Identification And Inference With Ranking Restrictions – Application Tables and Figures

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Abstract

This file contains results produced with the replication files (except for keeping the original tables). The results differ slightly from those in the actual paper because the results here are based on only 50, rather than 500, draws from the reduced form posterior.

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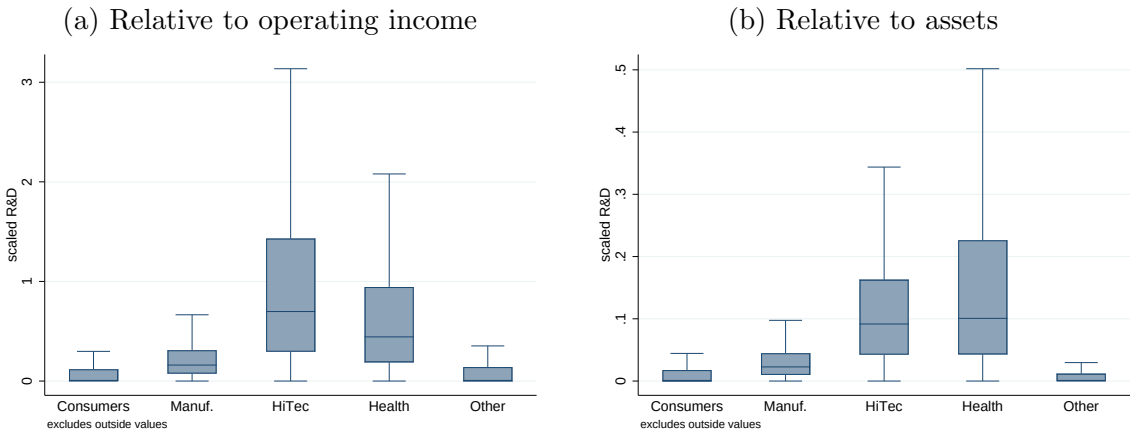
4 Application

4.1 Productivity news and industry data: Heterogeneity restrictions



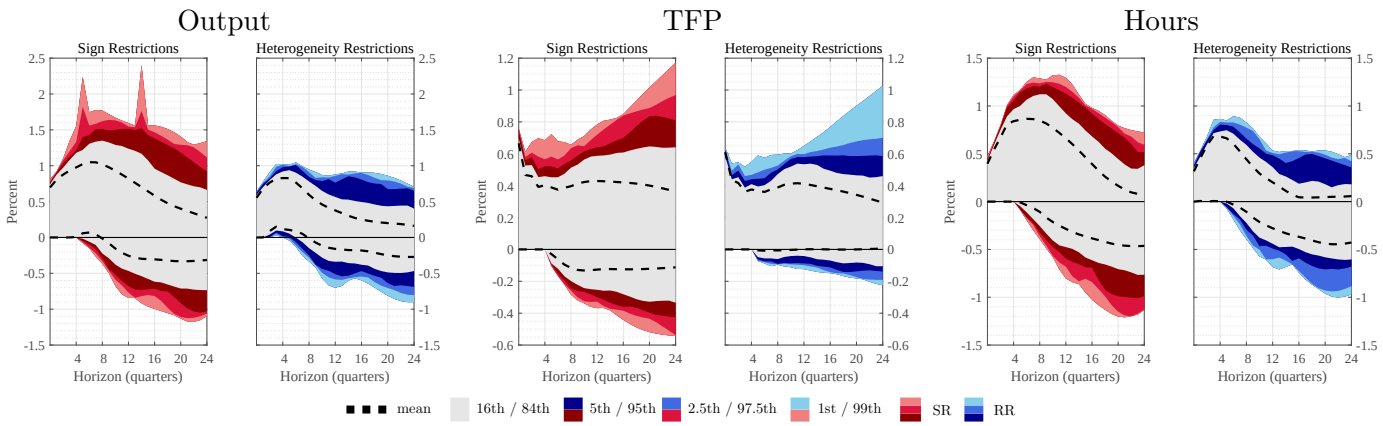
The boxes show the median along with the interquartile range (IQR) of the R&D intensity for the five industries in the coarsest **?** classification. The upper (lower) whiskers end in the values just above the 75th (25th) percentile plus (minus) 1.5 times the IQR. We measure firm size either as the lagged three-year moving average of operating income or total assets. Source: Compustat. U.S. firms, 1960–2015.

