets $\mathbf{Q}_j$. Assumption 6(ii) may be dropped at the cost of lacking sharpness in the identified sets in the rest of this section.

Partial identification of true local preferences and treatment effects occurs at the limit, as S_j approaches c_j, and is conditional on $\mathbf{Q}_j$. For this to work, we impose regularity conditions on the distribution of potential outcomes and $\mathbf{Q}_j$ conditional on S_j at the limit c_j.

Assumption 7 (Distribution of Local Preference Sets)**.** *Consider a pair* $(j,k) \in \mathcal{P}$ *and corresponding cutoff* c_j*. Assume that:*

(i) there exist a small $\varepsilon > 0$ *and collections of subsets of* $\mathcal{J}^0 \times \mathcal{J}^0$ *denoted* $\mathbf{\Lambda}_j^+$ *and* $\mathbf{\Lambda}_j^-$ *such that* $\mathbf{\Lambda}_j^+ = \mathbf{\Lambda}_j(c_j + e)\ \forall e \in [0,\varepsilon)$ *and* $\mathbf{\Lambda}_j^- = \mathbf{\Lambda}_j(c_j - e)\ \forall e \in (0,\varepsilon)$*; consistent with Definition 9, we define* $\mathbf{\Lambda}_j^{\cup+}$ *and* $\mathbf{\Lambda}_j^{\cup-}$ *as union sets of* $\mathbf{\Lambda}_j^+$ *and* $\mathbf{\Lambda}_j^-$*, respectively;*

(ii) for any $g \in \mathcal{G}$ *of Assumption 2 and any* $\underline{\tau}, \overline{\tau} \in \mathbb{R} \cup \{-\infty, +\infty\}$, $\underline{\tau} < \overline{\tau}$*, the side limits of the following expectations are well defined:* $\mathbb{E}\left[g(Y)\mathbb{I}\{\mathbf{Q}_j = A, \underline{\tau} < g(Y) < \overline{\tau}\} \mid S_j = c_j^+\right]\ \ \forall A \in \mathbf{\Lambda}_j^+$ *and* $\mathbb{E}\left[g(Y)\mathbb{I}\{\mathbf{Q}_j = A, \underline{\tau} < g(Y) < \overline{\tau}\} \mid S_j = c_j^-\right]\ \ \forall A \in \mathbf{\Lambda}_j^-$*.*

Assumption 7(i) concerns the distribution of $\mathbf{Q}_j$ conditional on S_j: the support set of $\mathbf{Q}_j$ is constant as $S_j = s$ approaches the cutoff c_j from either side of it. Part (ii) of the assumption concerns the joint distribution of potential outcomes and $\mathbf{Q}_j$ conditional on S_j. For example, Assumption 7(ii) implies that $\mathbb{P}\left[\mathbf{Q}_j = A \mid S_j = c_j^+\right]$ and $\mathbb{E}[Y \mid \mathbf{Q}_j = A, Y < \tau, S_j = c_j^+]$ are well-defined limits for any $A \in \mathbf{\Lambda}_j^+$ and $\tau \in \mathbb{R} \cup +\infty$ provided that $\mathbb{P}[\mathbf{Q}_j = A, Y < \tau \mid S_j = c_j^+] > 0$. The next result gives inequalities to construct bounds on $\mathbb{P}[Q_j = (a,b) | S_j = c_j]$ for any pair (a,b).

Proposition 4 (Sharp Set of Distributions of Local Preferences)**.** *Consider a pair* $(j,k) \in \mathcal{P}$*. Suppose Assumptions 2, 6, and 7 hold. Then, the sharp set of all possible discrete probability distributions of* Q_j *conditional on* $S_j = c_j$ *is characterized as follows. For every* $A \in \mathbf{\Lambda}_j^{\cup+} \cup \mathbf{\Lambda}_j^{\cup-}$*, each probability distribution in that set implies a value for* $\mathbb{P}\left[Q_j \in A | S_j = c_j\right]$ *that satisfies one of the three inequalities below:*

(i) if $A \in \mathbf{\Lambda}_j^{\cup+} \cap \mathbf{\Lambda}_j^{\cup-}$*,*

$$\mathbb{P}\left[Q_j \in A | S_j = c_j\right] \geq \max\left\{\ \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^+\right]\ ;\ \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^-\right]\ \right\};$$

(ii) if $A \in \mathbf{\Lambda}_j^{\cup+} \setminus \mathbf{\Lambda}_j^{\cup-}$*,*

$$\mathbb{P}\left[Q_j \in A | S_j = c_j\right] \geq \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^+\right];\ \ or$$

(iii) if $A \in \boldsymbol{\Lambda}_j^{\cup-} \setminus \boldsymbol{\Lambda}_j^{\cup+}$,

$$\mathbb{P}\left[Q_j \in A | S_j = c_j\right] \geq \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^-\right].$$

Proposition 4 provides a way to construct the sharp partially identified set of all possible distributions of Q_j conditional on $S_j = c_j$. A distribution of Q_j conditional on $S_j = c_j$ consists of values $p_{a,b} \in [0,1]$ for every $(a,b) \in \mathcal{J}^0 \times \mathcal{J}^0$ such that $\sum_{(a,b)\in\mathcal{J}^0\times\mathcal{J}^0} p_{a,b} = 1$, where $p_{a,b} = \mathbb{P}\left[Q_j = (a,b)|S_j = c_j\right]$. The sharp set is constructed by finding all values of $p_{a,b}$ where $\sum_{(a,b)\in A} p_{a,b}$ satisfies the inequalities of Proposition 4 for every $A \in \boldsymbol{\Lambda}_j^{\cup+} \cup \boldsymbol{\Lambda}_j^{\cup-}$.

Example (SD Example, Part V)**.** *Continue to assume that the four schools are acceptable for everyone. Suppose for a moment that all combinations of* (P, Q) *that satisfy Assumptions 3–5 exist in the economy, for both* $S_1 \geq c_4$ *and* $S_1 < c_4$*. Then, this is the list of all possible* $\mathbf{Q}_4$*:*

1. *if* $S_1 \geq c_4$, $\{(1,1)\}$, $\{(2,2)\}$, $\{(3,3)\}$, $\{(4,1)\}$, $\{(4,2)\}$, $\{(4,3)\}$, $\{(4,1),(4,2),(4,3)\}$, $\{(4,1),(4,3)\}$, *and* $\{(4,2),(4,3)\}$*;*

2. *if* $S_1 < c_4$, $\{(1,1)\}$, $\{(2,2)\}$, $\{(3,3)\}$, $\{(4,1)\}$, $\{(4,2)\}$, $\{(4,3)\}$, $\{(1,1),(4,1)\}$, $\{(2,2),(4,2)\}$, *and* $\{(3,3),(4,3)\}$.

Let us focus on the case that $S_1 \geq c_4$*. To keep things simple, suppose three types of* $\mathbf{Q}_4$ *occur with positive probability conditional on* $S_1 = s$ *for any* $s \geq c_4$*:* $\{(4,2)\}$ *with probability* 0.1, $\{(4,3)\}$ *with probability* 0.3, *and* $\{(4,2),(4,3)\}$ *with probability* 0.6*. It follows that* $\boldsymbol{\Lambda}_4(s) = \boldsymbol{\Lambda}_4^+ = \{\{(4,2)\}, \{(4,3)\}, \{(4,2),(4,3)\}\}$ *and* $\boldsymbol{\Lambda}_4^\cup(s) = \boldsymbol{\Lambda}_4^{\cup+} = \{\{(4,2)\},\{(4,3)\},\{(4,2),(4,3)\}\}$*. The lower bounds* $\mathbb{P}[\mathbf{Q}_4 \subseteq A|S_1 = c_4^+]$ *of Proposition 4 are as follows:* 0.1 *for* $A = \{(4,2)\}$*;* 0.3 *for* $A = \{(4,3)\}$*; and* 1 *for* $A = \{(4,2),(4,3)\}$*. Thus,* $\mathbb{P}[Q_4 = (4,2)|S_1 = c_4^+]$ *has lower bound* 0.1, $\mathbb{P}[Q_4 = (4,3)|S_1 = c_4^+]$ *has lower bound* 0.3*, and the sum of the two equals* 1*. When we look at each individual probability, the bounds are* $[0.1, 0.7]$ *on* $\mathbb{P}[Q_4 = (4,2)|S_1 = c_4^+]$ *and* $[0.3, 0.9]$ *on* $\mathbb{P}[Q_4 = (4,3)|S_1 = c_4^+]$*.*

Recall that the construction of the random set $\mathbf{Q}_j$ depends on assumptions regarding the behavior of agents when they submit P. For example, Section 4.1 characterizes $\mathbf{Q}_j$ by assuming weak partial order and cutoff characterization wrt P. Alternatively, the identification approach of Agarwal and Somaini (2018) makes different types of assumptions on agents' expectations and requires data variation in the choice environment. The theoretical credibility of these types of assumptions depends on the mechanism faced by agents; in practice, the assumptions have testable implications for what we should observe in the data. It is therefore useful to characterize a falsification test based on these implications to aid researchers in screening out assumptions rejected by the data. Such a test can rely on the fact that the right-hand sides of the inequalities in Proposition 4 must provide a lower bound for a probability mass function if the model assumptions are correct. For any partition $\mathcal{A}$ of $\mathcal{J}^0 \times \mathcal{J}^0$, we must have $\sum_{A\in\mathcal{A}} \mathbb{P}\left[Q_j \in A|S_j = c_j\right] = 1$ for any given distribution in the sharp set of Proposition 4. Thus, the same sum applied to the right-hand sides of the inequalities above must be less than or equal to one.

Corollary 2 (Model's Falsification Test)**.** *Assume the setup of Proposition 4, which presupposes that the model assumptions utilized to construct* $\mathbf{Q}_j$ *are true. Then, for any partition* $\mathcal{A}$ *of* $\mathcal{J}^0 \times \mathcal{J}^0$*, we have that*

$$\sum_{A \in \mathcal{A}} \Bigg\{ \mathbb{I}\left\{A \in \mathbf{\Lambda}_j^{\cup+} \cap \mathbf{\Lambda}_j^{\cup-}\right\} \max\left\{ \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^+\right] \; ; \; \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^-\right] \right\}$$
$$+ \mathbb{I}\left\{A \in \mathbf{\Lambda}_j^{\cup+} \setminus \mathbf{\Lambda}_j^{\cup-}\right\} \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^+\right]$$
$$+ \mathbb{I}\left\{A \in \mathbf{\Lambda}_j^{\cup-} \setminus \mathbf{\Lambda}_j^{\cup+}\right\} \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^-\right] \Bigg\} \leq 1.$$

Partial identification of the distribution of local preferences allows us to bound the fraction of individuals near cutoff c_j who have $Q_j = (j, k)$. The average outcome near the cutoff is a weighted average of the average outcomes from two different groups: first, individuals with $Q_j = (j, k)$, who interest us for the identification of treatment effects; and second, individuals with $Q_j \neq (j, k)$. The overall average is identified, but the average in each of the groups is not. A strictly positive lower bound on the fraction of individuals in the $Q_j = (j, k)$ group allows us to construct lower and upper bounds on the average outcome for that group.

Start with all individuals above and near cutoff c_j whose $\mathbf{Q}_j$ contain the comparable pair of interest, $(j, k) \in \mathcal{P}$. The fraction of those individuals who have $Q_j = (j, k)$ equals

$$\delta_{j,k}^+ = \frac{\mathbb{P}\left[Q_j = (j, k) | S_j = c_j\right]}{\mathbb{P}\left[\mathbf{Q}_j \cap \{(j, k)\} \neq \emptyset | S_j = c_j^+\right]},$$

where both numerator and denominator are strictly positive by virtue of (j, k) being a comparable pair (Definition 5) and of the sharpness of $\mathbf{Q}_j$ (Assumption 6). The denominator of $\delta_{j,k}^+$ is identified from the data, and Proposition 4 bounds the numerator. All we need for identification of treatment effects is a lower bound on $\delta_{j,k}^+$, which comes from a lower bound on its numerator. Let $\underline{p}_{j,k}$ denote the infimum over all probability values for $\mathbb{P}\left[Q_j = (j, k) | S_j = c_j\right]$ that belong to the partially identified set of Proposition 4. The sharp lower bound on $\delta_{j,k}^+$ equals

$$\underline{\delta}_{j,k}^+ = \frac{\underline{p}_{j,k}}{\mathbb{P}\left[\mathbf{Q}_j \cap \{(j, k)\} \neq \emptyset | S_j = c_j^+\right]}.$$

The denominator of $\underline{\delta}_{j,k}^+$ is strictly positive, but $\underline{p}_{j,k}$ may or may not be strictly positive.

