

**Supplemental Material 2:  
Additional Numerical Results**

**for**

**On Optimal Inference  
in the Linear IV Model**

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This file provides numerical results that supplement the results given in Tables I-IV in the main paper.

Table SM-I is analogous to Table I, but considers AR and POIS $_{\infty}$  CS's, in addition to the CLR and POIS2 $_{\infty}$  CS's. Here, POIS $_{\infty}$  denotes the CS obtained from the optimal one-sided invariant similar test as  $\beta_0 \rightarrow \pm\infty$  defined in (13.5) in Section 13.2 in Supplemental Material 1. Table SM-I reports probabilities of infinite length, as well as differences in probabilities of infinite length (DPIL's) for CLR and AR, AR and POIS2 $_{\infty}$ , and CLR and POIS2 $_{\infty}$  CS's. In addition, it reports simulation standard deviations for the first and third DPIL's.

The results for the DPIL's vary greatly with  $\rho_{uv}$ . When  $\rho_{uv} = 0$ , the AR CS is the same as the optimal POIS $_{\infty}$  CS and the CLR-AR DPIL's range over  $[\text{.001}, \text{.049}]$  as  $(k, \lambda)$  vary. On the other hand, when  $\rho_{uv} = .9$ , the AR CS is far from optimal and the CLR-AR DPIL's range over  $[-\text{.002}, -\text{.421}]$  as  $(k, \lambda)$  vary. In sum, when  $\rho_{uv} \geq .5$ , the AR CS can, and typically does, perform noticeably worse than the CLR CS in terms of DPIL's.

Table SM-II reports more detailed results than those given in Table II. Table SM-II reports the maximum power differences over  $\beta_0$  values between the POIS2 power envelope and the CLR test for a grid of  $(k, \rho_{uv}, \lambda)$  values. (In contrast, Table II reports maximum and average power differences over  $(\beta_0, \lambda)$  values for a grid of  $(k, \rho_{uv})$  values.) Table SM-II shows that the maximum (over  $\beta_0$ ) power differences vary substantially over  $\lambda$  values for  $\rho_{uv} \leq .7$  values and less so for  $\rho_{uv} = .9$ . For example, for  $k = 5$  and  $\rho_{uv} = .0, .3, .5, .7, .9$ , the power differences ranges (over  $\lambda$  values) are  $[\text{.004}, \text{.030}]$ ,  $[\text{.008}, \text{.034}]$ ,  $[\text{.007}, \text{.029}]$ ,  $[\text{.005}, \text{.033}]$ ,  $[\text{.001}, \text{.017}]$ , respectively.

Tables SM-III and SM-IV are the same as Table II except they consider the AR and LM tests, respectively, rather than the CLR test. As noted in the main paper, Tables SM-III and SM-IV show that the power of the AR and LM tests is much farther from the POIS2 power envelope than is the power of the CLR test. Table SM-III(a) shows that the maximum and average (over  $(\beta_0, \lambda)$ ) power differences for the AR test are increasing in  $k$  up to  $k = 20$ , but drop going from  $k = 20$  to 40. (This drop may be due to the choice of  $\lambda$  values considered. The choice yields the  $\lambda_{\max}$  value for the AR test to be on the upper bound of the values considered.) Table SM-III(b) shows that the maximum and average (over  $(\beta_0, \lambda)$ ) power differences for the AR test are increasing in  $\rho_{uv}$  for all values of  $k$ , which is the opposite of the pattern for the CLR test. Table SM-III(b) also shows that the  $\lambda_{\max}$  values are at the boundary of the grid of  $\lambda$  values considered for all  $k$ .

Table SM-IV(a) shows that the maximum and average power differences (over  $(\beta_0, \lambda)$ ) for the LM test are clearly increasing in  $k$ , except that for  $\rho_{uv} = 0, .3$  there is a drop from  $k = 20$  to 40 (which may be due to the choice of  $\lambda$  values considered, as for the AR test). Table SM-IV(b) shows that the maximum and average (over  $(\beta_0, \lambda)$ ) power differences for the LM test are decreasing in

$\rho_{uv}$  for all values of  $k$ , as for the CLR test. Table SM-IV(b) also shows that the  $\lambda_{\max}$  values decrease in  $\rho_{uv}$  for each  $k$ , as with the CLR test.

Table SM-V is the same as Table III except that it reports results for  $k = 2, 5, 10, 20$ , and  $40$ , rather than just  $k = 5$ . It also reports results for a finer grid of  $\beta_0$  values than in Table IV and it reports the power of the WAP2 test, in addition to the difference in power between the WAP2 and CLR tests.

AMS reports results for only two values of the correlation  $\rho_\Omega$  between the reduced-form errors, viz.,  $\rho_\Omega = .5$  and  $.95$ . However, this is not the reason that AMS did not detect scenarios where the CLR test's power is noticeably off the two-sided power envelope. Figure SM-I provides graphs that are the same as in AMS, but with  $\rho_\Omega = 0$ , rather than  $\rho_\Omega = .5$  or  $.95$ . Specifically, these graphs provide the power of the significance level  $.05$  CLR, LM, and AR tests and the POIS2 power envelope for fixed null value  $\beta_0 = 0$ , varying true value  $\beta^*$ ,  $k = 2, 5, 10$  and  $\lambda = 5, 20$ . The number of simulation repetitions used to construct the power functions is  $5,000$  and  $100,000$  repetitions are used to compute the null distribution of the POIS2 statistic to obtain its p-values.

Figure SM-I shows that the power of the CLR test is very close to the POIS2 power envelope in the scenarios considered. In fact, the maximum differences are  $.0074, .0040, .0110, .0062, .0102$ , and  $.0090$  in the six graphs in Figure SM-I. Note that  $\rho_\Omega = 0$  is the  $\rho_\Omega$  value that yields many of the largest differences between the power of the CLR test and the POIS2 power envelope when the true  $\beta^* = 0$  is fixed and the null value  $\beta_0$  varies, as shown in Tables II and SM-II.<sup>1</sup> The results in Figure SM-I show that standard power graphs with  $\beta_0 = 0$  fixed and true  $\beta^*$  varying, as in AMS, do not pick up the relatively large differences between the power of the CLR test and the POIS2 power envelope that appear in some  $\rho_\Omega = 0$  parameter configurations considered in Tables II and SM-II.

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<sup>1</sup>In Tables II and SM-II,  $\rho_{uv} = \rho_\Omega$  for all  $\beta_0$  values because the true value  $\beta^* = 0$ .

TABLE SM-I(a). Probabilities of Infinite-Length Confidence Intervals for  $\rho_{uv} = 0$ 

$k$	$\lambda$	AR	CLR	$POIS_{\infty}$	$POIS2_{\infty}$	CLR-AR	$SD$	AR- $POIS2_{\infty}$	CLR- $POIS2_{\infty}$	$SD$
2	1	.867	.868	.867	.867	.002	.0007	.000	.002	.0007
2	3	.680	.687	.680	.680	.007	.0009	.000	.007	.0009
2	5	.497	.508	.497	.497	.011	.0010	.000	.011	.0010
2	7	.345	.358	.345	.345	.013	.0010	.000	.013	.0010
2	10	.182	.195	.182	.182	.012	.0008	.000	.012	.0008
2	15	.056	.063	.056	.056	.007	.0006	.000	.007	.0006
2	20	.015	.017	.015	.015	.003	.0003	.000	.003	.0003
5	1	.902	.905	.902	.902	.003	.0008	.000	.003	.0008
5	3	.779	.789	.779	.779	.010	.0011	.000	.010	.0011
5	5	.639	.659	.639	.639	.020	.0012	.000	.020	.0012
5	7	.502	.529	.502	.502	.026	.0012	.000	.026	.0012
5	10	.323	.350	.323	.323	.027	.0012	.000	.027	.0012
5	12	.230	.257	.230	.230	.027	.0011	.000	.027	.0011
5	15	.132	.156	.132	.132	.023	.0009	.000	.023	.0009
5	20	.047	.059	.047	.047	.012	.0007	.000	.012	.0007
5	25	.015	.021	.015	.015	.006	.0004	.000	.006	.0004
10	1	.918	.920	.918	.918	.002	.0009	.000	.002	.0009
10	5	.733	.751	.733	.733	.018	.0012	.000	.018	.0012
10	10	.461	.496	.461	.461	.035	.0013	.000	.035	.0013
10	15	.242	.279	.242	.242	.037	.0012	.000	.037	.0012
10	17	.177	.212	.177	.177	.034	.0011	.000	.034	.0011
10	20	.109	.135	.109	.109	.026	.0010	.000	.026	.0010
10	25	.043	.060	.043	.043	.016	.0007	.000	.016	.0007
10	30	.016	.024	.016	.016	.008	.0005	.000	.008	.0005
20	1	.929	.932	.929	.929	.003	.0008	.000	.003	.0008
20	5	.806	.824	.806	.806	.017	.0012	.000	.017	.0012
20	10	.597	.632	.597	.597	.035	.0014	.000	.035	.0014
20	15	.393	.436	.393	.393	.043	.0014	.000	.043	.0014
20	20	.226	.268	.226	.226	.042	.0012	.000	.042	.0012
20	25	.116	.149	.116	.116	.033	.0010	.000	.033	.0010
20	30	.053	.077	.053	.053	.023	.0008	.000	.023	.0008
20	40	.010	.017	.010	.010	.007	.0004	.000	.007	.0004
40	1	.936	.938	.936	.936	.001	.0008	.000	.001	.0008
40	5	.861	.872	.861	.861	.011	.0011	.000	.011	.0011
40	10	.721	.750	.721	.721	.030	.0013	.000	.030	.0013
40	15	.553	.599	.553	.553	.046	.0014	.000	.046	.0014
40	20	.394	.443	.394	.394	.049	.0014	.000	.049	.0014
40	30	.155	.198	.155	.155	.043	.0012	.000	.043	.0012
40	40	.046	.068	.046	.046	.022	.0008	.000	.022	.0008
40	60	.002	.005	.002	.002	.003	.0003	.000	.003	.0003

TABLE SM-I(b). Probabilities of Infinite-Length Confidence Intervals for  $\rho_{uv} = .3$ 

$k$	$\lambda$	AR	CLR	$POIS_{\infty}$	$POIS2_{\infty}$	CLR-AR	$SD$	AR- $POIS2_{\infty}$	CLR- $POIS2_{\infty}$	$SD$
2	1	.867	.868	.859	.864	.001	.0007	.002	.003	.0007
2	3	.680	.684	.654	.676	.003	.0010	.004	.008	.0008
2	5	.497	.501	.461	.491	.004	.0011	.005	.010	.0009
2	7	.345	.347	.304	.338	.002	.0010	.008	.009	.0008
2	10	.182	.184	.150	.177	.001	.0009	.006	.007	.0007
2	15	.056	.056	.039	.052	.000	.0006	.003	.004	.0005
2	20	.015	.015	.010	.013	.001	.0004	.002	.002	.0003
5	1	.902	.905	.899	.903	.003	.0008	-.001	.002	.0008
5	3	.779	.785	.762	.779	.006	.0011	.000	.007	.0009
5	5	.639	.647	.607	.637	.008	.0013	.002	.010	.0010
5	7	.502	.510	.460	.498	.008	.0013	.005	.013	.0011
5	10	.323	.330	.279	.316	.007	.0013	.007	.014	.0010
5	12	.230	.235	.190	.222	.005	.0012	.008	.013	.0009
5	15	.132	.137	.101	.126	.004	.0010	.007	.011	.0007
5	20	.047	.048	.031	.042	.000	.0007	.005	.005	.0005
5	25	.015	.016	.009	.013	.001	.0004	.002	.003	.0003
10	1	.918	.920	.915	.918	.002	.0009	-.001	.002	.0008
10	5	.733	.744	.709	.733	.010	.0013	.001	.011	.0011
10	10	.461	.472	.414	.454	.011	.0014	.007	.018	.0011
10	15	.242	.248	.195	.231	.006	.0013	.011	.017	.0010
10	17	.177	.184	.139	.168	.007	.0012	.009	.016	.0009
10	20	.109	.114	.079	.100	.005	.0010	.009	.015	.0008
10	25	.043	.046	.028	.039	.003	.0007	.005	.008	.0005
10	30	.016	.017	.009	.013	.001	.0005	.003	.004	.0004
20	1	.929	.931	.928	.929	.002	.0008	-.001	.002	.0008
20	5	.806	.819	.789	.807	.012	.0012	-.000	.012	.0011
20	10	.597	.615	.558	.594	.018	.0015	.004	.021	.0012
20	15	.393	.406	.339	.383	.013	.0015	.010	.023	.0011
20	20	.226	.236	.180	.215	.009	.0013	.012	.021	.0010
20	25	.116	.122	.082	.106	.006	.0011	.010	.016	.0008
20	30	.053	.057	.035	.046	.004	.0008	.007	.011	.0006
20	40	.010	.011	.005	.008	.001	.0004	.002	.003	.0003
40	1	.936	.937	.934	.937	.001	.0008	-.000	.000	.0008
40	5	.861	.869	.849	.861	.008	.0011	-.000	.008	.0010
40	10	.721	.737	.690	.720	.016	.0014	.001	.016	.0012
40	15	.553	.572	.505	.548	.018	.0015	.006	.024	.0012
40	20	.393	.409	.336	.381	.015	.0015	.012	.028	.0012
40	30	.155	.164	.114	.142	.009	.0013	.013	.022	.0009
40	40	.046	.048	.027	.038	.002	.0008	.008	.010	.0006
40	60	.002	.002	.001	.001	.000	.0002	.001	.001	.0002