Figure 4.1: R&D intensity by industry in the five industry classification

	Benchmark data	**Sign restrictions**	**Heterogeneity restrictions**
Macro (common)	Real output	Real output ≥ 0	Real output ≥ 0
	TFP	TFP ≥ 0	TFP ≥ 0
	Confidence	Confidence ≥ 0	Confidence ≥ 0
	Hours worked	Hours worked ≥ 0	Hours worked ≥ 0
Industry	FF-5 Consumers	FF-5 Consumers ≥ 0	FF-5 Manu $\geq$ FF-5 Consumers ≥ 0
	FF-5 Manufacturing	FF-5 Manufacturing ≥ 0	FF-5 Manufacturing $\geq$ FF-5 Other ≥ 0
	FF-5 High Tech	FF-5 High Tech ≥ 0	FF-5 High Tech $\geq$ FF-5 Manu
	FF-5 Health	FF-5 Health ≥ 0	FF-5 Health $\geq$ FF-5 Manu
	FF-5 Other	FF-5 Other ≥ 0	FF-5 Other ≥ 0

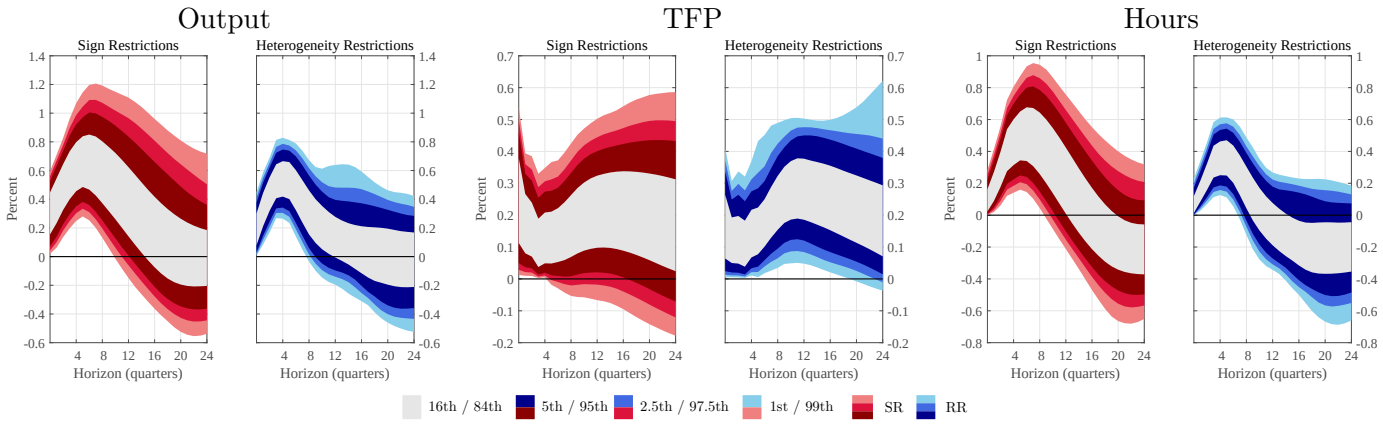
Table 4.1: Benchmark data and identifying restrictions in the VAR with industry-level data

4.2 Productivity news and macro data: Slope restrictions and multiple shocks



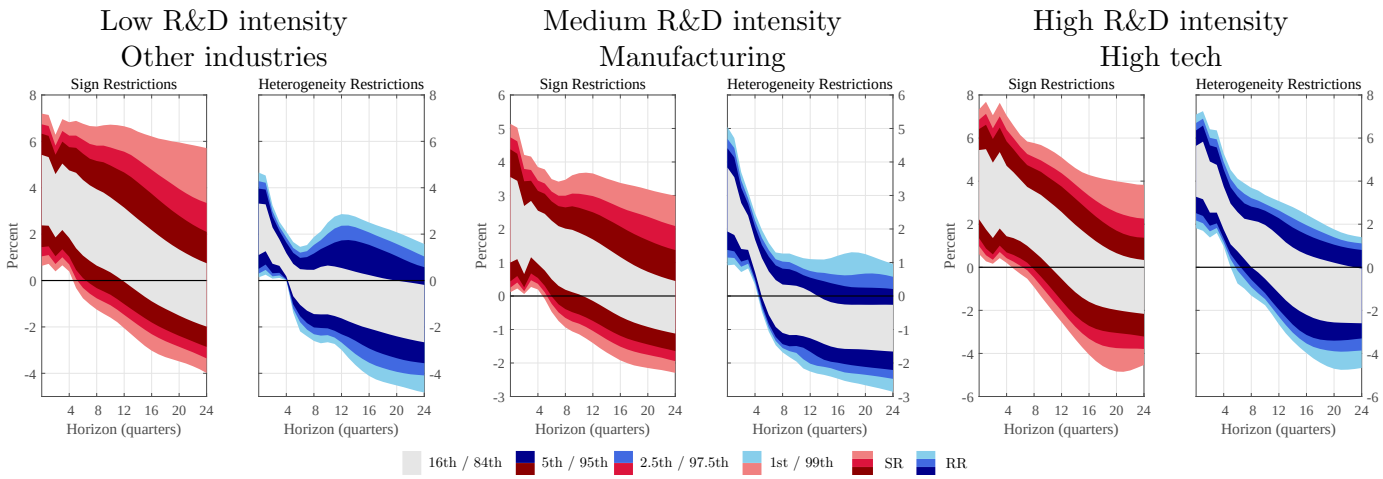
With parameter uncertainty we can only bound the responses after impact. Heterogeneity restrictions sharpen the bounds: The peak upper bounds with heterogeneity restrictions are a third smaller for output and hours at the 99th percentile. Heterogeneity restrictions cut the lowest lower bound for TFP also by half.

