ONLINE APPENDIX: REGIME-DEPENDENT EFFECTS OF UNCERTAINTY SHOCKS: A STRUCTURAL INTERPRETATION

Not for Publication

STÉPHANE LHUISSIER AND FABIEN TRIPIER

This Appendix consists of the following sections:

Appendix A. Data

All data are organized quarterly from the second Quarter of 1962 to the second Quarter of 2018. Most data comes from Federal Reserve Economic Database (FRED).

- gdp_t: output is the real GDP (GDPC1).
- vix_t: uncertainty is the Chicago Board of Options Exchange Market Volatility Index. From 1963 to 2009, we use the constructed index by Bloom (2009). Then, from 2009, we follow Stock and Watson (2012) and take a quarterly average of daily VIX.
- sp_t: credit spread is constructed as the difference between BAA corporate bond yields (BAA) and AAA corporate bond yields (AAA).

For inference, we use the natural log of output. Our spread and uncertainty variables remain unchanged.

Appendix B. Bayesian inference for MS-SVAR model

This section provides a detailed description of the Bayesian inference employed in this paper. More specifically, we closely follow Sims, Waggoner, and Zha (2008).

B.1. The posterior.

B.1. The posterior. Before describing the posterior distribution, we introduce the following notation: θ and q are vectors of parameters where θ contains all the parameters of the model (except those of the transition matrix) and $q = (q_{i,j}) \in \mathbb{R}^{h^2}$. $Y_t = (y_1, \ldots, y_t) \in (\mathbb{R}^n)^t$ are observed data with n denoting the number of endogenous variables and $S_t = (s_0, \ldots, s_t) \in H^{t+1}$ with $H \in \{1, \ldots, h\}$.

The log-likelihood function, $p(Y_T|\theta, q)$, is combined with the prior density functions, $p(\theta, q)$, to obtain the posterior density, $p(\theta, q|Y_T) = p(\theta, q)p(Y_T|\theta, q)$.

B.1.1. *The likelihood.* Following Hamilton (1989), Sims and Zha (2006), and Sims, Waggoner, and Zha (2008), we employ a class of Markov-switching structural VAR models of the following form:

$$y_t' A(s_t) = x_t' F(s_t) + \varepsilon_t' \Xi^{-1}(s_t), \tag{B.1}$$

with $x_t' = \begin{bmatrix} y_{t-1}' & \cdots & y_{t-\rho}' & 1 \end{bmatrix}$ and $F(s_t) = \begin{bmatrix} A_1(s_t) & \cdots & A_\rho(s_t) & C(s_t) \end{bmatrix}'$.

Let $a_j(k)$ be the jth column of $A(k)$, $f_j(k)$ be the jth column of $F(k)$, and $\xi_j(k)$ be the jth diagonal element of $\Xi(k)$. The conditional likelihood function is as follows:

$$p(y_t|s_t, Y_{t-1}) = |A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2} \left(y_t' a_j(s_t) - x_t' f_j(s_t) \right)^2 \right). \tag{B.2}$$

To simplify the Gibbs-sampling procedure described in the next section, it is preferable to rewrite the condition likelihood function with respect to free parameters from matrix $A(s_t)$ and $F(s_t)$:

$$|A(s_t)| \prod_{j=1}^{n} |\xi_j(s_t)| \exp\left(-\frac{\xi^2(s_t)}{2} \left((y_t' + x_t' W_j) U_j b_j(s_t) - x_t' V_j g_j(s_t) \right)^2 \right), \tag{B.3}$$

where $a_j(s_t) = U_j b_j(k)$ and $f_j(s_t) = V_j g_j - W_j U_j b_j(k)$ is a result from the linear restrictions $R_j \begin{bmatrix} a_j & f_j \end{bmatrix}' = 0$; and U_j and V_j are matrices with orthonormal columns and W_j is a matrix. See Waggoner and Zha (2003) for further details.

The log likelihood function is given by

$$p(Y_T|\theta, q) = \sum_{t}^{T} \ln \left\{ \sum_{s_t=1}^{h} p(y_t|s_t, Y_{t-1}) \Pr\left[s_t|Y_{t-1}\right] \right\}, \tag{B.4}$$

where

$$\Pr\left[s_t = i|Y_{t-1}\right] = \sum_{j=1}^{h} \Pr\left[s_t = i, s_{t-1} = j|Y_{t-1}\right] \tag{B.5}$$

$$= \sum_{j=1}^{h} \Pr\left[s_t = i|s_{t-1} = j\right] \Pr\left[s_{t-1} = j|Y_{t-1}\right]. \tag{B.6}$$

with $q_{i,j} = \Pr\left[s_t = i|s_{t-1} = j\right]$ are the transition probabilities from the $h \times h$ matrix Q

$$Q = \begin{bmatrix} q_{1,1} & \cdots & q_{1,j} \\ \vdots & \ddots & \vdots \\ q_{i,1} & \cdots & q_{i,j} \end{bmatrix} \tag{B.7}$$

The probability terms are updated as follows:

$$\Pr\left[s_t = j|Y_t\right] = \Pr\left[s_t = j|Y_{t-1}, y_t\right] = \frac{p(s_t = j, y_t|Y_{t-1})}{p(y_t|Y_{t-1})} \tag{B.8}$$

$$= \frac{p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}{\sum_{j=1}^{h} p(y_t|s_t = j, Y_{t-1}) \Pr[s_t = j|Y_{t-1}]}. \tag{B.9}$$

B.1.2. *The prior.* Following Sims and Zha (1998), we exploit the idea of a Litterman's random-walk prior from structural-form parameters. Note that dummy observations are not introduced as a component of the prior to keep in line with the original Litterman's prior. Using linear restrictions, the overall prior, $p(\theta, q)$, is given in the following way:

$$p(b_j(k)) = \text{normal}(b_j(k)|0, \bar{\Sigma}_{b_j}), \tag{B.10}$$

$$p(g_j(k)) = \text{normal}(g_j(k)|0, \bar{\Sigma}_{g_j}), \tag{B.11}$$

$$p(\xi_j^2(k)) = \text{gamma}(\xi_j^2(k)|\bar{\alpha}_j, \bar{\beta}_j), \tag{B.12}$$

$$p(q_j) = \text{dirichlet}(q_{i,j}|\alpha_{1,j}, \ldots, \alpha_{k,j}), \tag{B.13}$$

where $\bar{\Sigma}_{b_j}$, $\bar{\Sigma}_{\psi_j}$, and $\bar{\Sigma}_{\delta_j}$ denotes the prior covariance matrices and $\bar{\alpha}_j$ and $\bar{\beta}_j$ are set to one, allowing the standard deviations of shocks to have large values for some regimes.

