Supplement to "Endogenous sample selection: A laboratory study"

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Appendix SA

In this appendix, we report several robustness exercises to the findings presented in the main text.

Appendix to Section 3

Finding 1   We reported in the text that for those subjects who converge, the mean convergent threshold is 1.90 under No Selection and 2.29 under Selection; the median convergent thresholds are 1.75 and 2.38, respectively. If we consider all subjects, instead, the mean threshold for $T = 100$ is 1.88 under No Selection and 2.26 under Selection; the median thresholds are 1.75 and 2.25, respectively. If we consider all subjects, the differences across treatments in the mean (0.38) and the median (0.5) are still both statistically significant at the 1% level.[1]

Finding 2   Figure S1 updates Figure 6 by including all subjects. Since not all subjects have a convergent threshold, we focus on their round 100 choices. There are no qualitative differences, and thresholds in the Selection treatment still first-order stochastically dominate thresholds in the No Selection treatment.

Finding 3   Table S1 expands the information in Table 2. It includes standard errors and also includes all subjects, not just those for whom the threshold converges. When all subjects are included, there are only minor quantitative deviations, and the findings reported in the paper still hold. Figure S2 replicates Figure 7 of the paper, including all subjects.

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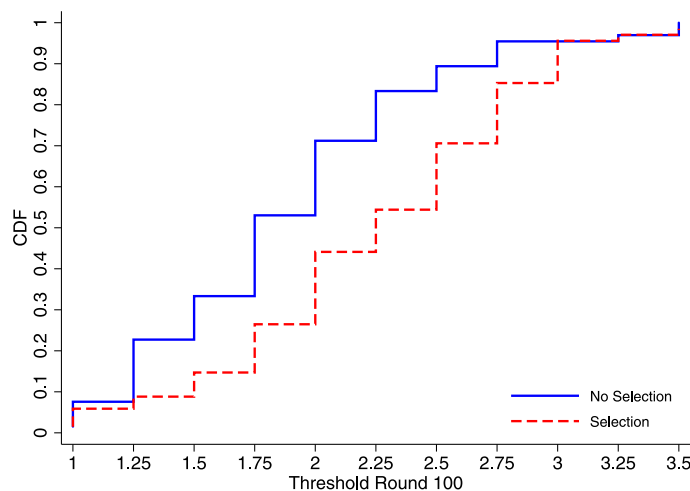[1]We use the same test described in footnote 21 of the paper.

FIGURE S1. Distribution of round 100 thresholds by treatment using all subjects.

Finding 4   Figure S3 updates Figure 8 by including all subjects and shows that, qualitatively, we reach the same conclusion as in the text.

Finding 5   Table S2 shows that the results reported in Finding 5 are unaffected if we extend the reduced-form analysis to include all subjects.

Appendix to Section 4

In the text, we focus on the median as a measure of central tendency for η. Table S3 reports on the distribution of the mean of η. Given that the distribution of η is truncated, the mean is expected to be higher than the median and this is verified in Table S3 for all cases. However, the main findings that we report for the median in the text hold if we focus on the mean.

Figure S4 shows the goodness of fit for the case where all subjects are included in the estimation.

Figure S5 presents the results of a different out-of-sample-prediction robustness exercise. Our data for the selection treatment was collected in five different sessions. We do five out-of-sample prediction exercises. In each case, we estimate the model described in Section 4, excluding one session from the data. We then use the estimated parameters to predict a distribution of thresholds under Selection. Finally, we contrast this prediction to the actual data from the excluded session. Figure S5 presents the findings when we constrain the data to subjects who converged at $T = 90$, but the patterns are unchanged if we include all subjects. When sessions 1, 3, 4, or 5 are excluded, we observe that the prediction is relatively close to the data from the excluded session. There is a larger difference in the case of session 2. Overall, the exercise illustrates that the model provides, in most cases, accurate out-of-sample predictions.

TABLE S1. Mean values in the data and reported beliefs by treatment for subjects who converge and all subjects.

	No Selection Treatment						Selection Treatment					
	Subjects Who Converged			All Subjects			Subjects Who Converged			All Subjects		
	Data (True)	Data (Naive)	Report	Data (True)	Data (Naive)	Report	Data (True)	Data (Naive)	Report	Data (True)	Data (Naive)	Report
% Good	25.0	24.9	30.6	24.9	24.5	29.3	25.6	56.1	48.4	25.5	56.2	47.8
	(4.3)	(6.6)	(15.3)	(4.3)	(6.5)	(14.8)	(4.6)	(7.5)	(18.1)	(4.7)	(7.9)	(18.4)
% Good|Piv	26.1	24.9	28.0	25.6	24.5	27.3	0	56.1	44.6	0	56.2	43.9
	(6.1)	(6.6)	(15.6)	(5.9)	(6.5)	(15.5)	(0)	(7.5)	(20.8)	(0)	(7.9)	(21.1)
% Mistake|good	49.7	49.9	43.4	50.4	50.2	43.9	0	50.1	36.1	0	50.5	35.4
	(7.5)	(3.1)	(16.4)	(7.3)	(3.2)	(17.3)	(0)	(4.3)	(18.7)	(0)	(4.3)	(18.4)
% Mistake|bad	50.0	50.0	49.1	49.9	49.7	49.1	32.6	49.9	40.4	32.3	49.5	39.3
	(4.4)	(3.1)	(17.1)	(4.1)	(3.1)	(18.9)	(3.9)	(4.3)	(17.9)	(3.8)	(4.3)	(17.8)