The same idea applies for individuals just below the cutoff. Select all individuals whose $\mathbf{Q}_j$ contain the comparable pair of interest, $(j, k) \in \mathcal{P}$. The fraction of those who have $Q_j = (j, k)$ equals

$$\delta_{j,k}^- = \frac{\mathbb{P}\left[Q_j = (j, k) | S_j = c_j\right]}{\mathbb{P}\left[\mathbf{Q}_j \cap \{(j, k)\} \neq \emptyset | S_j = c_j^-\right]},$$

and the sharp lower bound on $\delta^-_{j,k}$ is

$$\underline{\delta}^-_{j,k} = \frac{\underline{p}_{j,k}}{\mathbb{P}\left[\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset | S_j = c^-_j\right]}.$$

Example (SD Example, Part VI)**.** *We have that* $\mathbb{P}\left[\mathbf{Q}_4 \cap \{(4,2)\} \neq \emptyset | S_1 = c^+_4\right] = 0.7$ *and the bounds on* $\mathbb{P}\left[Q_4 = (4,2) | S_1 = c_4\right]$ *are* $[0.1, 0.7]$*. These imply* $\underline{p}_{4,2} = 0.1$ *and* $\underline{\delta}^+_{4,2} = 1/7$*.*

The following result utilizes the proportions $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ to partially identify average outcomes for individuals with $Q_j = (j,k)$ on either side of the cutoff. Taking differences of these bounds yield bounds for the averages of the treatment effects $Y(j) - Y(k)$.

Proposition 5. *Suppose Assumptions 2, 6, and 7 hold. Consider a pair* $(j,k) \in \mathcal{P}$ *such that* $\underline{p}_{j,k} > 0$*.*

(i) If $g(Y)$ *is a continuous random variable for* $g \in \mathcal{G}$ *of Assumption 2, then we have the following bounds on* $\mathbb{E}[g(Y(j))|Q_j = (j,k), S_j = c_j]$ *and* $\mathbb{E}[g(Y(k))|Q_j = (j,k), S_j = c_j]$*:*

$$\begin{aligned}
&\mathbb{E}\left[g(Y) \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) < F^{-1}_{j,k+}(\underline{\delta}^+_{j,k}), S_j = c^+_j\right] \\
&\quad\leq\ \mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = c_j\right]\ \leq \\
&\mathbb{E}\left[g(Y) \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) > F^{-1}_{j,k+}(1-\underline{\delta}^+_{j,k}), S_j = c^+_j\right],
\end{aligned}$$

and

$$\begin{aligned}
&\mathbb{E}\left[g(Y) \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) < F^{-1}_{j,k-}(\underline{\delta}^-_{j,k}), S_j = c^-_j\right] \\
&\quad\leq\ \mathbb{E}\left[g(Y(k))\,|Q_j = (j,k), S_j = c_j\right]\ \leq \\
&\mathbb{E}\left[g(Y) \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) > F^{-1}_{j,k-}(1-\underline{\delta}^-_{j,k}), S_j = c^-_j\right],
\end{aligned}$$

where $F^{-1}_{j,k+}(u) := \inf\left\{y : \mathbb{P}\left[g(Y) \leq y \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c^+_j\right] \geq u\right\}$ *and* $F^{-1}_{j,k-}(u) := \inf\left\{y : \mathbb{P}\left[g(Y) \leq y \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c^-_j\right] \geq u\right\}$*.*

(ii) If Y *is a binary random variable, then we have the following bounds on* $\mathbb{E}[Y(j)|Q_j = (j,k), S_j = c_j]$ *and* $\mathbb{E}[Y(k)|Q_j = (j,k), S_j = c_j]$*:*

$$\begin{aligned}
&\max\left\{1 - \frac{1}{\underline{\delta}^+_{j,k}}\mathbb{P}\left[Y = 0 \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c^+_j\right]\ ,\ 0\right\} \\
&\quad\leq\ \mathbb{E}\left[Y(j)\,|Q_j = (j,k), S_j = c_j\right]\ \leq \\
&\min\left\{\frac{1}{\underline{\delta}^+_{j,k}}\mathbb{P}\left[Y = 1 \middle| \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c^+_j\right]\ ,\ 1\right\},
\end{aligned}$$

and

$$\max\left\{1-\frac{1}{\underline{\delta}^{-}_{j,k}}\mathbb{P}\left[Y=0\,\middle|\mathbf{Q}_j\cap\{(j,k)\}\neq\emptyset, S_j=c_j^{-}\right]\ ,\ 0\right\}$$
$$\leq\ \mathbb{E}\left[Y(k)\,|Q_j=(j,k),S_j=c_j\right]\ \leq$$
$$\min\left\{\frac{1}{\underline{\delta}^{-}_{j,k}}\mathbb{P}\left[Y=1\,\middle|\mathbf{Q}_j\cap\{(j,k)\}\neq\emptyset, S_j=c_j^{-}\right]\ ,\ 1\right\}.$$

The bounds in Proposition 5 build on the work by Horowitz and Manski (1995). To see the intuition, take part (i) of the proposition, make $g(Y)=Y$, and focus on individuals just above the cutoff. Among all individuals in the subpopulation with $\mathbf{Q}_j\cap\{(j,k)\}\neq\emptyset$ and $S_j=c_j^{+}$, a fraction $\delta^{+}_{j,k}$ of them has $Q_j=(j,k)$ and $Y=Y(j)$ by the cutoff characterization. We do not know who these individuals are among those in the subpopulation. However, the lowest possible value for $\mathbb{E}[Y(j)|Q_j=(j,k),S_j=c_j]$ occurs if all such individuals are located at the lower tail of the distribution of outcomes in the subpopulation. Likewise, the highest possible value for $\mathbb{E}[Y(j)|Q_j=(j,k),S_j=c_j]$ occurs if all of that same fraction of individuals are located in the upper tail of the distribution of outcomes. We do not know $\delta^{+}_{j,k}$, but we do know that it is no smaller than $\underline{\delta}^{+}_{j,k}>0$. The bounds only grow wider as the fraction $\delta^{+}_{j,k}$ decreases, so the bounds evaluated at $\delta^{+}_{j,k}=\underline{\delta}^{+}_{j,k}$ take into account all possible values for $\delta^{+}_{j,k}$. The expressions for the bounds in the case of binary Y change relative to the case of continuous Y, but the derivation of the bounds follows the same intuition. We refer the reader to the proof in the appendix for details (Section A.8).

Although intuitive, these bounds are not necessarily sharp because $\underline{\delta}^{+}_{j,k}$ is not exogenously given as considered by Horowitz and Manski (1995); $\underline{\delta}^{+}_{j,k}$ is constructed from only the distribution of Q_j. Providing a complete characterization of the sharp bounds is complex because it involves deriving bounds on the joint distribution of potential outcomes and Q_j, which may not be practical when the potential outcomes are continuous. For the sake of simplicity, we relegate the sharp characterization to Section B in the appendix. The bounds of Proposition 4 contain the sharp bounds of Section B as long as our model assumptions are true. The lack of sharpness may not matter in practice when Proposition 4 yields tight bounds for a given dataset. However, if our assumptions are not true, the bounds of Proposition 4 may not contain the sharp bounds of Section B. That said, we may obtain tight bounds from the data using Proposition 4, but this does not mean they contain the true parameter (Kédagni et al., 2020). Therefore, it is advisable to assess the testable implications of Corollary 2 as a matter of routine.

5 Assignment to College Majors and Graduation in Chile

In this section, we illustrate our method using data from college applications in Chile. Before estimating bounds for the effect of assignment to a program on graduation outcomes, we document the presence of strategic behavior in this setting.

5.1 Data, Institutional Setting, and Evidence of Strategic Behavior

Centralized college application system in Chile. We use publicly available data on Chile's centralized college application and assignment system from 2004 to 2010 and on graduations from 2007 to 2020. The institutional setting has been described in detail by Hastings et al. (2013) and Larroucau and Rios (2020, 2021), among others. College choice in Chile is organized as a semicentralized system—a subset of universities participate in a centralized market in which a clearinghouse collects rank-order lists from applicants and determines assignments using a variant of the DA algorithm. Students can submit rank-order lists of up to eight major–university pairs ("programs") out of more than 1,000.[6] Priorities are program-specific and determined by a weighted average of scores obtained in a national standardized test (the PSU, for *prueba de selección universitaria*) and of high-school GPA.[7] Descriptive statistics about the sample of students and programs are shown in the left panel of Tables 3 and 4 in Appendix C.

Strategic behavior. That Chilean college applicants behave strategically has been thoroughly documented by Larroucau and Rios (2020, 2021). Using a 2014 survey linked to administrative data on applications, they show that listed programs often do not coincide with the truly preferred programs explicitly elicited by the survey. Focusing on applications to medicine programs, they find, for instance, that "among the 40,000 students who answered the survey, close to 10% (3,797) reported Medicine as their top preference, and 2,987 of these students ended up applying to the system. Among these, only 1,360 listed Medicine as their top preference." Moreover, students' probability of not including medicine at the top of their submitted list (while declaring it their most preferred program in the survey) increases as their application scores decrease, and the probability of applying to medicine drops after the 600–750 application score range, where the cutoffs for most medicine programs lie. This suggests that students tend to omit medicine as their admission chances fall, despite preferring it over other programs. These findings are confirmed in a 2019 survey, where the researchers find that "most students who did not include their first true preference in their application list expected a higher cutoff than the cutoff for their first listed preference."

Figure 1 provides additional evidence of strategic behavior. Consider a program j with cutoff c_j. Suppose that students tend to prefer programs of higher quality, consistent with what is found in the literature. If applicants behave strategically, one would expect applications to program j to peak among students with application scores close to c_j. If cutoffs tend to remain in the same neighborhood across years, students with application scores much higher than c_j can expect to be admissible to more selective, higher-quality programs than j, which they prefer over j. Hence, we expect very few of these students to include j on their list. As application scores drop and are closer to c_j, students' chances of admission to the most selective programs decrease, and program

[6] The cap was increased to ten in 2012.

[7] In 2014, students' relative rank within their high school was added as one of the "primary" scores to be averaged to construct priorities.

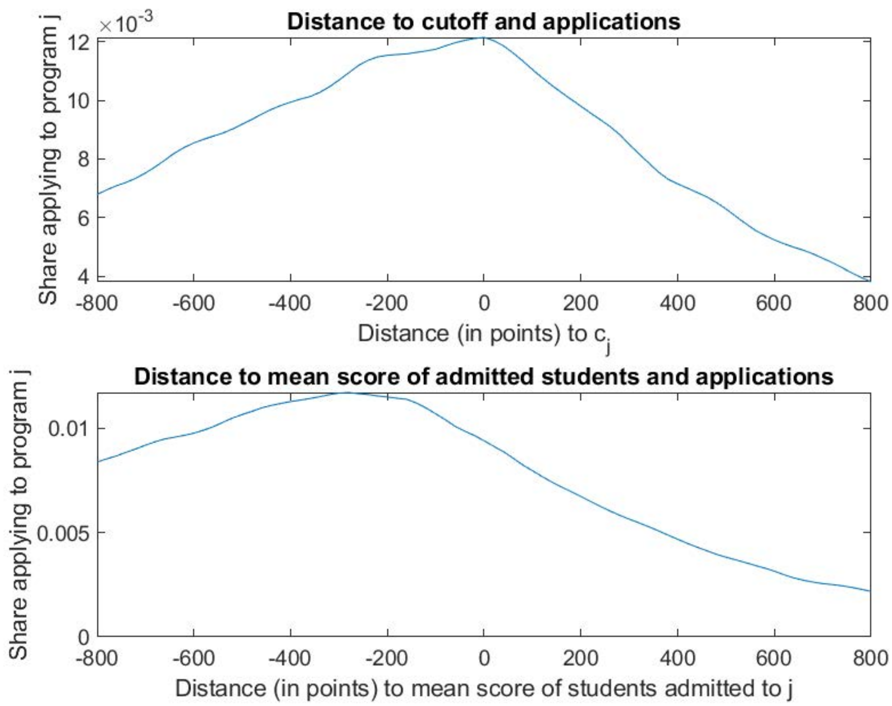
j becomes one of the most selective (desirable) programs among those for which they still have a high admission probability. Hence, we expect applications to c_j to increase as application scores decrease and draw closer to c_j. As application scores decrease below c_j, students realize that their probability of admission to program j is lower, and while program j remains a relatively desirable (selective) alternative, we expect these expectations to drive applications down. This application pattern, expected if students behave strategically, is exactly what we observe in the top panel of Figure 1. Pooling all programs j together, the top panel of Figure 1 shows the fraction of students listing program j in their rank-order lists (ROLs or P in terms of our notation), as a function of the distance between their priority score for program j and the cutoff c_j.

It may be difficult to disentangle the role of preferences from the role of expectations about admission probabilities when both may enter students' choice of which programs to include in their ROLs (Manski (2004); Agarwal and Somaini (2018)). The pattern observed in the top panel of Figure 1 could, alternatively, be consistent with students not behaving strategically but preferring programs that are a good fit in terms of quality, that is, programs in which their skill level would be close to the average skill level. If this were the case, applications should peak among students whose skills (proxied by application score) are close to the mean skill level in the program. This is not what we observe in the bottom panel of Figure 1. Pooling all programs j together, the bottom panel of Figure 1 shows the fraction of students including program j in their list, as a function of the distance between their priority score for program j and the mean application score among students admitted to j. Conditional on application score, the share of students applying to a program j is not the highest for application scores close to the mean score among students admitted to j. It peaks well below this level, showing that students do not systematically prefer programs in which they would be the "average" student. This is again consistent with the hypothesis that students behave strategically.

