Supplement to "Spatial interactions"

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Appendix A: Social capital fixed point

The fixed point in social capital can be computed by rewriting equation (12) as $n_{ij,r}s_{j,r} = (n_0 + \theta_{ij,r})s_{j,r} - cd_{ij,r}$, so that (13) becomes

$$s_{j,r} = 1 + \frac{\alpha}{N_r} \sum_{k=1,k\neq j}^{N_r} \left[(n_0 + \theta_{jk,r})s_{k,r}\right] - \frac{\alpha}{N_r} c \sum_{k=1,k\neq j}^{N_r} d_{jk,r}, \tag{A.1}$$

where the last term is $g_{j,r} = \sum_{k=1,k\neq j}^{N_r} c(d_{jk,r}) = c\sum_{k=1,k\neq j}^{N_r} d_{jk,r}$, the linear-cost equivalent of the access cost measure defined in (4) in the model. The system of linear equations (A.1) can be written in vector-matrix form as

$$\mathbf{s}_r = \mathbf{1}_r + \frac{\alpha}{N_r}(\mathbf{N}_{0,r} + \mathbf{\Theta}_r)\mathbf{s}_r - \frac{\alpha}{N_r} c\mathbf{D}_r \mathbf{1}_r, \tag{A.2}$$

where $\mathbf{s}_r = (s_{i,r})$ is a $(N_r \times 1)$ vector; $\mathbf{1}_r$ is the $(N_r \times 1)$ vector of 1; $\mathbf{N}_{0,r}$ is an $(N_r \times N_r)$ matrix in which the off-diagonal elements are n_0 and the diagonal elements are all zero; $\mathbf{\Theta}_r = (\theta_{ij,r}) = (x_{ij,r}^{\mathrm{T}}\beta + \varepsilon_{ij,r})$ is an $(N_r \times N_r)$ matrix; $\mathbf{D}_r = (d_{ij,r})$ is an $(N_r \times N_r)$ matrix.

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Namely,

$$\mathbf{D}_r = \begin{pmatrix} d_{11,r} & \cdots & d_{1i,r} & \cdots & d_{1N_r,r} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ d_{i1,r} & \cdots & d_{ii,r} & \cdots & d_{iN_r,r} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ d_{N_r1,r} & \cdots & d_{N_ri,r} & \cdots & d_{N_rN_r,r} \end{pmatrix} \quad \text{and}$$
$$\mathbf{\Theta}_r = \begin{pmatrix} \theta_{11,r} & \cdots & \theta_{1i,r} & \cdots & \theta_{1N_r,r} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \theta_{i1,r} & \cdots & \theta_{ii,r} & \cdots & \theta_{iN_r,r} \\ \vdots & \ddots & \vdots & \ddots & \vdots \\ \theta_{N_r1,r} & \cdots & \theta_{N_ri,r} & \cdots & \theta_{N_rN_r,r} \end{pmatrix}. \tag{A.3}$$

Solving the system of linear equations (A.1) leads to

$$\mathbf{s}_r^* = \left[\mathbf{I}_r - \frac{\alpha}{N_r}(\mathbf{N}_{0,r} + \mathbf{\Theta}_r)\right]^{-1}\left(\mathbf{I}_r - \frac{\alpha}{N_r} c\mathbf{D}_r\right)\mathbf{1}_r, \tag{A.4}$$

where $\mathbf{I}_r$ is the $(N_r \times N_r)$ identity matrix. The matrix $\mathbf{I}_r - \alpha(\mathbf{N}_{0,r} + \mathbf{\Theta}_r)$ is invertible if $\alpha < \frac{1}{\rho(\mathbf{N}_{0,r}+\mathbf{\Theta}_r)}$, where $\rho(\mathbf{N}_{0,r} + \mathbf{\Theta}_r)$ is the spectral radius of the matrix $\mathbf{N}_0 + \mathbf{\Theta}_r$. When this condition is satisfied, there is a unique solution to the system of linear equations (A.1).

Appendix B: Monte Carlo simulations

We carry out Monte Carlo simulation experiments to demonstrate that our structural estimation method can precisely capture the value of parameters in a complicated data generating process of social interactions among students. Each experiment is concerned with estimating the parameters in the model that we discussed in Section 6. That is,

$$n_{ij,r}^* = n_0 - \frac{cd_{ij,r}}{s_{j,r}^*} + \theta_{ij,r}, \tag{B.5}$$

and

$$s_{j,r}^* = 1 + \frac{\alpha}{N_r}\sum_{k=1}^{N_r} n_{jk,r}^* s_{k,r}^*, \tag{B.6}$$

where

$$\theta_{ij,r} = \beta_1|x_{i,r} - x_{j,r}| + \beta_2(x_{i,r} + x_{j,r}) + \varepsilon_{ij,r}. \tag{B.7}$$

We set the values of structural parameters as the ones we have estimated in our structural estimation. That is, $n_0 = 1.5$, $\alpha = 0.12$, and $c = 0.2$. We assign -0.3 for the parameter β_1 to assume homophily and 0.2 for β_2 to have positive the effect of combined levels on social interactions. The data generating processes for x_i and ε are the uniform distribution from the interval of $(0, 5)$ and the normal distribution with mean zero and standard deviation $\sigma_\varepsilon = 1.3$.

TABLE B.1. Monte Carlo simulation results.