The Gamma distribution is defined as follows:

$$\text{gamma}(x|\alpha, \beta) = \frac{1}{\Gamma(\alpha)} \beta^{\alpha} x^{\alpha-1} e^{-\beta x}. \tag{B.14}$$

Regarding the transition matrix, Q, suppose that $q_j = [q_{1,j}, \ldots, q_{h,j}]'$. The prior, denoted $p(q_j)$, follows a Dirichlet form as follows:

$$p(q_j) = \left(\frac{\Gamma\left(\sum_{i \in H} \alpha_{i,j}\right)}{\prod_{i \in H} \Gamma(\alpha_{i,j})} \right) \times \prod_{i \in H} (q_{i,j})^{\alpha_{i,j}-1}, \tag{B.15}$$

where Γ denotes the standard gamma function.

B.2. **Gibbs-sampling.** Following Kim and Nelson (1999) and Sims, Waggoner, and Zha (2008), a Markov Chain Monte Carlo (MCMC) simulation method is employed to approximate the joint posterior density, $p(\theta, q, S_T|Y_T)$. The advantage of using VARs is that conditional distributions like $p(S_T|Y_T, \theta, q)$, $p(q|Y_T, S_T, \theta)$, and $p(\theta|Y_T, q, S_T)$ can be obtained in order to exploit the idea of Gibbs-sampling by sampling alternatively from these conditional posterior distributions.

B.2.1. *Conditional posterior densities, $p(\theta|Y_T, q, S_T)$.* To simulate draws of $\theta \in \{b_j(k), g_j(k), \xi_j^2\}$ from $p(\theta|Y_T, S_t, q)$, one can start to sample from the conditional posterior

$$p(b_j(k)|y_t, S_t, b_i(k)) = \exp\left(-\frac{1}{2}b_j'(k)\bar{\Sigma}_{b_j}^{-1}b_j(k)\right) \times \prod_{t\in\{t:s_t=k\}}\left[|A(k)|\exp\left(-\frac{\xi^2(s_t)}{2}(y_t'a_j(k)-x_t'f_j(k))^2\right)\right], \quad \text{(B.16)}$$

using the Metropolis-Hastings (MH) algorithm. Then a multivariate normal distribution is employed to draw $g_j(k)$:

$$p(g_j(k)|y_t, S_t) = \text{normal}(g_j(k)|\tilde{\mu}_{g_j(k)}, \tilde{\Sigma}_{g_j(k)}). \quad \text{(B.17)}$$

The computational details of the posterior mean vectors and covariance matrices are given in Sims, Waggoner, and Zha (2008).

Disturbance variances ξ_j^2 are simulated from a gamma distribution

$$p(\xi_j^2(k)|y_t, S_t) = \text{gamma}(\xi_j^2(k)|\tilde{\alpha}_j(k), \tilde{\beta}_j(k)), \quad \text{(B.18)}$$

where $\tilde{\alpha}_j(k) = \bar{\alpha}_j + \frac{T_{2,k}}{2}$ and

$$\tilde{\beta}_j(k) = \bar{\beta}_j + \frac{1}{2}\sum_{t\in\{t:s_{2t}=k\}}(y_t'a_j(s_t) - x_t'f_j(s_t))^2, \quad \text{(B.19)}$$

with $T_{2,k}$ denoting the number of elements in $\{t : s_{2t} = k\}$.

B.2.2. *Conditional posterior densities, $p(S_T|Y_T, \theta, q)$.* A multi-move Gibbs-sampling is employed to simulate $S_t, t = 1, 2, ..., T$. First, draw s_t according to

$$p(s_t|y_t, S_t) = \sum_{s_{t+1}\in H} p(s_t|Y_T, \theta, q, s_{t+1})p(s_{t+1}|Y_T, \theta, q), \quad \text{(B.20)}$$

where

$$p(s_t|Y_t, \theta, q, s_{t+1}) = \frac{q_{s_{t+1},s_t}p(s_t|Y_t, \theta, q)}{p(s_{t+1}|Y_t, \theta, q)}. \quad \text{(B.21)}$$

Then, in order to generate s_t, one can use a uniform distribution between 0 and 1. If the generated number is less than or equal to the calculated value of $p(s_t|y_t, S_t)$, we set $s_t = 1$. Otherwise, s_t is set equal to 0.

B.2.3. *Conditional posterior densities, $p(q|Y_T, S_T, \theta)$.* The conditional posterior distribution of q_j is as follows:

$$p(q_j|Y_t, S_t) = \prod_{i=1}^{h} (q_{i,j})^{n_{i,j}+\beta_{i,j}-1}, \tag{B.22}$$

where $n_{i,j}$ is the number of transitions from $s_{t-1} = j$ to $s_t = i$.

Appendix C. Robustness results for MS-SVAR model

C.1. **The threshold of minimum contribution.** 60%, 70%, and 80%.