TABLE SM-I(c). Probabilities of Infinite-Length Confidence Intervals for  $\rho_{uv} = .5$ 

$k$	$\lambda$	AR	CLR	$POIS_{\infty}$	$POIS2_{\infty}$	CLR-AR	$SD$	AR- $POIS2_{\infty}$	CLR- $POIS2_{\infty}$	$SD$
2	1	.867	.867	.846	.864	.000	.0007	.003	.003	.0006
2	3	.680	.675	.615	.672	-.006	.0011	.009	.003	.0006
2	5	.497	.486	.405	.480	-.011	.0012	.017	.005	.0006
2	7	.345	.327	.250	.323	-.019	.0012	.022	.004	.0005
2	10	.182	.166	.111	.162	-.016	.0010	.020	.004	.0005
2	15	.056	.046	.025	.043	-.010	.0007	.012	.002	.0003
2	20	.015	.012	.005	.011	-.003	.0004	.004	.001	.0002
5	1	.902	.904	.890	.903	.001	.0009	-.000	.001	.0007
5	3	.779	.776	.729	.774	-.003	.0012	.005	.003	.0007
5	5	.639	.624	.551	.622	-.015	.0014	.017	.003	.0007
5	7	.502	.476	.392	.471	-.026	.0015	.031	.005	.0007
5	10	.323	.289	.213	.283	-.034	.0015	.040	.006	.0007
5	12	.230	.196	.133	.190	-.035	.0014	.041	.006	.0007
5	15	.132	.104	.063	.099	-.028	.0012	.033	.005	.0006
5	20	.047	.032	.016	.029	-.015	.0008	.018	.003	.0004
5	25	.015	.009	.004	.007	-.006	.0005	.008	.001	.0002
10	1	.918	.919	.908	.918	.002	.0009	-.000	.001	.0007
10	5	.733	.725	.662	.721	-.008	.0014	.013	.005	.0008
10	10	.461	.423	.333	.415	-.038	.0016	.046	.008	.0008
10	15	.242	.194	.130	.186	-.048	.0015	.056	.008	.0007
10	17	.177	.136	.085	.129	-.041	.0014	.048	.007	.0007
10	20	.109	.077	.043	.071	-.032	.0012	.038	.006	.0006
10	25	.043	.026	.012	.023	-.017	.0008	.020	.003	.0004
10	30	.016	.008	.003	.006	-.008	.0005	.009	.002	.0003
20	1	.929	.931	.922	.929	.002	.0009	-.001	.001	.0008
20	5	.806	.804	.756	.800	-.002	.0013	.006	.004	.0008
20	10	.597	.571	.482	.563	-.026	.0017	.034	.008	.0009
20	15	.393	.340	.251	.330	-.053	.0017	.063	.010	.0009
20	20	.226	.172	.111	.163	-.054	.0015	.063	.009	.0008
20	25	.116	.076	.042	.069	-.041	.0013	.047	.007	.0006
20	30	.053	.030	.014	.026	-.023	.0009	.027	.004	.0004
20	40	.010	.004	.002	.003	-.006	.0004	.007	.001	.0002
40	1	.936	.937	.932	.937	.000	.0009	-.000	.000	.0008
40	5	.861	.862	.825	.858	.001	.0012	.003	.005	.0008
40	10	.721	.706	.627	.700	-.015	.0016	.021	.006	.0009
40	15	.553	.513	.416	.502	-.041	.0018	.051	.011	.0009
40	20	.393	.335	.245	.322	-.059	.0018	.072	.013	.0009
40	30	.155	.103	.058	.093	-.052	.0014	.061	.010	.0007
40	40	.046	.022	.010	.018	-.023	.0009	.028	.004	.0004
40	60	.002	.001	.000	.000	-.002	.0002	.002	.000	.0001

TABLE SM-I(d). Probabilities of Infinite-Length Confidence Intervals for  $\rho_{uv} = .7$ 

$k$	$\lambda$	AR	CLR	$POIS_{\infty}$	$POIS2_{\infty}$	CLR-AR	$SD$	AR- $POIS2_{\infty}$	CLR- $POIS2_{\infty}$	$SD$
2	1	.867	.864	.823	.862	-.003	.0008	.004	.001	.0003
2	3	.680	.658	.558	.654	-.022	.0012	.026	.004	.0005
2	5	.497	.456	.343	.452	-.040	.0014	.044	.004	.0006
2	7	.345	.295	.199	.291	-.050	.0014	.054	.004	.0005
2	10	.182	.140	.082	.138	-.042	.0012	.044	.003	.0004
2	15	.055	.035	.017	.034	-.020	.0008	.021	.001	.0002
2	20	.015	.009	.004	.008	-.006	.0004	.006	.000	.0001
5	1	.902	.901	.872	.900	-.002	.0009	.002	.001	.0004
5	3	.779	.753	.667	.752	-.026	.0014	.027	.001	.0006
5	5	.639	.575	.461	.571	-.064	.0017	.069	.004	.0007
5	7	.502	.410	.297	.404	-.092	.0018	.098	.006	.0007
5	10	.323	.219	.139	.214	-.104	.0017	.109	.005	.0006
5	12	.230	.137	.081	.133	-.093	.0016	.097	.004	.0006
5	15	.132	.064	.033	.061	-.068	.0013	.071	.003	.0004
5	20	.047	.016	.007	.014	-.031	.0009	.033	.001	.0003
5	25	.015	.004	.002	.003	-.011	.0005	.012	.000	.0001
10	1	.918	.918	.895	.917	.000	.0009	.000	.001	.0005
10	5	.733	.676	.575	.673	-.057	.0016	.060	.003	.0008
10	10	.461	.322	.223	.317	-.139	.0019	.144	.005	.0008
10	15	.242	.115	.065	.110	-.127	.0017	.132	.005	.0006
10	17	.177	.073	.038	.069	-.104	.0015	.109	.004	.0005
10	20	.109	.034	.016	.033	-.075	.0013	.076	.002	.0004
10	25	.043	.009	.003	.008	-.034	.0009	.036	.001	.0002
10	30	.016	.002	.001	.002	-.014	.0005	.014	.000	.0001
20	1	.929	.930	.914	.930	.001	.0009	-.001	.000	.0006
20	5	.806	.771	.682	.768	-.036	.0015	.038	.003	.0008
20	10	.597	.470	.350	.462	-.127	.0020	.135	.008	.0009
20	15	.393	.220	.136	.211	-.174	.0020	.182	.009	.0008
20	20	.226	.083	.044	.079	-.143	.0018	.148	.005	.0006
20	25	.116	.027	.012	.024	-.089	.0014	.092	.003	.0004
20	30	.053	.008	.003	.007	-.045	.0010	.047	.002	.0002
20	40	.010	.001	.000	.001	-.009	.0004	.009	.000	.0001
40	1	.936	.936	.925	.936	-.001	.0009	.001	-.000	.0007
40	5	.861	.841	.772	.837	-.020	.0014	.024	.003	.0007
40	10	.721	.624	.505	.615	-.096	.0019	.106	.010	.0010
40	15	.553	.382	.269	.371	-.172	.0021	.182	.011	.0009
40	20	.393	.197	.118	.186	-.197	.0021	.207	.010	.0008
40	30	.155	.033	.015	.029	-.122	.0016	.125	.004	.0004
40	40	.046	.004	.001	.003	-.042	.0009	.043	.001	.0002
40	60	.002	.000	.000	.000	-.002	.0002	.002	.000	.0000

TABLE SM-I(e). Probabilities of Infinite-Length Confidence Intervals for  $\rho_{uv} = .9$ 

$k$	$\lambda$	AR	CLR	$POIS_{\infty}$	$POIS2_{\infty}$	CLR-AR	$SD$	AR- $POIS2_{\infty}$	CLR- $POIS2_{\infty}$	$SD$
2	1	.867	.854	.778	.851	-.013	.0010	.015	.002	.0006
2	3	.680	.617	.491	.614	-.063	.0015	.067	.004	.0005
2	5	.497	.410	.293	.407	-.087	.0016	.089	.002	.0003
2	7	.345	.258	.167	.256	-.087	.0015	.090	.003	.0003
2	10	.182	.119	.067	.117	-.063	.0013	.065	.002	.0002
2	15	.055	.029	.014	.029	-.026	.0008	.027	.001	.0001
2	20	.015	.006	.003	.006	-.008	.0004	.008	.000	.0001
5	1	.902	.887	.824	.884	-.016	.0011	.019	.003	.0007
5	3	.779	.675	.553	.670	-.104	.0018	.109	.005	.0007
5	5	.639	.462	.340	.459	-.177	.0021	.180	.004	.0005
5	7	.502	.297	.197	.295	-.205	.0021	.207	.002	.0004
5	10	.323	.140	.083	.139	-.182	.0019	.183	.001	.0003
5	12	.230	.083	.044	.082	-.148	.0017	.149	.001	.0003
5	15	.132	.036	.017	.035	-.097	.0014	.097	.000	.0002
5	20	.047	.008	.003	.008	-.039	.0009	.039	.000	.0001
5	25	.015	.002	.001	.002	-.013	.0005	.013	.000	.0001
10	1	.918	.907	.855	.904	-.011	.0011	.014	.003	.0007
10	5	.733	.533	.404	.526	-.201	.0022	.207	.007	.0006
10	10	.461	.176	.106	.173	-.285	.0022	.288	.002	.0004
10	15	.242	.047	.023	.046	-.196	.0018	.196	.001	.0003
10	17	.177	.026	.012	.026	-.151	.0016	.151	.000	.0002
10	20	.109	.011	.004	.011	-.098	.0014	.098	.000	.0002
10	25	.043	.002	.001	.002	-.041	.0009	.041	.000	.0001
10	30	.016	.000	.000	.000	-.015	.0006	.015	.000	.0000
20	1	.929	.923	.885	.921	-.006	.0010	.008	.002	.0007
20	5	.806	.625	.498	.617	-.181	.0021	.189	.008	.0007
20	10	.597	.243	.155	.240	-.354	.0024	.357	.003	.0005
20	15	.393	.072	.038	.070	-.321	.0022	.323	.002	.0003
20	20	.226	.019	.008	.018	-.207	.0018	.209	.001	.0002
20	25	.116	.004	.002	.004	-.112	.0014	.112	.000	.0001
20	30	.053	.001	.000	.001	-.052	.0010	.052	.000	.0001
20	40	.010	.000	.000	.000	-.010	.0004	.010	.000	.0000
40	1	.936	.932	.904	.932	-.005	.0010	.004	-.001	.0006
40	5	.861	.727	.607	.717	-.134	.0020	.144	.010	.0009
40	10	.721	.358	.249	.354	-.362	.0024	.366	.004	.0006
40	15	.553	.130	.074	.128	-.423	.0023	.426	.002	.0005
40	20	.393	.039	.019	.038	-.354	.0022	.356	.001	.0003
40	30	.155	.003	.001	.002	-.152	.0016	.152	.000	.0001
40	40	.046	.000	.000	.000	-.046	.0009	.046	.000	.0000
40	60	.002	.000	.000	.000	-.002	.0002	.002	.000	.0000



TABLE SM-II(a). Maximum Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and CLR Tests for Fixed Alternative  $\beta^* = 0$  for  $\rho_{uv} = 0.00$