5.2 Results

We are interested in identifying the effects of assignment to a given postsecondary program on college graduation. College returns are typically thought of as tied to college graduation, motivating our focus on graduation-related outcomes (see Kirkeboen et al. (2016) and Altonji et al. (2016) for further references). In addition, the extent to which students eventually graduate from the program to which they are assigned, rather than programs they switch to in later years, can be viewed as measure of performance of the assignment mechanism. However, for the average college program, initial enrollment and eventual graduation are far from being perfectly correlated (OECD 2019, Larroucau and Rios (2021)). Therefore, we first look at whether assignment to a given program increases one's probability of graduating from that same program. We also consider graduation from a top university as another outcome of interest. The choice of this second outcome is in line with the literature on returns to education and college choice, which highlights the role of institution quality in driving returns (see Kirkeboen et al. (2016) and Altonji et al. (2016) for more references). In our setting, the set of "top" universities consists of Pontificia Universidad Católica

Figure 1: Applications to a program peak among applicants with scores close to the cutoff



This figure provides evidence of strategic behavior in student applications. Pooling all programs j together, the top panel of the figure shows the fraction of students listing program j in their rank-order lists (ROLs or P in terms of our notation), as a function of the distance between their priority score for program j and the cutoff c_j. Pooling all programs j together, the bottom panel shows the fraction of students including program j in their list, as a function of the distance between their priority score for program j and the mean application score among students admitted to j.

de Chile in Santiago (PUC Santiago; hereafter PUC) and Universidad de Chile (UChile).

We present two types of results. First, we discuss the economic implications of our results by focusing on a single program j and popular next-best programs k associated with it. Then, we further illustrate the implementation of our method by estimating bounds on the treatment effects of interest for a large number of pairs (j, k) and presenting statistics across these pairs. The bounds we present throughout this section represent estimates—not inference bounds. Inference is beyond the scope of this paper.[8]

As noted earlier, the identification proposed in this paper resembles a sharp RD design and is therefore well suited for studying the effects of assignment to a program on graduation-related outcomes. In contrast, estimating the effects of graduation from a program on earnings typically involves a fuzzy design, which we study in separate ongoing work.[9]

[8]Beyond the fact that the admission cutoffs are estimated, the fact that the δs used in the construction of bounds are partially identified must be taken into account when confidence bands are derived—a task that we leave for future research.

[9]At the time of writing of the present paper, we do not have access to earnings data and therefore cannot provide

On the importance of preferences for graduation outcomes. We use medicine at PUC in Santiago as our illustrative program j. The choice of this program is driven by two considerations. First, to obtain reasonably precise estimates, we need sufficient observations, and this program is popular. Second, assignment to the program is interesting in itself in regard to our graduation outcomes of interest. The chosen program is a very popular program at a selective university, such that it is particularly relevant to look at its associated outcomes. Descriptive statistics about the program and its applicants are shown in the right panels of Tables 3 and 4 in Appendix C. We estimate the effects on graduation of being assigned to medicine at PUC instead of being assigned to a second-best option. We consider the five most popular second-best options to medicine at PUC—namely, medicine at UChile, medicine at University of Concepción, medicine at University of Santiago de Chile, science at PUC, and engineering at PUC. As a simple procedure to construct a sample local to the admission cutoff c_j of interest (medicine at PUC), we use a 30-point bandwidth on either side of c_j. The resulting estimation sample is therefore invariant across our two outcomes, that is, graduation from medicine at PUC and graduation from a top university.

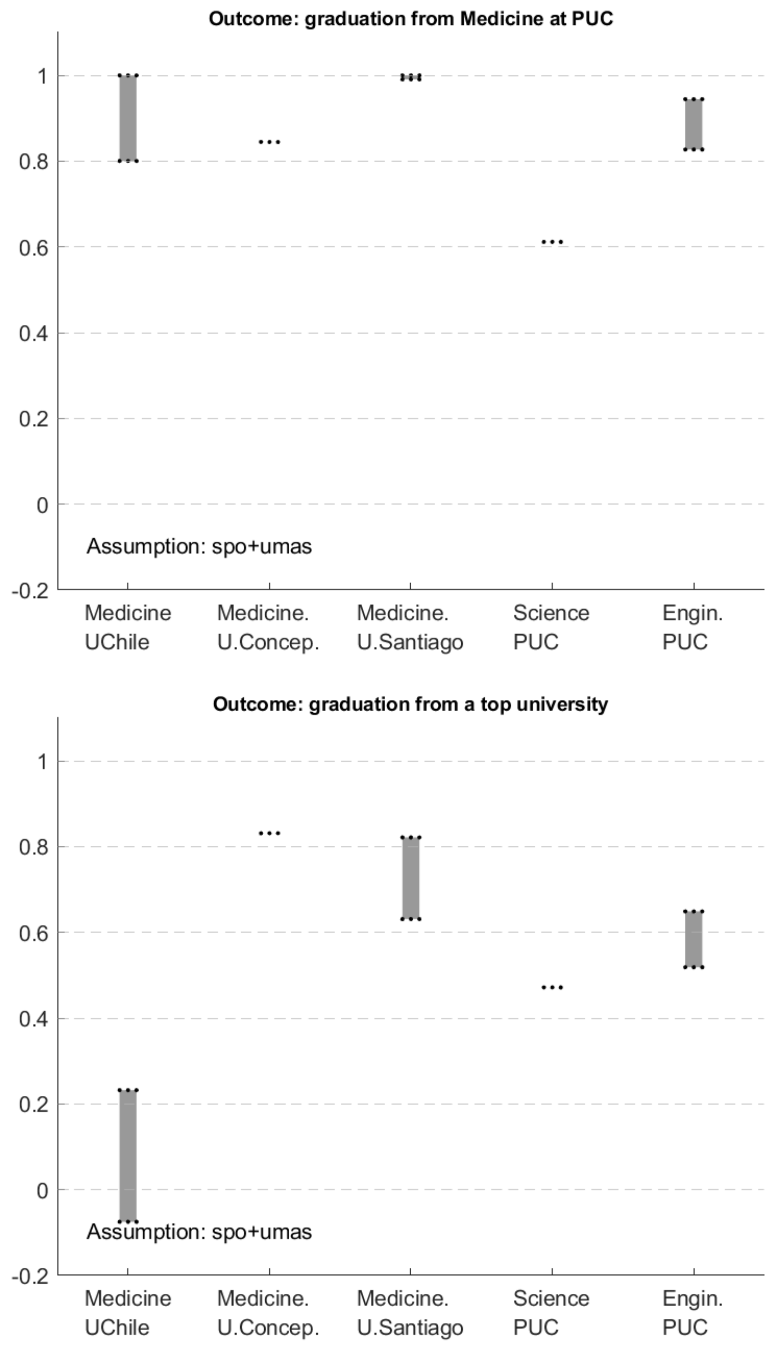
Given our choice of bandwidth, in estimating our bounds for the treatment effects of interest, the next step of the exercise is to recover local preferences Q_j for each student. Absent strategic behavior, students' next-best alternatives are straightforwardly given by their application lists—direct observation of P and placement scores allows to compute cutoffs, budget sets, and finally P_j, which equals Q_j under truth-telling. When we account for strategic behavior, Q_j is not fully revealed by the observed P_j. Section 4.1 shows how the set of possible Q_js, $\mathbf{Q}_j$, can be constructed under different sets of assumptions. The weak partial order (WPO) assumption implies that any submitted list is a subset of acceptable programs ordered just as in the student's true preferences. The strong partial order (SPO) assumption implies, in addition, that any student submitting a list strictly shorter than permitted reveals the student's complete true list of acceptable schools. Under the SPO assumption, true local preferences Q_j are observed for students submitting strictly fewer than eight choices (in the case of our empirical application). For students ranking eight choices, Q_j is observed only if $\mathbf{Q}_j$ is a singleton. The UMAS assumption (Assumption 5) can be used in combination with WPO and SPO to reduce the size of $\mathbf{Q}_j$ for those students for whom it is not already a singleton (Corollary 1). We first present the results obtained under our strongest set of assumptions (SPO in combination with UMAS) in Figure 2 and then discuss how the estimated bounds vary as we impose weaker assumptions (Figure 3).

Figure 2 shows the estimated bounds on the effect on graduation of being assigned to medicine at PUC instead of being assigned to a next-best alternative to medicine at PUC. The top panel shows the estimated effects on the probability of graduating from medicine at PUC Santiago and the bottom panel on the probability of graduating from any program at a top university.

Focusing first on the top panel, we can make several comments. First, reassuringly, the estimates show that assignment to the case-study program, medicine at PUC Santiago, tends to increase students' probability of graduating from that program. Indeed, our bounds exclude zero and

bounds here on the intent-to-treat (ITT) effects on earnings.

Figure 2: Estimated effects of assignment to medicine at PUC, under the SPO+UMAS assumptions



The figure shows estimated bounds for the effect of assignment to medicine at PUC relative to the effect of assignment to the five most frequent next-best alternatives on students' probability of graduating from medicine at PUC (top panel) and their probability of graduating from a top university, that is, PUC or UChile (bottom panel). The bounds on this picture are estimated under the SPO and UMAS assumptions and with a 30-point bandwidth on either side of the assignment cutoff of the program of interest (medicine at PUC). Note that the bounds represent bounds on estimates, not inference bounds.

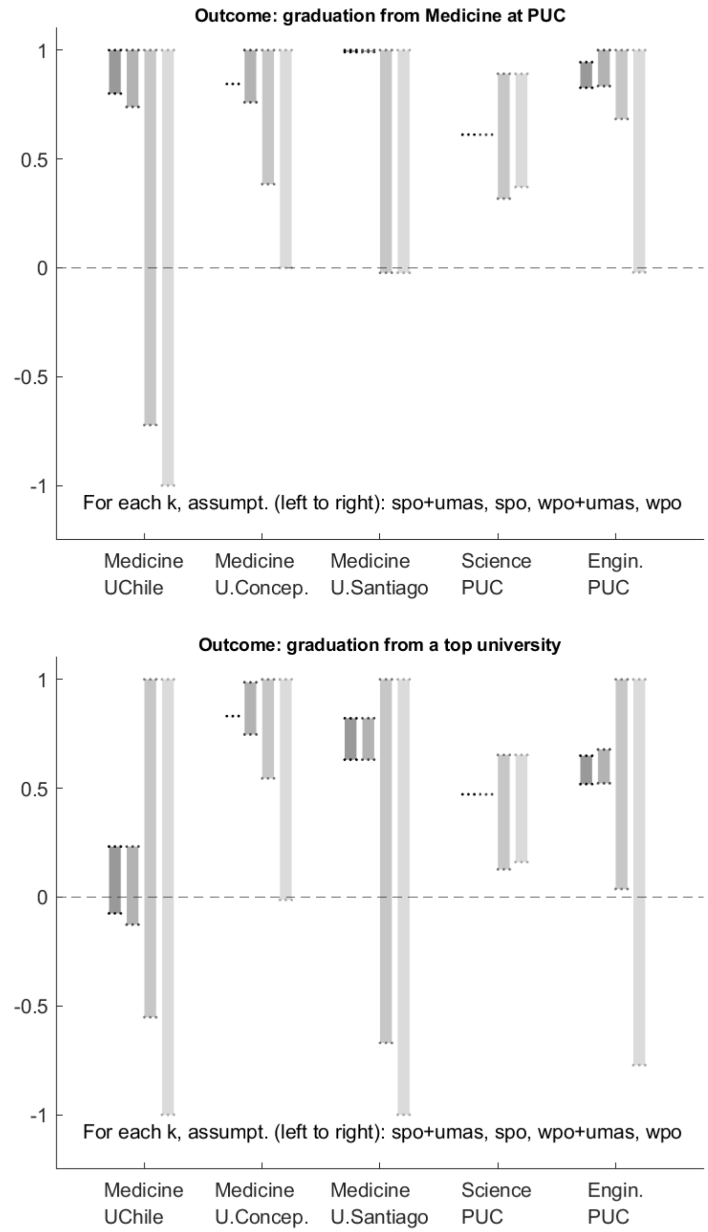
identify a positive treatment effect in all five cases. Our bounds also reveal that the effect is heterogeneous across the next-preferred alternatives. This is shown by the fact that, in some cases, the bounds do not overlap across the next-preferred alternatives. For instance, our bounds suggest that admission to medicine at PUC Santiago increases the probability of graduating from that program to a larger extent for students whose next-preferred alternative is medicine at University of Santiago de Chile than for those whose next-preferred alternative is medicine at University of Concepción, engineering at PUC, or science at PUC.

In the bottom panel, our estimates show that admission to the case-study program, medicine at PUC Santiago, tends to increase students' probability of graduating from a top university (i.e., PUC or UChile). For this outcome, our bounds exclude zero and identify a positive treatment effect in four of the five cases. Notably, though, the finding of a positive effect does not hinge on students' next-preferred option not being a top university. In fact, we find that admission to medicine at PUC Santiago increases the probability of graduating from a top university for students whose next-preferred option is not at a top university (such as medicine at University of Santiago de Chile or University of Concepción) and for students whose next-preferred option is at PUC, too (e.g., in science or engineering). Similarly to those for the previous outcome, our estimates reveal that the effect is heterogeneous across the next-preferred alternatives, as the bounds do not overlap across a number of next-preferred options.