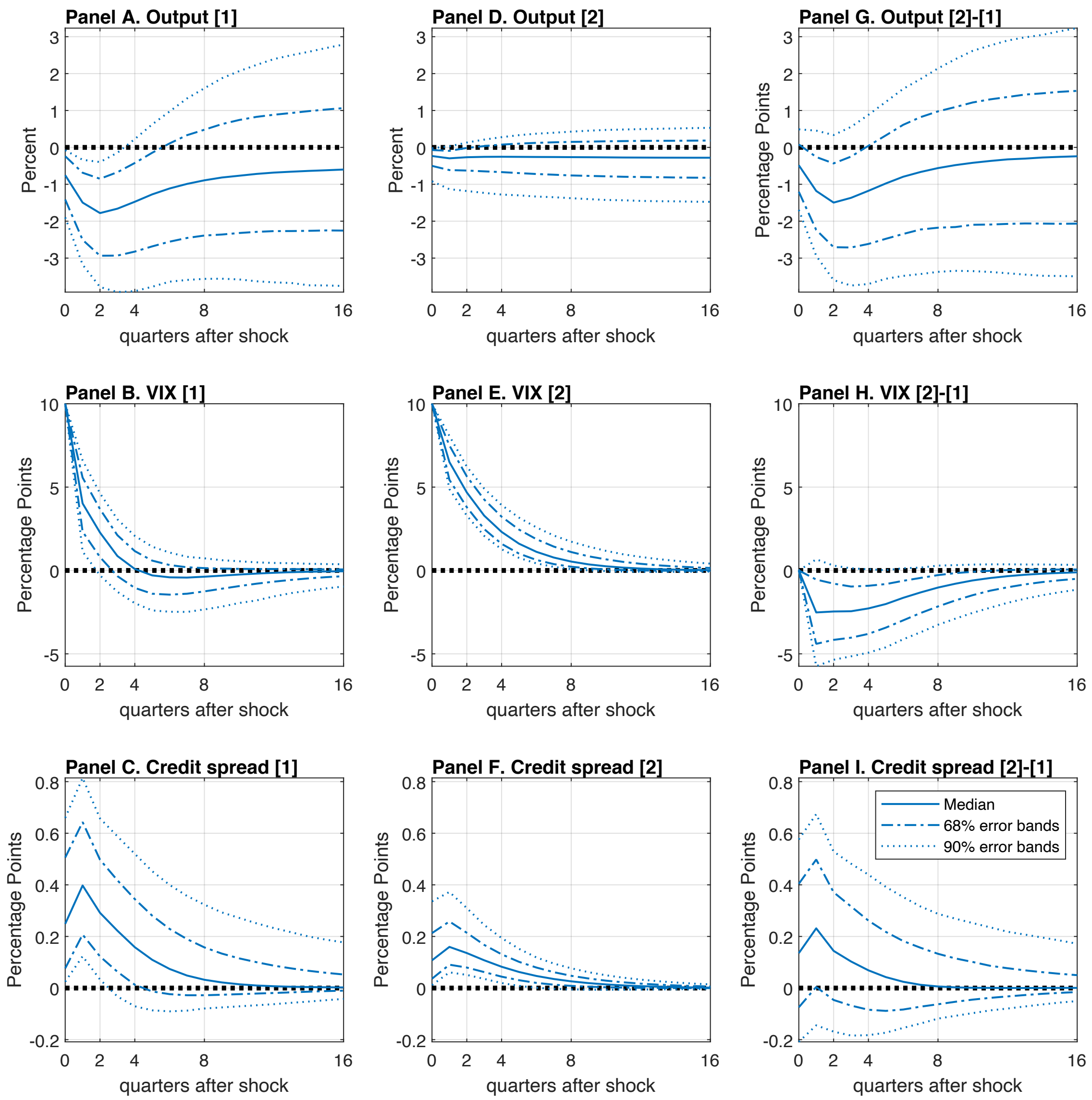

Figure 7. Impulse-response functions to uncertainty shock. Threshold of 60%.