Note: The label % Good denotes the percentage of times that project A was good; % Good|Piv: percentage of times that project A was good conditional on the subject being pivotal; % mistake | Good denotes the percentage of times a computer mistakenly votes for B when project A is good; % Mistake|bad denotes the percentage of times a computer mistakenly votes for A when project A is bad; Data (True)denotes the actual figure in the data; Data (Naive)denotes the actual figure a naive subject would report given the data; Report denotes the figure reported by subjects in Part II. Standard errors are given in parentheses.
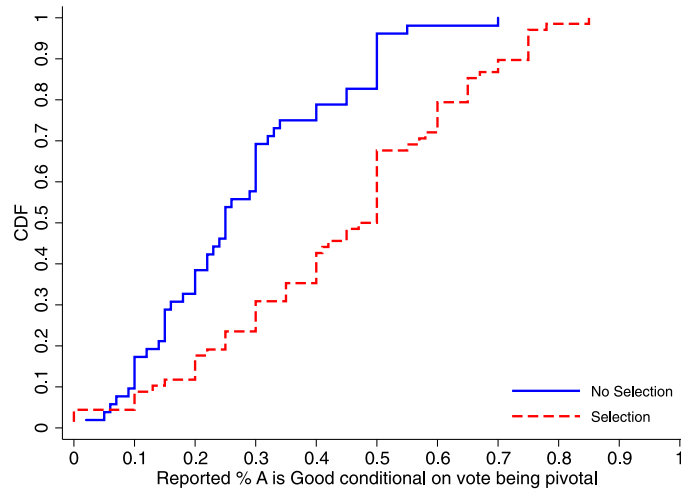
FIGURE S2. Distribution of reported beliefs on project A being good conditional on the recommendation being pivotal, by treatment. Includes all subjects.
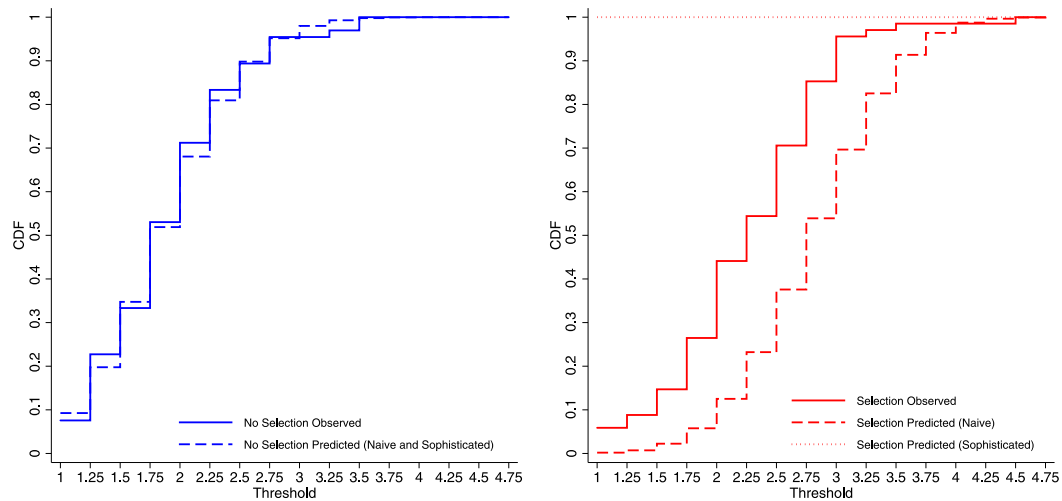

FIGURE S3. Predicted distribution of thresholds under the assumption of naive behavior versus observed distribution of convergent thresholds, by treatment. Includes all subjects.

Finally, we show results that correspond to several robustness exercises on the distribution that we assume for η. In the text, we assumed that η follows a truncated normal distribution, as we need η to have support in $[0, \infty)$. A first robustness exercise assumes, instead, that η follows a log-normal distribution, which has support in $[0, \infty)$. The first two columns of Table S4(a) allow us to compare the estimated distribution for the median of η depending on whether η follows a normal truncated or a log-normal distribution (focusing on $T = 90$). The median of η when the distribution is log normal is lower than when we use the truncated normal, but the overall message is similar: the values

TABLE S2. Reduced-form analysis: reaction in threshold to events in previous period (all subjects).

(a) The dependent variable and the controls are dummy variables[1]

Dep. Var.: $\mathbf{1}\{x_t \neq x_{t-1}\}$	Pooled	No Selection	Selection
Constant	0.083***	0.094***	0.074***
	(0.007)	(0.021)	(0.018)
Piv_{t-1}	0.005	−0.011	0.025**
	(0.008)	(0.008)	(0.010)
Info_{t-1}	0.022**	0.026**	0.019**
	(0.007)	(0.012)	(0.0010)
$\text{Piv}_{t-1} \times \text{Info}_{t-1}$	0.063***	0.056***	0.063***
	(0.013)	(0.020)	(0.022)

(b) All controls are dummy variables[2]

Dep. Var.: $x_t - x_{t-1}$	Pooled	No Selection	Selection
Constant	0.016***	0.019**	0.013**
	(0.004)	(0.008)	(0.004)
$(\text{Piv and good})_{t-1}$	0.020	−0.023	–
	(0.035)	(0.036)	–
$(\text{Piv and bad})_{t-1}$	−0.158***	−0.208***	−0.116***
	(0.031)	(0.054)	(0.031)
$(\text{Not Piv and good})_{t-1}$	−0.001	−0.016	−0.003
	(0.006)	(0.015)	(0.007)
$(\text{Not Piv and bad})_{t-1}$	−0.025***	−0.029*	−0.019
	(0.011)	(0.015)	(0.020)

Note: The asterisks *, **, and *** indicate significance at the 1, 5, and 10% levels, respectively. In both cases we report the results of fixed effects panel regressions and we cluster standard errors by subject. Both regressions include all 134 subjects and for each subject, we use the last 74 rounds of Part I (we lose one observation due to the lag). The regressions pool subjects from both treatments. Conclusions do not change if the analysis is conducted by treatment or if we add time dummies.[1] The variable $\mathbf{1}\{x_t \neq x_{t-1}\}$ takes value 1 if the threshold in period t is different than the threshold in period $t - 1$; Piv_{t-1} takes value 1 if the subject was pivotal in the previous period; Info_{t-1} takes value 1 if in the previous period the subject received feedback on whether project A was good or not. [2] The variable $(\text{Piv and Good})_{t-1}$ takes value 1 if the subject was pivotal, the company invested in A, and it turned out to be good. Other dummy variables are named accordingly. The excluded event is the case when the subject did not receive information in the previous period because the company invested in B.

of η are far from those corresponding to sophistication and are indicative of subjects giving a higher weight to pivotal relative to nonpivotal information.

The second two columns of Table S4(a) present a different estimation exercise. So as to estimate parameters of the distribution of η, we need to specify how to compute beliefs in the estimation procedure. As described in detail in the Appendix, to obtain the estimates that we have reported so far, we computed beliefs assuming that subjects are in steady state and using equilibrium predictions. For the alternative estimation exercise that we present now, we use data to compute beliefs.

Notice that the difference in this alternative approach is exclusive to the second stage of the estimation procedure, where to recover the parameters of the η distribution, we

TABLE S3. Mean of η using the bootstrap.

Percentile	$T = 70$	$T = 75$	$T = 80$	$T = 85$	$T = 90$	$T = 95$	$T = 100$
2.5	1.89	1.97	1.87	1.79	1.98	2.30	2.28
5	2.09	2.11	1.98	1.96	2.18	2.45	2.43
25	2.91	2.84	2.56	2.53	3.19	3.53	3.37
50	4.39	4.45	3.42	3.57	5.07	5.60	5.13
75	6.91	6.15	5.16	5.41	7.15	7.59	6.71
95	10.69	9.43	7.78	8.18	10.90	11.62	9.88
97.5	12.28	11.19	8.54	9.27	12.75	13.80	11.40