We interpret these results as evidence that students' preferences matter for their graduation outcomes, suggesting that preferences are correlated with ability or effort choices.

Figure 3 shows how the estimated bounds change across sets of assumptions for our two outcomes of interest. Just as Figure 2 does, Figure 3 shows the estimated effects of being assigned to medicine at PUC relative to the effects of being assigned to one of the five most frequent next-preferred alternatives. For each next-best alternative, four sets of bounds are shown: from left to right and from darkest to lightest gray, the bounds pictured are derived under SPO in combination with UMAS (thus coinciding with those pictured in Figure 2), SPO alone, WPO in combination with UMAS, and WPO alone. As we relax the assumptions we use to construct $\mathbf{Q}_j$, the estimated bounds become wider. Under SPO alone, we are still able to draw the same conclusions as above—namely, that students' preferences matter for their graduation outcomes, suggesting that preferences are correlated with ability or effort choices. Under SPO, we do find that assignment to medicine at PUC has a strong positive effect on the probability of graduating from the program for those students whose next-preferred alternative is any of the five we consider and that the magnitude of this effect differs across the next-preferred alternatives. For instance, the SPO bounds for the treatment effect on graduation from medicine at PUC for those whose next-preferred option is science at PUC do not overlap with the bounds for those whose next-preferred option is engineering at PUC or medicine at University of Santiago. The conclusions relative to our second outcome of interest—graduation from a top university—also continue to hold under SPO alone. The bounds derived under WPO, with and without UMAS, are much wider than those derived under SPO, demonstrating the identifying power of the SPO assumption. In particular, we do not recover

Figure 3: Estimated effects of assignment to medicine at PUC, across assumptions



The figure shows estimated bounds for the effect of assignment to medicine at PUC relative to the effect of assignment to the five most frequent next-best alternatives on students' probability of graduating from medicine at PUC (top panel) and their probability of graduating from a top university, that is, PUC or UChile (bottom panel). For each next-best alternative, four sets of bounds are shown: from left to right and from darkest to lightest gray, the bounds pictured are derived under SPO in combination with UMAS, SPO alone, WPO in combination with UMAS, and WPO alone. All sets of bounds are estimated with a 30-point bandwidth on either side of the assignment cutoff of the program of interest (medicine at PUC). Note that the bounds represent bounds on estimates, not inference bounds.

Table 1: Local samples and estimated $\underline{\delta}_{j,k}$s for j = medicine at PUC

	(1) SPO UMAS	(2) SPO	(3) WPO UMAS	(4) WPO
Next-preferred k: medicine at UChile				
Observations with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$	772	772	964	964
Share listing < 8 choices	.90	.90	.92	.92
$\underline{\delta}^+_{j,k}$	.55	.32	.07	.03
$\underline{\delta}^-_{j,k}$	.53	.53	.03	.03
Next-preferred k: medicine at U.Concepción				
Observations with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$	93	99	437	627
Share listing < 8 choices	.59	.55	.91	.92
$\underline{\delta}^+_{j,k}$	1	.46	.1	.03
$\underline{\delta}^-_{j,k}$	1	1	.92	.92
Next-preferred k: medicine at U.Santiago				
Observations with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$	63	75	161	475
Share listing < 8 choices	.69	.58	.88	.93
$\underline{\delta}^+_{j,k}$	.73	.52	.06	.03
$\underline{\delta}^-_{j,k}$	.84	.84	.16	.16
Next-preferred k: science at PUC				
Observations with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$	81	102	224	477
Share listing < 8 choices	.85	.67	.94	.93
$\underline{\delta}^+_{j,k}$	.92	.92	.49	.49
$\underline{\delta}^-_{j,k}$	1	1	.71	.71
Next-preferred k: engin. at PUC				
Observations with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$	97	113	1142	1362
Share listing < 8 choices	.51	.44	.95	.95
$\underline{\delta}^+_{j,k}$	1	.68	.22	.02
$\underline{\delta}^-_{j,k}$	.88	.88	.42	.42

The table describes the sample used in the estimation of the effect on outcomes of interest of assignment to medicine at PUC (j in the table) relative to the effect of assignment to several next-preferred options k, under different sets of assumptions used to construct $\mathbf{Q}_j$. Given each set of assumptions (shown at the top of each column), the table shows the number of students within a 30-point bandwidth around the cutoff to medicine at PUC for whom $\mathbf{Q}_j$ contains the pair (j,k) and the share of these students who listed strictly fewer than eight choices in their ROLs. For each pair (j,k) and each set of assumptions, the table also shows the estimated $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ used in the estimation of the bounds on the treatment effects.

heterogeneity in the treatment effects across the next-preferred alternatives. When we use WPO in combination with UMAS, our bounds still recover the positive sign of the treatment effect on each outcome for three next-preferred alternatives.

To illustrate how the different assumptions affect the derivation of estimates, Table 1 provides descriptive statistics on the sample used in the estimation of the bounds in each case and the implied (estimated) $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ for each pair (j,k) and each set of assumptions. The equations

Table 2: Summary of overall results

	Grad. from program j				Grad. from top university			
	SPO UMAS	SPO	WPO UMAS	WPO	SPO UMAS	SPO	WPO UMAS	WPO
Total pairs	309	309	309	309	309	309	309	309
Bounds identify sign	206	142	26	10	101	77	7	3
Naive est. is outside the bounds	76	37	7	1	176	152	76	69

The table summarizes the results of the implementation of our method for 309 pairs (j, k) and under different assumptions. For each of the two outcomes of interest and each of the four sets of assumptions, the table shows the numbers of pairs (j, k) for which our estimated bounds identify the sign of the treatment effect of assignment to j for those whose next-preferred option is k. It also shows the frequency at which naive RD estimates fall outside our bounds.

in Proposition 5 make clear how the different assumptions affect the values taken by our bounds. The different assumptions are used to construct $\mathbf{Q}_j$ for each individual and therefore determine the conditioning set $\{\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset\}$ and the values we estimate for $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$. Focusing first on the comparison of SPO and WPO, we note that an individual's set $\mathbf{Q}_j$ is larger when constructed under WPO than when constructed under SPO. This implies that the conditioning set $\{\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset\}$ is larger under WPO than it is under SPO—as seen in Table 1 when we compare the first row of each panel across Columns (1) and (3) or (2) and (4). Given a fixed bandwidth around the cutoff c_j, the set $\{\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset\}$ being larger under WPO than under SPO means that the denominator in the equations defining $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ is larger under WPO than under SPO. In addition, as the set $\mathbf{Q}_j$ is larger when constructed under WPO than when constructed under SPO, the share of individuals for whom Q_j is found to be a singleton is lower under WPO than it is under SPO. This means that the numerator in the equations defining $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ is smaller under WPO than under SPO. Overall, this leads to $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ being smaller under WPO than under SPO, as seen in Table 1. The increase in $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ as we impose SPO instead of WPO unambiguously tends to reduce the width of the bounds. The exact extent to which the width of the bounds shrinks as we impose SPO instead of WPO also depends on the joint distribution of the outcome Y and $\mathbf{Q}_j$, however. Turning to the role of the UMAS assumption (UMAS hereafter), we note that as we impose UMAS, certain elements are eliminated from individuals' sets $\mathbf{Q}_j$. As a consequence, following the same logic as above, $\{\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset\}$ is larger absent UMAS then under UMAS, and $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ are smaller absent UMAS than under UMAS. We can see this in Table 1 by comparing Columns (1) to (2) or (3) to (4). Again, the increased values of $\underline{\delta}^+_{j,k}$ and $\underline{\delta}^-_{j,k}$ are the key force that tends to reduce the width of the bounds as UMAS is imposed.

Results for a larger set of pairs. To further illustrate the informativeness and the relevance of our bounding approach, we look at 309 pairs (j, k), which we select as follows. For each postsecondary program j available on the platform at any point between 2004 and 2010 and each program k ranked right after j in a list, we compute the total number of students who listed (j, k) in their ROLs between 2004 and 2010 and had a j-specific application score within a 30-point bandwidth of the year-specific cutoff for j. We find 327 pairs (j, k) with at least 50 such applicants. These pairs involve 137 distinct programs j, each of them associated with a number of ks ranging from one to 18. We exclude the 18 pairs (j, k) associated with one of these programs because no graduates are recorded in the data for this program. This yields a final set of 309 pairs involving 136 distinct js.

As a simple procedure to construct a sample local to each admission cutoff c_j, we first use a 30-point bandwidth on either side of that cutoff. For a number of our js of interest, there exists another program ℓ that uses the same placement score as j and whose cutoff c_ℓ is within a 30-point distance of c_j. In these cases, we use a bandwidth smaller than 30 around the cutoff c_j of interest, so it does not include any such c_ℓ, and the bandwidth is possibly different across sides of c_j.

Table 2 provides statistics on the estimated bounds we obtain. The left panel shows statistics relative to our first outcome of interest, graduation from the treatment program j, while the right panel focuses on our second outcome of interest, graduation from a top university. Within each panel, statistics are shown for four different sets of assumptions: SPO with and without UMAS and WPO with and without UMAS. Under our strongest set of assumptions (SPO in combination with UMAS), our bounds identify the sign of the treatment effect on students' probability of graduating from program j (respectively, of graduating from a top university) for 206 (respectively, 101) out of the 309 pairs (j, k) we study. This shows that our bounding approach is informative enough to recover the sign of causal effects for potentially many cases of interest. As we relax the assumptions we use to construct $\mathbf{Q}_j$, the estimated bounds become less informative—they widen, and the frequency with which they recover the sign of the treatment effect decreases. This exercise can help empirical researchers evaluate and gauge the empirical content of the different assumptions they are willing to work with.

Finally, Table 2 also illustrates that naively ignoring strategic behavior in the identification of treatment effects can produce misleading results. First, recall that the naive estimand $\mathbb{E}[Y \mid P_j = (j, k), S_j = c_j^+] - \mathbb{E}[Y \mid P_j = (j, k), S_j = c_j^-]$ identifies our causal parameter of interest if we assume everyone is a truth-teller. One might expect naive estimands to always lie within our bounds since truth-telling is a special case of our behavioral assumptions. However, Table 2 shows this is not true. Under the SPO and UMAS assumptions, the naive estimates fall outside our bounds for approximately 25% of the pairs of interest for our first outcome and for more than 50% of the pairs of interest for our second outcome. That the naive estimates fall outside our bounds provides additional evidence consistent with behavior being strategic. Indeed, Part III of our SD example (Section 4.1, right after Proposition 2), shows that the distribution of the potential outcomes conditional on reported local preferences and the priority score can change discontinuously at the cutoff in the presence of strategic behavior. In such a case, the naive estimand $\mathbb{E}[Y \mid P_j = (j, k), S_j = c_j^+] - \mathbb{E}[Y \mid P_j = (j, k), S_j = c_j^-]$ no longer equals the average treatment effect and may fall outside our bounds. More broadly, this also suggests that caution is warranted in cases where most but not all students submit P with $|P| < K$ or very few students are assigned to their last-listed option. It may not be safe to simply assume that everyone is a truth-teller just because the cap does not bind for most people. In other words, naive estimates may be severely biased even if the share of constrained students is small.

6 Conclusion

Centralized mechanisms for the assignment of students to educational programs are growing in popularity across the world. These systems provide a valuable source of exogenous variation through the discontinuities that they generate around admission cutoffs. This along with individual-level data on admissions, preferences, and future outcomes allows researchers to identify a wide range of causal effects of education on outcomes. The variation is useful for identification as long as we control for students' true preferences; however, true preferences generally differ from what students report to the assignment system given that most real-world implementations of mechanisms generate incentives for students to behave strategically.

This paper provides a novel approach to partially identify the effects of mechanism assignment on future outcomes that is robust to strategic behavior. We illustrate our approach using data from a deferred-acceptance mechanism that assigns approximately 80,000 students to more than 1,000 university–major programs every year in Chile. First, we find substantial evidence of strategic behavior, confirming earlier findings from the literature on Chile. Second, we compute bounds on the average effect of program assignment on graduation outcomes. We do so for students whose scores are marginal to the cutoff for admission to a popular and selective program in the system: medicine at the Pontifical Catholic University in Santiago. Although admission to this program increases the likelihood of graduating from that same program, its effect on the likelihood of ever graduating from university varies with students' second-best option. In this paper, we illustrate our bounding approach for RD-like parameters that are local to 309 university–major pairs. However, in many contexts, researchers may want to focus on more aggregate average effects, either because of lack of data near the multitude of cutoffs or because policymakers are primarily concerned with the returns to a change in the field of knowledge instead of a change in the university–major program, e.g., the impact of pursuing a STEM degree vs. a non-STEM degree. The RD identification strategy we propose in this paper is valid at the most granular level because this is the level at which students make decisions and the matching algorithm is run. The same strategy is not automatically justified for aggregates of university–major pairs. Aggregation of local effects brings additional challenges that we address in separate ongoing work.

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A Proofs

A.1 Proof of Lemma 1

Consider a school j with cutoff c_j in the interior of the support $\mathcal{S}_j$. Once you fix an event $A \in \mathbf{A}_{-j}$, you fix the availability of those schools with non-j scores $\mathbf{S}_{-j}$. The right- and left-counterfactual budget sets $B_j^-(\mathbf{S})$ and $B_j^+(\mathbf{S})$ become fixed (i.e., nonrandom), regardless of the value of S_j. Once we fix S_j on top of that, the actual budget set $B(\mathbf{S})$ is also fixed.