$k$	$\lambda$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-CLR
2	1	-5.00	.98	.133	.009
2	3	100.00	-1.00	.314	.012
2	5	-10.00	1.00	.499	.019
<b>2</b>	<b>7</b>	<b>-10000.00</b>	<b>1.00</b>	<b>.663</b>	<b>.021</b>
2	10	7.50	-.99	.814	.018
2	15	10.00	-1.00	.950	.009
2	20	-1.25	.78	.923	.004
5	1	-10.00	1.00	.095	.004
5	3	-5.00	.98	.209	.012
5	5	-1000.00	1.00	.363	.017
5	7	-10.00	1.00	.501	.025
<b>5</b>	<b>10</b>	<b>-50.00</b>	<b>1.00</b>	<b>.680</b>	<b>.030</b>
5	15	-10000.00	1.00	.870	.025
5	20	-50.00	1.00	.953	.015
10	1	2.00	-.89	.078	.003
10	3	3.75	-.97	.161	.012
10	5	10.00	-1.00	.268	.021
10	7	100.00	-1.00	.379	.028
10	10	-10000.00	1.00	.540	.030
<b>10</b>	<b>15</b>	<b>-50.00</b>	<b>1.00</b>	<b>.760</b>	<b>.038</b>
10	20	50.00	-1.00	.888	.025
20	1	-2.75	.94	.064	.006
20	3	-100.00	1.00	.116	.007
20	5	100.00	-1.00	.180	.016
20	7	5.00	-.98	.252	.028
20	10	-100.00	1.00	.389	.040
<b>20</b>	<b>15</b>	<b>10.00</b>	<b>-1.00</b>	<b>.596</b>	<b>.042</b>
20	20	100.00	-1.00	.770	.040
20	22	100.00	-1.00	.820	.038
20	25	-10000.00	1.00	.878	.035
40	1	-0.25	.24	.054	.011
40	3	2.50	-.93	.095	.011
40	5	-7.50	.99	.149	.020
40	7	-50.00	1.00	.201	.024
40	10	-100.00	1.00	.287	.035
40	15	-7.50	.99	.441	.041
40	20	50.00	-1.00	.608	.058
<b>40</b>	<b>22</b>	<b>-50.00</b>	<b>1.00</b>	<b>.664</b>	<b>.059</b>
40	25	1000.00	-1.00	.742	.056

TABLE SM-II(b). Maximum Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and CLR Tests for Fixed Alternative  $\beta^* = 0$  for  $\rho_{uv} = 0.30$

$k$	$\lambda$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-CLR
2	1	10.00	-1.00	.140	.009
2	3	7.50	-.99	.335	.012
2	5	3.50	-.96	.543	.016
2	7	2.25	-.90	.692	.018
<b>2</b>	<b>10</b>	<b>3.75</b>	<b>-.96</b>	<b>.860</b>	<b>.019</b>
2	15	2.25	-.90	.966	.009
2	20	-1.75	.91	.920	.004
5	1	3.75	-.96	.102	.008
5	3	2.75	-.93	.239	.017
5	5	7.50	-.99	.387	.018
5	7	3.00	-.94	.546	.023
<b>5</b>	<b>10</b>	<b>3.50</b>	<b>-.96</b>	<b>.732</b>	<b>.034</b>
5	15	2.75	-.93	.901	.023
5	20	4.00	-.97	.971	.014
10	1	3.00	-.94	.090	.003
10	3	3.00	-.94	.181	.014
10	5	2.75	-.93	.296	.023
10	7	3.75	-.96	.417	.026
<b>10</b>	<b>10</b>	<b>3.00</b>	<b>-.94</b>	<b>.590</b>	<b>.032</b>
10	15	3.50	-.96	.806	.032
10	20	3.50	-.96	.921	.025
20	1	-50.00	1.00	.067	.006
20	3	3.50	-.96	.126	.009
20	5	4.00	-.97	.195	.019
20	7	3.00	-.94	.285	.030
20	10	3.25	-.95	.432	.038
<b>20</b>	<b>15</b>	<b>3.50</b>	<b>-.96</b>	<b>.655</b>	<b>.045</b>
20	20	2.75	-.93	.817	.038
20	22	3.00	-.94	.863	.034
20	25	3.75	-.96	.915	.031
40	1	-0.25	.50	.051	.008
40	3	1.50	-.78	.097	.008
40	5	2.75	-.93	.153	.014
40	7	5.00	-.98	.215	.020
40	10	4.00	-.97	.312	.036
40	15	3.00	-.94	.485	.042
40	20	3.75	-.96	.663	.059
<b>40</b>	<b>22</b>	<b>4.00</b>	<b>-.97</b>	<b>.724</b>	<b>.061</b>
40	25	3.75	-.96	.798	.052

TABLE SM-II(c). Maximum Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and CLR Tests for Fixed Alternative  $\beta^* = 0$  for  $\rho_{uv} = 0.50$

$k$	$\lambda$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-CLR
2	1	2.00	-.87	.162	.006
2	3	2.50	-.92	.399	.014
<b>2</b>	<b>5</b>	<b>2.00</b>	<b>-.87</b>	<b>.635</b>	<b>.016</b>
2	7	2.50	-.92	.782	.013
2	10	2.25	-.90	.922	.012
2	15	-10.00	1.00	.943	.004
2	20	-1.25	.90	.804	.003
5	1	1.75	-.82	.112	.007
5	3	2.00	-.87	.291	.019
5	5	2.50	-.92	.475	.022
5	7	2.50	-.92	.638	.028
<b>5</b>	<b>10</b>	<b>2.25</b>	<b>-.90</b>	<b>.821</b>	<b>.029</b>
5	15	1.75	-.82	.951	.014
5	20	1.00	-.50	.969	.007
10	1	2.00	-.87	.097	.003
10	3	1.75	-.82	.215	.014
10	5	2.00	-.87	.362	.028
10	7	1.75	-.82	.500	.028
<b>10</b>	<b>10</b>	<b>2.00</b>	<b>-.87</b>	<b>.697</b>	<b>.037</b>
10	15	2.00	-.87	.887	.023
10	20	2.25	-.90	.968	.018
20	1	5.00	-.98	.071	.008
20	3	2.00	-.87	.148	.011
20	5	3.00	-.94	.233	.024
20	7	2.00	-.87	.355	.034
<b>20</b>	<b>10</b>	<b>1.75</b>	<b>-.82</b>	<b>.533</b>	<b>.046</b>
20	15	2.00	-.87	.769	.040
20	20	2.00	-.87	.905	.031
20	22	2.25	-.90	.934	.025
20	25	2.25	-.90	.963	.014
40	1	-0.25	.65	.051	.007
40	3	1.50	-.76	.117	.008
40	5	2.00	-.87	.184	.014
40	7	3.25	-.95	.256	.026
40	10	2.00	-.87	.381	.035
<b>40</b>	<b>15</b>	<b>1.75</b>	<b>-.82</b>	<b>.594</b>	<b>.050</b>
40	20	1.75	-.82	.776	.049
40	22	2.00	-.87	.835	.049
40	25	2.00	-.87	.897	.040

TABLE SM-II(d). Maximum Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and CLR Tests for Fixed Alternative  $\beta^* = 0$  for  $\rho_{uv} = 0.70$

$k$	$\lambda$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-CLR
2	1	1.75	-.83	.214	.006
2	3	2.25	-.91	.528	.013
<b>2</b>	<b>5</b>	<b>1.50</b>	<b>-.75</b>	<b>.811</b>	<b>.016</b>
2	7	1.25	-.61	.927	.009
2	10	-2.75	.98	.672	.005
2	15	-4.00	.99	.896	.003
2	20	-1.00	.92	.671	.002
5	1	2.00	-.88	.142	.005
5	3	1.75	-.83	.413	.020
<b>5</b>	<b>5</b>	<b>1.50</b>	<b>-.75</b>	<b>.669</b>	<b>.033</b>
5	7	1.50	-.75	.834	.029
5	10	1.50	-.75	.949	.016
5	15	7.50	-.99	.971	.003
5	20	-2.50	.98	.897	.003
10	1	1.25	-.61	.117	.004
10	3	1.50	-.75	.320	.028
10	5	1.25	-.61	.521	.030
<b>10</b>	<b>7</b>	<b>1.50</b>	<b>-.75</b>	<b>.711</b>	<b>.036</b>
10	10	1.50	-.75	.878	.021
10	15	1.75	-.83	.979	.010
10	20	0.50	.27	.757	.007
20	1	2.00	-.88	.087	.007
20	3	1.50	-.75	.212	.025
20	5	1.50	-.75	.377	.038
<b>20</b>	<b>7</b>	<b>1.25</b>	<b>-.61</b>	<b>.544</b>	<b>.042</b>
20	10	1.50	-.75	.754	.036
20	15	1.50	-.75	.935	.024
20	20	-1.25	.94	.582	.007
20	22	-1.00	.92	.546	.007
20	25	-7.50	1.00	.952	.005
40	1	-100.00	1.00	.071	.006
40	3	2.25	-.91	.158	.015
40	5	1.75	-.83	.275	.030
40	7	1.25	-.61	.393	.038
40	10	1.50	-.75	.588	.049
<b>40</b>	<b>15</b>	<b>1.50</b>	<b>-.75</b>	<b>.837</b>	<b>.050</b>
40	20	1.50	-.75	.948	.026
40	22	1.50	-.75	.967	.017
40	25	-3.25	.98	.817	.009

TABLE SM-II(e). Maximum Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and CLR Tests for Fixed Alternative  $\beta^* = 0$  for  $\rho_{uv} = 0.90$

$k$	$\lambda$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-CLR
2	0.7	1.25	-.63	.359	.010
2	0.8	1.00	-.22	.412	.015
<b>2</b>	<b>0.9</b>	<b>1.25</b>	<b>-.63</b>	<b>.461</b>	<b>.017</b>
2	1	1.25	-.63	.505	.013
2	3	1.25	-.63	.947	.005
2	5	-7.50	1.00	.497	.002
2	7	-5.00	1.00	.600	.001
2	10	-3.50	1.00	.702	.001
2	15	-50.00	1.00	.967	.001
2	20	-3.75	1.00	.950	.001
5	0.7	1.00	-.22	.256	.013
5	0.8	1.00	-.22	.292	.017
<b>5</b>	<b>0.9</b>	<b>1.00</b>	<b>-.22</b>	<b>.331</b>	<b>.017</b>
5	1	1.00	-.22	.365	.015
5	3	1.00	-.22	.870	.015
5	5	1.00	-.22	.985	.004
5	7	0.75	.33	.975	.004
5	10	10.00	-1.00	.916	.002
5	15	-10.00	1.00	.934	.001
5	20	-1.75	.99	.815	.001
10	1	1.00	-.22	.273	.017
<b>10</b>	<b>3</b>	<b>1.25</b>	<b>-.63</b>	<b>.766</b>	<b>.027</b>
10	5	1.25	-.63	.956	.014
10	7	2.00	-.93	.964	.005
10	10	-50.00	1.00	.812	.004
10	15	-1.50	.98	.618	.003
10	20	-2.75	.99	.895	.003
20	1	1.00	-.22	.183	.015
<b>20</b>	<b>3</b>	<b>1.00</b>	<b>-.22</b>	<b>.607</b>	<b>.032</b>
20	5	1.25	-.63	.882	.022
20	7	5.00	-.99	.705	.008
20	10	3.00	-.98	.951	.005
20	15	-3.25	.99	.769	.003
20	20	-1.75	.99	.766	.003
20	22	-3.75	1.00	.931	.003
20	25	-3.25	.99	.946	.002
40	1	1.00	-.22	.146	.016
40	3	1.00	-.22	.440	.027
<b>40</b>	<b>5</b>	<b>1.25</b>	<b>-.63</b>	<b>.750</b>	<b>.040</b>
40	7	1.25	-.63	.919	.022
40	10	3.75	-.99	.844	.009
40	15	-3.00	.99	.674	.005
40	20	-2.25	.99	.770	.004
40	22	-2.75	.99	.857	.002
40	25	-0.25	.94	.151	.001

TABLE SM-III. Maximum and Average Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and AR Tests for Fixed Alternative  $\beta^* = 0$ 