Note: The bootstrap delivers 1000 estimations of the parameters of the model. For each repetition, we compute the mean of η and the table reports percentiles of the distribution of the mean and median. Each column indicates the rounds of Part I that were included in the estimation.
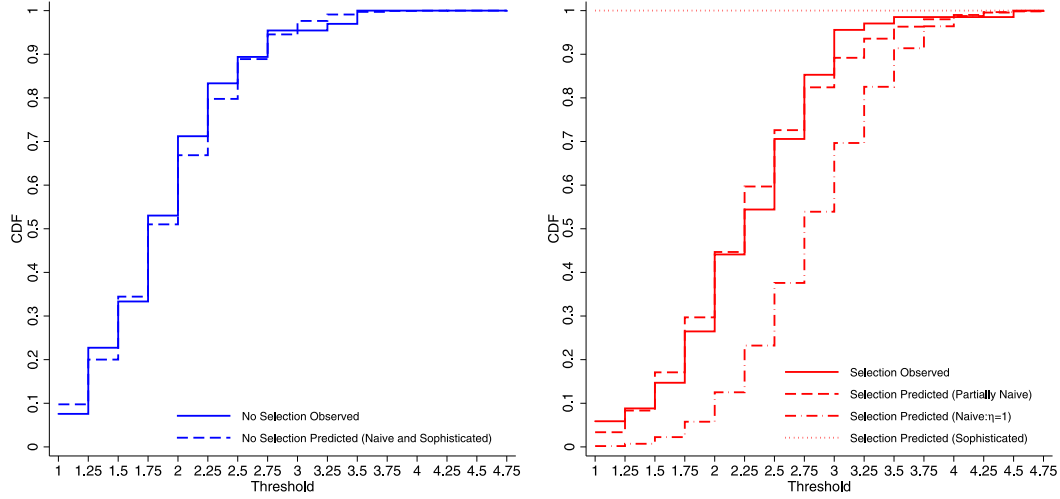
FIGURE S4. Goodness of fit using all subjects ($T = 100$).

need to compute beliefs. In particular, we assume that a subject's belief in round k is

$$z_{ik} = g(data_{ik}, \eta_i) + \nu_{ik},$$

where $g(data_{ik}, \eta_i)$ is the empirical counterpart of equation (5), which depends on the data observed by subject i up to round k and her parameter of naiveté η_i, and ν_{ik} denotes noise in the subject's estimation process in round k.[2] In this approach we use data from the last 10 rounds for subjects who converge at $T = 90$.[3]

Intuitively, data from the No Selection treatment are used to identify the belief noise ν (since the function $g(\cdot)$ is essentially constant in η under No Selection) and data from

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[2]Under this specification, z_{ik} might fall outside the $[0, 1]$ interval, in which case we set it equal to 0 or 1. This turns out not to be a serious constraint because the estimated variance of ν is fairly small.

[3]Notice that while the threshold does not change in the last 10 rounds, the information in $data_{ik}$ used to compute $g(\cdot, \eta_i)$ can change with k.
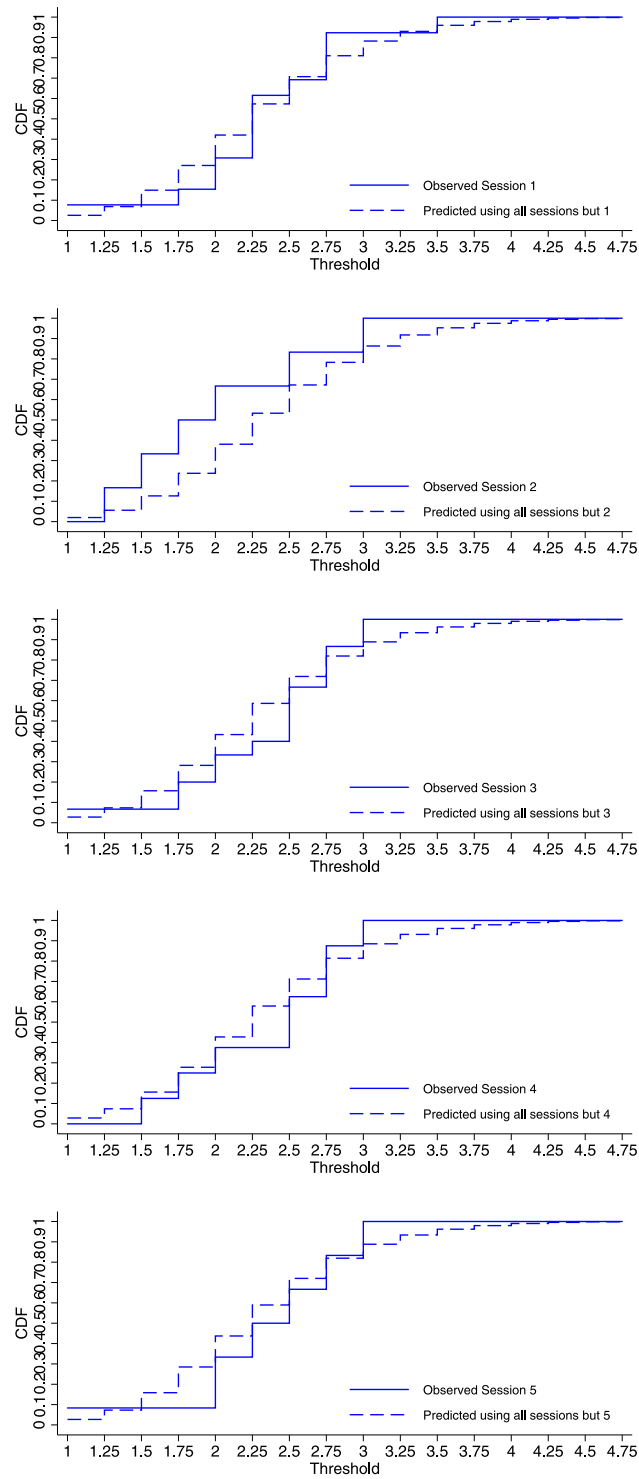
FIGURE S5. Out-of-sample predictions for each of the five sessions of the Selection treatment (using only those who converged at $T = 90$).

the Selection treatment are used to identify η.[4] We perform two exercises on the distribution of η, assuming that it follows either a normal truncated or a log-normal distribution. We also assume that the belief formation noise $\nu \sim N(\mu_\nu, \sigma_\nu^2)$ is normally distributed. We estimate the parameters using (simulated) maximum likelihood.

The last two columns of Table S4(a) show the output for the distribution of the median of η when we use observed data to compute beliefs. The main finding is that the median values of η that we report are comparable to what we report using equilibrium beliefs.

A final robustness exercise on the distribution of η is presented in Table S4(b). In this case, we assume that η follows a mixture between a truncated normal distribution and a mass point at value η^{MP}. This exercise allows us to check whether the assumption that η follows a truncated normal distribution that we report in the text implicitly discards a small mass of subjects with higher values of η (closer to sophistication). If we were discarding a few subjects with high values of η, the estimated weight on the mass point would be positive and the median distribution of η coming from the mixture may be higher.

The table reports statistics on the median distribution of η for cases where $\eta^{MP} \in \{15, 20, 25, 50\}$. In all cases, we find an estimated mass-point weight that is at the lower bound of 0. In all cases, the statistics for the distribution of η are comparable to the findings reported in the text, which, as a reference, are reported in the first column of Table S4(b).

