Supplement to "Bayesian estimation of a dynamic stochastic general equilibrium model with asset prices"

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A. Data

The estimation of the benchmark model (Uhlig (2007)) in Section 3 is based on six time series from 1963:qI to 2008:qII. All data are quarterly and in real terms. This supplement describes some modifications and, in particular, the source of the raw data.

We use the quarterly real gross domestic product (GDP) as a measure of aggregate output. We use civilian noninstitutional population over 16 years from the Bureau of Labor Statistics (BLS) as a proxy for population to calculate per capita time series. Finally, we calculate the first differences of the real logarithmic output per capita and afterward reduce the mean of this time series. This mean is used to calibrate the growth rate γ in the model. Consumption is expenditures on nondurables and services. Private investment is calculated as the sum of nominal gross private investment and personal durable consumption, both provided by the Bureau of Economic Analysis (BEA). Both time series—consumption and investment—are transformed into real and per capita terms by using the GDP deflator and the population series mentioned above. Finally, we calculate the demeaned log differences of consumption as well as the demeaned logarithm of the investment–output ratio for the estimation. Additionally, we include hours worked in the estimation. In particular, we use quarterly hours worked by employees working in private, nonfarm business excluding nonprofit business. This series is an updated version of that used by Francis and Ramey (2009). The final logarithmic time series is demeaned.

As a proxy for the riskless real interest rate, we use the quarterly returns calculated based on the monthly returns of the 3-month T-bill provided by the Board of Governors of the Federal Reserve System. The returns are calculated in real terms, too, by using the implicit inflation given by the GDP price deflator. Furthermore, the final logarithmic return series is demeaned. Finally, we also use excess returns as an observable variable. The excess returns are calculated as the log differences between the total returns of the

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S&P 500 and the 3-month T-bill returns. The sources for the aforementioned data are the following.

Real GDP: This series is *BEA NIPA Table 1.1.6, line 1 (A191RX1)*.

Nominal GDP: This series is *BEA NIPA Table 1.1.5, line 1 (A191RC1)*.

Implicit GDP Deflator: The implicit GDP deflator is calculated as the ratio of nominal GDP to real GDP.

Private Consumption: Real consumption expenditures for nondurables and services is the sum of the respective nominal values of the *BEA NIPA Table 1.1.5, line 5 (DND-GRC1)*, and *BEA NIPA Table 1.1.5, line 6 (DNDGRC1)*, and finally deflated by the deflator mentioned above.

Private Investment: Total real private investment is the sum of the respective nominal values of the series gross private investment *BEA NIPA Table 1.1.5, line 7 (A006RC1)*, and personal consumption expenditures: durable goods *BEA NIPA Table 1.1.5, line 4 (DDURRC1)*, and finally deflated by the deflator mentioned above.

Hours Worked: The series measures the hours worked of employees working in private nonfarm business excluding nonprofit business. This series is an updated version of that used by Francis and Ramey (2009) and is available on the authors' website. Source: http://weber.ucsd.edu/~vramey/.

Civilian Population: This series is calculated from monthly data of civilian noninstitutional population over 16 years (CNP16OV) from the U.S. Department of Labor, Bureau of Labor Statistics.

S&P 500: The total returns of the S&P 500 are calculated by the monthly values from the S&P price index and dividends calculated by Robert J. Shiller and provided on his website. Source: http://www.econ.yale.edu/~shiller/data.htm.

Risk-free Rate: The quarterly risk-free return is calculated from monthly returns of the 3-month Treasury bill: secondary market rate provided by the Board of Governors of the Federal Reserve System. The real returns are calculated with the implicit inflation rate of the price deflator series above.

B. Additional material for the Uhlig (2007) model

B.1 *Model solution*

B.1.1 *First-order necessary conditions* The economy described in the paper follows the trend γ. To write the equilibrium conditions in stationary terms, the set of variables has to be detrended by z_{t-1} as

$$\begin{aligned} \tilde{c}_t &= \frac{c_t}{e^{z_{P,t-1}}}, & \tilde{y}_t &= \frac{y_t}{e^{z_{P,t-1}}}, & \tilde{w}_t &= \frac{w_t}{e^{z_{P,t-1}}}, & \tilde{w}_t^f &= \frac{w_t^f}{e^{z_{P,t-1}}}, \\ \tilde{k}_{t-1} &= \frac{k_{t-1}}{e^{z_{P,t-1}}}, & \tilde{x}_t &= \frac{x_t}{e^{z_{P,t-1}}}, & \tilde{H}_t &= \frac{H_t}{e^{z_{P,t-1}}}, & \widetilde{\lambda}_t &= \frac{\lambda_t}{e^{-\eta z_{P,t-1}}}. \end{aligned} \tag{B-1}$$