$\rho_{uv}$	$k$	$\lambda_{\max}$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2-AR		
					POIS2	max	average
.0	2	20	0.50	-.45	.49	.079	.012
.0	5	20	0.75	-.60	.66	.151	.014
.0	10	20	-0.75	.60	.57	.183	.015
.0	20	25	0.75	-.60	.57	<b>.217</b>	<b>.016</b>
.0	40	25	-0.75	.60	.42	.161	.011
.3	2	20	0.50	-.21	.60	.084	.014
.3	5	20	-0.75	.74	.56	.163	.020
.3	10	20	-1.00	.81	.61	.201	.022
.3	20	25	0.50	-.21	.47	<b>.231</b>	<b>.024</b>
.3	40	25	-1.00	.81	.47	.194	.019
.5	2	20	-0.75	.82	.58	.090	.020
.5	5	20	0.50	.00	.66	.182	.031
.5	10	20	-1.00	.87	.59	.232	.038
.5	20	25	-1.00	.87	.60	<b>.285</b>	<b>.044</b>
.5	40	25	-1.25	.90	.54	.248	.040
.7	2	20	-0.75	.90	.54	.094	.030
.7	5	20	-1.25	.94	.72	.208	.054
.7	10	20	-1.25	.94	.67	.281	.069
.7	20	25	-1.25	.94	.71	<b>.361</b>	.084
.7	40	25	-2.00	.97	.73	.351	<b>.085</b>
.9	2	20	-1.00	.97	.64	.105	.033
.9	5	20	-1.25	.98	.70	.237	.068
.9	10	20	-1.50	.98	.75	.340	.133
.9	20	25	-1.25	.98	.76	.455	.160
.9	40	25	-1.75	.99	.82	<b>.513</b>	<b>.179</b>

k	$\rho_{uv}$	$\lambda_{\max}$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-AR	
						max	average
2	.0	20	0.50	-.45	.49	.079	.012
2	.3	20	0.50	-.21	.60	.084	.014
2	.5	20	-0.75	.82	.58	.090	.020
2	.7	20	-0.75	.90	.54	.094	.030
2	.9	20	-1.00	.97	.64	<b>.105</b>	<b>.033</b>
5	.0	20	0.75	-.60	.66	.151	.014
5	.3	20	-0.75	.74	.56	.163	.020
5	.5	20	0.50	.00	.66	.182	.031
5	.7	20	-1.25	.94	.72	.208	.054
5	.9	20	-1.25	.98	.70	<b>.237</b>	<b>.068</b>
10	.0	20	-0.75	.60	.57	.183	.015
10	.3	20	-1.00	.81	.61	.201	.022
10	.5	20	-1.00	.87	.59	.232	.038
10	.7	20	-1.25	.94	.67	.281	.069
10	.9	20	-1.50	.98	.75	<b>.340</b>	<b>.133</b>
20	.0	25	0.75	-.60	.57	.217	.016
20	.3	25	0.50	-.21	.47	.231	.024
20	.5	25	-1.00	.87	.60	.285	.044
20	.7	25	-1.25	.94	.71	.361	.084
20	.9	25	-1.25	.98	.76	<b>.455</b>	<b>.160</b>
40	.0	25	-0.75	.60	.42	.161	.011
40	.3	25	-1.00	.81	.47	.194	.019
40	.5	25	-1.25	.90	.54	.248	.040
40	.7	25	-2.00	.97	.73	.351	.085
40	.9	25	-1.75	.99	.82	<b>.513</b>	<b>.179</b>

TABLE SM-IV. Maximum and Average Power Differences over  $\lambda$  and  $\beta_0$  Values between POIS2 and LM Tests for Fixed Alternative  $\beta^* = 0$ 

(a) Across $k$ patterns for fixed $\rho_{uv}$								(b) Across $\rho_{uv}$ patterns for fixed $k$							
$\rho_{uv}$	$k$	$\lambda_{\max}$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-LM		k	$\rho_{uv}$	$\lambda_{\max}$	$\beta_{0,\max}$	$\rho_{uv,0}$	POIS2	POIS2-LM	
						max	average							max	average
.0	2	15	50.00	-1.00	.95	.312	.117	2	.0	15	50.00	-1.00	.95	<b>.312</b>	<b>.117</b>
.0	5	20	1000.00	-1.00	.95	.538	.173	2	.3	15	3.75	-.96	.97	.309	.103
.0	10	20	10000.00	-1.00	.89	.611	.174	2	.5	10	2.00	-.87	.92	.311	.077
.0	20	25	10000.00	-1.00	.88	<b>.687</b>	<b>.203</b>	2	.7	7	1.50	-.75	.93	.310	.041
.0	40	25	10000.00	-1.00	.74	.621	.173	2	.9	3	1.25	-.63	.95	.242	.010
.3	2	15	3.75	-.96	.97	.309	.103	5	.0	20	1000.00	-1.00	.95	.538	<b>.173</b>
.3	5	20	3.25	-.95	.97	.536	.155	5	.3	20	3.25	-.95	.97	.536	.155
.3	10	20	3.50	-.96	.92	.628	.161	5	.5	15	2.00	-.87	.95	<b>.538</b>	.121
.3	20	25	3.25	-.95	.91	<b>.710</b>	<b>.187</b>	5	.7	10	1.50	-.75	.95	.531	.070
.3	40	25	3.25	-.95	.80	.671	.168	5	.9	3	1.00	-.22	.87	.422	.016
.5	2	10	2.00	-.87	.92	.311	.077	10	.0	20	10000.00	-1.00	.89	.611	<b>.174</b>
.5	5	15	2.00	-.87	.95	.538	.121	10	.3	20	3.50	-.96	.92	.628	.161
.5	10	20	2.00	-.87	.97	.639	.133	10	.5	20	2.00	-.87	.97	<b>.639</b>	.133
.5	20	25	2.00	-.87	.96	.734	<b>.153</b>	10	.7	15	1.50	-.75	.98	.621	.084
.5	40	25	2.00	-.87	.90	<b>.750</b>	.152	10	.9	5	1.25	-.63	.96	.474	.023
.7	2	7	1.50	-.75	.93	.310	.041	20	.0	25	10000.00	-1.00	.88	.687	<b>.203</b>
.7	5	10	1.50	-.75	.95	.531	.070	20	.3	25	3.25	-.95	.91	.710	.187
.7	10	15	1.50	-.75	.98	.621	.084	20	.5	25	2.00	-.87	.96	<b>.734</b>	.153
.7	20	20	1.50	-.75	.99	.721	.089	20	.7	20	1.50	-.75	.99	.721	.089
.7	40	22	1.50	-.75	.97	<b>.784</b>	<b>.107</b>	20	.9	5	1.25	-.63	.88	.562	.021
.9	2	3	1.25	-.63	.95	.242	.010	40	.0	25	10000.00	-1.00	.74	.621	<b>.173</b>
.9	5	3	1.00	-.22	.87	.422	.016	40	.3	25	3.25	-.95	.80	.671	.168
.9	10	5	1.25	-.63	.96	.474	.023	40	.5	25	2.00	-.87	.90	.750	.152
.9	20	5	1.25	-.63	.88	.562	.021	40	.7	22	1.50	-.75	.97	<b>.784</b>	.107
.9	40	7	1.25	-.63	.92	<b>.620</b>	<b>.031</b>	40	.9	7	1.25	-.63	.92	.620	.031

TABLE SM-V(a). Average (over  $\lambda$ ) Power Differences for  $\lambda \in \{2.5, 5.0, \dots, 90.0\}$  between the WAP2 and CLR Tests for  $k = 2$

$\beta_0$	$\rho_{uv,0}$		WAP2						WAP2-CLR				
	$\rho_{uv} = 0$	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	
-10000.00	1.00	1.00	.946	.950	.951	.955	.958	.003	.002	.001	.001	<b>.001</b>	
-1000.00	1.00	1.00	.946	.950	.951	.955	.958	.003	.002	.001	.001	<b>.001</b>	
-100.00	1.00	1.00	.946	.949	.951	.954	.957	.003	.002	.001	.000	<b>.001</b>	
-50.00	1.00	1.00	.946	.949	.950	.953	.956	.003	.002	.001	.001	<b>.001</b>	
-10.00	1.00	1.00	.946	.945	.945	.947	.948	.003	.002	.001	.000	<b>.001</b>	
-7.50	.99	1.00	.945	.944	.943	.944	.945	.003	.002	.001	.000	.000	
-5.00	.98	1.00	.944	.941	.938	.938	.937	.003	.001	.001	.001	.000	
-4.00	.97	1.00	.943	.938	.935	.933	.931	.003	.001	.001	.001	.000	
-3.75	.97	1.00	.942	.937	.933	.931	.929	.003	.001	.001	.001	<b>.001</b>	
-3.50	.96	1.00	.941	.936	.932	.929	.927	.002	.001	.001	.001	<b>.001</b>	
-3.25	.96	.99	.940	.934	.930	.926	.923	.002	.001	.001	.001	<b>.001</b>	
-3.00	.95	.99	.940	.933	.927	.923	.920	.003	.001	.001	.001	.000	
-2.75	.94	.99	.938	.930	.925	.920	.916	.003	.001	.001	.000	.000	
-2.50	.93	.99	.937	.928	.922	.916	.911	.002	.001	.001	.001	.000	
-2.25	.91	.99	.935	.925	.918	.910	.904	.002	.001	.001	.001	.000	
-2.00	.89	.99	.932	.920	.912	.904	.896	.002	.002	.001	.001	<b>.001</b>	
-1.75	.87	.99	.928	.914	.904	.895	.885	.002	.001	.001	.001	.000	
-1.50	.83	.98	.921	.905	.893	.881	.870	.001	.001	.001	.001	.000	
-1.25	.78	.98	.910	.890	.876	.861	.847	.001	.002	.001	.001	<b>.001</b>	
-1.00	.71	.97	.895	.866	.845	.827	.809	.002	.002	.001	.001	<b>.001</b>	
-0.75	.60	.97	.851	.814	.788	.762	.738	.001	.002	<b>.002</b>	.001	<b>.001</b>	
-0.50	.45	.95	.736	.681	.646	.613	.584	.002	.002	<b>.002</b>	<b>.002</b>	<b>.001</b>	
-0.25	.24	.94	.366	.329	.310	.293	.279	.002	.002	<b>.002</b>	.001	<b>.001</b>	
0.25	-.24	.83	.366	.410	.447	.488	.544	.001	.001	<b>.002</b>	<b>.002</b>	<b>.001</b>	
0.50	-.45	.68	.737	.804	.848	.892	.937	.002	.002	.001	.001	.000	
0.75	-.60	.33	.851	.899	.928	.959	.989	.002	.001	.001	.001	.000	
1.00	-.71	-.22	.892	.928	.951	.975	.997	.001	.001	.001	.001	.000	
1.25	-.78	-.63	.911	.942	.959	.979	.997	.001	.002	.001	.001	.000	
1.50	-.83	-.81	.921	.948	.963	.980	.995	.001	.002	<b>.002</b>	.001	.000	
1.75	-.87	-.89	.928	.950	.965	.979	.993	.002	.002	<b>.002</b>	<b>.002</b>	.000	
2.00	-.89	-.93	.932	.953	.965	.978	.991	.001	.002	<b>.002</b>	<b>.002</b>	.000	
2.25	-.91	-.95	.935	.953	.965	.976	.989	.002	.002	<b>.002</b>	.001	.000	
2.50	-.93	-.96	.937	.954	.964	.975	.986	.002	<b>.003</b>	<b>.002</b>	.001	.000	
2.75	-.94	-.97	.938	.955	.964	.973	.985	.003	<b>.003</b>	<b>.002</b>	.001	.000	
3.00	-.95	-.98	.939	.955	.963	.973	.983	.002	<b>.003</b>	<b>.002</b>	.001	.000	
3.25	-.96	-.98	.940	.954	.963	.972	.982	.003	<b>.003</b>	<b>.002</b>	.001	.000	
3.50	-.96	-.99	.941	.954	.962	.970	.980	.003	<b>.003</b>	<b>.002</b>	.000	.000	
3.75	-.97	-.99	.942	.955	.962	.970	.979	.003	<b>.003</b>	<b>.002</b>	.001	.000	
4.00	-.97	-.99	.942	.954	.962	.969	.978	.003	.002	<b>.002</b>	.001	.000	
5.00	-.98	-.99	.944	.954	.960	.967	.974	.003	.002	<b>.002</b>	.001	.000	
7.50	-.99	-1.00	.945	.953	.957	.964	.970	.003	.002	.001	.001	.000	
10.00	-1.00	-1.00	.946	.952	.956	.961	.967	<b>.004</b>	.002	<b>.002</b>	.001	.000	
50.00	-1.00	-1.00	.946	.950	.952	.956	.960	.003	.002	.001	.000	<b>.001</b>	
100.00	-1.00	-1.00	.946	.950	.952	.955	.959	.003	.002	<b>.002</b>	.001	<b>.001</b>	
1000.00	-1.00	-1.00	.946	.950	.951	.955	.958	.003	.002	.001	.001	<b>.001</b>	
10000.00	-1.00	-1.00	.946	.950	.951	.955	.958	.003	.002	.001	.001	<b>.001</b>	



TABLE SM-V(b). Average (over  $\lambda$ ) Power Differences for  $\lambda \in \{2.5, 5.0, \dots, 90.0\}$  between the WAP2 and CLR Tests for  $k = 5$