<center>Appendix SB: Details on the structural estimation</center>

<center>*Stage 1: Estimation of the distribution of risk coefficients*</center>

In Part III of the experiment, we collect subjects' threshold choices for five known values of p. Indexing each problem by k, we have that the subject's belief of project A being good (z_k) is equal to the known value of p that corresponds to problem k. For each problem k, the optimal threshold is given by $x_{ik}^*(r_i) = u_{r_i}^{-1}(z_k \times u_{r_i}(5) + (1 - z_k) \times u_{r_i}(1))$. We assume that $r_i \sim N(\mu_r, \sigma_r^2)$, and recovering the parameters μ_r and σ_r^2 is the central goal of stage 1 of the estimation. We also assume that subjects can make mistakes when choosing their thresholds so that $x_{ik}^* = x_{ik}^*(r_i) + \varepsilon_{ik}$, where $\varepsilon_{ik} \sim N(\mu_\varepsilon, \sigma_\varepsilon^2)$. The noise guarantees that all possible threshold values have positive probability.

The probability of observing a discrete threshold $X^d = x^d$ is then computed as the probability that x_{ik}^* lies in an interval. For example, the probability that $X^d = 1$ (recall this codification means that the subject always chooses B) is given by $\Pr(x_{ik}^* \leq 1.25)$. Similarly, $\Pr(X^d = 1.25) = \Pr(x_{ik}^* \in (1.25, 1.50])$ and so on. Let $\Pr(X_{ik}^d = x_j^d)$ indicate the probability that subject i selects the discrete threshold x_j^d in problem k, where j indexes the 16 possible discrete thresholds that the subject can select. Let y_{ikj} be an indicator

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[4]Notice that in this alternative approach, we do not allow for the errors in the threshold to differ between stage 1 and stage 2. In the approach we present in the text, we estimated parameters of the distributions of ε and ε'. Since in this alternative estimation procedure we need to identify parameters of the distribution of ν, we cannot identify ε' and instead will assume that the distribution of errors in the threshold is not different in Part I and Part III.

TABLE S4. Robustness exercises: statistics of the median of η using the bootstrap.

(a) Robustness on the distribution of η and the computation of beliefs

Percentile	Using Equilibrium Beliefs		Using Data to Compute Beliefs	
	Normal Truncated	Log Normal	Normal Truncated	Log Normal
2.5	1.69	1.68	1.60	1.77
5	1.85	1.81	1.71	1.91
25	2.70	2.30	2.22	2.47
50	4.30	2.82	2.69	2.96
75	6.05	3.63	3.37	3.51
95	9.16	5.24	4.65	4.83
97.5	10.82	6.03	5.32	5.53

(b) Distribution of η as a mixture between the truncated normal and a mass points at different values of η

Mass Point at:	–	15	20	25	50
Estimated Mass-Point Weight	–	0.000	0.000	0.000	0.000
2.5	1.69	1.67	1.66	1.69	1.64
5	1.85	1.83	1.81	1.86	1.81
25	2.70	2.71	2.78	2.78	2.57
50	4.30	4.24	4.18	4.23	4.12
75	6.05	5.67	5.55	5.58	5.60
95	9.16	8.73	8.80	8.98	8.66
97.5	10.82	10.60	9.86	9.88	10.31

Note: The bootstrap delivers 1000 estimations of the parameters of the model. For each repetition, we compute the median of η and the table reports percentiles of the distribution. For all estimations, we use subjects who converged at $T = 90$.

variable that takes value 1 if subject i selects threshold x_j^d in problem k and value 0 otherwise. The contribution to the likelihood by subject i in decision problem k can be written as $L_{ik} = \prod_j \Pr(X_{ik}^d = x_j^d)^{y_{ikj}}$. The log-likelihood function can then be computed as $\log L(y_{ik}|\theta_1) = \sum_i \sum_k \log(L_{ik})$, where y_{ik} represents the data and $\theta_1 = (\mu_r, \sigma_r, \mu_\varepsilon, \sigma_\varepsilon)$ represents the parameters to be estimated in stage 1.

We look for $\hat{\theta}_1$ that maximizes the log-likelihood function given the data, where we simulate the distribution of r_i. We estimate the distribution of r_i by simulation by first selecting 1000 random numbers in [0, 1] using Halton draws (see Train (2009)). In each iteration of the maximization process, we transform the 1000 random draws in [0, 1] to compute draws of r_i.[5] For each draw h, we then compute the probabilities $L_{ik}(h)$ and then calculate L_{ik} by taking the average over all 1000 draws.

Stage 2: Estimation of the partial naiveté coefficients

In stage 2, we use estimates $\hat{\mu}_r$ and $\hat{\sigma}_r$ from stage 1 and data from Part I of the experiment to recover parameters of the distribution of η.[6] The subjects' threshold in period T—

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[5]The random value r_{ih} for random draw h is computed as the inverse of the cumulative normal, where the mean and the standard deviation are the values of μ_r^t and σ_r^t that correspond to iteration t.

[6]Alternatively, it is possible to estimate both stages at the same time, but we separate the estimation into two stages for computational ease.

per equation (8)—is assumed to be $x^* = x^*(r, \eta) + \varepsilon'$, where we drop the subscripts for convenience. In stage 2, we estimate four parameters related to the distributions of $\varepsilon' \sim N(\mu_{\varepsilon'}, \sigma_{\varepsilon'}^2)$ and $\eta \sim D(\delta_1, \delta_2)$, where D is a truncated normal distribution (as reported in the paper) or a log-normal distribution (as reported in the this Supplemental Appendix). The goal of stage 2 is to find the parameter value $\theta_2 = (\delta_1, \delta_2, \mu_{\varepsilon'}, \sigma_{\varepsilon'})$ that produces the best fit of the data to the theoretical model. We start by explaining how we compute the distribution of predicted choices for a fixed θ_2.

In the No Selection treatment, regardless of the value of η, the model predicts that the subject's belief of p converges to the true value of $\frac{1}{4}$. In the Selection treatment, we need to compute steady-state beliefs, which depend on η. As explained in the Appendix, the steady-state belief is unique and solves $q(z^*) = z^*$. Recall from the Appendix that $q(z) = p(x^*(z))$, where $x^*(z)$ is the optimal threshold given belief z. We now need to account for the fact that the threshold must be discrete, and so we must modify the formula for $p(\cdot)$ given in equation (10) in the Appendix. In particular, the term $\frac{(x_{\tau+1}-1)}{4}$ in the formula, which is the probability of voting for A when the threshold is $x_{\tau+1}$, must now be replaced by the probability for voting for A when the threshold $x_{\tau+1}$ is discrete and the payoff of the safe option is no longer uniform on $[1, 5]$ but rather a discrete random variable that takes 15 possible values, from 1.25 to 4.75, with each value being equally likely.

