Supplement to "Borrowing into debt crises"

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Appendix A: Formulation of the model with noise

Government The government that is current on its debt obligations has the general value function given by

$$W(B, H_{-1}, Y, \boldsymbol{\epsilon}) = \max_{d\in\{0,1\}} \big\{ d\big[V^D(H_{-1}, Y, \boldsymbol{\epsilon}) + \epsilon_{J+1}\big] + (1-d)V^R(B, H_{-1}, Y, \boldsymbol{\epsilon})\big\}, \tag{11}$$

where $\boldsymbol{\epsilon}$ is a $J+1$-dimensional vector of extreme value shocks. The distribution of these shocks is specified below.

The value of the government associated with repayment of debt ($d = 0$) is given by

$$V^R(B_i, H_{-1}, Y, \boldsymbol{\epsilon}) = \max_{j, C\geq 0, L\geq 0} \big\{ U\big(C^\alpha L^{1-\alpha}\big) + \beta \mathbb{E}_{Y'|Y}\mathbb{E}_{\boldsymbol{\epsilon}'} W\big(B_j, H, Y', \boldsymbol{\epsilon}'\big) + \epsilon_j \big\}, \tag{12}$$

where

$$C + wL = \tau Y - \big(\delta + (1-\delta)\kappa\big)B_i + Q(B_j, H, Y)\big(B_j - (1-\delta)B_i\big) + \phi_1 \min\left\{\frac{H}{H_{-1}} - 1, 0\right\},$$

$$H = \phi_0 C + (1-\phi_0)wL.$$

A sovereign who defaults ($d = 1$) is excluded from international credit markets and has probability θ of being readmitted every subsequent period. The associated value is

$$V^D(H_{-1}, Y, \boldsymbol{\epsilon}) = U\big(C^\alpha L^{1-\alpha}\big) + \beta \mathbb{E}_{Y'|Y}\mathbb{E}_{\boldsymbol{\epsilon}'}\big[\theta W\big(0, H, Y', \boldsymbol{\epsilon}'\big) + (1-\theta)V^D\big(H, Y', \boldsymbol{\epsilon}'\big)\big] + \epsilon_{J+1} \tag{13}$$

subject to

$$C + wL = \tau Y^d(Y)$$

$$H = \phi_0 C + (1-\phi_0)wL.$$

The two constraints are the budget constraint and the law of motion for legacy contracts. $Y - Y^d(Y)$ reflects the output cost of defaulting.

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International lenders The lenders are assumed to be risk-neutral and perfectly competitive. The actuarially fair bond price that compensates them for default risk is

$$Q(B', H, Y) = \frac{1}{1+r}\mathbb{E}_{Y'|Y}\mathbb{E}_{\boldsymbol{\epsilon}'}\big[\big(1 - d(B', H, Y', \boldsymbol{\epsilon}')\big) \times \big(\delta + (1-\delta)\kappa + (1-\delta)Q(B'', H', Y')\big)\big], \tag{14}$$

where

$$B'' = B'(B', H, Y', \boldsymbol{\epsilon}'),$$
$$H' = H(B', H, Y', \boldsymbol{\epsilon}').$$

The joint distribution of extreme value shocks takes the following standard form:

$$F(\boldsymbol{\epsilon}) = \exp\left[-\left(\sum_{j=1}^{J}\exp\left(\frac{\epsilon_j - \mu}{\rho\sigma}\right)^{\rho} + \exp\left(\frac{\epsilon_{J+1} - \mu}{\rho\sigma}\right)\right)\right],$$

where loosely σ determines the the variability of the shocks and ρ determines the correlation of the debt shocks. To determine the values of ρ and σ, we fix the former to 0.02 and find lowest σ for which our baseline specification converges. The resulting σ is 0.016. The same values are used in computing all other specifications. To assess the impact of the noise, we recalibrate the flexible parametrization for smaller values of sigma. We find that the impact of lower σ on standard deviation of spreads and other key moments is small. We further investigate the impact of the extreme value shocks in the next Appendix.