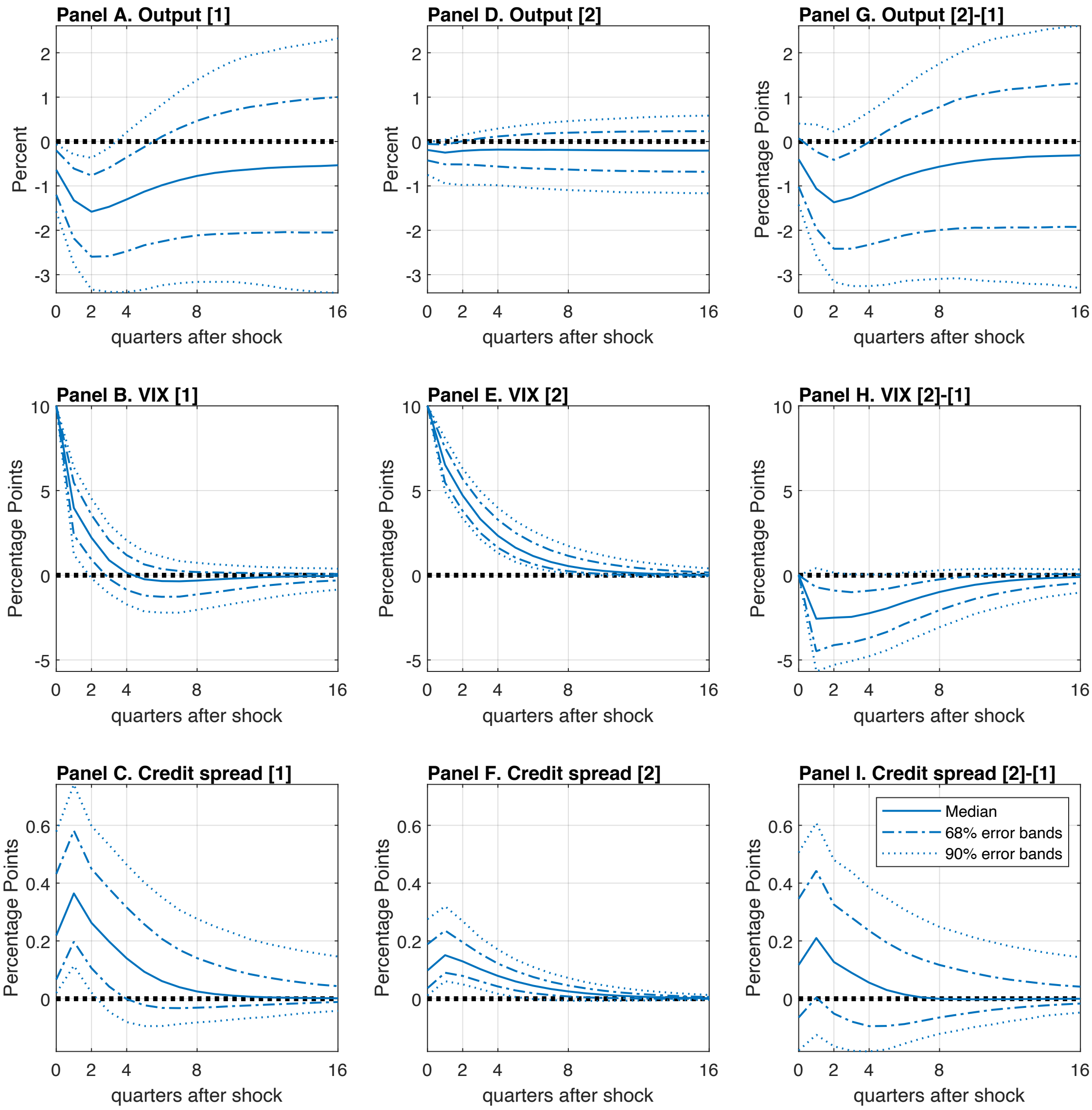

Figure 8. Impulse-response functions to uncertainty shock. Threshold of 70%.

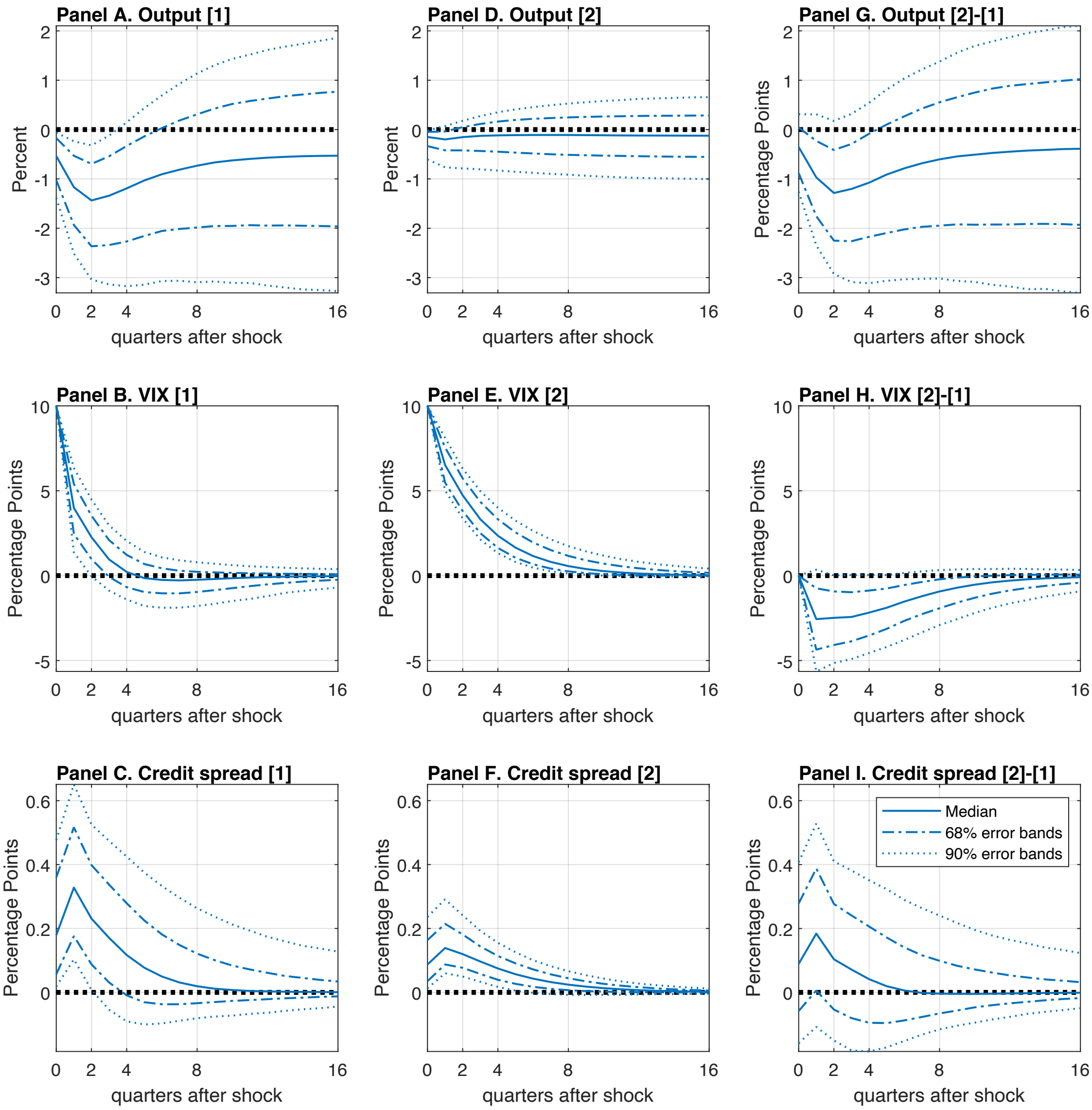

FIGURE 9. Impulse-response functions to uncertainty shock. Threshold of 80%.

C.2. **Alternative restrictions horizon.** Restrictions imposed for the next two quarters.