Formally, let $P_A(z)$ denote the probability that a subject who maximizes expected utility given belief z votes for A. In the expression for $p(\cdot)$, we now substitute $P_A(z)$ for $(x^*(z) - 1)/4$. The result is a function that maps P_A into a probability that project A is registered to be good (given that some information about A is registered), and we denote this function by $P_A \mapsto Z(P_A)$. We depict this function in Figure S6, where P_A, the probability of voting for A, is in the vertical axis and $z = Z(P_A)$, which represents the subject's belief, is in the horizontal axis. The function is smooth and decreasing in P_A.
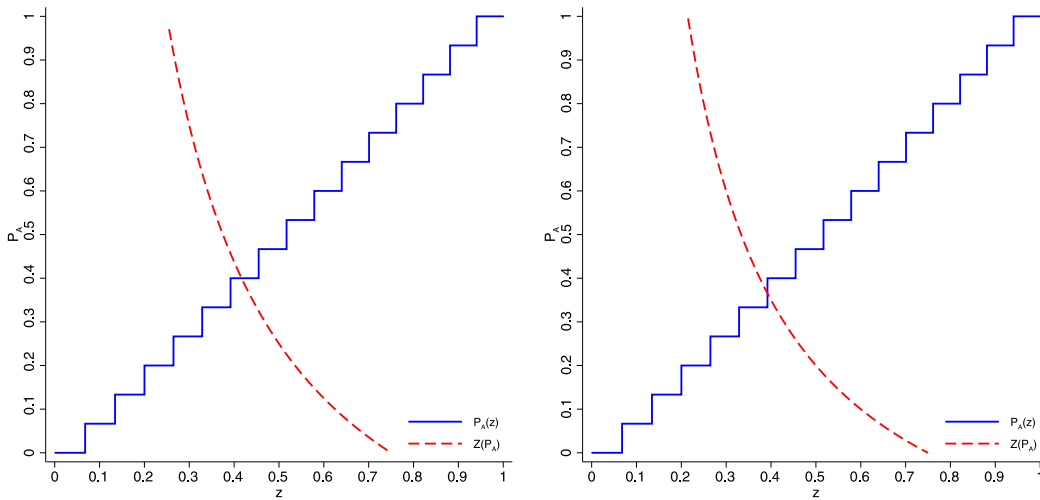

FIGURE S6. Determining steady-state beliefs.

The figure also plots $P_A(z)$, the probability of voting for A as a function of the belief. Note that it is a step function that jumps for those values of z where the subject is indifferent between two thresholds. A flat part represents a range of zs for which the subject strictly prefers a threshold. The equilibrium belief and the probability of voting for A are given by the intersection of $Z(\cdot)$ and $P_A(\cdot)$. The left panel shows a situation where the intersection occurs at one of the flat parts of $P_A(\cdot)$. A flat part is associated with a unique optimal threshold, and so in this case, the steady-state threshold is unique.

The right panel of Figure S6 shows an alternative situation where the intersection is where $P_A(\cdot)$ jumps, which reflects the case where the subject is indifferent between two different thresholds. In the steady state, the subject mixes between two contiguous thresholds, and the mixing probability is the one that guarantees that the probability of voting for A is indeed P_A^*, which denotes the probability at which $Z(P_A)$ intersects $P_A(z)$. In this case, the predicted threshold is a random variable.

From the previous analysis, it follows that, as a function of the risk aversion parameter r and the naiveté parameter η, $x^* = x^*(r, \eta) + \varepsilon'$ is the sum of a random variable (with support on at most two contiguous threshold values) plus noise. As in stage 1, the noise guarantees that all possible threshold values have positive probability. From here on, we can proceed as in stage 1; that is, the probability of observing a discrete threshold $X_d = x_d$ is computed as the probability that x^* lies in an interval and so on.

Because there are no closed-form equations to find steady-state thresholds, we proceed by simulating the distribution of η for estimation purposes. Specifically, in each stage t of the maximization routine, we draw 1000 values of η and r from distributions $D(\delta_1^t, \delta_2^t)$ and $N(\hat{\mu}_r, \hat{\sigma}_r)$, respectively, using Halton draws as described in stage 1.[7] For each pair (r, η), we follow the above procedure to compute the likelihood of selecting each possible threshold. The procedure looks for a $\hat{\theta}_2$ that maximizes the simulated likelihood function.

Standard errors

We estimate standard errors for the coefficients by bootstrap. Let N_{NS} and N_S denote the number of subjects from each treatment (NS, No Selection; S, Selection) that are included in the estimation. An observation for subject i consists of the five choices in Part III and the choice in round T of Part I. We start by taking a random sample of observations (with replacement) of N_{NS} and N_S subjects (i.e., we stratify by individual). For this random sample, we then estimate the eight coefficients as described above. We repeat the procedure 1000 times, which eventually provides us with a distribution for the estimates that we use to compute standard errors and confidence intervals.

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[7]Notice that if η follows a truncated normal distribution, μ_η and σ_η are derived as explained in footnote 32 of the paper. If η follows a log-normal distribution ($\log(\eta) \sim N(\mu_0, \sigma_0^2)$), then $\mu_\eta = e^{\mu_0 + \frac{\sigma_0^2}{2}}$ and $\sigma_\eta^2 = (e^{\sigma_0^2} - 1)e^{2\mu_0 + \sigma_0^2}$. In either case, the procedure provides estimates of μ_0 and σ_0, which then can be transformed to recover the parameters of the corresponding distribution of η.

APPENDIX SC: IDENTIFICATION OF PRIMITIVES

In this section, we show that the primitives of the model (p, m_G, m_B) can be identified from the observed data. Our model is one where the votes of the computers are i.i.d. conditional on the state of the world. Our instructions suggest that votes are conditionally i.i.d. (see Appendix SD), but, of course, it is not possible to precisely define conditional independence for subjects who may not have been formally exposed to the concept of a conditional probability. For this reason, we also consider a more general model where the votes of the computers can be correlated even conditional on the state of the world. We begin by showing that this more general model is not identified from the observed data. We then show that the model in the experiment, where votes are conditionally i.i.d., is identified. It is important to keep in mind that identification is not the ultimate objective of a subject. In the context we analyze, the subject only needs to learn the probability that project A is good conditional on being pivotal. Even in the more general model where votes are correlated conditional on the state, this conditional probability is easily identified from the data.

Let $X_i \in \{A, B\}$, $i = 1, 2$, denote the random variable representing computer i's vote and let $W \in \{\omega_G, \omega_B\}$ represent the state. The primitives are $p \equiv \Pr(W = \omega_G)$, $m_G \equiv \Pr(X_i = B \mid \omega_G)$, and $m_B \equiv \Pr(X_i = G \mid \omega_B)$ for $i = 1, 2$ and they satisfy $(p, m_G, m_B) = (1/4, 0, 1/3)$ in the Selection treatment.