Appendix B: Comparison of solution methods

In this Appendix, we show that incorporating extreme value shocks in our solution technique has little impact on both quantitative and qualitative results. We first compare the solution with extreme value shocks to the solutions without these shocks in the flexible and fixed specifications. To achieve convergence without the extreme value shocks, we use continuous choice of next period debt, numerical integration to calculate the expectations, and interpolation to evaluate off-grid points (Hatchondo and Martinez (2009)).[1] Importantly, we do not recalibrate the models but instead use the same parameter values as listed in Tables 3 and 5 in the main paper.[2] In the baseline specification, absence of extreme value shocks leads to nonconvergence. To assess the importance of the extreme value shocks in the baseline specification we follow (Dvorkin, Sánchez, Sapriza, and Yurdagul (2021)) in forcing the government to choose the most likely debt outcome, that is, the most desirable debt level.[3]

[1] We use 51 points for both the grid of assets and the grid of income. Off-grid points in the assets domain are evaluated using cubic spline interpolation, while off-grid income realizations are interpolated linearly.

[2] We recalibrate these models in Appendix C for the sake of evaluating the effects of global shocks.

[3] We thank one of our referees for suggesting this exercise.

TABLE 11. Simulated moments: noise versus continuous.

Statistic	Data	Flexible		Fixed	
		noise	*continuous*	*noise*	*continuous*
$ave(debt/Y)$	48.00	48.78	48.43	48.70	48.40
$ave(S)$	3.03	3.01	2.91	3.01	2.93
$std(S)$	2.21	0.83	0.79	1.34	1.34
$std(D)/std(Y)$	0.26	0.60	0.59	0.42	0.41
$corr(S,D)$	−0.58	−0.79	−0.86	−0.86	−0.89
$corr(Y,D)$	0.00	0.55	0.57	0.67	0.70
$corr(Y,S)$	−0.42	−0.83	−0.86	−0.90	−0.90
$std(C+L)/std(Y)$	1.57	1.40	1.44	1.29	1.34

Note: The empirical moments are calculated for Mexico's data covering 1994–2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.

Table 11 compares the relevant (targeted and untargeted) moments obtained with the two solution methods. We do this for both the flexible model, as well as the fixed labor variant (i.e., one that features a "minimum consumption" utility). As is evident, for both variants of the model, all moments are quantitatively close across the two solution methods. In particular, notice that the standard deviation of spread, the main object of interest in this paper, is 0.83% with noise and 0.79% with the exact computation in the flexible model, and it is 1.34% for both solution methods in the fixed labor model. Therefore, our extreme value shocks on debt choices add negligible volatility to the spread and have little impact on other moments in these two specifications.

Naturally, the question remains whether the inclusion of extreme value shocks generates extra spread volatility in the baseline specification. In this specification, the continuous choice and interpolation technique is numerically much less stable. This is why we rely on the extreme value shocks to robustly compute the equilibrium. To assess the impact of these shocks, we follow (Dvorkin et al. (2021)) in supposing the government each period disregards the extreme value shocks and selects the most desirable debt level. Crucially, creditors continue to perceive the government as impacted by the extreme value shocks.[4] This allows us to compute the equilibrium while at the same time removing the "noise" from the governments decision.[5]

The results from this exercise are reported in Table 12. Columns labeled "noise" correspond to the government being subjected to the extreme value shocks as in the main text. "Modal" refers to the exercise described above in which the government does not take the extreme value shocks into account. First, notice that the change in spread volatility between "noise" and "modal" is of the same magnitude as "noise" and "continuous" in the flexible and fixed specifications. This gives us confidence in this exercise

[4]In practice, this is implemented by solving the value function iteration and the resulting borrowing price assuming the extreme value shocks are in effect. However, in the simulations the debt level assigned the highest probability prior to the realization of the extreme value shocks is selected.

[5]This approach allows us to remove one of the impacts of the extreme value shocks, namely the realized "noise" in the government's decisions. However, we are not able to rule out the possibility that creditors' perception of noise and the resulting impact on the price schedule affects equilibrium outcomes.

TABLE 12. Simulated moments: noise versus modal.

		Flexible		Fixed		Baseline	
Statistic	Data	*noise*	*modal*	*noise*	*modal*	*noise*	*modal*
$ave(debt/Y)$	48.00	48.78	48.79	48.70	48.71	48.70	48.70
$ave(S)$	3.03	3.01	3.01	3.01	3.01	3.01	3.00
$std(S)$	2.21	0.83	0.81	1.34	1.33	1.82	1.81
$std(D)/std(Y)$	0.26	0.60	0.58	0.42	0.42	0.41	0.41
$corr(S,D)$	−0.58	−0.79	−0.86	−0.86	−0.88	−0.34	−0.34
$corr(Y,D)$	0.00	0.55	0.57	0.67	0.69	0.36	0.37
$corr(Y,S)$	−0.42	−0.83	−0.86	−0.90	−0.90	−0.73	−0.74
$std(C+L)/std(Y)$	1.57	1.40	1.39	1.29	1.29	1.36	1.36

Note: The empirical moments are calculated for Mexico's data covering 1994–2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.

as one in which we are correctly "removing the noise." Second, in line with the results above all moments are very similar in "noise" and "modal" for all three specifications. Summarizing, this exercise further reinforces our argument that extreme value shocks have little impact in the volatility of spreads.