Figure 4.2: Prior-robust responses of output, TFP, and hours worked to a one standard deviation productivity news shock



Our fully Bayesian posterior implies that TFP increases in a hump-shaped fashion after a productivity news shock. The hump shape in TFP is well defined only with heterogeneity restrictions. Heterogeneity restrictions also visibly reduce the credible sets for hours and TFP.

Figure 4.3: Fully Bayesian responses of output, TFP, and hours worked to a one standard deviation productivity news shock



Heterogeneity restrictions rank the responses of stock returns of industries from zero to four quarters according to their R&D intensity to sharpen inference about macro variables. Heterogeneity restrictions mechanically imply larger response in more R&D intensive industries. These differences are absent with only sign restrictions.

Figure 4.4: Fully Bayesian responses of (cumulative) industry returns to a one standard deviation productivity news shock.

Sample start	Restriction horizon	Only SR	No slope restrictions Actual HR	No slope restrictions Inverted HR	Added slope restrictions Actual HR	Added slope restrictions Inverted HR
1960	1	100%	100%	100%	99.8%	99.8%
1960	3	100%	100%	100%	98.4%	85.5%
1960	5	99.2%	99.8%	100%	84.7%	40.0%
1983	1	100%	100%	100%	>99.9%	98.3%
1983	3	100%	100%	98.8%	98.9%	34.1%
1983	5	100%	99.2%	71.2%	80.2%	0.6%

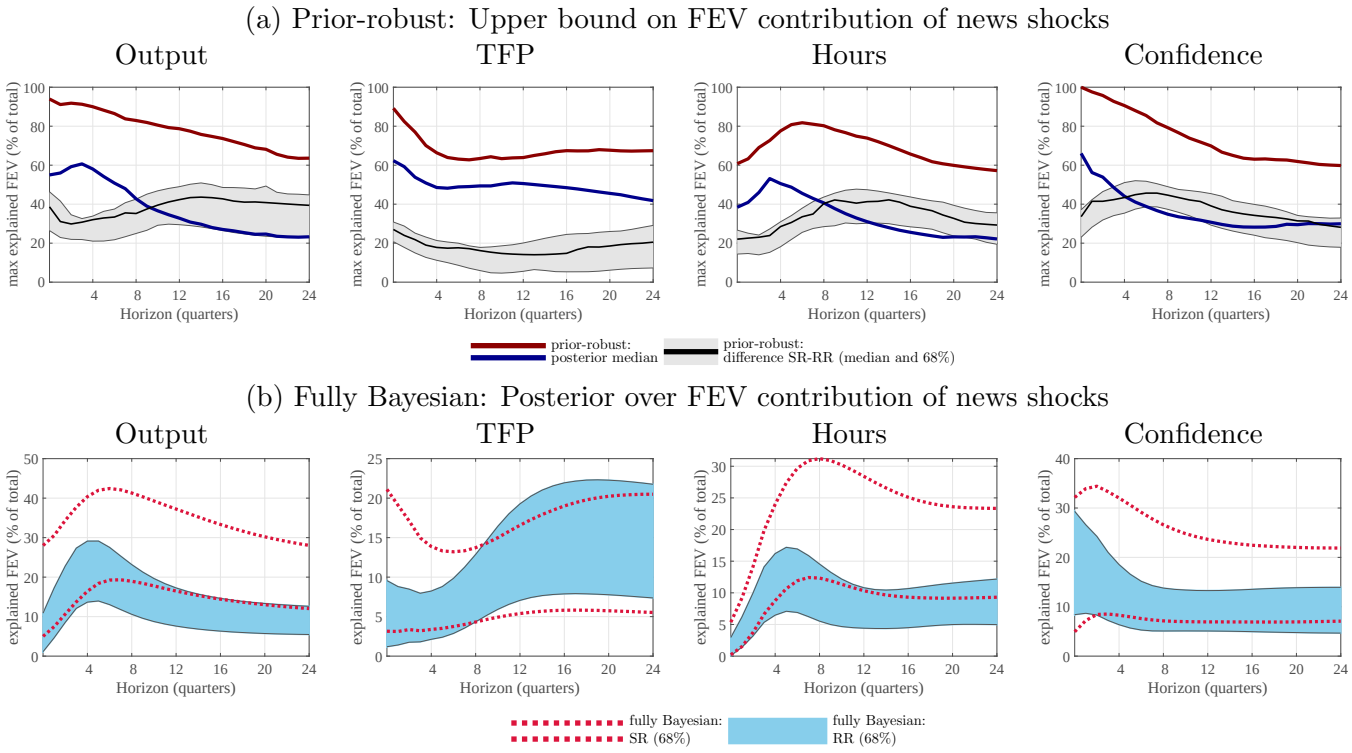
The table displays the posterior plausibilities of three different models, each for the full sample and the post 1983 sample. The results based on 2,500 draws. With added slope restrictions and in the late sub-sample, the data appear inconsistent with the inverted HR.