Following, the set of the stationary first-order necessary conditions of the equilibrium can be rewritten as

$$n_t = 1 - l_t, \tag{B-2}$$

$$R_t^k q_{t-1} = \frac{\theta \tilde{y}_t}{\tilde{k}_{t-1}} + \left(1 - \delta + g\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) q_t - \frac{\tilde{x}_t}{\tilde{k}_{t-1}}, \tag{B-3}$$

$$q_t e^{z_{I,t}} = \frac{1}{g'\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)}, \tag{B-4}$$

$$E\left[R_t^f\right] = E\left[\exp\left(-\log \bar{M} - E_t[\hat{M}_{t+1}] - \frac{\sigma_M^2}{2}\right)\right], \tag{B-5}$$

$$E\left[R_{t+1}^k\right] = E\left[\exp\left(-\log \bar{M} - E_t[\hat{M}_{t+1}] - \frac{\sigma_M^2}{2} + \sigma_{MR^k}\right)\right], \tag{B-6}$$

$$M_t = \beta \frac{\tilde{\lambda}_t}{\tilde{\lambda}_{t-1}} \exp\left(-\eta(\gamma + \varepsilon_{P,t-1})\right), \tag{B-7}$$

$$\tilde{\lambda}_t = (\tilde{c}_t - \tilde{H}_t)^{-\eta} \left(A + \left(e^{z_{L,t}} l_t - F_t\right)^{\nu}\right)^{1-\eta}, \tag{B-8}$$

$$\exp(\varepsilon_{P,t-1}) \tilde{H}_t = (1 - \rho_c) \chi \tilde{c}_{t-1} + \rho_c \tilde{H}_{t-1}, \tag{B-9}$$

$$F_t = (1 - \rho_l) \psi l_{t-1} + \rho_l F_{t-1}, \tag{B-10}$$

$$\tilde{w}_t^f = \frac{e^{z_{L,t}} \upsilon (\tilde{c}_t - \tilde{H}_t)}{A\left(e^{z_{L,t}} l_t - F_t\right)^{1-\nu} + e^{z_{L,t}} l_t - F_t}, \tag{B-11}$$

$$\tilde{w}_t = \frac{(1 - \theta) \tilde{y}_t}{n_t}, \tag{B-12}$$

$$\exp(\mu \varepsilon_{P,t-1}) \tilde{w}_t = (\tilde{w}_{t-1})^{\mu} \left(e^{\varpi + z_{W,t}} \tilde{w}_t^f\right)^{1-\mu}, \tag{B-13}$$

$$\tilde{y}_t = (\tilde{k}_{t-1})^{\theta} \left(\exp(\gamma + \varepsilon_{P,t}) n_t\right)^{1-\theta}, \tag{B-14}$$

$$\exp(\gamma + \varepsilon_{P,t}) \tilde{k}_t = \left(1 - \delta + g\left(e^{z_{I,t}} \frac{\tilde{x}_t}{\tilde{k}_{t-1}}\right)\right) \tilde{k}_{t-1}, \tag{B-15}$$

$$\tilde{y}_t = \tilde{c}_t + \tilde{x}_t + \bar{g} e^{z_{G,t}}. \tag{B-16}$$

The equilibrium is defined together with the exogenous variables $z_{L,t}$, $z_{I,t}$, $z_{W,t}$, and $z_{G,t}$.

B.1.2 *Steady state* To calculate the steady state, we take as given

$$\bar{z}_L = \bar{z}_I = 1 \quad \text{and} \quad \bar{q} = 1, \tag{B-17}$$

as well as that the steady-state ratio of government expenditures to output is 28%:

$$\frac{\bar{g}}{\bar{y}} = 0.28. \tag{B-18}$$

Furthermore, we can calculate the real depreciation rate:

$$\tilde{\delta} = e^{\gamma} + \delta - 1.$$

Remembering the previous discussion about the asset-pricing implications, we know that the Euler equation has to hold for any asset. This implies that (B-5) is equal to (B-6). Given a value for $\bar{R}^f$ and σ_M^2, we can solve for the steady-state pricing kernel:

$$\bar{M} = \exp\left(-\log\left(\bar{R}^f\right) - \frac{\sigma_M^2}{2}\right). \tag{B-19}$$

The return on capital is equal to

$$\bar{R}^k = \frac{1}{\bar{M}\exp\left(\frac{\sigma_M^2}{2} + \sigma_{MR^k}\right)}. \tag{B-20}$$