$\beta_0$	$\rho_{uv,0}$		WAP2					WAP2-CLR				
	$\rho_{uv} = 0$	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	$\rho_{uv} = 0$	.3	.5	.7	.9
-10000.00	1.00	1.00	.923	.924	.929	.939	.953	.005	.002	.001	.001	.000
-1000.00	1.00	1.00	.923	.924	.929	.939	.953	.005	.002	.001	.001	.000
-100.00	1.00	1.00	.923	.923	.929	.939	.952	.005	.002	.001	.001	.000
-50.00	1.00	1.00	.923	.923	.929	.938	.951	<b>.005</b>	.003	.001	.001	.000
-10.00	1.00	1.00	.922	.920	.924	.931	.942	.005	.002	.001	.000	.000
-7.50	.99	1.00	.921	.918	.922	.929	.938	.004	.002	.000	.000	.000
-5.00	.98	1.00	.919	.915	.917	.923	.931	.004	.001	.000	.000	.000
-4.00	.97	1.00	.918	.912	.913	.917	.924	.003	.001	.000	-.000	.000
-3.75	.97	1.00	.917	.911	.911	.915	.922	.003	.002	.000	-.000	-.000
-3.50	.96	1.00	.917	.910	.910	.913	.920	.003	.001	-.000	-.000	.000
-3.25	.96	.99	.916	.909	.908	.910	.917	.003	.001	.000	.000	-.000
-3.00	.95	.99	.916	.907	.906	.907	.914	.003	.001	.000	.000	-.000
-2.75	.94	.99	.914	.905	.903	.904	.910	.002	.001	.000	.000	-.000
-2.50	.93	.99	.913	.902	.899	.900	.904	.002	.001	.001	-.000	.000
-2.25	.91	.99	.910	.898	.895	.894	.897	.002	.000	.001	-.000	-.000
-2.00	.89	.99	.907	.893	.888	.887	.888	.002	.001	.000	.001	-.000
-1.75	.87	.99	.903	.886	.880	.877	.877	.001	.001	.001	.000	.000
-1.50	.83	.98	.896	.877	.868	.863	.863	.001	.001	.001	.000	.000
-1.25	.78	.98	.885	.861	.850	.842	.839	.002	.001	.001	.000	.000
-1.00	.71	.97	.865	.836	.820	.808	.800	.001	.000	-.000	-.000	-.000
-0.75	.60	.97	.823	.783	.760	.741	.727	.000	-.000	.001	-.000	-.000
-0.50	.45	.95	.705	.649	.618	.592	.572	-.000	-.000	-.001	-.001	-.000
-0.25	.24	.94	.340	.311	.294	.282	.272	-.001	-.001	-.001	-.000	-.000
0.25	-.24	.83	.347	.390	.428	.476	.536	-.000	-.001	-.001	-.000	-.001
0.50	-.45	.68	.711	.777	.825	.877	.931	.001	.000	.000	.000	.000
0.75	-.60	.33	.827	.873	.908	.944	.985	.000	.001	.001	.001	.000
1.00	-.71	-.22	.868	.904	.930	.960	.994	.002	.001	.001	.001	.000
1.25	-.78	-.63	.887	.915	.940	.966	.994	.001	.001	.004	.003	<b>.001</b>
1.50	-.83	-.81	.897	.922	.943	.966	.992	.001	.002	.003	<b>.003</b>	.001
1.75	-.87	-.89	.904	.926	.945	.965	.989	.002	.003	.004	.003	-.000
2.00	-.89	-.93	.907	.928	.945	.963	.986	.002	.003	<b>.004</b>	.002	.000
2.25	-.91	-.95	.910	.930	.945	.961	.984	.001	.004	.004	.001	.000
2.50	-.93	-.96	.912	.931	.945	.960	.981	.001	<b>.005</b>	.004	.001	-.000
2.75	-.94	-.97	.914	.931	.944	.958	.979	.003	.005	.004	.001	.000
3.00	-.95	-.98	.915	.931	.943	.957	.977	.003	.005	.003	.001	.000
3.25	-.96	-.98	.916	.931	.942	.956	.976	.003	.004	.003	.001	.000
3.50	-.96	-.99	.917	.931	.942	.955	.974	.003	.005	.003	.001	-.000
3.75	-.97	-.99	.918	.931	.941	.954	.973	.003	.004	.002	.001	.000
4.00	-.97	-.99	.919	.931	.940	.954	.972	.004	.005	.002	.001	.000
5.00	-.98	-.99	.920	.930	.939	.951	.968	.004	.005	.002	.000	.000
7.50	-.99	-1.00	.922	.929	.936	.948	.963	.005	.004	.001	.001	.000
10.00	-1.00	-1.00	.922	.928	.935	.946	.960	.005	.003	.001	.001	.000
50.00	-1.00	-1.00	.923	.925	.930	.941	.955	.005	.003	.001	.000	-.000
100.00	-1.00	-1.00	.923	.924	.930	.940	.954	.005	.003	.001	.000	.000
1000.00	-1.00	-1.00	.923	.924	.929	.939	.953	.005	.002	.001	.001	.000
10000.00	-1.00	-1.00	.923	.924	.929	.939	.953	.005	.002	.001	.001	.000

TABLE SM-V(c). Average (over  $\lambda$ ) Power Differences for  $\lambda \in \{2.5, 5.0, \dots, 90.0\}$  between the WAP2 and CLR Tests for  $k = 10$

$\beta_0$	$\rho_{uv,0}$		WAP2					WAP2-CLR				
	$\rho_{uv} = 0$	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	$\rho_{uv} = 0$	.3	.5	.7	.9
-10000.00	1.00	1.00	.901	.903	.910	.924	.946	<b>.011</b>	.006	.003	.001	.000
-1000.00	1.00	1.00	.901	.903	.910	.924	.946	<b>.011</b>	.006	.003	.001	.000
-100.00	1.00	1.00	.901	.902	.910	.924	.945	<b>.011</b>	.006	.003	.002	.001
-50.00	1.00	1.00	.901	.902	.909	.923	.944	<b>.011</b>	.006	.003	.002	.001
-10.00	1.00	1.00	.900	.898	.904	.916	.935	<b>.011</b>	.005	.003	.001	.001
-7.50	.99	1.00	.900	.896	.902	.913	.932	.010	.005	.003	.002	.001
-5.00	.98	1.00	.897	.892	.897	.907	.924	.008	.004	.003	.002	.001
-4.00	.97	1.00	.895	.889	.893	.902	.918	.007	.004	.002	.002	.001
-3.75	.97	1.00	.894	.888	.891	.900	.916	.007	.004	.002	.001	.001
-3.50	.96	1.00	.893	.886	.889	.898	.914	.007	.004	.002	.002	.001
-3.25	.96	.99	.892	.884	.888	.896	.911	.006	.003	.003	.002	.001
-3.00	.95	.99	.891	.882	.885	.893	.908	.006	.004	.002	.001	.001
-2.75	.94	.99	.890	.881	.882	.889	.904	.007	.005	.002	.001	.001
-2.50	.93	.99	.888	.878	.879	.886	.899	.006	.004	.002	.002	.001
-2.25	.91	.99	.885	.874	.874	.881	.892	.006	.003	.002	.002	.001
-2.00	.89	.99	.881	.869	.868	.873	.884	.004	.003	.002	.002	.001
-1.75	.87	.99	.876	.862	.860	.863	.872	.005	.003	.003	.002	.001
-1.50	.83	.98	.869	.853	.847	.848	.857	.005	.003	.002	.001	.002
-1.25	.78	.98	.858	.837	.829	.827	.834	.003	.003	.003	.002	.002
-1.00	.71	.97	.838	.809	.799	.794	.796	.003	.003	.002	.002	.002
-0.75	.60	.97	.793	.756	.738	.727	.724	.003	.003	.003	.002	.002
-0.50	.45	.95	.676	.623	.596	.580	.572	.004	.004	.003	.003	<b>.004</b>
-0.25	.24	.94	.323	.294	.282	.274	.272	.003	.004	.003	.002	.003
0.25	-.24	.83	.326	.370	.408	.460	.530	.004	.004	.004	.003	.002
0.50	-.45	.68	.674	.743	.796	.857	.923	.003	.003	.002	.002	.001
0.75	-.60	.33	.794	.845	.885	.929	.979	.003	.003	.003	.002	.001
1.00	-.71	-.22	.838	.878	.910	.948	.989	.003	.004	.003	.003	.002
1.25	-.78	-.63	.857	.891	.920	.953	.989	.003	.005	.005	<b>.005</b>	.002
1.50	-.83	-.81	.870	.899	.927	.955	.986	.004	.005	.007	<b>.005</b>	.001
1.75	-.87	-.89	.877	.904	.929	.953	.983	.005	.007	<b>.008</b>	.004	.001
2.00	-.89	-.93	.881	.906	.929	.951	.980	.005	.007	<b>.008</b>	.004	.001
2.25	-.91	-.95	.884	.909	.929	.950	.978	.005	.009	<b>.008</b>	.003	.001
2.50	-.93	-.96	.887	.910	.928	.948	.975	.006	.009	<b>.008</b>	.003	.001
2.75	-.94	-.97	.888	.910	.928	.946	.973	.006	.009	<b>.008</b>	.003	.001
3.00	-.95	-.98	.890	.911	.927	.945	.971	.006	<b>.010</b>	<b>.008</b>	.002	.001
3.25	-.96	-.98	.892	.911	.925	.944	.969	.007	<b>.010</b>	.007	.002	.000
3.50	-.96	-.99	.893	.912	.925	.943	.968	.007	<b>.010</b>	.007	.002	.001
3.75	-.97	-.99	.893	.912	.924	.942	.967	.007	<b>.010</b>	.006	.002	.000
4.00	-.97	-.99	.894	.912	.923	.942	.966	.008	<b>.010</b>	.006	.002	.000
5.00	-.98	-.99	.896	.910	.921	.939	.962	.009	.009	.005	.002	.001
7.50	-.99	-1.00	.899	.908	.918	.934	.957	.009	.008	.004	.002	.000
10.00	-1.00	-1.00	.900	.907	.916	.932	.954	<b>.011</b>	.008	.004	.002	.000
50.00	-1.00	-1.00	.901	.903	.912	.926	.948	.010	.006	.003	.002	.001
100.00	-1.00	-1.00	.901	.903	.911	.925	.947	<b>.011</b>	.006	.003	.002	.001
1000.00	-1.00	-1.00	.901	.903	.910	.925	.946	<b>.011</b>	.006	.003	.001	.000
10000.00	-1.00	-1.00	.901	.903	.910	.924	.946	<b>.011</b>	.006	.003	.001	.000

TABLE SM-V(d). Average (over  $\lambda$ ) Power Differences for  $\lambda \in \{2.5, 5.0, \dots, 90.0\}$  between the WAP2 and CLR Tests for  $k = 20$