Table 4.2: Posterior probability of non-empty identified sets with reversed rankings and added slope restrictions.

Variable	Shock 1 "TFP news"	Shock 2 "TFP surprise"	Shock 3 "Monetary policy"
		Sign restrictions	
Output	+ (0, . . . , 4)	+ (0, . . . , 4)	– (0, . . . , 4)
TFP	+ (0, . . . , 4)	+ (0, . . . , 4)	n/a
SP500	+ (0, . . . , 4)	n/a	n/a
Nominal rate	n/a	n/a	+ (0, . . . , 2)
CPI level	n/a	– (0, . . . , 4)	– (0, . . . , 2)
		Ranking restrictions	
TFP slope	+ (0 vs 1 and 1 vs 2)	– (0 vs 1 and 1 vs 2)	n/a
Nominal rate slope	n/a	n/a	– (0 vs 1 and 1 vs 2)

Restriction horizon in parenthesis.

Table 4.3: Data and identifying restrictions in the 5-variable VAR



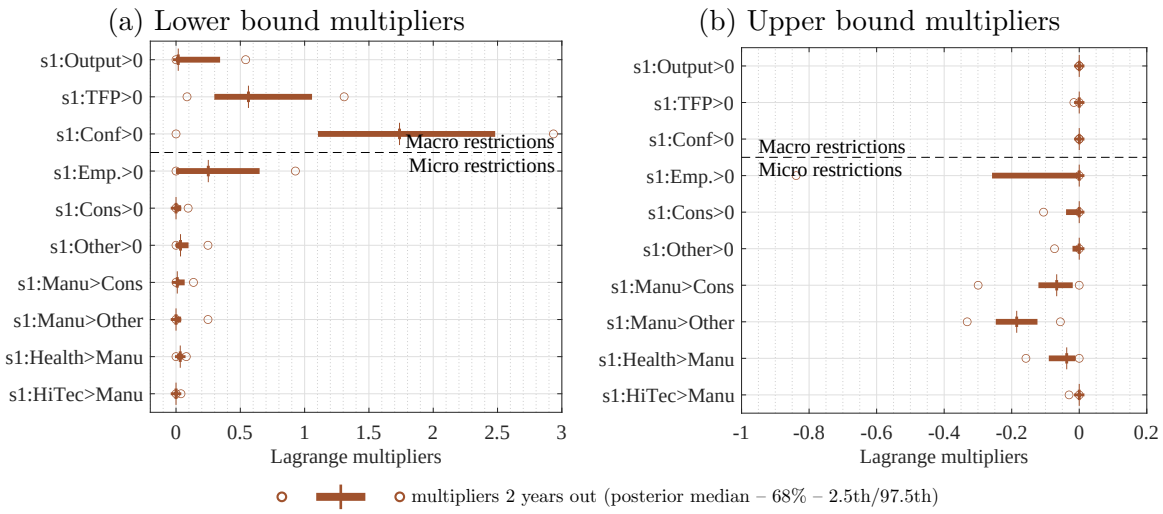
FEV contributions are relative to the total FEV. Heterogeneity restrictions reduce the maximum role of news shocks (upper panel): With sign restrictions alone, news could explain all of the initial output and confidence FEV. Heterogeneity restrictions shrink the maximum FEV by 20–50pp for output with 68% posterior probability. The reduction in the bound for TFP is up to 30pp. The bounds remain wide in the short-run. With conditionally uniform beliefs (lower panel), the importance of news peaks at below 30% for output after one year with 68% probability, compared to the prior-robust result that with 50% posterior probability the response could be as high as 60%.