Now we can also solve for the discount rate:

$$\beta = \bar{m}\exp(\eta\gamma). \tag{B-21}$$

Now we can also solve for

$$\frac{\bar{\tilde{x}}}{\bar{\tilde{y}}} = \frac{\theta\tilde{\delta}}{\bar{R}^k + \delta - 1} \tag{B-22}$$

and

$$\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} = \frac{\bar{R}^k + \delta - 1}{\theta}, \tag{B-23}$$

and, because $\bar{\tilde{x}}/\bar{\tilde{k}} = \tilde{\delta}$, for

$$\frac{\bar{\tilde{c}}}{\bar{\tilde{k}}} = \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}} - \frac{\bar{\tilde{x}}}{\bar{\tilde{k}}} - \frac{\bar{\tilde{g}}}{\bar{\tilde{y}}} \cdot \frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}. \tag{B-24}$$

Given the assumption that steady-state leisure is twice as high as labor, $\bar{l} = 2/3$ and

$$\bar{n} = 1 - \bar{l}, \tag{B-25}$$

we can solve for the steady-state capital

$$\bar{\tilde{k}} = \left[\frac{\bar{\tilde{y}}}{\bar{\tilde{k}}}\right]^{1/(\theta-1)} \bar{n}e^{\gamma}; \tag{B-26}$$

this allows us now to solve for steady-state value $\bar{\tilde{y}}$, $\bar{\tilde{x}}$, $\bar{\tilde{g}}$, and $\bar{\tilde{c}}$.

As shown in Section 5, we use the condition of the Frisch elasticity (τ) to re-solve for the remaining steady states and parameters. In the case of wage rigidities, the steady-state relationship between the market wage and the frictionless wage (marginal rate of substitution),

$$\bar{\tilde{w}} = \bar{\tilde{w}}^{f} e^{\varpi}, \tag{B-27}$$

holds, where the market wage is determined by the condition

$$\bar{\tilde{w}} = (1-\theta)\frac{\bar{\tilde{y}}}{\bar{n}}. \tag{B-28}$$

Now we define the parameter κ as

$$\kappa = \frac{e^{\varpi}}{1-\theta}\frac{1-\bar{l}}{\bar{l}}\frac{\bar{c}}{\bar{y}}. \tag{B-29}$$

Given the Frisch elasticity τ, the following equality has to hold:

$$\Xi = \frac{\bar{l}}{1-\bar{l}}\frac{1}{\tau} - \left(2-\frac{1}{\eta}\right)\frac{1}{(1-\chi)\kappa}. \tag{B-30}$$

Afterward, we can resolve for the remaining parameters by solving the equations

$$\nu = 1-(1-\psi)\Xi, \tag{B-31}$$

$$\alpha = \frac{\kappa\nu(1-\chi)}{1-\psi} - 1, \tag{B-32}$$

$$A = \alpha(1-\psi)^{\nu}\bar{l}^{\nu}. \tag{B-33}$$

Given these remaining parameters, we can solve for the steady-state values of the remaining variables.

B.1.3 *Log linearization* For readability, from now on, we drop $\sim$ as a special mark for detrended variables, drop all steady states marked by $-$, and drop all log deviations from steady states marked by $\wedge$:

$$\hat{l}_t = -\frac{\bar{n}}{1-\bar{n}}\hat{n}_t, \tag{B-34}$$

$$\hat{r}^k_t + \hat{q}_{t-1} = \left[\frac{\bar{R}^k - 1 + \delta}{\bar{R}^k}\right](\hat{y}_t - \hat{k}_{t-1}) + \frac{e^{\gamma}}{\bar{R}^k}\hat{q}_t + \frac{\tilde{\delta}}{\bar{R}^k} z_{I,t}, \tag{B-35}$$

$$\hat{q}_t = \frac{1}{\zeta}\hat{x}_t + \left(\frac{1}{\zeta}-1\right) z_{I,t} - \frac{1}{\zeta}\hat{k}_{t-1}, \tag{B-36}$$

$$\hat{w}_t = \hat{y}_t - \hat{n}_t, \tag{B-37}$$

$$\hat{w}^f_t = z_{L,t} + \hat{c}^d_t + \left[\frac{\nu\alpha}{1+\alpha} - 1\right]\hat{l}^d_t, \tag{B-38}$$

$$\hat{\lambda}_t = -\eta \hat{c}_t^d + \left[\frac{\nu(1-\eta)}{1+\alpha}\right]\hat{l}_t^d, \tag{B-39}$$