Finally, we turn to our second result of increasing borrowing during crises. A natural concern in our baseline specification might be that "borrowing into debt crises" may be a result of the government swayed by the extreme value shocks. To address this concern, we consider the same crises as in the main text and compare the simulated debt paths of the baseline model to the "modal" ones specified above. Figure 11 shows that the result of increasing debt during crises does not hinge on the extreme value shocks.

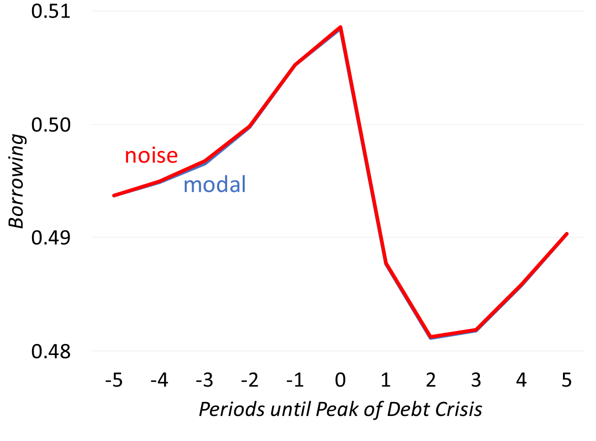


FIGURE 11. Simulated behavior of borrowing. Note: The figure presents simulated paths for borrowing during crises (as defined in the main text) in the baseline specification with ("noise") and without ("modal") noise.

Appendix C: Standard model with global shocks

In this section, we augment a standard sovereign default model with shocks to foreign investors' risk aversion. Empirical literature in economics has shown that global factors are an important driver of sovereign spreads (González-Rozada and Levy Yeyati (2008)). The aim of this section is to investigate the contribution of such shocks to the volatility of the spreads.

Consider the standard sovereign default model (corresponding to the "flexible" version in our paper). To introduce global risk premium shocks, we closely follow Bianchi, Hatchondo, and Martinez (2018) by imposing the following expression for the foreign investors' stochastic discount factor:

$$m_{t,t+1} = e^{-r-(\kappa_t \varepsilon_{t+1} + 0.5\kappa_t^2 \sigma^2)}, \quad \kappa_t \geq 0. \tag{15}$$

In expression (15), κ_t represents the two-state risk premium shock. In particular, this formulation implies that cash flows tend to be more valuable to foreign investors in the states of low income shocks, implying a positive risk premium. Similar to Bianchi, Hatchondo, and Martinez (2018), we normalize $\kappa_L = 0$, which represents normal times when creditors are risk-neutral, and set $\kappa_H = 23$. κ_t follows a Markov process with transition probabilities $\pi_{LL} = 0.85$ and $\pi_{HH} = 0.2$.

Table 13 presents the simulated moments without and with shocks to the lenders' stochastic discount factor, for the two variants of the standard model: flexible and fixed labor.[6] It is immediate to notice that while the inclusion of such shocks does increase the volatility of bond spread, the magnitude of this increase is small (from 0.82% to 0.85% for the flexible model, and from 1.35 to 1.40% for the fixed labor model).

The results shown in Table 13 indicate that while global factors do play a role in driving sovereign spreads (the models in the table had to be recalibrated to hit the targets),

Table 13. Simulated moments: models with and without global shocks.