So as to consider a more general model where the votes of the computers could be correlated conditional on the state, let $\{AA, BB, AB\}$ denote the set of profiles of votes of the computers, where AA stands for "both computers vote A", BB stands for "both computers vote B," and AB stands for "one computer votes A and the other votes B." The primitives of the more general model are $p \equiv \Pr(W = \omega_G)$, $(\Pr(AA \mid \omega_G), \Pr(AB \mid \omega_G))$, and $(\Pr(AA \mid \omega_B), \Pr(AB \mid \omega_B))$.

The decision maker observes the unconditional probabilities $\Pr(AA)$, $\Pr(AB)$ and also the proportion of projects A that are good conditional on project A being implemented when the subject votes for A.[8] We denote the latter probability by Z_A and note that

$$Z_A = \frac{\Pr(AB, AA \mid \omega_G)p}{\Pr(AB, AA)}. \tag{S1}$$

According to the primitives of the selection treatment, $\Pr(AB, AA \mid \omega_G) = 1$, $p = 1/4$, and $\Pr(AB, AA) = 2/3$. Therefore, the subject observes $Z_A = 3/8$ and $\Pr(AB, AA) = 2/3$ from the data. The subject can then use equation (SS1) to identity $\Pr(AB, AA \mid \omega_G)p = 1/4$.

In addition, conditional on observing ω_G, the subject observes AA with probability 1 (because $m_G = 0$). Thus, if the subject votes for A with positive probability (which is the case for the subjects in our experiment), then she can learn that $\Pr(AB \mid \omega_G) = 0$. To see this last equality, note that if $\Pr(AB \mid \omega_G) > 0$, then, since the subject votes for A with positive probability, there would be positive probability of observing the event

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[8]The subject also observes this proportion when she votes for B or, more generally, for any mixed strategy she may use. But no additional information about the primitives can be inferred from it.

($\omega_G AB$). But the probability of this event is 0 given the true primitives. Thus, the subject can identify $\Pr(AB, AA \mid \omega_G)p = \Pr(AA \mid \omega_G)p = 1/4$.

In general, the subject cannot identify p. Intuitively, the subject cannot infer $\Pr(BB \mid \omega)$ for any state ω from the data, because if both computers vote for B, then the state is never observed.[9] But in the case where the votes of the computers are conditionally i.i.d., the facts that $\Pr(AB \mid \omega_G) = 0$ and that the event ($\omega_G AA$) has positive probability imply that $\Pr(X_i = B \mid \omega_G) = 0$, which then implies that the subject can identify $m_G \equiv \Pr(X_i = B \mid \omega_G) = 0$ and, therefore, $\Pr(AA \mid \omega_G) = 1$. And so the subject can identify $p = 1/4$. Once the subject identifies p, she can identify m_B using the fact that $\Pr(B) = (1 - m_B)(1 - p)$, where $\Pr(B)$ is observed from the data and p is identified as described above.

Finally, note that even if the subject holds the more general model, where votes are possibly correlated conditional on the state, she can easily identify the probability that A is good conditional on being pivotal, provided that she votes for A with positive probability. The reason is that she observes whether or not she is pivotal, and that every time she is pivotal and votes for A, she observes whether or not A is good or bad. Formally, the subject can identify $\Pr(AB \mid \omega_G) = 0$, as explained earlier, and $\Pr(AB \mid \omega_B) > 0$, simply from the fact that she observes the event ($\omega_B AB$) with positive probability. This is all the information she needs to conclude that it is optimal to always choose B.

APPENDIX SD: INSTRUCTIONS[10]

Welcome

You are about to participate in a session on decision-making, and you will be paid for your participation with cash, privately at the end of the session. What you earn depends partly on your decisions and partly on chance.

The entire session will take place through computer terminals and there will be no interaction with participants seated at other terminals. Please do not communicate with other participants during the session. Please turn off cell phones now.

Please remember that the experiment will last 90 minutes. Remember also that you will be compensated for being in the lab for the next 90 minutes, and that the better your performance, then the higher the amount of money you are likely to earn.

We will start with a brief instruction period. Please pay attention. When I finish reading the instructions, you will be asked questions regarding these instructions. If you have any questions, please wait until we finish reading the instructions.

Instructions: Part 1

You work for a company and your job is to help decide which investment projects to undertake. Projects come from two industries: industry A and industry B.

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[9]Simple calculations yield that the following primitives are all the primitives that are consistent with the data in the more general model: For any $p \in [1/4, 7/12]$, $\Pr(AA \mid \omega_A) = 1/4p$, $\Pr(AB \mid \omega_A) = 0$, $\Pr(AA \mid \omega_B) = 1/(12(1 - p))$, and $\Pr(AB \mid \omega_B) = 1/(3(1 - p))$.

[10]Part 1 in the instructions corresponds to what we call Part I in the text, Part 2 corresponds to Part II, and Parts 3 and 4 correspond to Part III.

In every round, the company has the option of investing in a project from industry A or in a project from industry B. The company will invest in one of these two projects, but not in both of them. For simplicity, we will often refer to these projects as project A and project B, respectively. You will help the company decide between investing in project A or B.

The projects from industry A can be either good or bad. The company does not know the chance that a project from industry A is good.

Investing in a good project A results in a payoff of 5 points and investing in a bad project A results in a payoff of 1 point. The payoff from investing in project B is higher than 1 point but lower than 5 points. Therefore, if project A is good, it is best to invest in project A; and, if project A is bad, it is best to invest in project B.

You can perfectly assess the performance of a project from industry B. However, you do not know whether a particular project from industry A is good or bad.

The company has programmed two computers, Computer 1 and Computer 2, to evaluate whether the project from industry A is good or bad. Each computer performs its own evaluation of the project from industry A and submits a recommendation. If a computer assesses project A to be good, then it recommends project A. If a computer assesses project A to be bad, then it recommends project B.

The software of each computer is in beta mode, so it is possible that the computers make mistakes in their recommendations. Each computer can potentially make two types of mistakes when assessing whether project A is good or bad: it can mistakenly recommend project A when project A is bad, and it can mistakenly recommend project B when project A is good. The company does not know the rates of either type of mistake. However, it is known that Computer 1 and Computer 2 have the same rate for each of these two types of mistakes, although of course they might make different recommendations.

The company wants you to submit a recommendation for project A or a recommendation for project B. You will make this recommendation without knowledge of the recommendation of the computers. Together with the recommendations of the two computers, the company will then have received a total of 3 recommendations. The company will invest in project A if a majority of the recommendations are for project A (that is, 2 or 3 of the 3 recommendations are for project A). And the company will invest in project B if a majority of the recommendations are for project B (that is, 2 or 3 of the 3 recommendations are for project B). In other words, the company will follow the recommendation of the majority.

Your payoffs in the round are given by the following table:

	Project A is good	Project A is bad
Majority recommends Project A	5 points	1 point
Majority recommends Project B	x points	x points

In other words, if a majority recommends project A and project A turns out to be good, then your payoff for the round is 5 points. If a majority recommends project

A and project A turns out to be bad, then your payoff for the round is 1 point. Finally, if a majority recommends project B, then your payoff for the round is x points. Here, x represents the payoff of investing in project B and x is equally likely to take values from 1.25 to 4.75, with increments in quarter points, that is 1.25, 1.50, 1.75, 2, and so on, all the way to 4.75. In each round, you will know the value of x before making your decision. The computers, on the other hand, have no information about industry B, and, therefore, their recommendations will not depend on the value of x.