$$\hat{H}_t + \varepsilon_{P,t-1} = (1-\rho_c)\hat{c}_{t-1} - \rho_c \hat{H}_{t-1}, \tag{B-40}$$

$$\hat{F}_t = (1-\rho_l)\hat{l}_{t-1} - \rho_l \hat{F}_{t-1}, \tag{B-41}$$

$$(1-\chi)\hat{c}_t^d = \hat{c}_t - \chi \hat{H}_t, \tag{B-42}$$

$$(1-\psi)\hat{l}_t^d = z_{L,t} + \hat{l}_t - \psi \hat{F}_t, \tag{B-43}$$

$$0 = E_t\left[\hat{r}_{t+1}^k + \hat{M}_{t+1}\right], \tag{B-44}$$

$$0 = E_t[\hat{M}_{t+1}] + \hat{r}_t^f, \tag{B-45}$$

$$\hat{M}_t = \hat{\lambda}_t + \hat{\lambda}_{t-1} - \eta \varepsilon_{P,t-1}, \tag{B-46}$$

$$\hat{y}_t = \theta \hat{k}_{t-1} + (1-\theta)\hat{n}_t + (1-\theta)\varepsilon_{P,t}, \tag{B-47}$$

$$\hat{w}_t = \mu \hat{w}_{t-1} + (1-\mu)\hat{w}_t^f + (1-\mu) z_{W,t} - \mu \varepsilon_{P,t-1}, \tag{B-48}$$

$$e^{\gamma}\hat{k}_t = (1-\delta)\hat{k}_{t-1} + \tilde{\delta}\hat{x}_t + \tilde{\delta} z_{I,t} - e^{\gamma}\varepsilon_{P,t}, \tag{B-49}$$

$$\bar{y}\hat{y}_t = \bar{c}\hat{c}_t + \bar{x}\hat{x}_t + \bar{g} z_{G,t}, \tag{B-50}$$

$$z_{W,t} = \pi_W z_{W,t-1} + \varepsilon_{W,t}, \tag{B-51}$$

$$z_{L,t} = \pi_L z_{L,t-1} + \varepsilon_{L,t}, \tag{B-52}$$

$$z_{I,t} = \pi_I z_{I,t-1} + \varepsilon_{I,t}, \tag{B-53}$$

$$z_{G,t} = \pi_G z_{G,t-1} + \varepsilon_{G,t}. \tag{B-54}$$

B.2 *Tables and figures*

B.2.1 *Prior and posterior distribution*

TABLE S1. Prior distribution for the model parameter and the additional parameter. Para(1) and Para(2) correspond to means and standard deviations for the Beta, Gamma, inverted Gamma, and Normal distribution, while for the uniform distribution, these values correspond to the lower and upper bounds. The acronym s.s. indicates steady-state values.

		Domain	Density	Para(1)	Para(2)
Model parameter					
μ	Wage rigidity	$[0, 1)$	Beta	0.75	0.1
η	Power utility parameter	$\mathbb{R}^+$	Uniform	1	200
χ	Consumption habit	$[0, 1)$	Beta	0.5	0.23
ψ	Leisure habit	$[0, 1)$	Beta	0.5	0.23
ρ_c	Consumption habit	$[0, 1)$	Beta	0.5	0.23
ρ_l	Leisure habit	$[0, 1)$	Beta	0.5	0.23
δ	Depreciation rate	$[0, 1)$	Beta	0.02	0.005
ζ	Investment adjustment costs	$\mathbb{R}$	Normal	4.0	1.0
$1/\tau$	Inverse Frisch elasticity (s.s.)	$\mathbb{R}^+$	Gamma	1.00	0.750
$\log \bar{R}^f$	Risk-free return (s.s.)	$\mathbb{R}^+$	InvGam	0.005	0.01
Υ	Leverage	$[0, 1)$	Beta	0.5	0.23
Autoregressive parameter and s.d. of shocks					
π_G	AR government shock	$[0, 1)$	Beta	0.85	0.1
π_W	AR wage mark-up shock	$[0, 1)$	Beta	0.85	0.1
π_I	AR investment shock	$[0, 1)$	Beta	0.85	0.1
π_L	AR labor supply shock	$[0, 1)$	Beta	0.85	0.1
σ_P	s.d. technology shock	$\mathbb{R}^+$	InvGam	0.01	4.0
σ_W	s.d. wage mark-up shock	$\mathbb{R}^+$	InvGam	0.01	4.0
σ_I	s.d. investment shock	$\mathbb{R}^+$	InvGam	0.01	4.0
σ_L	s.d. labor supply shock	$\mathbb{R}^+$	InvGam	0.01	4.0
σ_G	s.d. government shock	$\mathbb{R}^+$	InvGam	0.01	4.0
σ_Q	s.d. excess return shock	$\mathbb{R}^+$	InvGam	0.01	4.0

TABLE S2. MCMC results.