Figure 4.5: Forecast error variance contribution of productivity news shock: Macro variables.

	SR	SR + RR	SR	SR + RR	SR + RR vs. only SR
Prior	Posterior probability $\mathbb{E}_T^{\boldsymbol{\theta}}[\mathbb{1}\{\circ\}]$		Prior probability $\mathbb{E}_0^{\boldsymbol{\theta}}[\mathbb{1}\{\circ\}]$		$2\ln$ Bayes factor
Flat	100.0%	90.4%			
Weak ($\phi = \frac{1}{4}$)	100.0%	91.2%	80.2%	41.0%	1.16
Intermediate ($\phi = \frac{1}{2}$)	100.0%	94.4%	92.6%	73.0%	0.36
Stronger ($\phi = 1$)	100.0%	95.6%	98.6%	87.2%	0.16

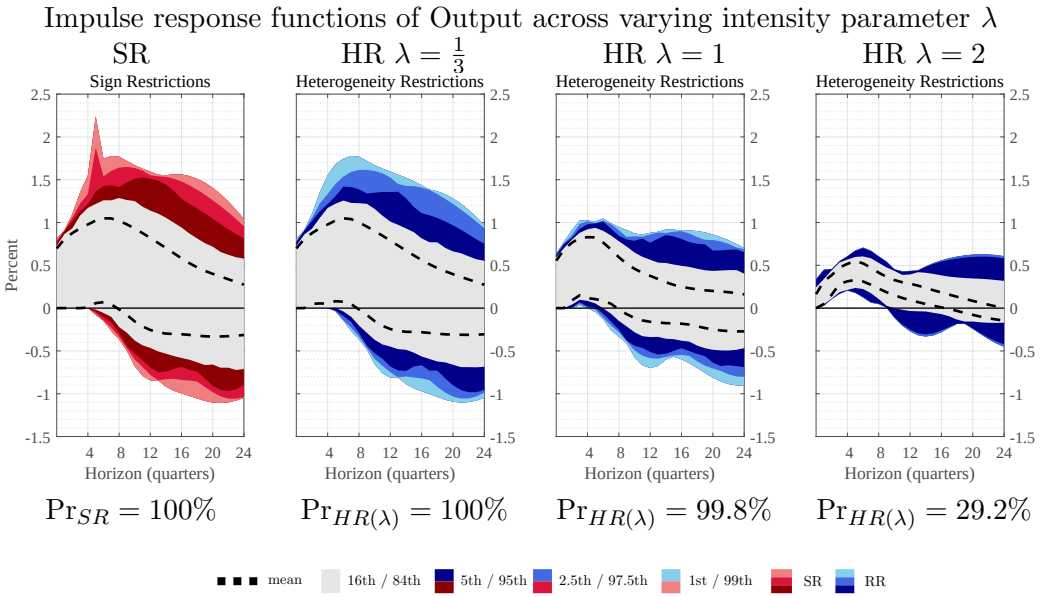
Estimated standard errors for prior and posterior probabilities are below 0.1% and below 0.03 for Bayes factors. For various priors over the reduced form parameters and a weak, proper prior over the covariance matrix, the data are consistent with the ranking restrictions.