$\beta_0$	$\rho_{uv,0}$		WAP2					WAP2-CLR				
	$\rho_{uv} = 0$	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	$\rho_{uv} = 0$	.3	.5	.7	.9
-10000.00	1.00	1.00	.862	.865	.878	.901	.934	<b>.013</b>	.008	.004	.001	.000
-1000.00	1.00	1.00	.862	.865	.878	.900	.934	<b>.013</b>	.008	.004	.001	.000
-100.00	1.00	1.00	.862	.864	.877	.900	.933	<b>.013</b>	.007	.003	.002	.000
-50.00	1.00	1.00	.862	.864	.876	.899	.932	<b>.013</b>	.007	.003	.002	.000
-10.00	1.00	1.00	.860	.860	.870	.892	.923	.012	.007	.002	.002	.001
-7.50	.99	1.00	.859	.858	.867	.889	.919	.012	.007	.002	.002	.000
-5.00	.98	1.00	.856	.853	.862	.881	.911	.010	.006	.003	.001	.000
-4.00	.97	1.00	.854	.849	.858	.876	.905	.009	.005	.003	.002	.000
-3.75	.97	1.00	.853	.847	.856	.875	.902	.009	.005	.003	.003	-.000
-3.50	.96	1.00	.852	.845	.854	.872	.900	.009	.004	.003	.003	.000
-3.25	.96	.99	.851	.843	.852	.869	.897	.009	.004	.003	.002	-.000
-3.00	.95	.99	.849	.841	.849	.866	.893	.008	.004	.003	.002	.000
-2.75	.94	.99	.847	.839	.846	.862	.889	.008	.005	.003	.001	-.000
-2.50	.93	.99	.845	.836	.842	.857	.884	.007	.004	.003	.002	.000
-2.25	.91	.99	.842	.832	.837	.851	.877	.006	.004	.002	.002	.000
-2.00	.89	.99	.838	.826	.830	.843	.869	.006	.004	.003	.002	.000
-1.75	.87	.99	.833	.818	.820	.832	.857	.005	.004	.003	.002	.000
-1.50	.83	.98	.825	.807	.806	.817	.841	.005	.005	.003	.002	.000
-1.25	.78	.98	.811	.789	.787	.794	.816	.005	.004	.004	.002	.001
-1.00	.71	.97	.787	.760	.752	.757	.776	.004	.004	.003	.001	.001
-0.75	.60	.97	.739	.701	.688	.688	.704	.004	.002	.002	.001	.000
-0.50	.45	.95	.615	.567	.549	.543	.556	.002	.002	.001	.001	.001
-0.25	.24	.94	.286	.263	.258	.257	.263	.001	.001	.002	.001	.000
0.25	-.24	.83	.286	.328	.367	.428	.512	.002	.003	.001	.001	.001
0.50	-.45	.68	.617	.692	.757	.829	.912	.004	.003	.003	.002	.000
0.75	-.60	.33	.743	.800	.849	.906	.971	.004	.004	.003	.003	.001
1.00	-.71	-.22	.790	.837	.877	.927	.983	.004	.005	.004	.003	<b>.002</b>
1.25	-.78	-.63	.812	.853	.889	.936	.983	.004	.005	.005	<b>.007</b>	<b>.002</b>
1.50	-.83	-.81	.824	.861	.897	.936	.980	.004	.006	.008	<b>.007</b>	.001
1.75	-.87	-.89	.832	.867	.900	.934	.976	.006	.007	<b>.010</b>	.006	.001
2.00	-.89	-.93	.838	.870	.900	.931	.972	.006	.009	<b>.010</b>	.004	.001
2.25	-.91	-.95	.842	.872	.900	.930	.969	.007	.010	<b>.010</b>	.004	.001
2.50	-.93	-.96	.845	.873	.899	.927	.967	.008	.011	<b>.010</b>	.003	.001
2.75	-.94	-.97	.848	.874	.897	.925	.965	.008	.011	.009	.002	.001
3.00	-.95	-.98	.849	.875	.896	.924	.963	.008	.011	.008	.002	.001
3.25	-.96	-.98	.851	.875	.895	.922	.961	.009	<b>.012</b>	.008	.002	.000
3.50	-.96	-.99	.852	.875	.894	.921	.960	.009	<b>.012</b>	.007	.002	.001
3.75	-.97	-.99	.853	.875	.893	.920	.958	.009	<b>.012</b>	.007	.002	.001
4.00	-.97	-.99	.854	.875	.893	.919	.957	.009	<b>.012</b>	.007	.002	.001
5.00	-.98	-.99	.857	.874	.890	.916	.954	.010	<b>.012</b>	.006	.002	.001
7.50	-.99	-1.00	.859	.872	.887	.911	.948	.012	.010	.005	.002	.001
10.00	-1.00	-1.00	.860	.870	.885	.909	.945	.012	.010	.005	.002	.001
50.00	-1.00	-1.00	.862	.866	.879	.902	.936	<b>.013</b>	.008	.004	.002	.001
100.00	-1.00	-1.00	.862	.865	.879	.901	.935	<b>.013</b>	.008	.004	.001	.000
1000.00	-1.00	-1.00	.862	.865	.878	.901	.934	<b>.013</b>	.008	.004	.001	.000
10000.00	-1.00	-1.00	.862	.865	.878	.901	.934	<b>.013</b>	.008	.004	.001	.000

TABLE SM-V(e). Average (over  $\lambda$ ) Power Differences for  $\lambda \in \{2.5, 5.0, \dots, 90.0\}$  between the WAP2 and CLR Tests for  $k = 40$

$\beta_0$	$\rho_{uv,0}$		WAP2					WAP2-CLR				
	$\rho_{uv} = 0$	.9	$\rho_{uv} = 0$	.3	.5	.7	.9	$\rho_{uv} = 0$	.3	.5	.7	.9
-10000.00	1.00	1.00	.817	.819	.835	.869	.919	<b>.024</b>	.013	.006	.004	.001
-1000.00	1.00	1.00	.817	.819	.835	.869	.919	.023	.013	.006	.004	.001
-100.00	1.00	1.00	.817	.818	.834	.867	.917	.023	.013	.006	.004	.001
-50.00	1.00	1.00	.817	.817	.833	.867	.916	.023	.012	.005	.004	.001
-10.00	1.00	1.00	.814	.810	.826	.858	.905	.022	.010	.005	.003	.000
-7.50	.99	1.00	.812	.807	.823	.853	.901	.020	.010	.005	.003	.001
-5.00	.98	1.00	.808	.802	.816	.845	.892	.018	.009	.005	.003	.000
-4.00	.97	1.00	.804	.797	.810	.838	.884	.016	.008	.004	.003	.000
-3.75	.97	1.00	.802	.796	.808	.836	.882	.015	.008	.004	.004	.000
-3.50	.96	1.00	.801	.794	.806	.834	.879	.014	.008	.004	.003	-.000
-3.25	.96	.99	.799	.792	.803	.831	.876	.013	.008	.004	.003	.000
-3.00	.95	.99	.797	.790	.800	.828	.872	.012	.008	.004	.004	.000
-2.75	.94	.99	.795	.786	.796	.823	.867	.012	.007	.004	.003	.001
-2.50	.93	.99	.792	.783	.792	.817	.861	.011	.007	.005	.002	.001
-2.25	.91	.99	.788	.777	.786	.811	.854	.011	.006	.004	.003	.000
-2.00	.89	.99	.783	.772	.778	.802	.846	.009	.007	.005	.004	.001
-1.75	.87	.99	.777	.761	.767	.790	.833	.009	.006	.004	.003	-.000
-1.50	.83	.98	.768	.749	.754	.773	.816	.008	.006	.004	.003	.001
-1.25	.78	.98	.752	.730	.730	.750	.791	.007	.006	.004	.003	.000
-1.00	.71	.97	.727	.698	.694	.709	.751	.007	.007	.004	.003	-.000
-0.75	.60	.97	.672	.633	.624	.632	.675	.006	.005	.004	.001	-.000
-0.50	.45	.95	.536	.490	.480	.489	.524	.004	.004	.002	.001	-.002
-0.25	.24	.94	.233	.217	.215	.221	.242	.001	.001	-.000	-.001	-.002
0.25	-.24	.83	.237	.275	.318	.383	.491	.001	.000	.000	-.002	-.002
0.50	-.45	.68	.539	.621	.697	.788	.892	.004	.004	.004	.003	.001
0.75	-.60	.33	.672	.741	.804	.876	.958	.004	.005	.005	.004	.001
1.00	-.71	-.22	.727	.784	.837	.901	.974	.005	.006	.006	.006	.003
1.25	-.78	-.63	.754	.803	.853	.911	.974	.006	.008	.010	.009	<b>.004</b>
1.50	-.83	-.81	.769	.815	.861	.913	.969	.007	.012	.014	<b>.011</b>	.002
1.75	-.87	-.89	.779	.824	.865	.910	.965	.008	.016	.016	.008	.001
2.00	-.89	-.93	.785	.828	.866	.905	.961	.009	.017	.016	.006	.001
2.25	-.91	-.95	.790	.831	.866	.903	.958	.010	.019	<b>.017</b>	.006	.001
2.50	-.93	-.96	.794	.833	.864	.901	.955	.011	.020	.016	.005	.001
2.75	-.94	-.97	.797	.834	.862	.898	.953	.012	.020	.015	.004	.001
3.00	-.95	-.98	.799	.834	.861	.896	.950	.013	<b>.021</b>	.014	.004	.001
3.25	-.96	-.98	.801	.834	.859	.894	.949	.014	<b>.021</b>	.013	.004	.001
3.50	-.96	-.99	.803	.835	.858	.893	.947	.015	<b>.021</b>	.013	.004	.001
3.75	-.97	-.99	.805	.834	.857	.892	.946	.016	<b>.021</b>	.013	.004	.001
4.00	-.97	-.99	.806	.834	.855	.890	.944	.016	<b>.021</b>	.012	.004	.001
5.00	-.98	-.99	.810	.831	.852	.887	.940	.018	.019	.011	.004	.001
7.50	-.99	-1.00	.814	.828	.847	.881	.933	.021	.017	.009	.004	.001
10.00	-1.00	-1.00	.815	.826	.844	.878	.929	.022	.016	.009	.004	.002
50.00	-1.00	-1.00	.817	.820	.837	.871	.921	.023	.014	.007	.004	.001
100.00	-1.00	-1.00	.817	.820	.836	.870	.920	.023	.014	.007	.005	.001
1000.00	-1.00	-1.00	.817	.819	.836	.869	.919	<b>.024</b>	.013	.006	.004	.001
10000.00	-1.00	-1.00	.817	.819	.835	.869	.919	<b>.024</b>	.013	.006	.004	.001

TABLE SM-VI(a). Differences in Probabilities of Infinite-Length Confidence Intervals between  $CLR2_n$  and CLR for  $\rho_{uv} = .0$  and  $n = 100, 500$  and  $1000$

$k$	$\lambda$	CLR	$CLR2_{100}$ -CLR	SD	$CLR2_{500}$ -CLR	SD	$CLR2_{1000}$ -CLR	SD
2	1	.868	-.0011	.0001	-.0013	.0001	-.0014	.0000
2	3	.687	-.0021	.0002	-.0025	.0001	-.0025	.0001
2	5	.508	-.0019	.0002	-.0025	.0001	-.0028	.0001
2	7	.358	-.0009	.0002	-.0023	.0001	-.0025	.0001
2	10	.195	-.0011	.0002	-.0017	.0001	-.0019	.0001
2	15	.063	.0002	.0002	-.0005	.0001	-.0006	.0001
2	20	.017	.0001	.0001	-.0002	.0000	-.0002	.0000
5	1	.905	.0002	.0002	.0001	.0001	-.0001	.0001
5	3	.789	.0017	.0003	.0005	.0001	.0002	.0001
5	5	.659	.0028	.0003	.0008	.0002	.0005	.0001
5	7	.529	.0031	.0004	.0005	.0002	.0003	.0001
5	10	.350	.0041	.0003	.0009	.0002	.0004	.0001
5	12	.257	.0043	.0003	.0011	.0001	.0007	.0001
5	15	.156	.0029	.0003	.0007	.0001	.0004	.0001
5	20	.059	.0020	.0002	.0005	.0001	.0002	.0001
5	1	.905	.0002	.0002	.0001	.0001	-.0001	.0001
5	3	.789	.0017	.0003	.0005	.0001	.0002	.0001
5	5	.659	.0028	.0003	.0008	.0002	.0005	.0001
5	7	.529	.0031	.0004	.0005	.0002	.0003	.0001
5	10	.350	.0041	.0003	.0009	.0002	.0004	.0001
5	12	.257	.0043	.0003	.0011	.0001	.0007	.0001
5	15	.156	.0029	.0003	.0007	.0001	.0004	.0001
5	20	.059	.0020	.0002	.0005	.0001	.0002	.0001
5	25	.021	.0011	.0002	.0001	.0001	.0000	.0000
10	1	.920	.0006	.0003	.0003	.0001	.0002	.0001
10	5	.751	.0042	.0004	.0009	.0002	.0006	.0001
10	10	.496	.0068	.0005	.0011	.0002	.0005	.0001
10	15	.279	.0077	.0004	.0016	.0002	.0007	.0002
10	17	.212	.0067	.0004	.0013	.0002	.0006	.0002
10	20	.135	.0065	.0004	.0013	.0002	.0006	.0001
10	25	.060	.0039	.0003	.0008	.0001	.0003	.0001
10	30	.024	.0020	.0002	.0003	.0001	.0001	.0000
20	1	.932	.0009	.0003	.0004	.0002	.0003	.0001
20	5	.824	.0044	.0005	.0007	.0002	.0006	.0001
20	10	.632	.0099	.0006	.0023	.0003	.0014	.0002
20	15	.436	.0126	.0006	.0030	.0003	.0020	.0002
20	20	.268	.0141	.0006	.0031	.0003	.0015	.0002
20	25	.149	.0107	.0005	.0024	.0002	.0012	.0001
20	30	.077	.0067	.0004	.0014	.0002	.0006	.0001
20	40	.017	.0025	.0002	.0005	.0001	.0003	.0001
40	1	.938	.0011	.0004	.0004	.0002	.0003	.0001
40	5	.872	.0057	.0005	.0010	.0002	.0007	.0002
40	10	.750	.0125	.0006	.0027	.0003	.0017	.0002
40	15	.599	.0201	.0007	.0033	.0003	.0021	.0002
40	20	.443	.0235	.0008	.0047	.0003	.0028	.0002
40	30	.198	.0194	.0006	.0039	.0003	.0021	.0002
40	40	.068	.0113	.0005	.0020	.0002	.0012	.0002
40	60	.005	.0013	.0002	.0002	.0001	.0001	.0001

TABLE SM-VI(b). Differences in Probabilities of Infinite-Length Confidence Intervals between  $CLR2_n$  and CLR for  $\rho_{uv} = .3$  and  $n = 100, 500$  and  $1000$