Parameter	Unconstrained Prior: Posterior Mean	Unconstrained Prior: HPD 5%	Unconstrained Prior: HPD 95%	Constrained Prior: Posterior Mean	Constrained Prior: HPD 5%	Constrained Prior: HPD 95%
Model parameter						
μ	0.27	0.14	0.39	0.29	0.16	0.42
η	4.95	2.11	8.09	108.18	84.19	134.30
χ	0.87	0.81	0.93	0.84	0.78	0.91
ψ	0.86	0.80	0.92	0.84	0.79	0.90
ρ_c	0.61	0.50	0.72	0.66	0.55	0.76
ρ_l	0.07	0.00	0.13	0.07	0.00	0.14
Υ	0.12	0.01	0.23	0.11	0.01	0.21
ζ	7.87	6.76	9.01	7.84	6.80	8.96
δ	0.02	0.01	0.02	0.02	0.01	0.02
$1/\tau$	5.53	3.61	7.44	7.11	5.19	9.05
$\log(\bar{R}^f) \times 100$	0.47	0.27	0.65	0.32	0.17	0.46
Autoregressive parameter and s.d. of shocks						
π_G	0.91	0.88	0.95	0.93	0.89	0.96
π_I	0.71	0.64	0.77	0.70	0.64	0.76
π_W	0.60	0.48	0.71	0.93	0.89	0.97
π_L	0.92	0.88	0.97	0.65	0.54	0.77
$\sigma_P \times 100$	0.91	0.83	0.98	0.90	0.83	0.97
$\sigma_I \times 100$	1.35	1.07	1.62	1.32	1.08	1.56
$\sigma_L \times 100$	0.31	0.27	0.34	0.29	0.26	0.32
$\sigma_W \times 100$	2.05	1.49	2.56	2.11	1.55	2.65
$\sigma_G \times 100$	1.97	1.71	2.24	1.90	1.69	2.11
$\sigma_Q \times 100$	7.65	6.96	8.31	7.38	6.77	8.02
Log marginal density	3439.74			3437.86		

TABLE S3. Distributions of implicit model parameter and steady-state values.

Parameter	Unconstrained Prior: Posterior Mean	Unconstrained Prior: HPD 5%	Unconstrained Prior: HPD 95%	Constrained Prior: Posterior Mean	Constrained Prior: HPD 5%	Constrained Prior: HPD 95%
β	1.016	1.004	1.029	1.044	0.954	1.139
ν	5.831	3.440	8.134	5.109	2.935	7.441
α	0.595	0.392	0.792	0.607	0.325	0.880
$\bar{x}/\bar{y}$	0.323	0.294	0.351	0.313	0.294	0.333
$\bar{c}/\bar{y}$	0.397	0.369	0.426	0.407	0.387	0.426
RRA	39.20	15.26	63.95	719.25	419.23	1032.3

B.2.2 *Business-cycle facts*

TABLE S4. HP-filtered ($\lambda = 1600$) simulated and empirical moments. The simulated moments are based on 500 parameter vector draws from the posterior. For each draw, we simulate 100 time series for each variable of interest. After removing a burn-in for each simulation, the final simulated time series have the same length as the vector of observable. The numbers in brackets indicate 5% and 95% probabilities.

		Unconstrained Prior	Constrained Prior	Data
Standard Deviation of Output				
Output × 100	$\hat{y}$	1.26	1.24	1.69
		[1.06; 1.50]	[1.05; 1.48]	
Relative Standard Deviation to Output				
Consumption	$\hat{c}$	1.04	1.04	0.50
		[0.85; 1.25]	[0.85; 1.26]	
Investment	$\hat{x}$	2.97	3.12	3.75
		[2.51; 3.50]	[2.66; 3.67]	
Hours worked	$\hat{n}$	1.14	1.12	1.24
		[0.99; 1.30]	[0.96; 1.27]	
Correlation with Output				
Consumption	$\hat{c}$	0.64	0.61	0.76
		[0.46; 0.77]	[0.41; 0.75]	
Investment	$\hat{x}$	0.75	0.76	0.84
		[0.63; 0.84]	[0.64; 0.85]	
Hours worked	$\hat{n}$	0.78	0.77	0.89
		[0.67; 0.85]	[0.66; 0.85]	