Table 4.4: Posterior model probabilities and model comparison



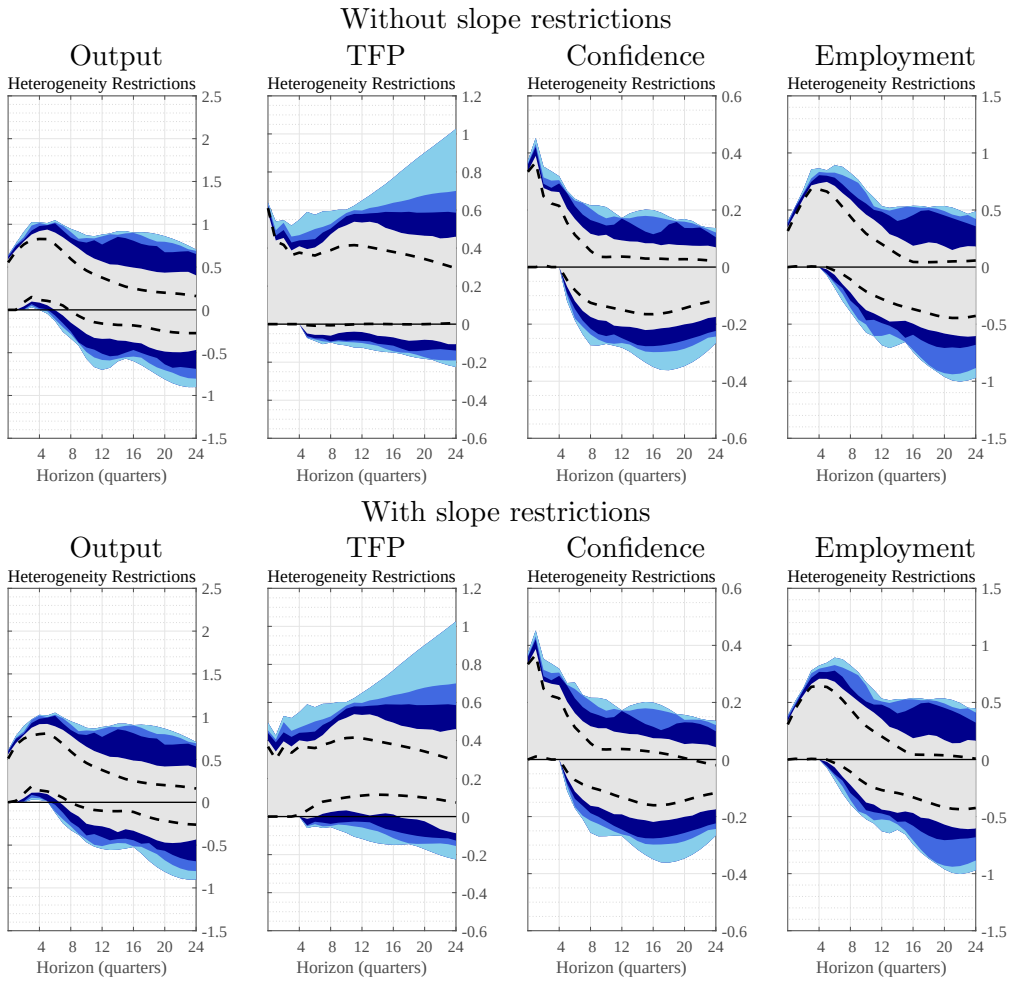
This figure quantifies the importance of all sign and heterogeneity restrictions for the lower and upper bound of the output response at the two-year horizon. It shows the distribution of Lagrange multipliers on all restrictions across reduced-form draws. The multipliers are summed across the restriction horizons $0, \ldots, \bar{H}$. Multipliers on upper bounds are negative as tighter restrictions lower the bound. For pinning down the upper bound, the heterogeneity restrictions on stock return industry data matter more than the macro sign restrictions. Restrictions on manufacturing are particularly important. In contrast, for the lower bound, sign restrictions on macro variables dominate.

Figure 4.6: Lagrange multipliers on restrictions for output responses to news shocks after two years.



Intensities of λ above the benchmark value of $\lambda = 1$ yield to an increasingly smaller posterior plausibility $\Pr_{HR(\lambda)}$, but also narrower prior-robust credible sets. $\lambda = 1$ yields narrower credible sets than $\lambda = \frac{1}{3}$ with essentially the same posterior plausibility.

Figure 4.7: Prior-robust responses of output to a one standard deviation productivity news shock across varying values for intensity parameter $\lambda = \{0, \frac{1}{3}, 1, 2\}$



The model contrasts the prior-robust 68% and 98% credible sets for the benchmark model with HR (shown in solid and dashed lines) with an extension that also includes slope restrictions on TFP (shown as shaded areas). Slope restrictions narrow the credible sets both for TFP and the other macro variables.