$k$	$\lambda$	CLR	$CLR2_{100}$ -CLR	SD	$CLR2_{500}$ -CLR	SD	$CLR2_{1000}$ -CLR	SD
2	1	.868	-.0015	.0002	-.0016	.0001	-.0017	.0000
2	3	.684	-.0023	.0002	-.0026	.0001	-.0026	.0001
2	5	.501	-.0024	.0002	-.0030	.0001	-.0029	.0001
2	7	.347	-.0016	.0002	-.0025	.0001	-.0027	.0001
2	10	.184	-.0004	.0002	-.0014	.0001	-.0015	.0001
2	15	.056	-.0001	.0001	-.0006	.0001	-.0006	.0001
2	20	.015	-.0001	.0001	-.0002	.0000	-.0002	.0000
5	1	.905	.0001	.0002	.0001	.0001	.0001	.0001
5	3	.785	.0010	.0003	.0003	.0001	.0002	.0001
5	5	.647	.0019	.0003	.0007	.0002	.0003	.0001
5	7	.510	.0028	.0003	.0007	.0002	.0003	.0001
5	10	.330	.0034	.0004	.0007	.0001	.0003	.0001
5	12	.235	.0029	.0003	.0006	.0002	.0001	.0001
5	15	.137	.0020	.0003	.0001	.0001	.0000	.0001
5	20	.048	.0015	.0002	.0001	.0001	.0000	.0001
5	25	.016	.0007	.0002	.0002	.0001	.0001	.0000
10	1	.920	.0000	.0003	.0001	.0001	.0000	.0001
10	5	.744	.0030	.0004	.0004	.0002	.0001	.0001
10	10	.472	.0055	.0005	.0005	.0002	-.0000	.0001
10	15	.248	.0052	.0004	.0008	.0002	.0003	.0001
10	17	.184	.0049	.0004	.0006	.0002	.0000	.0001
10	20	.114	.0038	.0003	.0007	.0001	.0002	.0001
10	25	.046	.0018	.0002	.0003	.0001	.0002	.0001
10	30	.017	.0013	.0002	.0002	.0001	.0002	.0001
20	1	.931	.0007	.0004	-.0000	.0001	.0001	.0001
20	5	.819	.0032	.0005	.0013	.0002	.0008	.0001
20	10	.615	.0080	.0006	.0017	.0002	.0012	.0002
20	15	.406	.0104	.0006	.0019	.0003	.0010	.0002
20	20	.236	.0100	.0005	.0017	.0002	.0007	.0002
20	25	.122	.0075	.0004	.0018	.0002	.0010	.0001
20	30	.057	.0050	.0003	.0010	.0002	.0007	.0001
20	40	.011	.0015	.0002	.0004	.0001	.0001	.0001
40	1	.937	.0009	.0004	.0002	.0002	.0003	.0001
40	5	.869	.0046	.0005	.0014	.0002	.0009	.0002
40	10	.737	.0114	.0007	.0027	.0003	.0016	.0002
40	15	.572	.0164	.0007	.0033	.0003	.0019	.0002
40	20	.409	.0189	.0007	.0036	.0003	.0019	.0002
40	30	.164	.0147	.0006	.0023	.0003	.0011	.0002
40	40	.048	.0079	.0004	.0015	.0002	.0008	.0001
40	60	.002	.0010	.0001	.0002	.0001	.0001	.0000

TABLE SM-VI(c). Differences in Probabilities of Infinite-Length Confidence Intervals between  $CLR2_n$  and CLR for  $\rho_{uv} = .5$  and  $n = 100, 500$  and  $1000$

$k$	$\lambda$	CLR	$CLR2_{100}$ -CLR	SD	$CLR2_{500}$ -CLR	SD	$CLR2_{1000}$ -CLR	SD
2	1	.867	-.0019	.0001	-.0018	.0001	-.0018	.0001
2	3	.675	-.0025	.0002	-.0027	.0001	-.0028	.0001
2	5	.486	-.0024	.0002	-.0028	.0001	-.0029	.0001
2	7	.327	-.0013	.0002	-.0020	.0001	-.0021	.0001
2	10	.166	-.0012	.0002	-.0018	.0001	-.0018	.0001
2	15	.046	-.0004	.0001	-.0007	.0001	-.0007	.0000
2	20	.012	-.0001	.0001	-.0001	.0000	-.0002	.0000
5	1	.904	.0003	.0002	.0000	.0001	.0001	.0001
5	3	.776	.0010	.0003	-.0000	.0001	-.0001	.0001
5	5	.624	.0010	.0004	.0001	.0002	-.0000	.0001
5	7	.476	.0013	.0004	.0000	.0002	-.0000	.0001
5	10	.289	.0008	.0004	-.0000	.0001	-.0001	.0001
5	12	.196	.0009	.0003	.0000	.0001	-.0001	.0001
5	15	.104	.0012	.0003	-.0002	.0001	-.0003	.0001
5	20	.032	.0008	.0002	.0001	.0001	-.0001	.0001
5	25	.009	.0003	.0001	.0000	.0000	-.0000	.0000
10	1	.919	.0004	.0003	.0000	.0001	-.0001	.0001
10	5	.725	.0008	.0004	.0001	.0002	.0001	.0001
10	10	.423	.0026	.0005	.0001	.0002	-.0002	.0001
10	15	.194	.0030	.0004	.0001	.0002	-.0001	.0001
10	17	.136	.0026	.0004	.0002	.0002	-.0001	.0001
10	20	.077	.0022	.0003	.0004	.0001	.0001	.0001
10	25	.026	.0012	.0002	.0001	.0001	-.0000	.0001
10	30	.008	.0007	.0001	.0003	.0001	.0000	.0000
20	1	.931	.0006	.0004	.0003	.0002	.0003	.0001
20	5	.804	.0033	.0005	.0011	.0002	.0009	.0002
20	10	.571	.0055	.0006	.0012	.0003	.0010	.0002
20	15	.340	.0065	.0006	.0017	.0003	.0011	.0002
20	20	.172	.0059	.0005	.0013	.0002	.0005	.0001
20	25	.076	.0046	.0004	.0007	.0002	.0004	.0001
20	30	.030	.0027	.0003	.0005	.0001	.0003	.0001
20	40	.004	.0005	.0001	.0001	.0001	.0001	.0000
40	1	.937	.0007	.0004	-.0001	.0002	.0001	.0001
40	5	.862	.0032	.0005	.0009	.0002	.0005	.0002
40	10	.706	.0059	.0007	.0012	.0003	.0008	.0002
40	15	.513	.0094	.0007	.0013	.0003	.0011	.0002
40	20	.335	.0099	.0007	.0021	.0003	.0013	.0002
40	30	.103	.0085	.0005	.0011	.0002	.0007	.0002
40	40	.022	.0036	.0003	.0006	.0001	.0002	.0001
40	60	.001	.0003	.0001	.0000	.0000	.0000	.0000

TABLE SM-VI(d). Differences in Probabilities of Infinite-Length Confidence Intervals between  $CLR2_n$  and CLR for  $\rho_{uv} = .7$  and  $n = 100, 500$  and  $1000$

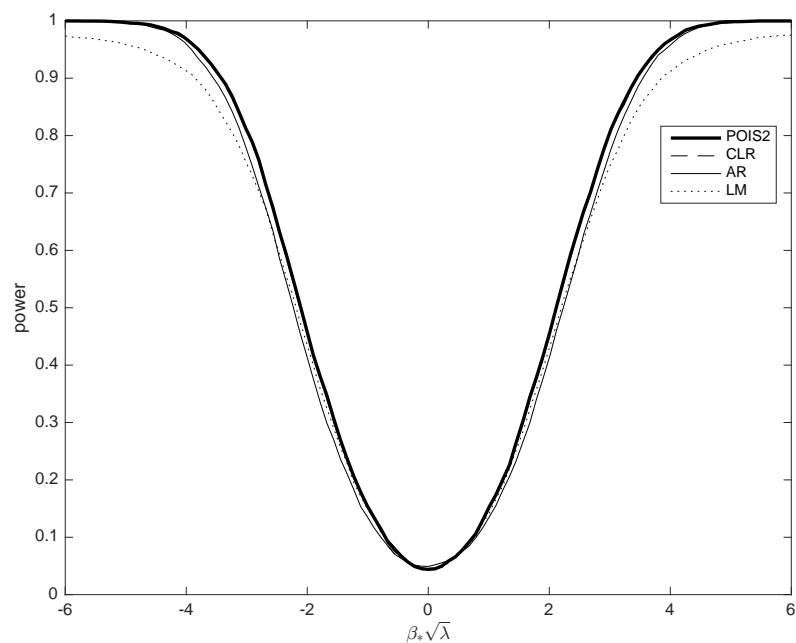
$k$	$\lambda$	CLR	$CLR2_{100}$ -CLR	SD	$CLR2_{500}$ -CLR	SD	$CLR2_{1000}$ -CLR	SD
2	1	.864	-.0017	.0002	-.0018	.0001	-.0018	.0000
2	3	.658	-.0026	.0003	-.0028	.0001	-.0028	.0001
2	5	.456	-.0026	.0003	-.0029	.0001	-.0030	.0001
2	7	.295	-.0026	.0002	-.0025	.0001	-.0025	.0001
2	10	.140	-.0009	.0002	-.0012	.0001	-.0013	.0001
2	15	.035	-.0002	.0001	-.0004	.0000	-.0004	.0000
2	20	.009	-.0000	.0001	-.0000	.0000	-.0001	.0000
5	1	.901	-.0000	.0002	-.0001	.0001	.0000	.0001
5	3	.753	.0001	.0003	.0000	.0001	-.0003	.0001
5	5	.575	-.0010	.0004	-.0007	.0002	-.0006	.0001
5	7	.410	-.0011	.0004	-.0015	.0002	-.0014	.0001
5	10	.219	.0003	.0003	-.0006	.0002	-.0009	.0001
5	12	.137	-.0001	.0003	-.0006	.0001	-.0008	.0001
5	15	.064	.0005	.0002	-.0001	.0001	-.0002	.0001
5	20	.016	.0003	.0002	.0000	.0001	-.0001	.0001
5	25	.004	.0001	.0001	.0000	.0000	-.0000	.0000
10	1	.918	-.0002	.0003	.0000	.0001	-.0001	.0001
10	5	.676	-.0008	.0005	-.0006	.0002	-.0008	.0002
10	10	.322	.0004	.0005	-.0006	.0002	-.0008	.0002
10	15	.115	.0010	.0004	-.0005	.0002	-.0005	.0001
10	17	.073	.0007	.0003	-.0002	.0001	-.0003	.0001
10	20	.034	.0009	.0002	.0000	.0001	-.0000	.0001
10	25	.009	.0004	.0001	-.0001	.0001	-.0001	.0001
10	30	.002	.0002	.0001	.0000	.0000	.0000	.0000
20	1	.930	-.0005	.0004	.0001	.0001	.0001	.0001
20	5	.771	.0001	.0005	-.0002	.0002	-.0002	.0002
20	10	.470	.0025	.0006	.0002	.0003	-.0001	.0002
20	15	.220	.0013	.0005	-.0004	.0002	-.0004	.0002
20	20	.083	.0027	.0004	.0004	.0002	.0002	.0001
20	25	.027	.0019	.0003	.0002	.0001	.0001	.0001
20	30	.008	.0009	.0002	.0001	.0001	.0000	.0000
20	40	.001	.0001	.0001	.0000	.0000	.0000	.0000
40	1	.936	.0008	.0004	.0004	.0002	.0003	.0001
40	5	.841	-.0005	.0006	.0002	.0003	.0000	.0002
40	10	.624	-.0003	.0008	-.0006	.0003	-.0002	.0002
40	15	.382	.0042	.0008	.0003	.0004	-.0001	.0003
40	20	.197	.0052	.0006	.0005	.0003	.0000	.0002
40	30	.033	.0036	.0003	.0005	.0001	.0001	.0001
40	40	.004	.0009	.0001	.0001	.0001	.0001	.0001
40	60	.000	.0000	.0000	.0000	.0000	.0000	.0000