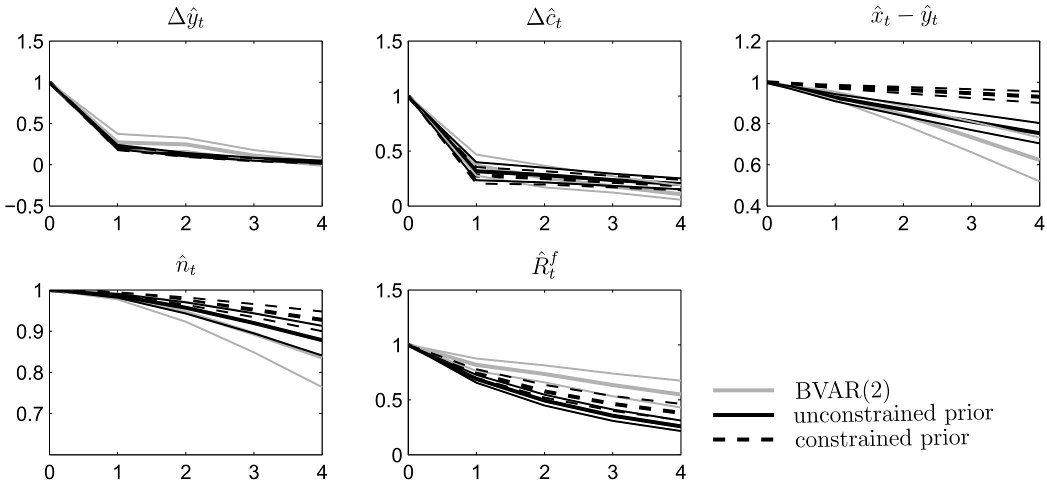


FIGURE S1. Predicted autocorrelation (population moments) of observable variables of the DSGEs and the BVAR(2).

B.3 *Historical and variance decomposition*

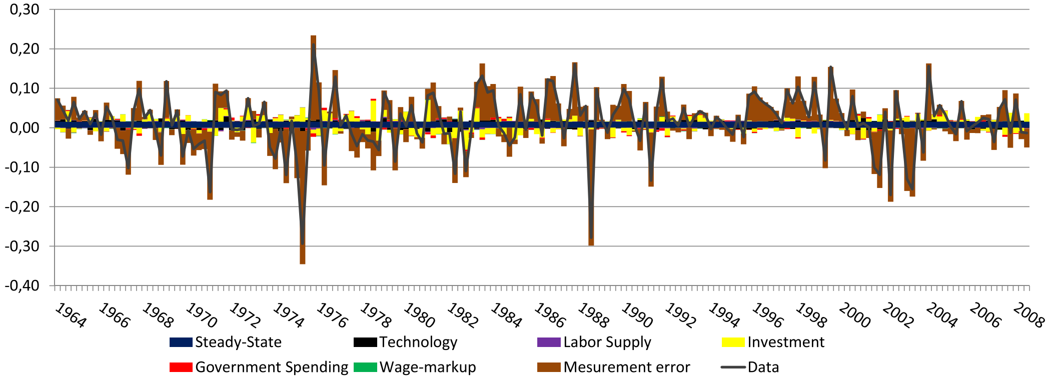


FIGURE S2. Historical decomposition of excess returns (individual shocks) calculated at the posterior mode of the estimation with a constrained prior.

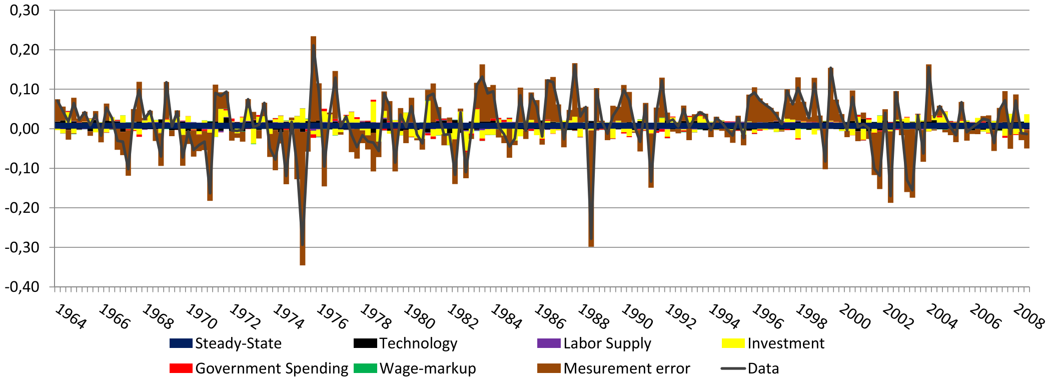


FIGURE S3. Historical decomposition of excess returns (grouped shocks) calculated at the posterior of the estimation with a constrained prior mode.