Figure 4.8: The effect of slope restrictions on TFP on the prior-robust credible sets of macro variables.

(a) prior robust upper bounds

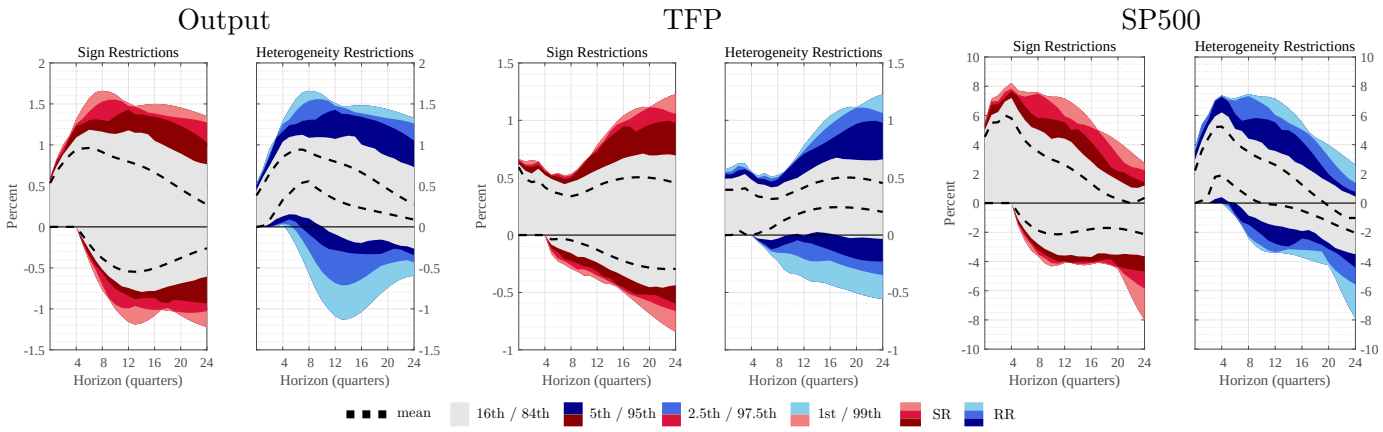
	SR	RR	Bound reduction	
On impact	100.0	32.1	67.5	(36.4,87.4)
4 qtrs out	92.2	58.6	30.3	(12.7,59.4)
8 qtrs out	88.0	74.0	14.2	(2.7,36.6)
24 qtrs out	76.9	70.0	3.6	(0.1,18.6)

(b) fully Bayesian 68% sets

	SR	RR
On impact	(2.0,52.6)	(0.1,13.3)
4 qtrs out	(6.4,50.0)	(12.2,47.3)
8 qtrs out	(5.6,47.6)	(25.9,63.8)
24 qtrs out	(6.4,36.2)	(24.7,62.8)

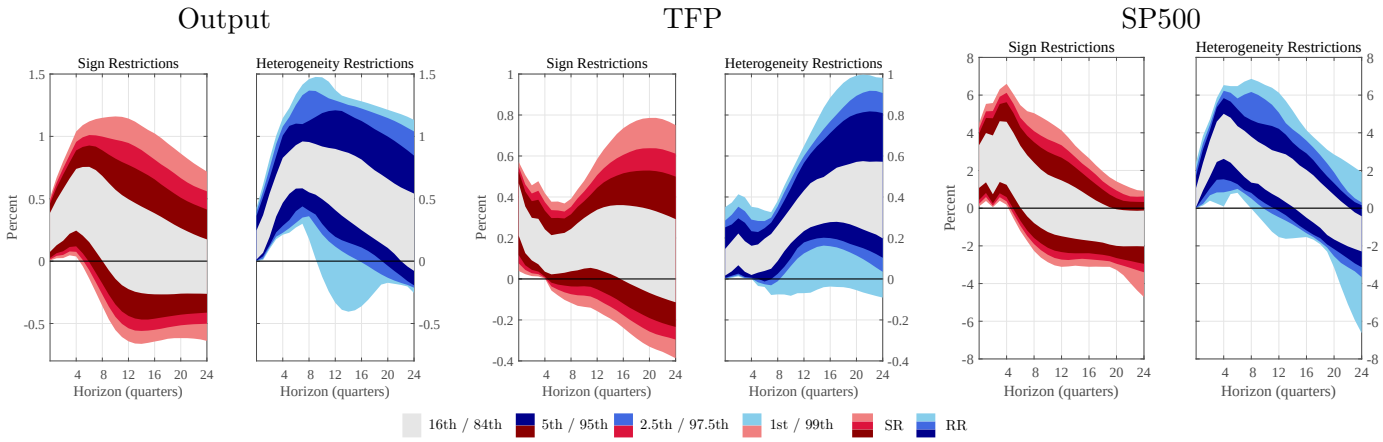
Posterior medians (and 68% credible sets) of forecast error variance decompositions (FEVD), relative to total FEVD at given horizon. The set reduction is computed for draw by draw, for draws accepted under ranking restrictions.