Statistic	Data	Flexible		Fixed	
		no SDF shocks	*SDF shocks*	*no SDF shocks*	*SDF shocks*
$ave(debt/Y)$	48.00	47.93	48.13	48.08	47.95
$ave(S)$	3.03	3.03	3.03	3.03	3.03
$std(S)$	2.21	0.82	0.85	1.35	1.40
$std(D)/std(Y)$	0.26	0.60	0.66	0.42	0.46
$corr(S,D)$	−0.58	−0.86	−0.86	−0.89	−0.88
$corr(Y,D)$	0.00	0.57	0.50	0.70	0.60
$corr(Y,S)$	−0.42	−0.86	−0.80	−0.90	−0.84
$std(C+L)/std(Y)$	1.57	1.44	1.47	1.34	1.34

Note: The empirical moments are calculated for Mexico's data covering 1994–2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico. All models are solved using continuous choice methods as described in Appendix B. The flexible model without and with SDF shocks is calibrated using the following parameters, respectively: $\beta = 0.794$, $\hat{y} = 0.833$ and $\beta = 0.796$, $\hat{y} = 0.829$. The fixed labor model without and with SDF shocks is calibrated using the following parameters, respectively: $\beta = 0.781$, $\hat{y} = 0.851$ and $\beta = 0.788$, $\hat{y} = 0.846$.

[6] All four variants are recalibrated here to match the average debt and spread targets.

alone they cannot raise the volatility of the bond spread. This is because the government in the model responds to such shocks in a similar way as it does to the local income shocks by orchestrating a quick current account reversal and reducing its debt aggressively.

Appendix D: Calibration with high government debt

In this section, we present a calibration of our model that matches a higher level of government debt. The purpose of this exercise is to explore the limits of our mechanism.

We calibrate the model to match average external *debt service* to government revenues (25.8%), rather than external debt as in our baseline parametrization. We also abandon the assumption that only a fraction of the debt observed in the data is unsecured, originally due to Chatterjee and Eyigungor (2012). Table 14 summarizes the calibration of our baseline model, along with a fully flexible version. The resulting debt stock to revenues comes out to 80%, considerably higher than 48% in our preferred calibration. This increase in the debt stock comes at the expense of extremely low discount factor of 0.575. In addition, this calibration is a worse match for the average labor share observed in the data.

Table 15 reports the nontargeted moments in both models. As can be seen, the standard deviation of the spreads in this calibration declines for both, the baseline and the flexible variants of the model. Interestingly, this moment declines in both variants proportionally, as a result of which the standard deviation of the spread still more than doubles in our baseline model, as opposed to the flexible model. The decline in the standard deviation of the spreads is mirrored by the increase in the variability of the deficit (0.41 to 0.54 in the baseline and 0.60 to 0.80 in the flexible). We conclude that our key quantitative result is robust to a calibration with higher government debt.

Figure 12 highlights a main point of departure from our baseline results that follows the increased debt target. Specifically, the ‘borrowing into debt crises” behavior presented in Section 3.4 of the main paper is moderated. In particular, the government still

Table 14. Calibration of structural parameters: baseline versus flexible.

Parameter	Baseline	Flexible
Discount factor, β	0.575	0.710
Max default endowment, $\hat{Y}$	0.812	0.786
Interm. consumption weight, α	0.488	0.488
Adjustment weight, ϕ_0	0.482	0.000
Adjustment scale, ϕ_1	0.474	1.000

Target	Data	Baseline	Flexible
Avg. debt service/revenues (%)	25.80	25.83	25.86
Avg. spread (%)	3.03	3.05	3.04
Avg. labor share (%)	63.00	51.28	51.16
Elasticity of wL w.r.t. C in crises	0.24	0.24	–
Avg. ratio st. dev. of inputs (%)	164.00	161.80	–

TABLE 15. Untargeted moments: baseline versus flexible.

Statistic	Mexico Data	Baseline	Flexible
$std(S)$	2.21	1.40	0.58
$std(D)/std(Y)$	0.26	0.54	0.80
$corr(S, D)$	−0.58	−0.62	−0.82
$corr(Y, D)$	0.00	0.58	0.53
$corr(Y, S)$	−0.42	−0.69	−0.79
$std(C+L)/std(Y)$	1.57	1.67	1.53
$corr(Y, Cost)$	–	−0.36	–
$corr(S, Cost)$	–	0.81	–
avg. cost (% of avg. revenues)	–	1.10	–

Note: The empirical moments are calculated for Mexico's data covering 1994–2019. The bond spread is the EMBI index, while government final consumption expenditure and output are taken from National Accounts. Government deficit data is acquired from Banco de Mexico.

borrows up to the eve of the crisis, but then chooses to reduce its debt when confronted with the worst spread. It should be emphasized that this debt reduction is slower and less drastic than in the baseline model. This behavior is a direct result of the discount factor falling so significantly. Due to a higher discounting of the future, the government no longer maintains a buffer for crises and is more forced to reduce its debt if one occurs. We conclude that our main qualitative result, on borrowing into debt crises, is sensitive to the value of the discount factor, and thus the targeted debt level in the calibration.