TABLE S5. Variance decomposition of selected model variables and conditional decomposition of selected asset-pricing facts calculated at the corresponding posterior mode.

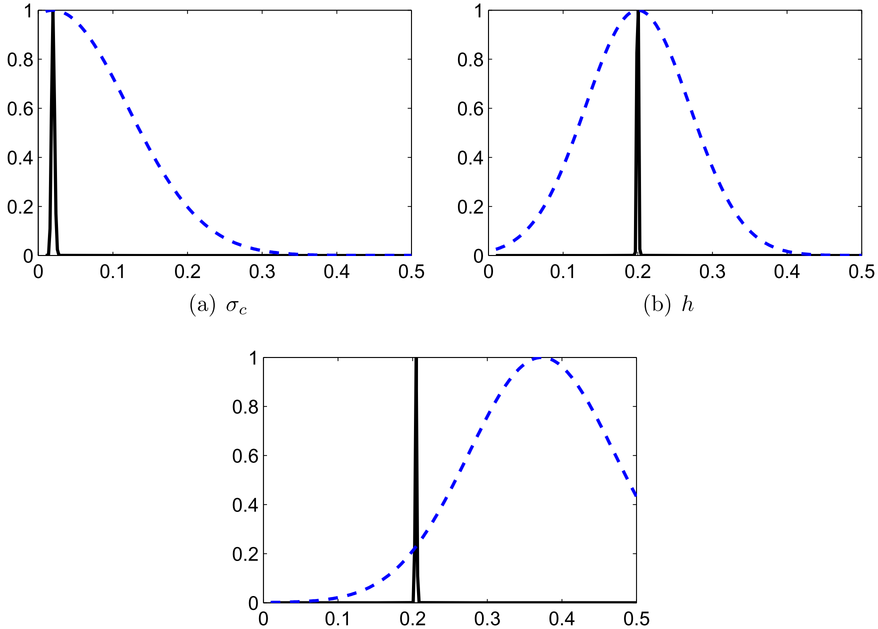
Variable	Symbol	ε_P	ε_L	ε_I	ε_G	ε_W	ε_Q
Unconstrained prior							
Excess returns	$\hat{R}_t^q$	0.26	0.03	4.79	0.11	0.00	94.81
Stochastic discount factor	$\hat{m}_t$	53.75	37.91	5.13	3.20	0.00	0.00
Return on capital	$\hat{R}_t^k$	3.96	3.97	89.14	2.89	0.05	0.00
Risk-free return	$\hat{R}_t^f$	3.68	14.69	79.79	1.82	0.02	0.00
Risk premium	$-\sigma_{MR^k}$	0.009	0.005	0.012	0.002	0.00	0.00
Sharpe ratio	ω	0.75	0.38	0.98	0.16	0.00	0.00
In percent		*32.94*	*16.60*	*43.37*	*7.07*	*0.02*	*0.00*
Constrained prior							
Excess returns	$\hat{R}_t^q$	0.26	0.02	4.88	0.12	0.00	94.71
Stochastic discount factor	$\hat{m}_t$	99.18	0.39	0.00	0.42	0.01	0.00
Return on capital	$\hat{R}_t^k$	5.03	3.10	87.53	3.98	0.36	0.00
Risk-free return	$\hat{R}_t^f$	8.91	7.77	75.05	6.52	1.74	0.00
Risk premium	$-\sigma_{MR^k}$	0.2167	0.009	0.001	0.013	0.000	0.000
Sharpe ratio	ω	18.40	0.79	0.06	1.09	0.01	0.00
In percent		*90.44*	*3.90*	*0.28*	*5.35*	*0.03*	*0.00*

B.4 *Robustness exercises*

We investigated in more detail the sensitivity of our estimation results regarding the tightness of our chosen prior for $g(\omega|X)$ and $f(\omega)$: both set implicitly the tightness of the transformation function $h(\cdot)$.

Figure S4 shows the probability density function (pdf) for the benchmark choice (black solid line) as well as the pdf for an alternative specification of g and f (blue dashed line). In particular, we choose the distribution f to be close to the data by assuming $f \sim N(0.2, 0.07^2)$. Hence, to ensure that the tails of h die out and that we therefore guarantee a proper prior, we pick a looser distribution for $g \sim N(0.02, 0.1^2)$. Given that we want to abstract from negative Sharpe ratios for economic reason, we discard all draws that imply a negative Sharpe ratio.