Fix an event $A \in \mathbf{A}_{-j}$ and S_j in a small neighborhood of c_j. Pick schools $k, l \in \mathcal{J}^0$ such that $\mathbb{P}[Q_j = (k, l)|S_j = c_j] > 0$. Define $\mathbb{Q}(A)$ to be the set of all preference relations $Q \in \mathcal{Q}$ such that $Q(B_j^+(\mathbf{S})) = k$ and $Q(B_j^-(\mathbf{S})) = l$. Note that $\mathbb{Q}(A)$ is a fixed set of preferences (i.e., nonrandom).

Enumerate the mutually exclusive events in $\mathbf{A}_{-j}$ as $A_1, \ldots, A_M$, where $M = 2^{J-1}$. We have that $Q_j = (k, l)$ is equivalent to $Q \in \mathbb{Q}(A_1)$ if A_1, ..., $Q \in \mathbb{Q}(A_M)$ if A_M. Therefore, for s in a small neighborhood of c_j,

$$
\begin{aligned}
&\mathbb{E}[g(Y(d))\mathbb{I}\{Q_j = (k,l)\}|S_j = s] \\
&= \sum_{l=1}^{M} \mathbb{E}[g(Y(d))\mathbb{I}\{Q_j = (k,l), \mathbf{S}_{-j} \in A_l\}|S_j = s] \\
&= \sum_{l=1}^{M} \mathbb{E}[g(Y(d))\mathbb{I}\{Q \in \mathbb{Q}(A_l), \mathbf{S}_{-j} \in A_l\}|S_j = s] \\
&= \sum_{l=1}^{M} \sum_{Q_0 \in \mathbb{Q}(A_l)} \mathbb{E}[g(Y(d))\mathbb{I}\{Q = Q_0, \mathbf{S}_{-j} \in A_l\}|S_j = s].
\end{aligned}
$$

It follows that $\mathbb{E}[g(Y(d))\mathbb{I}\{Q_j = (k,l)\}|S_j = s]$ is a continuous function of s at $s = c_j$ because $\mathbb{E}[g(Y(d))\mathbb{I}\{Q = Q_0, \mathbf{S}_{-j} \in A_l\}|S_j = s]$ is continuous at $s = c_j$ for every l and Q_0 by assumption.

Likewise,

$$
\begin{aligned}
&\mathbb{P}[Q_j = (k,l)|S_j = s] \\
&= \sum_{l=1}^{M} \mathbb{P}[Q_j = (k,l), \mathbf{S}_{-j} \in A_l|S_j = s] \\
&= \sum_{l=1}^{M} \mathbb{P}[Q \in \mathbb{Q}(A_l), \mathbf{S}_{-j} \in A_l|S_j = s] \\
&= \sum_{l=1}^{M} \sum_{Q_0 \in \mathbb{Q}(A_l)} \mathbb{P}[Q = Q_0, \mathbf{S}_{-j} \in A_l|S_j = s],
\end{aligned}
$$

which is continuous at $s = c_j$ for every l and Q_0 by assumption.

Therefore,

$$
\begin{aligned}
&\mathbb{E}[g(Y(d))|Q_j = (k,l), S_j = s] \\
&= \frac{\mathbb{E}[g(Y(d))\mathbb{I}\{Q_j = (k,l)\}|S_j = s]}{\mathbb{P}[Q_j = (k,l)|S_j = s]}
\end{aligned}
$$

is continuous at $s = c_j$

□

A.2 Proof of Proposition 1

Take $(j,k) \in \mathcal{P}$. Start with school j, $s \geq c_j$,

$$\begin{aligned}
\lim_{s \downarrow c_j} \mathbb{E}[g(Y)|P_j = (j,k), S_j = s] &= \lim_{s \downarrow c_j} \mathbb{E}[g(Y)|Q_j = (j,k), S_j = s] \\
&= \lim_{s \downarrow c_j} \mathbb{E}[g(Y(j))|Q_j = (j,k), S_j = s] \\
&= \mathbb{E}[g(Y(j))|Q_j = (j,k), S_j = c_j].
\end{aligned}$$

For school k, $s < c_j$,

$$\begin{aligned}
\lim_{s \uparrow c_j} \mathbb{E}[g(Y)|P_j = (j,k), S_j = s] &= \lim_{s \uparrow c_j} \mathbb{E}[g(Y)|Q_j = (j,k), S_j = s] \\
&= \lim_{s \uparrow c_j} \mathbb{E}[g(Y(k))|Q_j = (j,k), S_j = s] \\
&= \mathbb{E}[g(Y(k))|Q_j = (j,k), S_j = c_j].
\end{aligned}$$

□

A.3 Proof that Partial Orders Dominate Nonpartial Orders

First, we state some definitions. The strategy for each student ω is an ordered list of schools $P(\omega) \subseteq \mathcal{J}$ that has at least one and at most $K < J$ schools in it. The strategy profile of the economy is a correspondence $P : \Omega \rightrightarrows \mathcal{J}$ (random set). The score profile of individuals in the economy is denoted by the random vector $\mathbf{S} : \Omega \to \mathcal{S}$. A mechanism φ takes the whole correspondence P and function $\mathbf{S}$ as givens and produces school assignments for each individual $\omega \in \Omega$, such that $\varphi(P, \mathbf{S}) : \Omega \to \mathcal{J}^0$. The assignment of student ω for profiles $(P, \mathbf{S})$ is $\varphi(P, \mathbf{S})[\omega]$.

For this proof, it is convenient to focus on the assignment of an individual ω_0 as a function of her individual ranking submission $p \subseteq \mathcal{J}$, the ranking submissions of others $P^{-\omega_0} : \Omega \setminus \{\omega_0\} \rightrightarrows \mathcal{J}$, and everyone's scores $\mathbf{S} : \Omega \to \mathcal{S}$. We write this assignment as $\varphi((p, P^{-\omega_0}), \mathbf{S})[\omega_0]$.

For individual ω_0 with true preferences $Q(\omega_0)$, we say p' **weakly dominates** p if $\varphi((p', P^{-\omega_0}), \mathbf{S})[\omega_0]\ \bar{Q}(\omega_0)\ \varphi((p, P^{-\omega_0}), \mathbf{S})[\omega_0]$ for every $P^{-\omega_0}$. In addition, we say φ is strategy proof with unrestricted lists if submitting the true list of acceptable schools weakly dominates submitting anything else for every individual ω.

Lemma 2. *Assume φ is strategy proof with unrestricted lists. Consider student ω with true preference $Q(\omega)$. Fix an arbitrary ranking of schools $p \subseteq \mathcal{J}$. For this student, p is weakly dominated by any weak partial order p' of $Q(\omega)$ that contains all the acceptable schools in p. In turn, p' is weakly dominated by any strong partial order p'' of $Q(\omega)$ that contains all the acceptable schools in p'.*

Moreover, suppose the number of acceptable schools in $Q(\omega)$ is less than or equal to K. Then, the dominant strategy is to submit the unique strong partial order of $Q(\omega)$ that equals the true list of acceptable schools.

Proof of Lemma 2:

This proof is from Haeringer and Klijn (2008), Lemma 4.2. We expand it here in terms of our framework and definitions of partial order.

From the main text, everyone has at least one acceptable school. Take student ω, and consider an arbitrary list of schools p that has at least one acceptable school for that student. If p does not have any acceptable schools, then it is clearly dominated by any weak partial order.

First, remove the unacceptable schools from p (if any), keep the relative ordering of the acceptable schools, and call the resulting list $\bar{p}$. It follows that $\bar{p}$ weakly dominates p for student ω.

Second, let p' be a weak partial order of $Q(\omega)$ that contains the schools listed in $\bar{p}$. Construct a new "true" preference ranking $q \subseteq \mathcal{J}^0$ as follows: (a) take the acceptable schools of p', and place them first in q in the same order as they appear in p'; (b) add a 0 to q after the last school in part (a); (c) fill the remaining positions below 0 in q with the schools not listed in p', in any order. Note that p' equals the true list of acceptable schools from q, p' is a weak partial order of q, and p' is the unique strong partial order of q.

Third, suppose for a moment that the true preference of individual ω were q instead of $Q(\omega)$. In that case, strategyproofness of φ implies that

$$\varphi((p', P^{-\omega}), \mathbf{S})[\omega] \ \bar{q} \ \varphi((\bar{p}, P^{-\omega}), \mathbf{S})[\omega] \text{ for every } P^{-\omega}.$$

Given that p' is a weak partial order of both $Q(\omega)$ and q, for any two options d, d' in p', we have $d' \ \bar{q} \ d$ implies $d'\bar{Q}(\omega)d$. Therefore,

$$\varphi((p', P^{-\omega}), \mathbf{S})[\omega] \ \bar{Q}(\omega) \ \varphi((\bar{p}, P^{-\omega}), \mathbf{S})[\omega] \text{ for every } P^{-\omega}.$$

It then follows that p' weakly dominates $\bar{p}$, which weakly dominates p, so p' weakly dominates p.

It follows that any strong partial order p'' of $Q(\omega)$ that contains the same acceptable schools as p' weakly dominates p'. To see this, repeat the argument above by replacing p with p' and replacing p' with p''.

The second claim of the lemma follows from the strategyproofness of φ since the cap constraint is not binding for an individual whose number of acceptable schools is less than or equal to K and submission of the true list of acceptable schools is feasible.

□

A.4 Proof of Proposition 2

The true preference list Q is unobserved. The submitted preference list P is observed, and P is a weak partial order of Q. Note that all the schools listed in P appear in Q ranked before 0 (i.e., as acceptable schools). There may be other elements in Q not listed in P. These remaining schools might appear anywhere in Q as long as the relative ordering of schools in P is preserved in Q. Our focus is on students with $P_j = (a, b)$, so we know that $a\bar{Q}b$, where aQb if $a \neq b$.

We consider all possibilities of Q that are consistent with the observed P and assumptions and that affect Q_j. In such cases, the acceptable schools in Q include all the schools in P and possibly more; in fact, since $|P| \leq K < J$, the $J - |P| > 0$ unlisted schools in P may appear as acceptable in Q. The additional acceptable schools in Q may be schools that are feasible or infeasible within the budget set of the individual.

The proposition defines two sets of unlisted feasible schools for an individual with scores $\mathbf{S}$ and submitted preferences P: $N_j^+ = B_j^+(\mathbf{S}) \setminus \{P \cup \{0\}\}$ and $N_j^- = B_j^-(\mathbf{S}) \setminus \{P \cup \{0\}\}$. The only difference between B_j^+ and B_j^- is the set of schools whose priority scores equal S_j and cutoffs equal c_j.

The rest of the proof builds on the following reasoning. Ignore Assumption 4 for a moment. The first coordinate of Q_j depends on P_j and the set N_j^+. In fact, the first coordinate of P_j is the best option in $B_j^+(\mathbf{S})$ according to P: that is option a. The best option in $B_j^+(\mathbf{S})$ according to Q may also be a as long as there are no unlisted options in P that are available in $B_j^+(\mathbf{S})$ and rank higher than a in Q. A similar argument applies to the second coordinate of Q: it is a function of P_j and the set N_j^-. Now, Assumption 4 further restricts Q_j because it implies that $P(B(\mathbf{S})) = Q(B(\mathbf{S}))$ and $P(B(\mathbf{S}))$ is observed.

Regarding the outside option 0, the only way that Q_j will have a zero is if P_j has a zero. In fact, if $a \neq 0$ and $b \neq 0$, we have that $a\bar{P}bP0$, which implies $a\bar{Q}bQ0$, so none of the coordinates of Q_j will be zero. This is why the sets of unlisted feasible options, N_j^+ and N_j^-, do not contain zero.

Case 1: $S_j \geq c_j$.

By Assumption 4, we have that the mechanism assignment μ equals the best option according to P in the set $B(\mathbf{S}) = B_j^+(\mathbf{S})$. An individual with $P_j = (a, b)$ has $a = P(B_j^+(\mathbf{S})) = P(B(\mathbf{S}))$; therefore, $\mu = a$. The cutoff characterization dictates that $a = Q(B(\mathbf{S})) = Q(B_j^+(\mathbf{S}))$, so the first coordinate of Q_j equals a. It remains to us to determine the second coordinate of Q_j, which depends on P_j and the set N_j^-.

Case 1.1: $N_j^- = \emptyset$.

None of the unlisted schools in P are feasible in the counterfactual below the cutoff. These unlisted schools may rank higher than b in Q, but none of them will ever be the best feasible option in the counterfactual below the cutoff. Thus, the 2nd coordinate of Q_j equals b, and $\mathbf{Q}_j = \{P_j\}$.

Case 1.2: $N_j^- \neq \emptyset$.

For any option $d \in N_j^-$, we have that $d \neq a$, $d \neq b$, $d \in N_j^+$, and aQd.

Case 1.2.1: If $a \neq b$, we have aQb. We can always find a Q such that dQb and the second coordinate of Q_j equals d; and we can always find another Q such that bQd and the second coordinate of Q_j equals b. Therefore, $\mathbf{Q}_j = \{P_j\} \cup \{(a \times N_j^-)\}$.