		Number of Networks (R)		
		50 Networks	100 Networks	150 Networks
n_0	Average	1.5269	1.5264	1.5275
(True value = 1.5)	Bias	0.0269	0.0264	0.0275
	RMSE	0.0448	0.042	0.0388
α	Average	0.1204	0.1208	0.1205
(True value = 0.12)	Bias	0.0004	0.0008	0.0005
	RMSE	0.0026	0.0023	0.0026
c	Average	0.2006	0.1994	0.20005
(True value = 0.2)	Bias	0.0006	0.0006	0.00005
	RMSE	0.0045	0.005	0.0041
β_1	Average	−0.2987	−0.2991	−0.2995
(True value = −0.3)	Bias	0.0013	0.0009	0.0005
	RMSE	0.0053	0.0065	0.0063
β_2	Average	0.2022	0.2027	0.2024
(True value = 0.2)	Bias	0.0022	0.0027	0.0024
	RMSE	0.0052	0.0053	0.0043
σ_ε	Average	1.3150	1.3100	1.3134
(True value = 1.3)	Bias	0.0150	0.0100	0.0134
	RMSE	0.0298	0.0218	0.0283

Note: A total of 100 simulations for each experiment.

We generate $R = 50$, 100, and 150 networks, which correspond to connected components as in our empirical setup. Each network has four to ten individuals. Using the social interaction and social capital fixed points, that is, equations (16) and (17), we generate $n^*_{ij,r}$ for all networks and all pairs.

We generate $H = 100$ sets of generated sample of R networks. For each set of generated data, we run the I-I estimation method. Each hth estimation requires the estimation of the weight matrix A in equation (22) using a bootstrap method and the generation of additional $T = 100$ sets of simulation errors. Although the dimension of the parameter vector is smaller than that in the empirical analysis, this Monte Carlo simulation is also computationally heavy. Hence, to facilitate the computation, we reduce the size of the bootstrap sample for the weight matrix estimation from 3000 in the empirical analysis to 100.

The results of the Monte Carlo simulations are displayed in Table B.1. We report the averages of the estimate, bias, and the Root Mean Squared Error (RMSE) for each method. In general, regardless of the number of networks, our structural estimation method that employs indirect inference captures accurately the value of true parameters in the data generating process. In particular, we succeed to estimate the most important structural parameters, α and c, very precisely.

Appendix C: Calibration in the policy exercises

Consider equations (31) and (35) in Section 7 and denote them as follows:

$$n_{ij,r} = n_0 + \theta_{ij,r} - \frac{\sigma_r - (1-\tau_r)cd_{ij,r}}{s_{j,r}}, \tag{C.8}$$

and

$$s_{j,r} = 1 + \frac{\alpha}{N_r} \sum_{k=1,k\neq j}^{N_r} n_{jk,r} s_{k,r},$$

where we implement together the two policies. The first equation can be written as

$$n_{ij,r} s_{j,r} = (n_0 + \theta_{ij,r}) s_{j,r} + \sigma_r - (1-\tau_r)cd_{ij,r},$$

so that the second equation becomes

$$s_{j,r} = 1 + \frac{\alpha}{N_r} \sum_{k=1,k\neq j}^{N_r} \big[(n_0 + \theta_{jk,r}) s_{k,r}\big] - \frac{\alpha}{N_r} \sum_{k=1}^{N_r} \big[\sigma_r - (1-\tau_r)cd_{jk,r}\big]. \tag{C.9}$$

Denote by $\mathbf{s}_r = (s_{1,r}, \ldots, s_{n,r})^{\mathrm{T}}$ the $(N_r \times 1)$ vector of social capital. Thus, in vector-matrix form, (C.9) can be written as

$$\mathbf{s}_r = \mathbf{1}_r + \alpha(\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r)\mathbf{s}_r + \alpha\sigma_r N_r \mathbf{1}_r - \alpha(1-\tau_r)c\mathbf{D}_r\mathbf{1}_r.$$

Solving this equation leads to

$$\mathbf{s}_r = \big[\mathbf{I}_r - \alpha(\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r)\big]^{-1} \big[(1 + \alpha\sigma_r N_r)\mathbf{1}_r - \alpha(1-\tau_r)c\mathbf{D}_r\mathbf{1}_r\big],$$

or, equivalently,

$$\mathbf{s}_r = \big[\mathbf{I}_r - \alpha(\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r)\big]^{-1} \big[(1 + \alpha\sigma_r N_r)\mathbf{I}_r - \alpha(1-\tau_r)c\mathbf{D}_r\big]\mathbf{1}_r. \tag{C.10}$$

The matrix $\mathbf{I}_r - \alpha(\mathbf{N}_0 + \boldsymbol{\Theta}_r)$ is invertible if $\alpha < \frac{1}{\rho(\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r)}$, where $\rho(\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r)$ is the spectral radius of the matrix $\mathbf{N}_{0,r} + \boldsymbol{\Theta}_r$. Consequently, we could solve the model using (C.8) and (C.10). Observe that $n_{ij,r} > 0$ if $(1 + \theta_{ij,r}) s_{j,r} > (1-\tau_r)cd_{ij,r}$, $\forall i, j$. A sufficient condition is

$$s_{j,r} > \max_i \frac{(1-\tau_r)cd_{ij,r} - \sigma_r}{(1 + \theta_{ij,r})}.$$

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