Table 4.5: Forecast error variance decomposition for output: TFP news shock



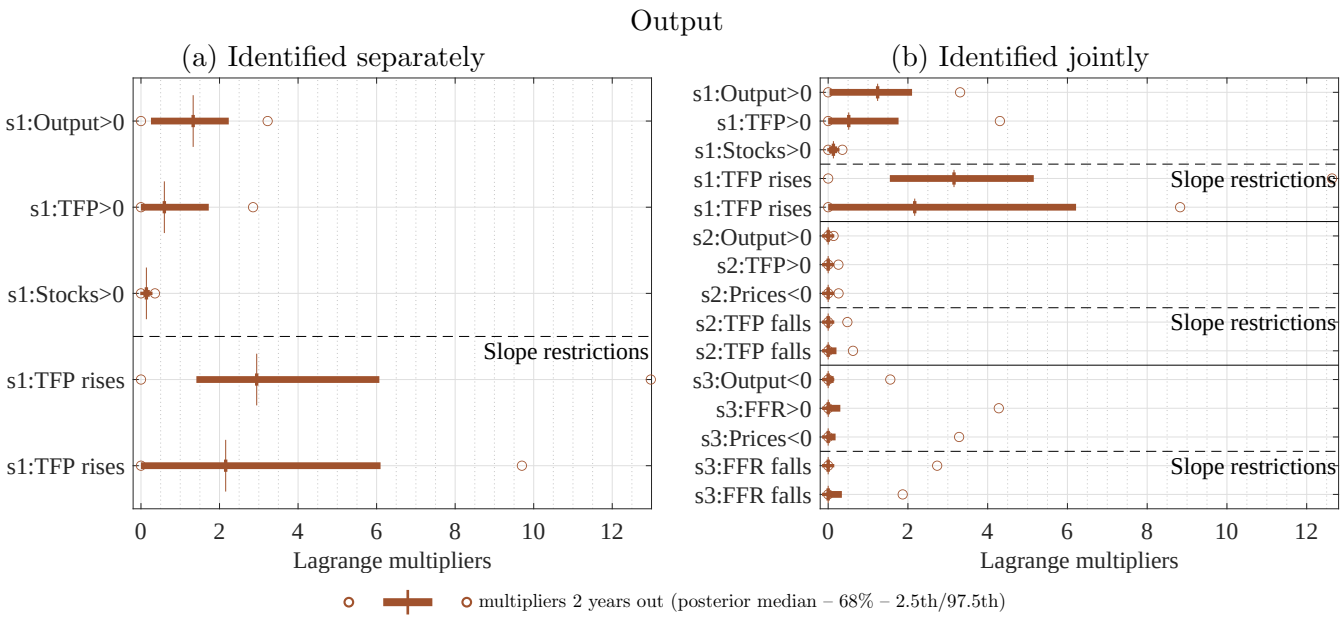
Slope restrictions shrink the size of the identified set at the posterior mean significantly. For the three real variables, this is robust to parameter uncertainty, yielding narrower credible sets. The responses of output, TFP, and the SP500 are strictly positive with at least 84% posterior probability with slope restrictions even at unrestricted horizons.

Figure 4.9: Prior-robust posterior over responses to a one standard deviation TFP news shock.



Slope restrictions lead to longer-lasting and stronger increase in output, TFP, and the SP500 than pure sign restrictions.

Figure 4.10: Fully Bayesian posterior over responses to a one standard deviation TFP news shock.



"s1" refers to TFP news shocks, "s2" to TFP surprises, and "s3" to monetary policy shock restrictions. The multipliers for standard sign restrictions are summed across restriction horizons. Slope restriction have the largest multipliers. Multipliers on the restrictions on other shocks are zero for almost all reduced form draws, so that the identified set is pinned down from the restrictions on the productivity news shock itself.

Figure 4.11: Lagrange Multipliers for the lower bound of the two-year output response, comparing joint and separate identification.