Case 1.2.2: If $a = b$, then we have bQd because aQd. Thus, $\mathbf{Q}_j = \{P_j\}$.

Case 2: $S_j < c_j$.

By Assumption 4 we have that the mechanism assignment μ equals the best option according to P in the set $B(\mathbf{S}) = B_j^-(\mathbf{S})$. An individual with $P_j = (a, b)$ has $b = P(B_j^-(\mathbf{S})) = P(B(\mathbf{S}))$; therefore, $\mu = b$. The cutoff characterization dictates that $b = Q(B(\mathbf{S})) = Q(B_j^-(\mathbf{S}))$, so the second coordinate of Q_j equals b. It remains to us to determine the first coordinate of Q_j, which depends on P_j and the set N_j^+.

Case 2.1: $N_j^+ = \emptyset$.

None of the unlisted options in P are feasible in the counterfactual above the cutoff. These unlisted options may rank higher than a in Q, but none of them will ever be the best feasible option in the counterfactual above the cutoff. Thus, the first coordinate of Q_j equals a, and $\mathbf{Q}_j = \{P_j\}$.

Case 2.2: $N_j^+ \neq \emptyset$

For any option $d \in N_j^+ \cap N_j^-$, we have that $d \neq a$, $d \neq b$, and d is in $B_j^-(\mathbf{S})$, but bQd since the 2nd coordinate of Q_j equals b. We have that $a\bar{Q}b$, and it follows that aQd. In this case, we can never find a Q such that the best choice in $B_j^+(\mathbf{S})$ is d.

Consider there is an option $d \in N_j^+ \setminus N_j^-$. We have that $d \neq a$, $d \neq b$, d is in $B_j^+(\mathbf{S})$ but not in $B_j^-(\mathbf{S})$.

It is possible to find Q such that dQa, in which case the best choice in $B_j^+(\mathbf{S})$ is d. It is also possible to find another Q such that aQd, in which case the best choice in $B_j^+(\mathbf{S})$ is a. Therefore, $\mathbf{Q}_j = \{P_j\} \cup ((N_j^+ \setminus N_j^-) \times \{b\})$.

Moreover, assume P is a strong partial order of Q. This implies that $|P| = \min\{K, |\{d \in Q : dQ0\}|\}$ in addition to P being a subset of $\{d \in Q : dQ0\}$. If $|P| < K$, then $|P| = |\{d \in Q : dQ0\}|$, and P equals the true list of acceptable schools in Q. Therefore, $Q_j = P_j$, and $\mathbf{Q}_j = \{P_j\}$. On the other hand, if $|P| = K$, P may or may not be the true list of acceptable schools in Q, and $\mathbf{Q}_j$ continues to be as defined in the case of weak partial order.

Finally, the proof above is constructive as it considers all possibilities of Q given P and $\mathbf{S}$ that are consistent with the assumptions. Thus, it leads to the sharp set of possible Q_js under the full support assumption.

□

A.5 Proof of Proposition 3

Before the matching mechanism is run, the agent knows her placement scores $\mathbf{s} = (s_1, \ldots, s_J)$ and her true preferences Q but does not know what the admission cutoffs will be after the matching is run. She sees the admission cutoffs as random variables $(C_1, \ldots, C_J)$. The strict preference relation Q is represented by a vector of distinct utility values $(U_0, U_1, ..., U_J)$ so that $aQb \ \Leftrightarrow \ U_a > U_b$ for any $a, b \in \mathcal{J}^0$. We normalize $U_0 = 0$ for simplicity.

The agent has to decide on a ranking of acceptable schools P to submit. The number of schools ranked in P is $|P|$, and a feasible ranking has $1 \leq |P| \leq K$. The set of all feasible rankings is defined as ΔP. We let P^u denote the u-th school listed in P, for $u = 1, \ldots, |P|$. We define L_u^P as the agent's expected probability of being assigned to school u when submitting ranking P, $u = 1, \ldots, |P|$. L_0^P denotes the expected probability of her remaining unassigned, that is, of her taking the outside option. Naturally, $L_u^P \geq 0$ for every u, and $\sum_{u=0}^{|P|} L_u^P = 1$. Cutoff characterization wrt P (Assumption 4) implies:

$$
\begin{aligned}
L_0^P &= \mathbb{P}\left[\cap_{v=1}^{|P|} \{s_{P^v} < C_{P^v}\}\right], \\
L_u^P &= \mathbb{P}\left[\cap_{v=1}^{u-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_{P^u} \geq C_{P^u}\}\right], \quad u = 1, \ldots, |P|,
\end{aligned}
$$

where we adopt the convention that $\cap_{v=1}^{u-1}\{s_{P^v} < C_{P^v}\} \ \cap A = A$ for any measurable set A if $u = 1$; in other words, if the intersection $\cap_{v \in V}$ is to be computed over an empty set of indices, $V = \emptyset$, then $\mathbb{P}\left[\cap_{v \in V}\{s_{P^v} < C_{P^v}\} \ \cap \ A\right] = \mathbb{P}\left[A\right]$, for any measurable set A.

The agent's optimal ranking to be submitted is the solution to the following problem,

$$
\max_{P \in \Delta P} \sum_{u=1}^{|P|} U_{P^u} L_u^P.
$$

From now on, let P be the optimal choice of this agent. Suppose there is a pair of schools $(d, e) \in UMAS \cap (P \times P^c)$. Let n be the position in P where d appears, i.e., $P^n = d$. Construct $\widetilde{P}$ by taking P and replacing option d with option e. This implies that $\widetilde{P}^u = P^u$ for every $u \neq n$ and $\widetilde{P}^n = e$.

Suppose $eQd \Leftrightarrow U_e > U_d$ by contradiction. In what follows, we compare the expected utility of submitting P to the expected utility of submitting $\widetilde{P}$ for this agent and show a contradiction. To do this, we first establish some implications of Assumption 5 for the probabilities of admission:

$$
\begin{aligned}
L_u^P = L_u^{\widetilde{P}} \text{ for any } u \text{ such that } 1 \leq u < n \text{ if } n > 1, \qquad (6)\\
L_n^P \leq L_n^{\widetilde{P}}, \qquad (7)\\
L_u^P \geq L_u^{\widetilde{P}} \text{ for any } u \text{ such that } n < u \leq |P| \text{ if } n < |P|, \qquad (8)\\
L_0^P \geq L_0^{\widetilde{P}}, \qquad (9)
\end{aligned}
$$

where at least one of the inequalities (7)–(9) is strict if $n < |P|$; or, if $n = |P|$ and at least one of the inequalities (7) and (9) is strict. Below, we prove (6)–(9).

Equation 6 comes from the fact that $\widetilde{P}^u = P^u$ for every $1 \le u < n$ if $n > 1$. The admission probability depends on u being the lowest ranked school for which the agent qualifies. Thus,

$$\mathbb{P}\left[\cap_{v=1}^{u-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_{P^u} \ge C_{P^u}\}\right] = \mathbb{P}\left[\cap_{v=1}^{u-1}\{s_{\widetilde{P}^v} < C_{\widetilde{P}^v}\} \ \cap \ \{s_{\widetilde{P}^u} \ge C_{\widetilde{P}^u}\}\right],$$

for every $1 \le u < n$ if $n > 1$.

For Equation 7, we have that $\mathbb{P}[s_d \ge C_d] \le \mathbb{P}[s_e \ge C_e]$ is implied by the first condition of Assumption 5. This further implies that

$$\mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_d \ge C_d\}\right] \le \mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_e \ge C_e\}\right],$$

which is equivalent to $L_n^P \le L_n^{\widetilde{P}}$.

For Equation 8, note that the first condition of Assumption 5 implies $\mathbb{P}[s_d < C_d] \ge \mathbb{P}[s_e < C_e]$. This further implies that

$$\begin{aligned} L_u^P =& \mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_d < C_d\} \ \cap_{v=n+1}^{u-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_{P^u} \ge C_{P^u}\}\right] \\ &\ge \\ &\mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_e < C_e\} \ \cap_{v=n+1}^{u-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_{P^u} \ge C_{P^u}\}\right] = L_u^{\widetilde{P}} \end{aligned}$$

for u such that $n < u \le |P|$ if $n < |P|$.

For Equation 9, again we have that $\mathbb{P}[s_d < C_d] \ge \mathbb{P}[s_e < C_e]$ implies

$$\begin{aligned} L_0^P =& \mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_d < C_d\} \ \cap_{v=n+1}^{|P|}\{s_{P^v} < C_{P^v}\}\right] \\ &\ge \\ &\mathbb{P}\left[\cap_{v=1}^{n-1}\{s_{P^v} < C_{P^v}\} \ \cap \ \{s_e < C_e\} \ \cap_{v=n+1}^{|P|}\{s_{P^v} < C_{P^v}\}\right] = L_0^{\widetilde{P}}. \end{aligned}$$

Finally, the second condition in Assumption 5 (relevance condition) implies that at least one of the inequalities (7)–(9) is strict if $n < |P|$ or, if $n = |P|$, at least one of the inequalities (7) and (9) is strict. Having established these facts, we now move on to compare the expected utility of submitting P to the expected utility of submitting $\widetilde{P}$.

Define $\epsilon = U_e - U_d = U_{\widetilde{P}^n} - U_{P^n} > 0$. Note that $L_0^P = 1 - \sum_{u=1}^{|P|} L_u^P$ and $L_0^P - L_0^{\widetilde{P}} = \sum_{u=1}^{|P|}\left(L_u^{\widetilde{P}} - L_u^P\right)$ $= \left(L_n^{\widetilde{P}} - L_n^P\right) + \sum_{u=n+1}^{|P|}\left(L_u^{\widetilde{P}} - L_u^P\right)$, where we adopt the convention that a sum over an empty set of indices equals zero, i.e., $\sum_{u=n+1}^{|P|}\left(L_u^{\widetilde{P}} - L_u^P\right) = 0$ if $n+1 > |P|$. This leads to $\left(L_n^{\widetilde{P}} - L_n^P\right) = \left(L_0^P - L_0^{\widetilde{P}}\right) - \sum_{u=n+1}^{|P|}\left(L_u^{\widetilde{P}} - L_u^P\right)$. Next, we combine these definitions with the inequalities in Equations 7–9 to evaluate the difference between the expected utility of submitting P and the expected utility of submitting $\widetilde{P}$:

$$\begin{aligned} &\sum_{u=1}^{|P|} U_{P^u} L_u^P - U_{\widetilde{P}^u} L_u^{\widetilde{P}} = U_{P^n} L_n^P - U_{\widetilde{P}^n} L_n^{\widetilde{P}} + \sum_{u=n+1}^{|P|} U_{P^u}(L_u^P - L_u^{\widetilde{P}}) \\ &= U_{P^n}(L_n^P - L_n^{\widetilde{P}}) - \epsilon L_n^{\widetilde{P}} + \sum_{u=n+1}^{|P|} U_{P^u}(L_u^P - L_u^{\widetilde{P}}) \\ &= U_{P^n}\left[-\left(L_0^P - L_0^{\widetilde{P}}\right) + \sum_{u=n+1}^{|P|}\left(L_u^{\widetilde{P}} - L_u^P\right)\right] - \epsilon L_n^{\widetilde{P}} + \sum_{u=n+1}^{|P|} U_{P^u}(L_u^P - L_u^{\widetilde{P}}) \end{aligned}$$

$$= -U_{P^n}\left(L_0^P - L_0^{\widetilde{P}}\right) - U_{P^n}\sum_{u=n+1}^{|P|}\left(L_u^P - L_u^{\widetilde{P}}\right) - \epsilon L_n^{\widetilde{P}} + \sum_{u=n+1}^{|P|} U_{P^u}\left(L_u^P - L_u^{\widetilde{P}}\right)$$

$$= -U_{P^n}\left(L_0^P - L_0^{\widetilde{P}}\right) - \epsilon L_n^{\widetilde{P}} - \sum_{u=n+1}^{|P|}\left(U_{P^n} - U_{P^u}\right)\left(L_u^P - L_u^{\widetilde{P}}\right) < 0,$$

where we use the fact that weak partial order (Assumption 3) implies $U_{P^1} > \ldots > U_{P^{|P|}} > 0$, $\epsilon > 0$, and at least one of $\left(L_0^P - L_0^{\widetilde{P}}\right)$, $L_n^{\widetilde{P}}$, and $\left(L_u^P - L_u^{\widetilde{P}}\right)$ for $u > n$ is strictly positive if $n < |P|$. If $n = |P|$, at least one of $\left(L_0^P - L_0^{\widetilde{P}}\right)$ and $L_n^{\widetilde{P}}$ is strictly positive. The inequality above shows that submitting $\widetilde{P}$ increases the expected utility relative to that from submitting P, which contradicts P being the optimal choice. Therefore, dQe.

□

A.6 Proof of Corollary 1

Let (d, e) be an arbitrary pair of distinct schools such that e is uniformly more accessible than school d; $d \in P$, $e \notin P$. Call $(a, b) = P_j$. Proposition 3 implies dQe. This fact weakly decreases the set of possible Qs each individual has and thus potentially affects only the nonsingleton cases in the definition of $\mathbf{Q}_j$ in Proposition 2. These correspond to cases 1.2.1 and 2.2 in the proof of Proposition 2. We re-examine these cases below for the arbitrary pair (d, e).