APPENDIX E: DECOMPOSITION OF GOVERNMENT CONSUMPTION

We use the OECD Government Accounts decomposition of government expenditure by transaction, which distinguishes the following components (classification code in parenthesis):

- Final consumption expenditure (P.3);

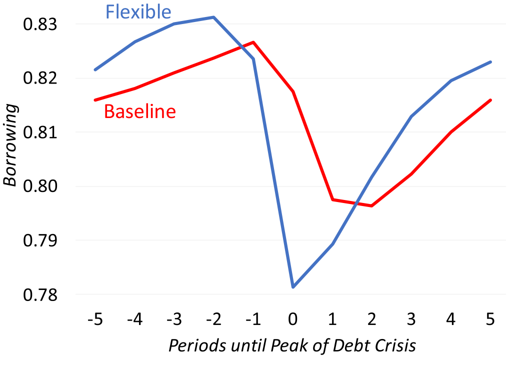


FIGURE 12. Simulated behavior of borrowing. Note: The figure presents simulated paths for borrowing during crises (as defined in the main text) in the baseline and flexible specifications when we target a higher average debt level.

- Gross capital formation (P.5);
- Acquisitions less disposals of nonproduced nonfinancial assets (K.2);
- Subsidies (D.3);
- Property income (D.4);
- Other current taxes (D.5);
- Social benefits other than social transfers in kind (D.62);
- Current transfers (D.7);
- Capital transfers (D.9).

We focus on the final consumption expenditure of the government because our model is not suited to capture elements such as investment or redistribution across households. For this reason, we focus exclusively on final consumption expenditure of the government, which is calculated as follows:

$$\begin{aligned}\text{Final consumption expenditure (P.3)} = {} & \text{Intermediate consumption (P.2)}\\ & + \text{Compensation of employees (D.1)}\\ & + \text{Consumption of fixed capital (K.1)}\\ & + \text{Other taxes on production (D.29)}\\ & - \text{Market output (P.11)}\\ & - \text{Output for own final use (P.12)}\\ & - \text{Payments for nonmarket output (P.131)}\\ & + \text{Social benefits in kind (D.631)}\end{aligned}$$

In our analysis, for simplicity we focus on intermediate consumption and compensation of employees, and ignore the other elements that contribute to the final consumption expenditure. We choose to do so because the data on other elements is not always complete and consistent across countries and time periods. This simplification is reasonable because the share of the sum of intermediate consumption and compensation of employees in final consumption expenditure is 92.6% on average across all countries and all time periods, with the minimum of 71.8%. In other words, these two elements are responsible for a vast majority of the final government consumption expenditure.

Appendix F: Resource costs of adjustment

In this Appendix, we further investigate the impact of the realized adjustment costs, $\phi_1 \min\{H/H_{-1} - 1, 0\}$. In particular, we are interested in identifying the extent to which these costs are behind the higher standard deviation of spreads. As seen in Figure 13, the average adjustment cost is largest at the peak of the debt crisis. We assess the impact of

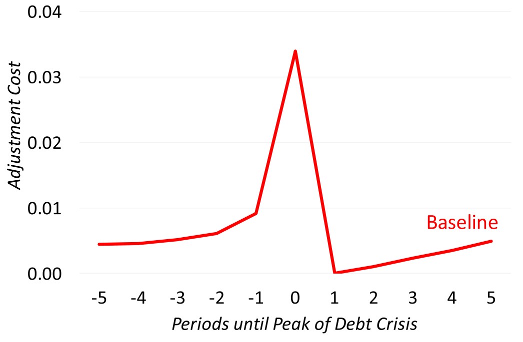


FIGURE 13. Simulated behavior of adjustment costs. Note: The figure presents simulated paths for adjustment costs during crises (as defined in the main text) in the baseline specification. The units are in terms of average revenues, for example, 0.01 is 1% of average revenues.