The results for this alternative constraint is quite different. The approach with $g \sim N(0.02, 0.1^2)$ and $f \sim N(0.2, 0.07^2)$ generates a far smaller Sharpe ratio of 0.105, which is approximately half of the mean of the targeted distribution f. The reduction of the Sharpe ratio by 1/2 goes along with a smaller preference parameter η of around 52, which is also half of the value from the benchmark exercise. Hence, the constraint imposed is not informative enough to ensure that we just sample from an area of the posterior that would give that $g(\omega|X) \approx f(\omega)$.



(a) σ_c (b) h (c) σ_l

FIGURE S4. Distributions of $g(\omega|X)$, $f(\omega)$, and $h(\omega)$ for different assumptions about the underlying distribution.

C. ADDITIONAL MATERIAL FOR THE SMETS AND WOUTERS MODEL

Table S6 shows the estimation results for the Smets and Wouters (2007) model. The parameters are the same as in the original paper (https://www.aeaweb.org/articles.php?doi=10.1257/aer.97.3.586). To follow our changes as described in Section 4 of the paper, we present a comprehensive list of equations together with the replications files.

TABLE S6. MCMC results for the Smets and Wouters (2007) model.

Parameter	Unconstrained Prior Posterior Mean	Unconstrained Prior HPD 5%	Unconstrained Prior HPD 95%	Constrained Prior Posterior Mean	Constrained Prior HPD 5%	Constrained Prior HPD 95%
Model parameter						
μ_p	0.69	0.51	0.87	0.81	0.68	0.94
μ_w	0.84	0.75	0.92	0.88	0.82	0.94
φ	5.17	3.36	6.92	6.87	5.30	8.61
σ_c	**1.59**	**1.23**	**1.97**	**8.99**	**6.91**	**10.81**
h	0.59	0.51	0.67	0.46	0.40	0.52
ξ_w	**0.68**	**0.58**	**0.79**	**0.81**	**0.75**	**0.87**
σ_l	**1.55**	**0.78**	**2.45**	**2.11**	**1.22**	**2.94**
ξ_p	**0.66**	**0.57**	**0.75**	**0.82**	**0.78**	**0.85**
ι_w	0.61	0.42	0.80	0.51	0.30	0.70
ι_p	0.23	0.08	0.37	0.22	0.08	0.35
Ψ	0.46	0.29	0.61	0.22	0.11	0.32
Φ	1.57	1.45	1.68	1.69	1.57	1.79
r_π	2.02	1.74	2.28	1.95	1.66	2.21
ρ	0.81	0.77	0.85	0.88	0.85	0.91
r_y	0.09	0.06	0.13	0.12	0.07	0.16
$r_{\Delta y}$	0.25	0.20	0.29	0.17	0.13	0.20
$\bar{\pi}$	0.79	0.62	0.94	0.76	0.60	0.92
$100(R^f - 1)$	0.65	0.49	0.79	0.64	0.36	0.96
$\bar{l}$	0.06	−1.70	1.73	−0.72	−2.63	1.02
$\bar{\gamma}$	0.43	0.40	0.45	0.43	0.40	0.46
α	0.25	0.22	0.30	0.24	0.19	0.28
ρ_{ga}	0.56	0.43	0.70	0.49	0.31	0.64
Υ	0.11	0.01	0.22	0.12	0.01	0.24
Autoregressive parameter and s.d. of shocks						
ρ_a	**0.95**	**0.93**	**0.97**	**0.98**	**0.96**	**0.99**
ρ_b	0.39	0.20	0.59	0.90	0.87	0.93
ρ_g	0.98	0.96	0.99	0.99	0.99	1.00
ρ_I	0.77	0.67	0.86	0.57	0.50	0.65
ρ_r	0.12	0.03	0.20	0.11	0.03	0.19
ρ_p	0.89	0.81	0.97	0.90	0.84	0.96
ρ_w	0.96	0.93	0.98	0.94	0.91	0.97
σ_a	0.45	0.40	0.50	0.45	0.41	0.49
σ_b	**0.20**	**0.15**	**0.24**	**0.03**	**0.02**	**0.04**
σ_g	0.51	0.47	0.56	0.57	0.52	0.62
σ_I	0.41	0.34	0.48	0.58	0.47	0.68
σ_r	0.25	0.22	0.27	0.23	0.21	0.25
σ_P	0.14	0.10	0.16	0.12	0.09	0.16
σ_w	0.25	0.21	0.29	0.22	0.19	0.26
$\sigma_Q \times 0.1$	0.79	0.71	0.85	0.79	0.71	0.87
Log marginal density	−1481.65			−1538.21		

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