Case 1.2.1: $S_j \geq c_j$, $N_j^- \neq \emptyset$, and $a \neq b$.

We have aQb. For any option $f \in N_j^-$, we have that $f \neq a$, $f \neq b$, $f \in B_j^-(\mathbf{S})$, and $f \in B_j^+(\mathbf{S})$.

Case 1.2.1(a): Suppose $b\bar{P}d$.

$b\bar{P}d$ implies that $b\bar{Q}d$. Given that dQe, we have bQe. In case $e \in N_j^-$, it is no longer true that we can construct Q such that fQl for any $f \in N_j^-$. We can do so only for $f \neq e$. Therefore, $Q_j = (a, e)$ does not belong to $\mathbf{Q}_j$.

Case 1.2.1(b): Suppose dPb.

This implies that dQb. The fact that dQe does not restrict us from having two possibilities: $dQeQb$ or $dQbQe$. It is again possible to construct Q such that fQb for any $f \in N_j^-$, including $f = e$ if $e \in N_j^-$. Therefore, $Q_j = (a, e)$ does belong to $\mathbf{Q}_j$ in this case.

Case 2.2: $S_j < c_j$, $N_j^+ \neq \emptyset$

Consider there is an option $f \in N_j^+ \setminus N_j^-$. We have that $f \neq a$, $f \neq b$, f is in $B_j^+(\mathbf{S})$ but not in $B_j^-(\mathbf{S})$.

Case 2.2(a): Suppose $a\bar{P}d$.

$a\bar{P}d$ implies that $a\bar{Q}d$. Given that dQe, we have that aQe. If $f = e$, we cannot construct Q such that fQa. Thus, it is no longer true that we can find Q such that fQa for every $f \in N_j^+ \setminus N_j^-$; this is true only for $f \neq e$. Therefore, $Q_j = (e, b)$ does not belong to $\mathbf{Q}_j$ in this case.

Case 2.2(b): Suppose dPa.

dPa implies dQa. The fact that dQe does not restrict us from having two possibilities: $dQeQa$ or $dQaQe$. It is again possible to construct Q such that fQa for every $f \in N_j^+ \setminus N_j^-$, including $f = e$ if $e \in N_j^+ \setminus N_j^-$. Therefore, $Q_j = (e, b)$ does belong to $\mathbf{Q}_j$ in this case.

Sharpness follows because we remove all Qs that violate the implication of Proposition 3.

□

A.7 Proof of Proposition 4

The random set $\mathbf{Q}_j$ is a measurable map from Ω to $\mathcal{J}^0 \times \mathcal{J}^0$, so it is compact valued. Following Definition A.1 by Molinari (2020), we say that $\mathbf{Q}_j$ is a random closed set because for every compact set $\mathcal{K} \in \mathbb{R}^2$, the set $\{\omega \in \Omega : \mathbf{Q}_j(\omega) \cap \mathcal{K} \neq \emptyset\}$ is a measurable event. By Assumption 6(i), the random vector Q_j and the random set $\mathbf{Q}_j$ are measurable maps on the same probability space, and $\mathbb{P}\left[Q_j \in \mathbf{Q}_j | S_j = s\right] = 1$ for any $s \in \mathcal{S}_j$.

Artstein's inequality (Theorem A.1 by Molinari (2020)) characterizes all possible probability mass functions $\mathbb{P}\left[Q_j = (a,b) | S_j = s\right]$ over $(a,b) \in \mathcal{J}^0 \times \mathcal{J}^0$ for any Q_j such that $\mathbb{P}\left[Q_j \in \mathbf{Q}_j | S_j = s\right] = 1$ for any $s \in \mathcal{S}_j$. Since $\mathbf{Q}_j$ is sharp (Assumption 6(ii)), the Artstein's inequality yields the sharp set of all probability mass functions $\mathbb{P}\left[Q_j = (a,b) | S_j = s\right]$. For any $s \in \mathcal{S}_j$, the inequality says that

$$\mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = s\right] \leq \mathbb{P}\left[Q_j \in A | S_j = s\right] \quad \forall A \in 2^{\mathcal{J}^0 \times \mathcal{J}^0}, \tag{10}$$

where $2^{\mathcal{J}^0 \times \mathcal{J}^0}$ denotes the power set of $\mathcal{J}^0 \times \mathcal{J}^0$.

Lemma 1 of Chesher and Rosen (2017) applied to our case shows that the inequalities (10) are equivalent to:

$$\mathbb{P}\left[\mathbf{Q}_j \subseteq A \mid S_j = s\right] \leq \mathbb{P}\left[Q_j \in A \mid S_j = s\right] \quad \forall A \in \mathbf{\Lambda}_j^{\cup}(s).$$

Next, consider any $s \in [c_j, c_j + \varepsilon)$ for ε of Assumption 7(i). We have that

$$\mathbb{P}\left[\mathbf{Q}_j \subseteq A \mid S_j = s\right] \leq \mathbb{P}\left[Q_j \in A \mid S_j = s\right] \quad \forall A \in \mathbf{\Lambda}_j^{\cup +},$$

and taking limits on both sides as $s \downarrow c_j$ leads to

$$\mathbb{P}\left[\mathbf{Q}_j \subseteq A \mid S_j = c_j^+\right] \leq \mathbb{P}\left[Q_j \in A \mid S_j = c_j\right] \quad \forall A \in \mathbf{\Lambda}_j^{\cup +}, \tag{11}$$

where we use the continuity of $\mathbb{P}\left[Q_j \in A \mid S_j = s\right]$ wrt s (Assumption 2) and the existence of side limit $\mathbb{P}\left[\mathbf{Q}_j \subseteq A \mid S_j = c_j^+\right]$ (Assumption 7(ii)). Applying an analogous argument to the left of the cutoff c_j leads to

$$\mathbb{P}\left[\mathbf{Q}_j \subseteq A \mid S_j = c_j^-\right] \leq \mathbb{P}\left[Q_j \in A \mid S_j = c_j\right] \quad \forall A \in \mathbf{\Lambda}_j^{\cup -}. \tag{12}$$

If there is $A \in \mathbf{\Lambda}_j^{\cup +} \cap \mathbf{\Lambda}_j^{\cup -}$, then both (11) and (12) are true, which leads to

$$\max\left\{\ \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^+\right]\ ;\ \mathbb{P}\left[\mathbf{Q}_j \subseteq A | S_j = c_j^-\right]\ \right\} \leq \mathbb{P}\left[Q_j \in A \mid S_j = c_j\right]. \tag{13}$$

In summary, we have an inequality for every $A \in \mathbf{\Lambda}_j^{\cup +} \cup \mathbf{\Lambda}_j^{\cup -}$. There are three possibilities: $A \in \mathbf{\Lambda}_j^{\cup +} \cap \mathbf{\Lambda}_j^{\cup -}$ (Inequality 13), $A \in \mathbf{\Lambda}_j^{\cup +} \setminus \mathbf{\Lambda}_j^{\cup -}$ (Inequality 11), and $A \in \mathbf{\Lambda}_j^{\cup -} \setminus \mathbf{\Lambda}_j^{\cup +}$ (Inequality 12).
□

A.8 Proof of Proposition 5

Define

$$\delta_{j,k}(s) = \frac{\mathbb{P}\left[Q_j = (j,k) | S_j = s\right]}{\mathbb{P}\left[\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset | S_j = s\right]},$$

where we know $\delta_{j,k}(s)$ is well defined for s in a neighborhood of c_j because (j,k) is a comparable pair (Definition 5) and because of Assumption 6.

By Assumptions 2 and 7, the side limits of $\delta_{j,k}(s)$ as $s \downarrow c_j$ and $s \uparrow c_j$ are well defined and equal to $\delta^+_{j,k}$ and $\delta^-_{j,k}$, respectively.

Take $g \in \mathcal{G}$ of Assumption 2. For $s \geq c_j$,

$$\begin{aligned}
&\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = s\right] \\
&= \delta_{j,k}(s)\ \mathbb{E}\left[g(Y)\,\middle|Q_j = (j,k), \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = s\right] \\
&\quad + (1-\delta_{j,k}(s))\ \mathbb{E}\left[g(Y)\,\middle|Q_j \neq (j,k), \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = s\right] \\
&= \delta_{j,k}(s)\ \mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = s\right] \\
&\quad + (1-\delta_{j,k}(s))\ \mathbb{E}\left[g(Y)\,\middle|Q_j \neq (j,k), \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = s\right],
\end{aligned}$$

where we use the cutoff characterization and the fact that $\{Q_j = (j,k)\} \subseteq \{\mathbf{Q}_j \cap \{(j,k)\}\}$. Taking the limit as $s \downarrow c_j$,

$$\begin{aligned}
&\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^+\right] \\
&= \delta^+_{j,k}\ \mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = c_j\right] \\
&\quad + (1-\delta^+_{j,k})\ \mathbb{E}\left[g(Y)\,\middle|Q_j \neq (j,k), \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^+\right],
\end{aligned}$$

where again all limits are well defined by Assumptions 2 and 7. Repeating the derivation for $s < c_j$ and making $s \uparrow c_j$,

$$\begin{aligned}
&\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^-\right] \\
&= \delta^-_{j,k}\ \mathbb{E}\left[g(Y(k))\,|Q_j = (j,k), S_j = c_j\right] \\
&\quad + (1-\delta^-_{j,k})\ \mathbb{E}\left[g(Y)\,\middle|Q_j \neq (j,k), \mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^-\right].
\end{aligned}$$

We know the expectations on the left-hand sides of the last two equations, but we do not know the δs or the expectations on the right-hand sides. Above the cutoff, the goal is to partially identify $\mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = c_j\right]$ using the distribution of $g(Y)$ conditional on $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$ and $S_j = c_j^+$ plus knowledge of a strictly positive lower bound on $\delta^+_{j,k}$, i.e., $\underline{\delta}^+_{j,k}$. Likewise, below the cutoff, the goal is to partially identify $\mathbb{E}\left[g(Y(k))\,|Q_j = (j,k), S_j = c_j\right]$ using the distribution of $g(Y)$ conditional on $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$ and $S_j = c_j^-$ plus $\underline{\delta}^-_{j,k}$. This problem fits the setting of Horowitz and Manski (1995), specifically, Propositions 1–4 and Corollary 4.1 of their paper.

Part (i)

Assume $g(Y)$ is a continuous random variable. Assume for the time being that we know the fraction of individuals with $Q_j = (j,k)$, that is, $\delta^+_{j,k} > 0$. The lowest possible value for the mean $\mathbb{E}[g(Y(j))|Q_j = (j,k), S_j = c_j]$ occurs when all individuals with $Q_j = (j,k)$ are in the lower tail of the distribution of $g(Y)$ conditional on $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$ and $S_j = c_j^+$. This gives the lower bound

$$\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) < F^{-1}_{j,k+}(\delta^+_{j,k}), S_j = c_j^+\right].$$

On the other hand, the highest possible value for the mean $\mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = c_j\right]$ occurs when all individuals with $Q_j = (j,k)$ are in the upper tail of the distribution of $g(Y)$ conditional on $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$

and $S_j = c_j^+$. This gives the upper bound

$$\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) > F^{-1}_{j,k+}(1-\delta^+_{j,k}), S_j = c_j^+\right].$$

We do not know $\delta^+_{j,k}$, but we know that $0 < \underline{\delta}^+_{j,k} \leq \delta^+_{j,k}$, so all values $\delta^+_{j,k} \in [\underline{\delta}^+_{j,k}, 1]$ are possible. As the fraction $\delta^+_{j,k}$ decreases, the lower bound derived above decreases, and the upper bound increases. Thus, the widest bounds occur when $\delta^+_{j,k} = \underline{\delta}^+_{j,k}$, that is,

$$\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) < F^{-1}_{j,k+}(\underline{\delta}^+_{j,k}), S_j = c_j^+\right]$$

and

$$\mathbb{E}\left[g(Y)\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, g(Y) > F^{-1}_{j,k+}(1-\underline{\delta}^+_{j,k}), S_j = c_j^+\right].$$

The bounds for $\mathbb{E}\left[g(Y(k))\,|Q_j = (j,k), S_j = c_j\right]$ are derived in an analogous fashion.

Part (ii)

Assume $g(Y) = Y$ and Y is binary. The expressions for the bounds in Part (i) are not valid in this case. To see this, take the lower bound for the mean of $Y(j)$ as an example. The quantile function $F^{-1}_{j,k+}(\underline{\delta}^+_{j,k})$ equals either 0 or 1, so that the conditioning event $Y < F^{-1}_{j,k+}(\underline{\delta}^+_{j,k})$ is either empty or has probability that is generally different from $\underline{\delta}^+_{j,k}$.

To start, note that

$$\mathbb{E}\left[g(Y(j))\,|Q_j = (j,k), S_j = c_j\right] = \mathbb{P}\left[Y(j) = 1\,|Q_j = (j,k), S_j = c_j\right].$$

Define

$$q^+_{j,k} := \mathbb{E}\left[Y\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^+\right] = \mathbb{P}\left[Y = 1\,\middle|\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset, S_j = c_j^+\right].$$

Assume for the time being that we know $\delta^+_{j,k}$.