This decision problem will be repeated for a total of 100 rounds. In each round, you will have to decide between a new project from industry A and a new project from industry B. The chance that a project from industry A is good is fixed between 0 and 100 percent. The chance that the computers make the first type of mistake (recommend A when A is bad) is fixed between 0 and 100 percent, and the chance that the computers make the second type of mistake (recommend B when A is good) is fixed between 0 and 100 percent.

You do not know the chance that a project from industry A is good, the chance that the computers make the first type of mistake, and the chance that the computers make the second type of mistake. But these chances are fixed and will not change throughout the experiment. In particular, notice that the problem that you face is the same in every round. In every round, the interface will display on the screen information from past rounds, including the round number, the recommendation of the computers, your recommendation, the majority recommendation (which is the project in which the company invests), whether the project turned out to be good or bad in those cases where the company invested in a project from industry A, and the payoff for that round.

Your total payment for Part 1 will be as follows. We will randomly select 12 out of 100 rounds (with each round having equal chance of being chosen) and we will pay you the total number of points that you made in these 12 rounds. We will then convert points into dollars at the rate of $1 dollar for every 3 points.

Here is a brief reminder of what happens in each of the 100 rounds:

1. You will help decide between a new project from industry A and a new project from industry B. The chance that a project from industry A is good is fixed between 0 and 100 percent and will not change throughout the experiment.

2. Each of two computers evaluates whether the project from industry A is good or bad and submits a recommendation for project A or project B. Computer 1 and Computer 2 make the same rates of mistakes. The chance that the computers make the first type of mistake is fixed between 0 and 100 percent and will not change throughout the experiment. The chance that the computers make the second type of mistake is fixed between 0 and 100 percent and will not change throughout the experiment.

3. The interface draws a value of x (all values from 1.25 to 4.75, with increments in quarter points, are equally likely) that represents the payoff if the company invests in the project from industry B. You, but not the computers, will observe the value of x. You will then submit a recommendation for project A or B.

4. The payoffs for the round are given by the following table:

	Project A is good	Project A is bad
Majority recommends Project A	5 points	1 point
Majority recommends Project B	x points	x points

Now, you will be asked to answer some questions about these instructions. If you have any questions, please raise your hand.

Questions on the instructions [Subjects read and answer these questions on the screen. After subjects submit answers to each set of questions, they are provided with feedback. For each question they are informed if their answer was correct or not and the interface highlights the correct answer. If all answers are correct, subjects start with part 1 of the experiment. If any answer is incorrect, subjects face the set of questions that corresponds to the incorrect answer again until they answer correctly.]

Set of questions 1[11] :

1. The chance that project from industry A is good:

(a) is the same in every round.

(b) can be different in each round.

2. Each computer can potentially make two types of mistakes when assessing whether project A is good or bad: it can mistakenly recommend project A when project A is bad, and it can mistakenly recommend project B when project A is good. Select all true statements:

(a) Computer 1 and Computer 2 have the same rates of mistakes.

(b) Computer 1 and Computer 2 can end up making different recommendations.

(c) The rates of mistakes of the computers are the same in every round.

3. The recommendations of the computers:

(a) cannot depend on the value of x.

(b) can depend on the value of x.

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[11]The correct answers is the first alternative for the first and third questions in this set, and all alternatives should be selected for the second question. Out of 134 subjects, 48, 56, and 21 answer correctly on their first, second, and third tries, respectively. Fewer subjects needed more tries: 3 subjects needed 5 tries, 4 subjects needed 5, one subject needed 6, and one subject needed 8. The findings reported in the paper do not change if we exclude subjects who needed several tries to answer this set of questions correctly.

Set of questions 2:[12]

1. Suppose that Computer 1 recommends Project A and Computer 2 recommends Project A. Suppose that you recommend Project B. What is the recommendation of the majority?

(a) Project A.

(b) Project B.

2. Suppose that Computer 1 recommends Project B and Computer 2 recommends Project A. Suppose that you recommend Project B. What is the recommendation of the majority?

(a) Project A.

(b) Project B.

Set of questions 3:[13]

1. Suppose that you are in a round in which the majority recommends Project A and Project A is Good. Your payoff in that round is:

(a) 5 points.

(b) 1 point.

(c) x points.

2. Suppose that you are in a round in which the majority recommends Project A and Project A is Bad. Your payoff in that round is:

(a) 5 points.

(b) 1 point.

(c) x points.

Instructions: Part 1, rounds 26–100

The company introduces these additional instructions with the purpose of helping you make better decisions.

The problem in rounds 26 through 100 is exactly the same as before, with the only difference that now the company decides to introduce an additional task. This task should help you make better choices.

At the beginning of each round, the company will ask you to submit a decision rule. A decision rule indicates which option you would recommend for each possible value of x from 1.25 to 4.75.

You will submit your decision rule in each round as follows. On your screen, you will see a slider with the numbers from 1 through 5, with increments in quarter points. You will then click on the slider. Your choice is interpreted in the following way:

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[12]Out of 134 subjects, 129 answer correctly on the first try. Four subjects need 2 tries and 1 subject needed 3.

[13]Out of 134 subjects, 132 answered correctly on the first try. The rest needed 2 tries.

- *For any value of x lower than the value on which you clicked, you would recommend project A.*

- *For any value of x higher than the value on which you clicked, you would recommend project B.*

Once you submit your decision rule, the round continues exactly in the same way as you played rounds 1 through 25. In particular, the interface will draw a value of x and you will observe the value of x. You will then be asked to recommend project A or B. If your recommendation goes against the decision rule that you submitted in the round, you will be alerted and you will be asked to make a recommendation that is consistent with your decision rule. If you would like to change your decision rule, you will be able to do so in the following round. At the beginning of each round, the interface will display the decision rule you selected in the previous round. You are free to change the decision rule as desired.

Please click on Continue to begin playing round 26, but please wait for my instruction before submitting your choice. Please go ahead and click on the slider to practice. Once you click, you will see a clear indication of the values of x for which you would recommend project A or B. You can adjust your decision rule by clicking on a new value on the slider. Please go ahead and try adjusting your choice by clicking on the slider. Once you click the submit bottom, your decision will be final.

Company report[14]

When you finish playing 100 rounds, the company will ask you to report how you came to your decisions. Your report will explain the following:

1. A description of the aspects of the data that informed your decision.

2. A quantitative assessment of those aspects of the data that informed your decision.

3. Using your responses to (1), (2), and any additional argument you find relevant, you will be asked to justify your decision rule in round 100.

In order to make better decisions and earn more money in the following rounds, it is important that you pay attention to the data displayed on your screen. For this reason, we have provided you with scrap paper in case you want to take notes about your data. Paying attention to the data on your screen will also help you write a better report. The data on your screen will not be available to you when you write the report, you will only have your notes. All of you will have the chance to make an additional $8 from the report, and one of you will have the chance to make up to an additional $20.