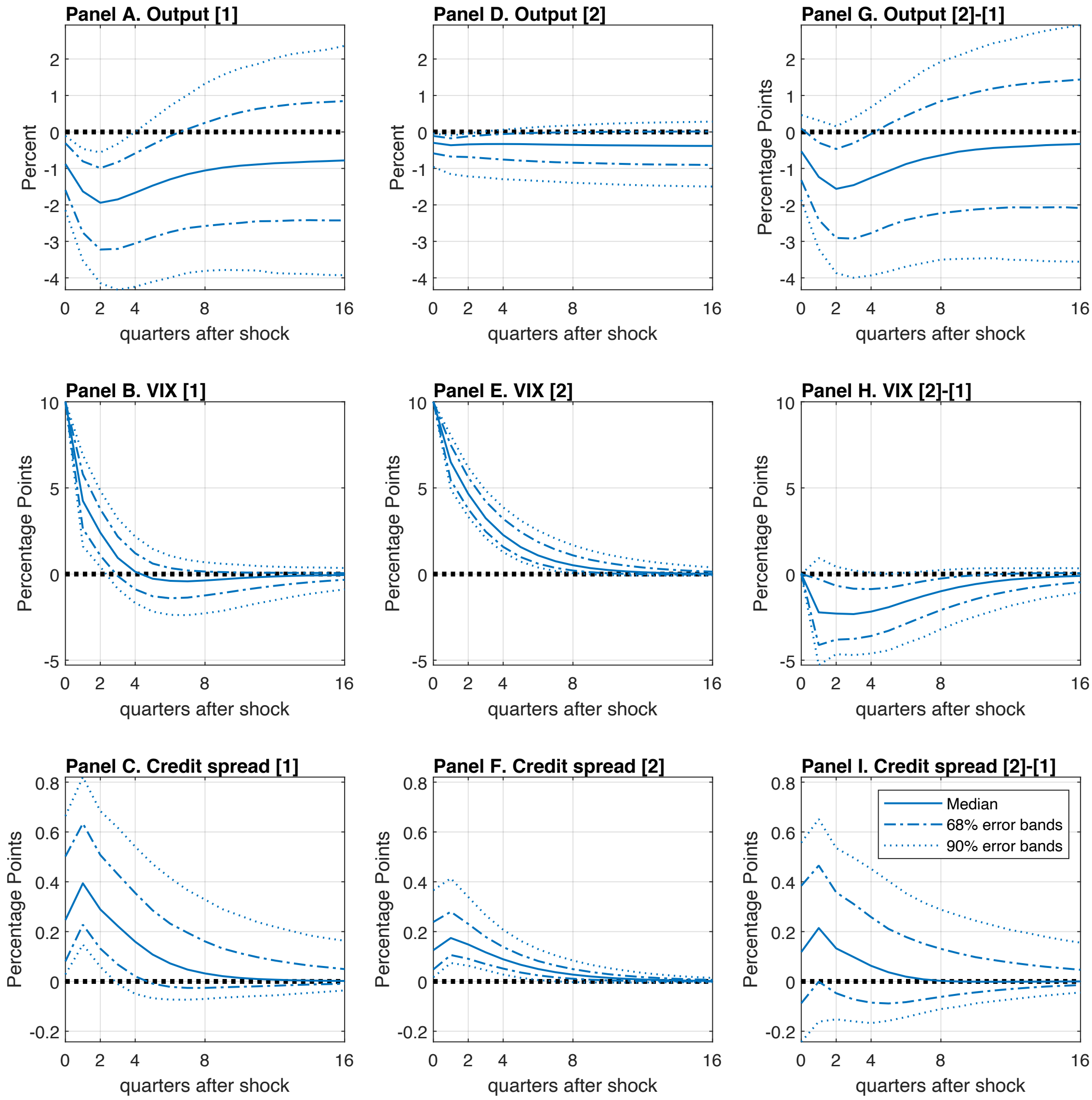

FIGURE 10. Impulse-response functions to uncertainty shock.

C.3. **Alternative restriction on forecast error variance.** Identification strategy in the spirit of Uhlig (2003).