Case (a): Suppose $1-q^+_{j,k} \geq \delta^+_{j,k}$, that is, that the fraction of individuals with $Y=0$ in the population of individuals with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$ and $S_j = c_j^+$ is greater than or equal to the fraction of individuals with $Q_j = (j,k)$ in that same population. In this case, the lowest possible value for $\mathbb{P}\left[Y(j) = 1\,|Q_j = (j,k), S_j = c_j\right]$ occurs when all $\delta^+_{j,k}$ individuals have Y equal to 0. That lowest value is zero.

Case (b): Now, suppose $1-q^+_{j,k} < \delta^+_{j,k}$. It is no longer possible to have all $\delta^+_{j,k}$ individuals with Y equal to 0. The lowest possible value for $\mathbb{P}\left[Y(j) = 1\,|Q_j = (j,k), S_j = c_j\right]$ has as many of $\delta^+_{j,k}$ individuals as we can with $Y=0$, that is, $1-q^+_{j,k}$ of them. The remaining $\delta^+_{j,k} - (1-q^+_{j,k})$ individuals must have $Y=1$. That lowest possible value is $(1-q^+_{j,k})/\delta^+_{j,k} \times 0 \; + \; [\delta^+_{j,k} - (1-q^+_{j,k})]/\delta^+_{j,k} \times 1 = 1 - (1-q^+_{j,k})/\delta^+_{j,k}$.

The lower-bound expression for $\mathbb{P}\left[Y(j) = 1\,|Q_j = (j,k), S_j = c_j\right]$ that covers both cases (a) and (b) is as follows:

$$\max\left\{1 - \frac{1-q^+_{j,k}}{\delta^+_{j,k}}\;,\;0\right\}.$$

Next, we consider the upper bound for $\mathbb{P}\left[Y(j) = 1\,|Q_j = (j,k), S_j = c_j\right]$.

Case (c): Suppose $q^+_{j,k} \geq \delta^+_{j,k}$, that is, that the fraction of individuals with $Y=1$ in the population of individuals with $\mathbf{Q}_j \cap \{(j,k)\} \neq \emptyset$ and $S_j = c_j^+$ is greater than or equal to the fraction of individuals with

$Q_j = (j,k)$ in that same population. The highest possible value for $\mathbb{P}\left[Y(j) = 1 \,|Q_j = (j,k), S_j = c_j\right]$ occurs when all $\delta^+_{j,k}$ individuals have $Y = 1$. That highest value is 1.

Case (d): Now, suppose $q^+_{j,k} < \delta^+_{j,k}$. It is no longer possible to have all $\delta^+_{j,k}$ individuals with Y equal to 1. The highest possible value for $\mathbb{P}\left[Y(j) = 1 \,|Q_j = (j,k), S_j = c_j\right]$ has as many of the $\delta^+_{j,k}$ individuals as we can with $Y = 1$, that is, $q^+_{j,k}$ of them. The remaining $\delta^+_{j,k} - q^+_{j,k}$ individuals must have $Y = 0$. That highest possible value is $q^+_{j,k}/\delta^+_{j,k} \times 1 \ + \ (\delta^+_{j,k} - q^+_{j,k})/\delta^+_{j,k} \times 0 = q^+_{j,k}/\delta^+_{j,k}$.

The upper-bound expression for $\mathbb{P}\left[Y(j) = 1 \,|Q_j = (j,k), S_j = c_j\right]$ that covers both cases (c) and (d) is as follows:

$$\min\left\{\frac{q^+_{j,k}}{\delta^+_{j,k}}\ ,\ 1\right\}.$$

We do not know $\delta^+_{j,k}$; we know only the lower bound $\underline{\delta}^+_{j,k} > 0$. As in Part (i) above, we compute the bounds at $\underline{\delta}^+_{j,k}$ because they grow weakly wider as $\delta^+_{j,k}$ decreases.

The bounds for $\mathbb{P}[Y(k) = 1|Q_j = (j,k), S_j = c_j]$ are derived in an analogous fashion.

□

B Sharp Bounds on Treatment Effects

In this section, we utilize Artstein's inequality (Theorem A.1 from Molinari (2020)) to characterize sharp bounds on the joint distribution of potential outcomes and true local preferences. That set of distributions produces sharp bounds on the averages of the treatment effects $Y(j) - Y(k)$ conditional on $Q_j = (j,k)$ and $S_j = c_j$ for any comparable pair $(j,k) \in \mathcal{P}$. Recall that we denote the space of possible outcomes Y as $\mathcal{Y}$. Let $2^{\mathcal{J}^0 \times \mathcal{J}^0}$ denote the power set of $\mathcal{J}^0 \times \mathcal{J}^0$ and $2^{\mathcal{Y}}$ denote the power set of $\mathcal{Y}$.

Theorem 1. *Suppose Assumptions 2, 6, and 7 hold. Assume $\mathcal{Y}$ is compact. Consider a pair $(j,k) \in \mathcal{P}$. Then, the inequalities below characterize the sharp set of all probability values of $\mathbb{P}\left[Y(d) \in A, Q_j = (b,b')|S_j = c_j\right]$ for $A \subseteq \mathcal{Y}$, $(b,b') \in \mathcal{J}^0 \times \mathcal{J}^0$, and $d \in \{b,b'\}$:*

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B|S_j = c^+_j\right] \leq \sum_{(b,b')\in B} \mathbb{P}\left[Y(b) \in A, Q_j = (b,b')|S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}},\ \ B \in \mathbf{\Lambda}^{\cup+}_j,$$

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B|S_j = c^-_j\right] \leq \sum_{(b,b')\in B} \mathbb{P}\left[Y(b') \in A, Q_j = (b,b')|S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}},\ \ B \in \mathbf{\Lambda}^{\cup-}_j.$$

Proof of Theorem 1: The random set $(\{Y\} \times \mathbf{Q}_j)$ is a measurable map from Ω to $\mathcal{Y} \times \mathcal{J}^0 \times \mathcal{J}^0$, so it is compact valued. Following Definition A.1 from Molinari (2020), we say that $(\{Y\} \times \mathbf{Q}_j)$ is a random closed set because, for every compact set $\mathcal{K} \in \mathbb{R}^3$, the set $\{\omega \in \Omega : (\{Y(\omega)\} \times \mathbf{Q}_j(\omega)) \cap \mathcal{K} \neq \emptyset\}$ is a measurable event. By Assumption 6(i), the random vector (Y, Q_j) and the random set $(\{Y\} \times \mathbf{Q}_j)$ are measurable maps on the same probability space, and $\mathbb{P}\left[(Y, Q_j) \in (\{Y\} \times \mathbf{Q}_j)|S_j = s\right] = 1$ for any $s \in \mathcal{S}_j$.

Artstein's inequality (Theorem A.1 from Molinari (2020)) characterizes the sharp set of all possible probability distributions for (Y, Q_j) that are consistent with our observation of $(\{Y\} \times \mathbf{Q}_j)$ and the fact that $\mathbb{P}[(Y, Q_j) \in (\{Y\} \times \mathbf{Q}_j)|S_j = s] = 1$. For any $s \in \mathcal{S}_j$, the inequality says that

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B|S_j = s\right] \leq \mathbb{P}\left[Y \in A, Q_j \in B|S_j = s\right] \quad \forall A \in 2^{\mathcal{Y}},\ \ B \in 2^{\mathcal{J}^0 \times \mathcal{J}^0}. \tag{14}$$

Next, we use the cutoff characterization and rewrite the right-hand side of (14) as a sum. For $s \geq c_j$,

$$\begin{aligned} \mathbb{P}\left[Y \in A, Q_j \in B | S_j = s\right] &= \sum_{(b,b') \in B} \mathbb{P}\left[Y \in A, Q_j = (b,b') | S_j = s\right] \\ &= \sum_{(b,b') \in B} \mathbb{P}\left[Y(b) \in A, Q_j = (b,b') | S_j = s\right]. \end{aligned} \tag{15}$$

Substitute (15) into (14) and take the limit as $s \downarrow c_j$ on both sides,

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B | S_j = c_j^+\right] \leq \sum_{(b,b') \in B} \mathbb{P}\left[Y(b) \in A, Q_j = (b,b') | S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}}, \;\; B \in 2^{\mathcal{J}^0 \times \mathcal{J}^0}, \tag{16}$$

where the limits of the left- and right-hand sides of the inequality are well defined by Assumptions 7 and 2, respectively.

Similarly, for $s > c_j$,

$$\begin{aligned} \mathbb{P}\left[Y \in A, Q_j \in B | S_j = s\right] &= \sum_{(b,b') \in B} \mathbb{P}\left[Y \in A, Q_j = (b,b') | S_j = s\right] \\ &= \sum_{(b,b') \in B} \mathbb{P}\left[Y(b') \in A, Q_j = (b,b') | S_j = s\right]. \end{aligned} \tag{17}$$

Use (17) into (14), and take the limit as $s \uparrow c_j$ on both sides,

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B | S_j = c_j^-\right] \leq \sum_{(b,b') \in B} \mathbb{P}\left[Y(b') \in A, Q_j = (b,b') | S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}}, \;\; B \in 2^{\mathcal{J}^0 \times \mathcal{J}^0}. \tag{18}$$

Lemma 1 of Chesher and Rosen (2017) applied to our case shows that the inequalities (16) and (18) are equivalent to:

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B | S_j = c_j^+\right] \leq \sum_{(b,b') \in B} \mathbb{P}\left[Y(b) \in A, Q_j = (b,b') | S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}}, \;\; B \in \mathbf{\Lambda}_j^{\cup +},$$

$$\mathbb{P}\left[Y \in A, \mathbf{Q}_j \subseteq B | S_j = c_j^-\right] \leq \sum_{(b,b') \in B} \mathbb{P}\left[Y(b') \in A, Q_j = (b,b') | S_j = c_j\right] \quad \forall A \in 2^{\mathcal{Y}}, \;\; B \in \mathbf{\Lambda}_j^{\cup -}.$$

□

Theorem 1 characterizes the sharp set of all possible probability values of $\mathbb{P}\left[Y(d) \in A, Q_j = (b,b') | S_j = c_j\right]$ for $A \subseteq \mathcal{Y}$, $(b,b') \in \mathcal{J}^0 \times \mathcal{J}^0$, and $d \in \{b, b'\}$. For a fixed $g \in \mathcal{G}$ of Assumption 2, that set of distributions allows us to define the sharp set of all possible means of potential outcomes,

$$\mathbb{E}\left[g(Y(d)) | Q_j = (j,k), S_j = c_j\right],$$

for $(j,k) \in \mathcal{P}$ such that $\underline{p}_{j,k} > 0$ and $d \in \{j,k\}$. The set of possible means in turn allows us to define the sharp set of all average treatment effects of the form

$$\mathbb{E}\left[g(Y(j)) - g(Y(k)) | Q_j = (j,k), S_j = c_j\right].$$

In case of continuous Y, it is impossible to directly evaluate all inequalities of Theorem 1 because there are uncountably many sets $A \in 2^{\mathcal{Y}}$. This is one of the drawbacks of the Artstein inequality approach, which has been extensively discussed by Beresteanu et al. (2012). In case Y takes finitely many values, e.g., when Y is binary, the number of such inequalities is feasible to evaluate because $2^{\mathcal{Y}} \times \mathbf{\Lambda}_j^{\cup+}$ (or $2^{\mathcal{Y}} \times \mathbf{\Lambda}_j^{\cup-}$) has finitely many elements. In fact, the number of inequalities is slightly higher than the number of inequalities in Proposition 4 of the main text, which we utilize to compute lower bounds on $\delta_{j,k}^{+}$ and $\delta_{j,k}^{-}$.

C Empirical Appendix

Table 3: Descriptive statistics: Students

	All students		Applicants to medicine at PUC Santiago	
	Mean	Std. dev.	Mean	Std. dev.
Number of programs in ROL	4.86	2.20	5.18	2.01
ROL strictly shorter than permitted	.80	.40	.77	.42
Assigned (to any prog.)	.65	.48	.68	.46
Rank of assigned program, cond. on assigned	2.24	1.59	2.58	1.68
Reapplies	.19	.39	.25	.43
Graduates from assigned prog., cond. on assigned	.22	.41	.32	.48
Graduates from any program	.77	.42	.89	.31
Number of students	519,409		9,398	

The left panel of the table provides descriptive statistics on the population of participants to the centralized college assignment mechanism in Chile between 2004 and 2010. The right panel provides descriptive statistics on the subpopulation of students who included medicine at PUC Santiago (our program j of interest in Section 5.2) in their rank-ordered list (ROL).

Table 4: Descriptive statistics: Programs

	All programs		Medicine at PUC Santiago	
	Mean	Std. dev.	Mean	Std. dev.
Number of applicants (per year)	474	351	1,094	138
Number of admitted students (per year)	66	48	90	.90
Year-to-year absolute change in cutoff (points)	.85	6.9	2.5	11.5
Cutoff			720	10
Number of programs	1,191			

The left panel of the table provides descriptive statistics on all programs involved in the centralized college assignment mechanism in Chile between 2004 and 2010. The right panel provides descriptive statistics on the medicine program at PUC Santiago (our program j of interest in Section 5.2).