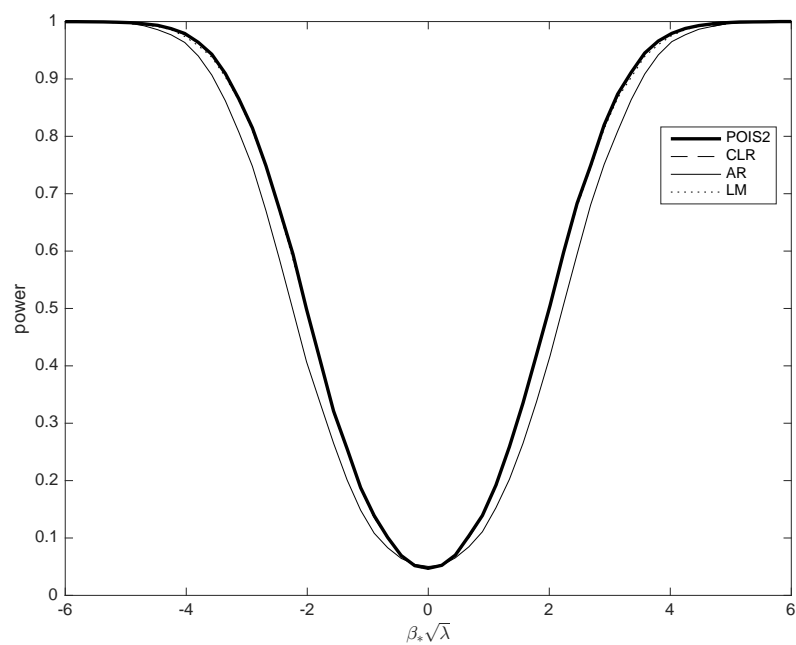
TABLE SM-VI(e). Differences in Probabilities of Infinite-Length Confidence Intervals between  $CLR2_n$  and CLR for  $\rho_{uv} = .9$  and  $n = 100, 500$  and  $1000$

$k$	$\lambda$	CLR	$CLR2_{100}$ -CLR	SD	$CLR2_{500}$ -CLR	SD	$CLR2_{1000}$ -CLR	SD
2	1	.854	-.0023	.0002	-.0020	.0001	-.0020	.0001
2	3	.617	-.0026	.0003	-.0028	.0001	-.0027	.0001
2	5	.410	-.0029	.0003	-.0027	.0001	-.0025	.0001
2	7	.258	-.0019	.0003	-.0019	.0001	-.0020	.0001
2	10	.119	-.0011	.0002	-.0011	.0001	-.0011	.0000
2	15	.029	-.0003	.0001	-.0004	.0000	-.0004	.0000
2	20	.006	-.0001	.0000	-.0001	.0000	-.0001	.0000
5	1	.887	-.0006	.0003	-.0004	.0001	-.0003	.0001
5	3	.675	-.0026	.0004	-.0018	.0002	-.0016	.0001
5	5	.462	-.0026	.0004	-.0018	.0002	-.0018	.0001
5	7	.297	-.0022	.0004	-.0020	.0002	-.0021	.0001
5	10	.140	-.0002	.0003	-.0011	.0001	-.0011	.0001
5	12	.083	.0003	.0003	-.0005	.0001	-.0007	.0001
5	15	.036	-.0001	.0002	-.0003	.0001	-.0004	.0001
5	20	.008	.0000	.0001	-.0000	.0000	-.0001	.0000
5	25	.002	.0000	.0000	-.0000	.0000	-.0000	.0000
10	1	.907	-.0008	.0003	-.0002	.0001	-.0002	.0001
10	5	.533	-.0035	.0005	-.0026	.0002	-.0024	.0002
10	10	.176	-.0004	.0004	-.0014	.0002	-.0016	.0001
10	15	.047	.0006	.0002	-.0003	.0001	-.0004	.0001
10	17	.026	.0001	.0002	-.0002	.0001	-.0003	.0001
10	20	.011	.0002	.0002	-.0001	.0001	-.0002	.0000
10	25	.002	-.0000	.0001	-.0000	.0000	-.0000	.0000
10	30	.000	.0000	.0000	-.0000	.0000	-.0000	.0000
20	1	.923	-.0005	.0004	-.0001	.0002	-.0001	.0001
20	5	.625	-.0016	.0006	-.0012	.0003	-.0009	.0002
20	10	.243	.0019	.0005	-.0010	.0002	-.0012	.0002
20	15	.072	.0024	.0004	-.0006	.0002	-.0005	.0001
20	20	.019	.0006	.0002	-.0002	.0001	-.0003	.0001
20	25	.004	.0004	.0001	-.0000	.0000	-.0001	.0000
20	30	.001	.0001	.0001	.0000	.0000	.0000	.0000
20	40	.000	.0000	.0000	.0000	.0000	.0000	.0000
40	1	.932	.0004	.0005	.0004	.0002	.0002	.0001
40	5	.727	-.0035	.0007	-.0012	.0003	-.0011	.0002
40	10	.358	.0019	.0007	-.0013	.0003	-.0016	.0002
40	15	.130	.0048	.0006	.0000	.0002	-.0004	.0002
40	20	.039	.0030	.0004	.0000	.0001	-.0000	.0001
40	30	.003	.0004	.0001	.0001	.0000	.0000	.0000
40	40	.000	.0000	.0000	.0000	.0000	.0000	.0000
40	60	.000	.0000	.0000	.0000	.0000	.0000	.0000

FIGURE SM-I(a). Power of the significance level .05 CLR, LM, and AR tests and the POIS2 power envelope for fixed null value  $\beta_0 = 0$ , varying true value  $\beta_*$ ,  $k = 2$ ,  $\rho_\Omega = 0$ , and  $\lambda = 5, 20$

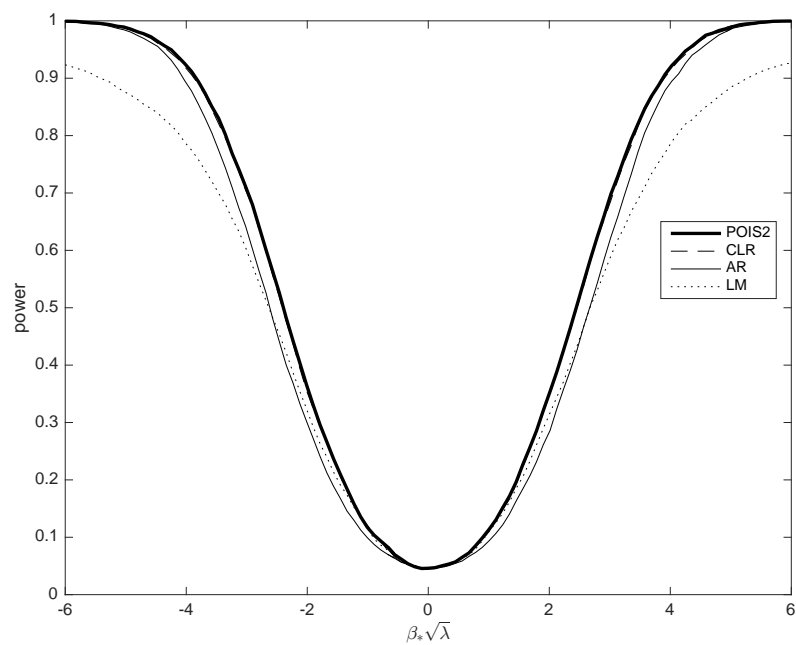


(i)  $\lambda = 5$

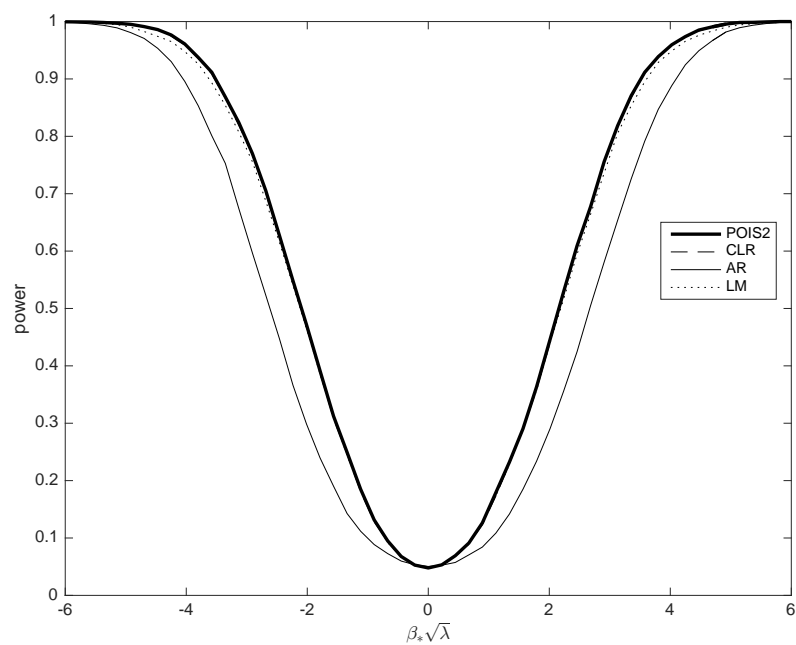


(ii)  $\lambda = 20$

FIGURE SM-I(b). Power of the significance level .05 CLR, LM, and AR tests and the POIS2 power envelope for fixed null value  $\beta_0 = 0$ , varying true value  $\beta_*$ ,  $k = 5$ ,  $\rho_\Omega = 0$ , and  $\lambda = 5, 20$

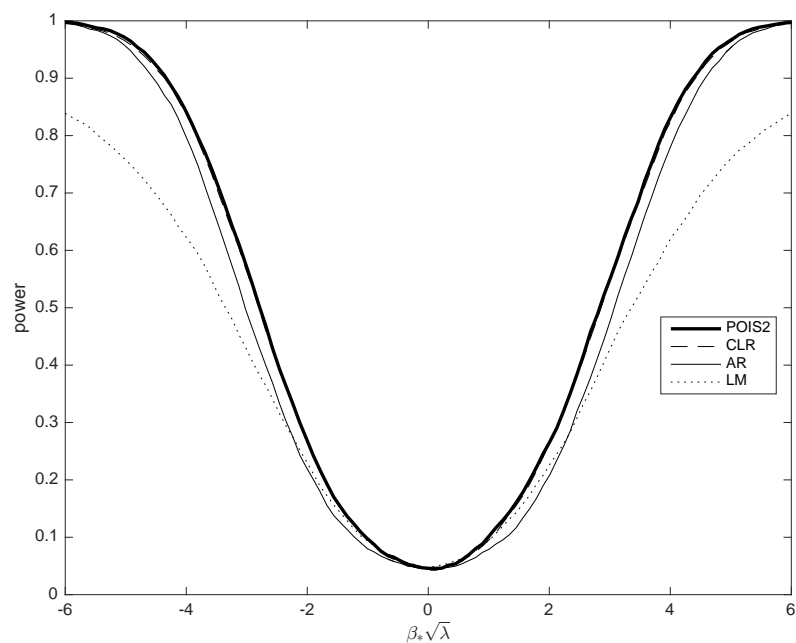


(i)  $\lambda = 5$

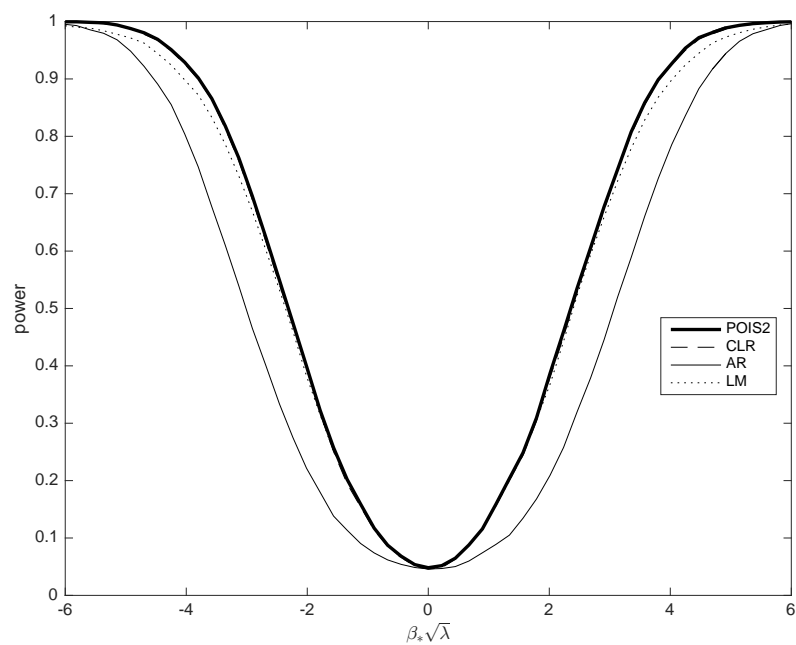


(ii)  $\lambda = 20$

FIGURE SM-I(c). Power of the significance level .05 CLR, LM, and AR tests and the POIS2 power envelope for fixed null value  $\beta_0 = 0$ , varying true value  $\beta_*$ ,  $k = 10$ ,  $\rho_\Omega = 0$ , and  $\lambda = 5, 20$



(i)  $\lambda = 5$



(ii)  $\lambda = 20$