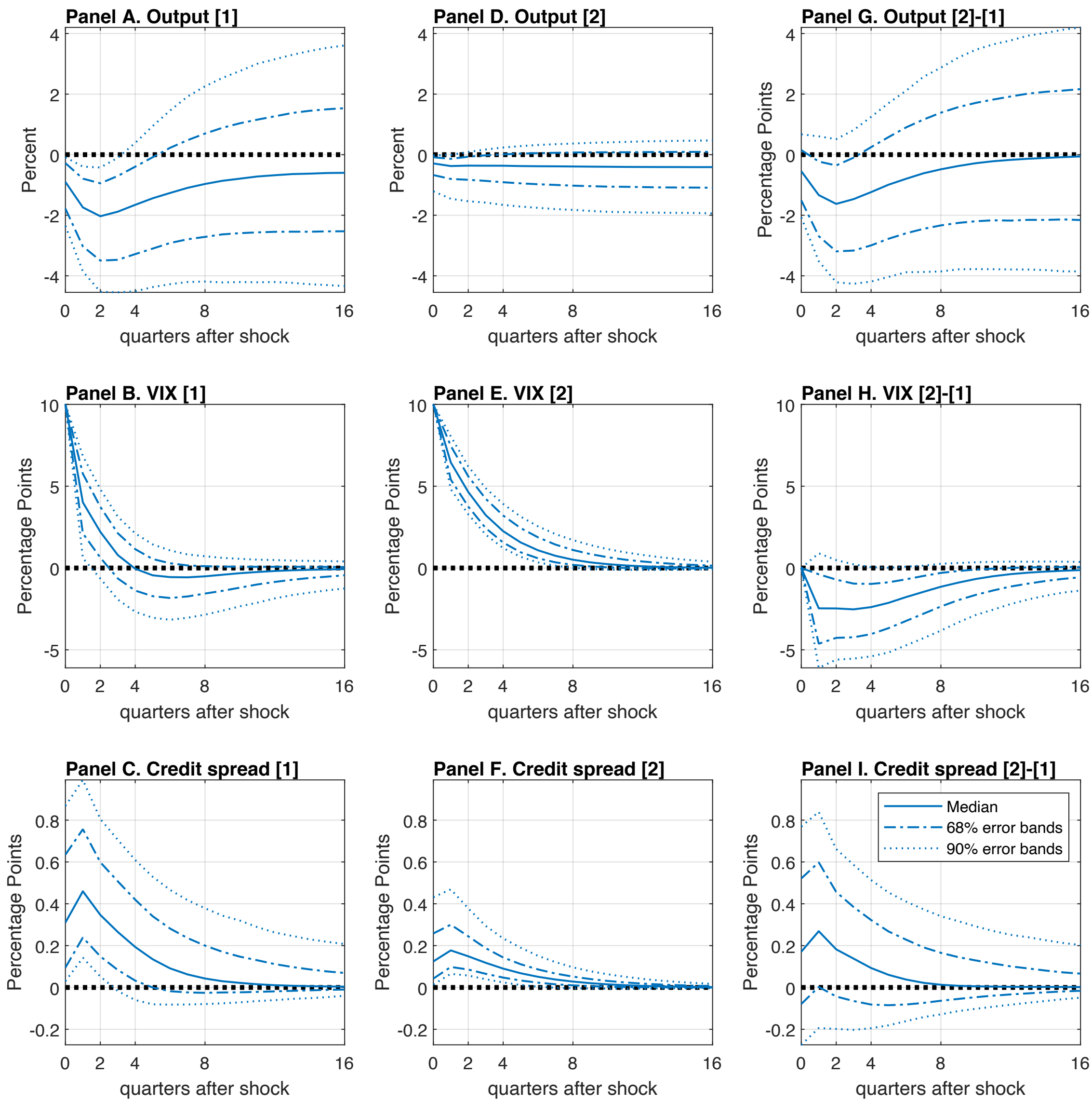

FIGURE 11. Impulse-response functions to uncertainty shock.

Appendix D. Equilibrium Conditions of MS-DSGE model

To solve our model, we require that the variables be stationary. The level of neutral and investment-specific technology have a unit root. The composite trend is then $z_t^* = z_t \Upsilon^{(\frac{\alpha}{1-\alpha})t}$ with the following steady state growth rate:

$$z^* = z\Upsilon^{(\frac{\alpha}{1-\alpha})}. \tag{D.1}$$

Several variables are then transformed to induce stationarity as follows:

$$c_t = \frac{C_t}{z_t^*}, \qquad y_t = \frac{Y_t}{z_t^*}, \qquad i_t = \frac{I_t}{z_t^* \Upsilon^t}, \qquad k_t = \frac{K_t}{z_{t-1}^* \Upsilon^{t-1}}, \qquad n_t = \frac{N_t}{P_{t-1} z_{t-1}^*} \tag{D.2}$$

$$g_t = \frac{G_t}{z_t^*}, \qquad w_t = \frac{W_t}{z_t^* P_t}, \qquad \mu_{z,t}^* = \frac{z_t*}{z_{t-1}^*}, \qquad gdp_t = \frac{GDP_t}{z_t^*}, \qquad w_t^e = \frac{W_t^e}{P_t z_t^*}. \tag{D.3}$$

The nominal rental rate on capital ($\tilde{r}_t^k P_t$) and the market price of capital are transformed to induce stationarity as well

$$r_t^k = \Upsilon^t \tilde{r}_t^k, \qquad q_t = \Upsilon^t \frac{Q_{K,t}}{P_t}. \tag{D.4}$$

We now re-write the model in a stationary form. The pricing equation by monopolistic producers is

$$p_t^* = \left((1-\xi_p) \left(\frac{K_{p,t}}{F_{p,t}} \right)^{\frac{\lambda_f}{1-\lambda_f}} + \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} p_{t-1}^* \right)^{\frac{\lambda_f}{1-\lambda_f}} \right)^{\frac{1-\lambda_f}{\lambda_f}} \tag{D.5}$$

with

$$F_{p,t} = \lambda_{z,t} y_t + \beta \mathrm{E}_t \left\{ F_{p,t+1}\, \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{1}{1-\lambda_f}} \right\}, \quad \text{and} \tag{D.6}$$

$$K_{p,t} = y_t \lambda_{z,t} \lambda_f s_t + \beta \mathrm{E}_t \left\{ \xi_p \left(\frac{\tilde{\pi}_{t+1}}{\pi_{t+1}} \right)^{\frac{\lambda_f}{1-\lambda_f}} K_{p,t+1} \right\}, \tag{D.7}$$

which satisfy

$$K_{p,t} = F_{p,t} \left(\frac{1 - \xi_p \left(\frac{\tilde{\pi}_t}{\pi_t} \right)^{\frac{1}{1-\lambda_f}}}{1-\xi_p} \right)^{1-\lambda_f} \tag{D.8}$$

The inflation indexation rule is

$$\tilde{\pi}_t = \pi^{*\iota} \pi_{t-1}^{1-\iota} \tag{D.9}$$

The wage equation setting by labor contractor is

$$w_t^* = \left((1-\xi_w) \left(\frac{1-\xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}}\right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{\lambda_w} + \xi_w \left(\mu_z \frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} w_{t-1}^* \right)^{\frac{\lambda_w}{1-\lambda_w}} \right)^{\frac{1}{\frac{\lambda_w}{1-\lambda_w}}} \tag{D.10}$$

with

$$F_{w,t} = \frac{h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} \lambda_{z,t} \left(1-\tau_l\right)}{\lambda_w} + \beta \mathrm{E}_t \left\{ F_{w,t+1} \frac{\xi_w \mu_z^{*\frac{\lambda_w}{1-\lambda_w}} \tilde{\pi}_{w,t+1}^{\frac{1}{1-\lambda_w}}}{\pi_{t+1}} \left(\frac{1}{\tilde{\pi}_{w,t+1}} \right)^{\frac{\lambda_w}{1-\lambda_w}} \right\}, \tag{D.11}$$