Please go ahead now and make your decision for round 26 and then continue to play all rounds until round 100. If you have any questions, please raise your hand.

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[14]We mention that we will ask for a report later to highlight incentives for paying attention to the data that subjects gather as Part 1 of the experiment evolves.

Instructions: Part 2 (10 minutes)[15] [Subjects are asked to provide a report on paper. After providing the report they answer questions on beliefs on the screen]

Please take the next few minutes to complete the report on the back of this page.

At the end of the experiment, we will randomly select one of you to play the role of the CEO of the company. For the rest of you, we will randomly select 1 out of every 4 reports. This means that if you are not selected as the CEO, your report has a 25% chance of being selected.

The CEO will then be given these selected reports and he/she will have to pick what he/she considers to be the best report. The experimenter will publicly announce the best report among the reports that were selected. The person who wrote this best report will get an additional payment of $12 and the person selected as the CEO will get an additional payment of $5.

Please enter Lab # _____

Please answer the following questions in the space provided.

Question 1: Your decision rule in round 100:

Please draw a circle around the values of x for which you recommend project A:
1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75
Please draw a circle around the values of x for which you recommend project B:
1.25 1.5 1.75 2 2.25 2.5 2.75 3 3.25 3.5 3.75 4 4.25 4.5 4.75
(Please check that every value of x is circled exactly once; that is, for each value of x you must indicate a recommendation for either A or B, but not both)

Question 2: Please explain the logic behind your decision rule in round 100 by answering the following questions.

(a) Describe which aspects of the data informed your decision and provide a quantitative assessment of what you believe to be the relevant aspects of the data.

(b) Use your answer to part (a) and any other relevant arguments to provide a justification for your choice of decision rule in round 100.

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[15]The report had two main objectives. First, as mentioned earlier, we use it as a way to incentivize subjects to pay attention to the data in Part 1. Second, it serves as a device to check the reasons for their choices. Given that the answers subjects provide are free form, they are challenging to codify and we do not base any findings on these data. However, some points are worth mentioning. First, 50 out of 66 subjects in the No Selection treatment explicitly mention that they experienced project A not to be good a majority of the rounds as the basis for their choices. In the case of the Selection treatment, only three subjects provide a correct explanation of optimal behavior. The vast majority (58 subjects) provide an explanation that either explicitly mentions a computed probability of A being good as the basis of their choice or implicitly assumes the probability to be 50% and makes an argument for the choice based on risk aversion (18 of these 50 subjects acknowledge pivotality, but their choices are not based on optimal behavior). Finally, there are 7 subjects whose report does not fall in either of these classifications.

Questions on Beliefs [Subjects read and answer the following questions on the screen]

Question 1: What is the chance that a project from industry A is good? Enter a number between 0 and 100. You will receive $2 if your answer is within 5 points of the correct percentage.

Question 2: Computers' Mistakes.

1. Suppose a Project from Industry A is Good. What is the chance that a computer will mistakenly recommend project B? Enter a number between 0 and 100. You will receive $2 if your answer is within 5 points of the correct percentage.

2. Suppose a Project from Industry A is Bad. What is the chance that a computer will mistakenly recommend Project A? Enter a number between 0 and 100. You will receive $2 if your answer is within 5 points of the correct percentage.

Question 3: Suppose that one computer recommends project A and the other recommends project B. Suppose that you recommend project A, which implies that the company will then invest in project A. What is the chance that this investment in project A turns out to be good? Enter a number between 0 and 100. You will receive $2 if your answer is within 5 points of the correct percentage.

Instructions: Part 3 (10 minutes)
In Part 3, you will participate in an environment that is very similar to the 100 rounds in Part 1, but with the following differences:

1. You will face 5 different cases. The chance that a project from industry A is good will now be 10%, 30%, 50%, 70%, or 90%, depending on the case. You will now know which case you are facing, so you will know the chance that project A is good. For each case, you will have to choose a decision rule by clicking on the slider below the case.

2. The second difference is that there are no computers submitting recommendations. Therefore, your recommendation is the only one that counts.

How do you make your choice for each case? In the same screen, you will see all 5 cases at once. You will then have to select a decision rule for each of the 5 cases. The interpretation is the same as in Part 1:
For any value of x lower than the value on which you clicked, you recommend project A for that case.
For any value of x higher than the value on which you clicked, you recommend project B for that case.
Once you make your choices for each of the 5 cases, you can submit your choices by clicking on the "Submit" button. You can change your choices as many times as you want before clicking the "Submit" button.

How your payment in Part 3 is determined: Your payment will be determined by selecting one of the 5 cases with equal probability. We will then select a value of x (with any

number between 1.25 and 4.75 with equal probability) and use your decision rule in the selected case to determine your vote. We will then pay you according to the following table, which is similar to the payoff table in Part 1 of the experiment, except that now there are no computers submitting recommendations and your recommendation alone determines the project in which the company invests.

	Project A is good	Project A is bad
You recommend project A	5 points	1 point
You recommend project B	x points	x points

You will now be asked some brief questions about these instructions. Please raise your hand if you have any questions.

[Subjects read and answer the following questions on the screen. The questions are repeated until all are answered correctly.]

Question 1: In Part 3 you have to submit 5 decision rules. True or False?

Question 2: In Part 3 your recommendation and the recommendations of two computers will determine your payoffs. True or False?

Question 3: The chance that a project from industry A is good will be the same in all cases of Part 3. True or False?

Instructions: Part 4 (final part of the experiment, 5 minutes)
This final part of the experiment is NOT related to any of the previous parts. In these instructions, we will explain how to answer Part 4.
To illustrate, consider a choice between Column A and Column B in the table below.

Column A You will receive		Column B You will receive
$2 with chances 50/100; $1.6 with chances 50/100		$3.85 with chances 50/100; $0.1 with chances 50/100

If you choose Column A, then your payoff will be $2 with chances 50/100 and $1.6 with chances 50/100. If you choose Column B, then your payoff will be $3.85 with chances 50/100 and $0.1 with chances 50/100.
In Part 4, you will actually observe a table with 10 rows. Each row of the table contains a choice between Column A and Column B. The full table will contain 10 rows. For each row, you have to choose whether you prefer the lottery described in Column A or the lottery described in Column B. If you prefer the lottery in Column A, you must click on the center column next to Column A; if you prefer the lottery in Column B, you must click on the center column next to Column B. Only one option can be selected for each row, and you can change your selection as many times as you would like before clicking the "Submit" button.

How your payment in Part 4 is determined: Your payment will be determined by selecting one of the 10 rows in the table with equal probability. We will pay you according to the choice you made for that row (either Column A or Column B).

Please raise your hand if you have any questions.

REFERENCE

Train, K. E. (2009), *Discrete Choice Methods With Simulation*. Cambridge University Press. [9]