these realized costs by removing them from the budget constraint in the following way. After the country has decided how much it wants to borrow b', the price of borrowing is set to q, and consumption has taken place we reduce the market value of debt by the realized adjustment cost. That is, we reduce future debt to $\hat{b}'$ found by solving the following equation:

$$q\hat{b}' = qb' - \phi_1 \min\left\{\frac{H}{H_{-1}} - 1, 0\right\}.$$

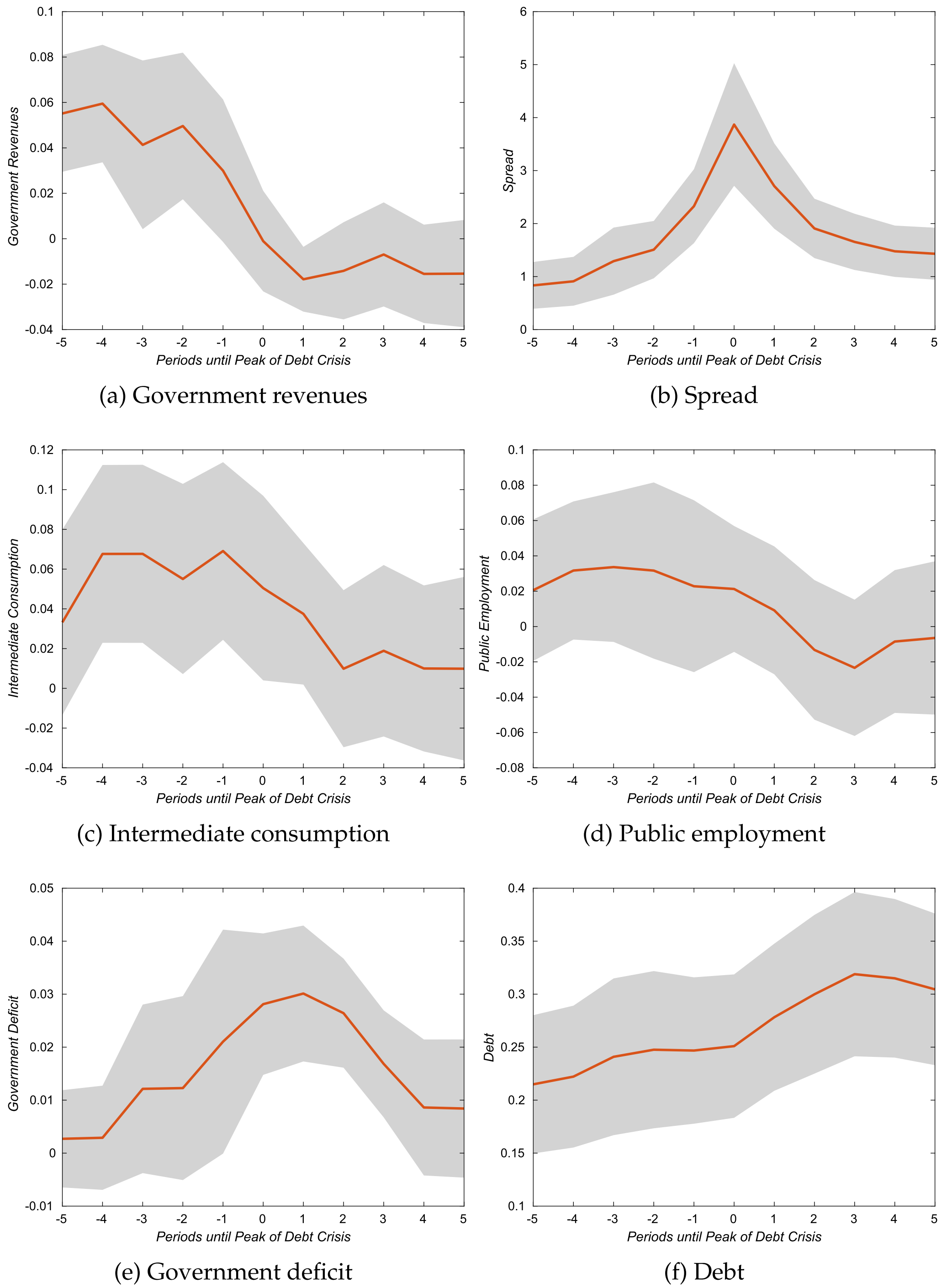
We then recalculate the moments implied by the simulations.[7] The resulting moments can be seen in Table 16. Not surprisingly, we see that average debt and spreads decline slightly. Further, the standard deviation of the spreads also declines but not drastically. In particular, the coefficient of variation for the spreads is 0.595 whereas in the baseline specification the same statistic was 0.605, a small decline. We conclude that the realized adjustment costs have a small impact in the standard deviation of spreads.