$$K_{w,t} = \left(h_t \; \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}} \right)^{1+\sigma_L} + \beta \mathrm{E}_t \left\{ \xi_w \left(\frac{\tilde{\pi}_{w,t+1}}{\pi_{w,t+1}} \mu_z^* \right)^{\frac{\lambda_w}{1-\lambda_w}(1+\sigma_L)} K_{w,t+1} \right\}, \tag{D.12}$$

which satisfy

$$K_{w,t} = \left(\frac{1-\xi_w \left(\frac{\tilde{\pi}_{w,t}}{\pi_{w,t}} \mu_{z,t}^* \right)^{\frac{1}{1-\lambda_w}}}{1-\xi_w} \right)^{1-\lambda_w(1+\sigma_L)} \tilde{w}_t \frac{F_{w,t}}{\psi_L}. \tag{D.13}$$

The wage inflation equation is

$$\pi_{w,t} = \pi_t \mu_z^* \frac{\tilde{w}_t}{\tilde{w}_{t-1}}, \tag{D.14}$$

and the indexation rule is

$$\tilde{\pi}_{w,t} = \left(\pi_t^{target}\right)^{\iota_w} \pi_{t-1}^{1-\iota_w}. \tag{D.15}$$

The efficiency condition for setting capital utilization is

$$r_t^k = \exp\left(\sigma_a \left(u_t - 1\right)\right) \bar{r}^k, \tag{D.16}$$

where the rental rate on capital is

$$r_t^k = \alpha \left(\frac{\Upsilon \mu_z h_t \left(w_t^*\right)^{\frac{\lambda_w}{\lambda_w - 1}}}{u_t k_{t-1}} \right)^{1-\alpha} s_t. \tag{D.17}$$

The capital utilization costs are given by

$$a_t = \frac{\bar{r}^k \; \left(\exp\left(\sigma_a \left(u_t - 1\right)\right) - 1\right)}{\sigma_a}, \tag{D.18}$$

and the capital adjustment costs are as follows

$$S_t = \exp\left[\sqrt{\frac{S''\left(\chi_t\right)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z \right) \right] + \exp\left[-\sqrt{\frac{S''\left(\chi_t\right)}{2}} \left(\Upsilon \mu_z \frac{i_t}{i_{t-1}} - \Upsilon \mu_z \right) \right] - 2 \tag{D.19}$$

The level of output is given by

$$y_t = p_t^{*\frac{\lambda_f}{\lambda_f - 1}} \left(\left(\frac{u_t k_{t-1}}{\Upsilon \mu_z} \right)^{\alpha} \left(h_t w_t^{*\frac{\lambda_w}{\lambda_w - 1}} \right)^{1-\alpha} - \phi \right) \tag{D.20}$$

and the marginal cost of production is

$$s_t = \left(\frac{r_t^k}{\alpha} \right)^{\alpha} \left(\frac{\tilde{w}_t}{1-\alpha} \right)^{1-\alpha} \tag{D.21}$$

The stationary household first-order conditions are

$$\lambda_{z,t} = \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1}}{\mu_{z,t}^* \pi_{t+1}} (1 + R_t) \right\}, \tag{D.22}$$

$$(1+\tau_c)\, \lambda_{z,t} = \frac{\mu_{z,t}^*}{\mu_{z,t}^* c_t - b c_{t-1}} - \beta \mathrm{E}_t \left\{ \frac{b}{\mu_{z,t}^* c_{t+1} - b c_t} \right\}, \tag{D.23}$$

$$\lambda_{z,t} = \lambda_{z,t} q_t \left(-S_t' \frac{i_t}{i_{t-1}} \Upsilon \mu_{z,t}^* + 1 - S_t \right) + \beta \mathrm{E}_t \left\{ \frac{\lambda_{z,t+1} q_{t+1} S_{t+1}'}{\Upsilon \mu_{z,t}^*} \left(\frac{\Upsilon \mu_{z,t}^* i_{t+1}}{i_t} \right)^2 \right\}, \tag{D.24}$$

where (D.22), (D.23), and (D.24) are with respect to risk-free bonds, consumption, and investment, respectively.

Regarding the entrepreneurs, the zero profit condition is as follows

$$\frac{q_t k_{t+1}}{n_{t+1}} \frac{R_{t+1}^k}{R_t} \left[\Gamma_t(\omega_{t+1}) - \mu(\chi_t) G_t(\omega_{t+1}) \right] - \frac{q_t k_{t+1}}{n_{t+1}} + 1 = 0, \tag{D.25}$$

where

$$G_t(\omega_{t+1}) \equiv \int_0^{\omega_{t+1}} \omega dF_t(\omega), \qquad \text{and} \qquad \Gamma_t(\omega_{t+1}) \equiv \omega_{t+1} \left[1 - F_t(\omega_{t+1}) \right] + G_t(\omega_{t+1}). \tag{D.26}$$

The stationary entrepreneur first-order condition with respect to capital is given by

$$\begin{aligned} 0 =& \mathrm{E}_t \left\{ \frac{(1 - \Gamma_t(\omega_{t+1}))\, R_{t+1}^k}{R_t} \right. \\ &+ \left. \frac{\Gamma_t(\omega_{t+1})'}{\Gamma_t(\omega_{t+1})' - \mu(\chi_t)\, G_t(\omega_{t+1})'} \left(\frac{R_{t+1}^k}{R_t} \left(\Gamma_t(\omega_{t+1}) - \mu(\chi_t)\, G_t(\omega_{t+1}) \right) - 1 \right) \right\}, \end{aligned} \tag{D.27}$$

where $\Gamma_t(\omega_{t+1})' = 1 - F_t(\omega_{t+1})$.

The return of capital for entrepreneurs is

$$R_t^k = \frac{(1-\tau_k) \left(r_t^k u_t - a_t \right) + (1-\delta)\, q_t}{\Upsilon q_{t-1}} \pi_t + \delta \tau_k, \tag{D.28}$$

The law of motion of entrepreneurial net worth is given by

$$n_{t+1} = q_t k_{t-1} \frac{\gamma}{\pi_t \, \mu^*_{z,t}} \left\{ R^k_t - R_{t-1} - \mu(\chi_t) \int_0^{\omega_t} \omega dF_{t-1}(\omega) R^k_t \right\} + n_t \left(\frac{R_{t-1}}{\pi_t \mu^*_{z,t}} \right) \gamma + W^e. \quad \text{(D.29)}$$

The monetary policy rule is as follows

$$\begin{aligned} \log\left(\frac{R_t}{R}\right) &= \rho_R \log\left(\frac{R_{t-1}}{R}\right) + \phi_\pi (1-\rho_R) \frac{\pi^*}{R} \log\left(\frac{\pi_{t+1}}{\pi^*}\right) \\ &\quad + \phi_y (1-\rho_R) \frac{\mu_z}{4R} \left(\frac{c_R \log\left(\frac{c_t}{c_{t-1}}\right) + i_R \log\left(\frac{i_t}{i_{t-1}}\right)}{(c_R + i_R)/(1-\eta_g)} \right), \end{aligned} \quad \text{(D.30)}$$

where c_R and i_R coefficients account for the share of consumption and investment in GDP.

The stationary resource constraint of the economy is

$$y_t = c_t + g_t + i_t + a_t \frac{k_t}{\Upsilon \mu^*_z} + d_t + \Theta \frac{(1-\gamma)}{\gamma} (n_{t+1} - W^e). \quad \text{(D.31)}$$

where Θ is the share of assets consumed by dying entrepreneurs and d_t the monitoring costs, which are given by

$$d_t = \frac{\mu(\chi_t) G(\overline{\omega}_t) R^k_t q_{t-1} k_t}{\pi_t \mu^*_z}. \quad \text{(D.32)}$$

The law of motion for physical capital is

$$k_{t+1} = (1-\delta) \frac{k_t}{\Upsilon \mu^*_{z,t}} + \left(1 - S\left(\frac{i_t \mu^*_{z,t} \Upsilon}{i_{t-1}}, \chi_t\right)\right) i_t, \quad \text{(D.33)}$$

The law of motion for the uncertainty shock is as follows

$$\log \sigma_t = (1 - \rho_\sigma(\chi_t)) \log \sigma + \rho_\sigma(\chi_t) \log \sigma_{t-1} + \varepsilon_{\sigma,t}, \quad \text{(D.34)}$$

with $\varepsilon_{\sigma,t} = \text{normal}(\varepsilon_{\sigma,t} | 0, \sigma_\sigma(\chi_t))$.

Appendix E. Results for alternative specifications of MS-DSGE model

Table 5. Posterior distribution, alternative specifications

		Model			
		(1)	(2)	(3)	(4)
Coefficient	Description	Benchmark	Taylor rule LWZ (2011)	Nominal rigidities LWZ (2011)	Flexible prices and wages
p_{11}	Transition matrix	0.9170 [0.7971;0.9600]	0.9134 [0.7997;0.9566]	0.9139 [0.8066;0.9558]	0.9079 [0.7851;0.9440]
p_{22}	Transition matrix	0.9684 [0.8385;0.9868]	0.9794 [0.8105;0.9856]	0.9670 [0.8368;0.9867]	0.9989 [0.7851;0.9996]
κ	Measurement VIX	0.9142 [0.5849;1.5301]	0.9516 [0.6492;1.5800]	0.7720 [0.5271;1.3030]	0.7305 [0.5234;1.2774]
$S''(\chi_t = 1)$	Investment adj. costs	0.5993 [0.2820;1.2952]	0.6453 [0.2799;1.4609]	0.5337 [0.2419;1.3725]	0.9779 [0.7734;1.7522]
$S''(\chi_t = 2)$	Investment adj. costs	1.3437 [0.7985;1.9746]	1.4032 [0.8543;2.0745]	0.9059 [0.4270;1.6728]	2.0424 [1.6096;2.6620]
$\mu(\chi_t = 1)$	Monitoring costs	0.2090 [0.1125;0.3231]	0.2076 [0.1130;0.3198]	0.2169 [0.1264;0.3345]	0.1631 [0.1059;0.2656]
$\mu(\chi_t = 2)$	Monitoring costs	0.0615 [0.0326;0.1290]	0.0597 [0.0387;0.1385]	0.0662 [0.0402;0.1303]	0.0682 [0.0503;0.1326]
$\rho_\sigma(\chi_t = 1)$	Persistence shock	0.5509 [0.2962;0.6603]	0.5156 [0.2714;0.6305]	0.5980 [0.3374;0.7116]	0.5227 [0.2235;0.7610]
$\rho_\sigma(\chi_t = 2)$	Persistence shock	0.7689 [0.6902;0.8103]	0.7493 [0.6846;0.7971]	0.7730 [0.7011;0.8095]	0.7425 [0.6211;0.7972]
$\sigma_\sigma(\chi_t = 1)$	Std. Dev. shock	0.4096 [0.3197;0.5682]	0.4003 [0.3213;0.5650]	0.4018 [0.3118;0.5418]	0.3805 [0.2809;0.4944]
$\sigma_\sigma(\chi_t = 2)$	Std. Dev. shock	0.4204 [0.3045;0.5588]	0.4227 [0.3041;0.5389]	0.4528 [0.3316;0.5708]	0.3130 [0.2369;0.3620]
Log Marginal Likelihood		-22.5363	-22.6416	-22.4009	-44.1524

Note: Posterior modes and 90% probability intervals are reported. LWZ (2011) stands for Liu, Waggoner, and Zha (2011).