TABLE 16. Untargeted moments: habit formation model.

	Baseline	
Statistic	*with cost*	*without cost*
$avg(B)$	48.70	47.76
$avg(S)$	3.01	2.82
$std(S)$	1.82	1.68

Note: Both moments are calculated in the baseline specification. The first column (with cost) presents simulated moments from the baseline model in the main text. In contrast, the second column (without cost) removes the realized adjustment cost from the budget constraint as described in this Appendix.

[7]Note that the value function iteration remains the same, that is, the government expects to have to pay these adjustment costs. The reduction in adjustment cost takes place when simulating the model.



(a) Government revenues

(b) Spread

(c) Intermediate consumption

(d) Public employment

(e) Government deficit

(f) Debt

FIGURE 14. Behavior of endogenous variables around debt crises: data. Notes: Solid lines present averages, while the shaded areas span the 5% and 95% confidence intervals. The government revenues, intermediate consumption and public employment series are detrended using a common GDP trend and demeaned.

Appendix G: Further comparison of model and data

In this section, we turn back to our original data set in order to further investigate how the behavior of the model's endogenous variables compares to the data. We define a crisis episode here in the same way as in Section 5 of the main paper.

Figure 14, which is a direct counterpart of Figure 2 from the model (in the main paper), presents the dynamics of government accounts around debt crises in the data, averaged across countries.[8] The decline in government revenue (Figure 14a) coincides with an increase in bond spread toward its peak (Figure 14b). Further, while both types of government expenditure decline (Figures 14c–14d), the decline in public employment is smoother and smaller in size. We can also observe that the government deficit is positive throughout and goes up at the peak of the crisis, a behavior that contrast with the results of our model.[9] Most importantly, government debt *increases* throughout the episode (Figure 14f), with the pace of the increase accelerating at the peak of the crisis, only to start declining 3 years after the peak.

The main discrepancy between Figures 14 and 2 stems from the path of government revenues following the peak of the crisis. In our model (Figure 2a), the revenue recovers promptly as a result of imposing an AR(1) income process, a reasonable assumption for our calibration target, Mexico. By contrast, in the data (Figure 14a), the recovery is very sluggish, which further affects the paths of the two components of government spending (Figures 14c–14d), and overall debt (Figure 14f). The lack of recovery in government revenues arises from the fact that our data consists of OECD member states, mostly advanced countries. The simple AR(1) is not the best assumption for an income process in that case, especially in the aftermath of the European debt crisis (Paluszynski (2023)).

References

Bianchi, Javier, Juan Carlos Hatchondo, and Leonardo Martinez (2018), "International reserves and rollover risk." *American Economic Review*, 108, 2629–2670. [5]

Chatterjee, Satyajit and Burcu Eyigungor (2012), "Maturity, indebtedness, and default risk." *American Economic Review*, 102, 2674–2699. [6]

Dvorkin, Maximiliano, Juan M. Sánchez, Horacio Sapriza, and Emircan Yurdagul (2021), "Sovereign debt restructurings." *American Economic Journal: Macroeconomics*, 13, 26–77. [2, 3]

[8]Series on government revenues, intermediate consumption and public employment come from the OECD Government Accounts. Data on government deficit and bond spreads are acquired from OECD, with the latter being partly supplemented with EMBI when observations are missing. Government debt denotes the sum of external debt securities to GDP and the data is obtained from the Quarterly External Debt Statistics (QEDS). Because some values are missing in the data, we only consider the debt crisis events around which the entire path of data ($\pm$5 years) is available.

[9]The opposite sign of the deficit variable stems from the fact that in the data economies tend to grow continually, while growth is not present in our model. As a result, governments are able to run permanent deficits while maintaining sustainable debts. Although quantitative sovereign default models without growth are not capable of replicating the increase in deficit at the height of the crisis, we note that our baseline model makes progress in this dimension by dampening and delaying the reduction in deficit.

González-Rozada, Martín and Eduardo Levy Yeyati (2008), "Global factors and emerging market spreads." *Economic Journal*, 118, 1917–1936. [5]

Hatchondo, Juan Carlos and Leonardo Martinez (2009), "Long-duration bonds and sovereign defaults." *Journal of International Economics*, 79, 117–125. [2]

Paluszynski, Radoslaw (2023), "Learning about debt crises." *American Economic Journal: Macroeconomics*, 15, 106–134. [11]

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