Long-term government debt and household portfolio composition

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Formal dynamic analyses of household portfolio choice in the literature focus on holdings of equity and a risk-free asset or bonds of different maturities, neglecting the interdependence of the decisions to invest in equity, short-term and long-term bonds made by households. Data from the Survey of Consumer Finances is used to derive stylized facts about participation in the long-term government-debt market and conditional portfolio shares. To explain the mechanisms underlying these facts, I draw on a life-cycle model in which investors have access to three financial assets—equity, long-term debt, and a riskless short-term bond—and are exposed to uninsurable idiosyncratic risk through nonfinancial income as well as aggregate risk through the asset returns. An application shows that the low Treasury returns observed in the US between 2009 and 2013 have quantitatively significant yet transitory effects on the composition of household portfolios. In combination with the observed rise in stock returns, they lead to persistent changes in the participation rate, the conditional portfolio shares, and the distribution of wealth.

Keywords. Dynamic portfolio choice, life cycle, long-term government debt, asset-market participation, survey of consumer finances.

JEL classification. D10, D15, E21, G11.

1. Introduction

Analyses of portfolio choice over the life cycle generally focus on holdings of stock and a risk-free asset, not taking into account the significant positions of long-term government debt that can be found in household portfolios. With short-term nominal interest rates close to the zero lower bound in Europe and the US in the aftermath of the the Financial Crisis of 2007 to 2009, central banks have purchased large amounts of long-term debt as a part of their unconventional monetary policy programs. In this context, the role that long-term bonds play in household portfolios and the motives for rebalancing portfolios in response to return shocks have become of considerable interest. Using a life-cycle model in which agents can invest in three financial assets—stocks, long-term government debt, and a riskless asset—this paper studies the decision to participate in

financial markets and the composition of household portfolios over the course of the life cycle with a focus on the role of long-term government debt.

Stylized facts are derived based on a data set constructed from seven consecutive waves of the Survey of Consumer Finances (SCF). The joint existence of birth cohort, time, and age influences on the participation rate and the respective portfolio shares conditional on participation result in a well-known identification problem.[1] Using three different identification strategies, two from the literature and one novel, a latent variable model with a participation equation is estimated. The standard approach based on cohort restrictions performs the least well. The remaining two, although distinct, give nearly identical results. Similar to participation in the stock market, participation in the market for long-term debt takes an inverse U-shape. While the conditional portfolio share of stocks is declining with age, the conditional share of long-term government debt is moderately increasing until the age of around 55 and significantly lowered from about 65 onwards.[2] Long-term bond holdings and holdings of the riskless asset differ with respect to their elasticities of substitution with equity, suggesting that the shares of long-term debt and the riskless asset are rebalanced in distinctive ways in response to wealth and return shocks.

The theoretical analysis is based on a model in which agents adjust consumption and holdings of the three financial assets facing uninsurable labor or retirement income risk as well as random stock and long-term debt returns. Fixed participation costs prevent agents from investing in stocks and government debt at young age. The participation rate first rises as they accumulate wealth and later declines due to agents retiring and running down their savings. The average long-term debt and stock share conditional on participation respectively increases and decreases with age during the employment stage in line with the data. This is the case, since the portfolio income of market participants grows on average, implying that the ratio of portfolio to labor income rises. As a result, agents with CRRA utility rebalance their portfolios to reduce their risk exposure. Long-term debt plays an important part in this process, because its return is less volatile than that of stocks but higher in expectation than that of the risk-free asset. Incompleteness of financial markets gives rise to a nondegenerate distribution of wealth. Agents that consistently participate in the markets for equity and debt at a young age accumulate wealth faster than those that enter later or remain stuck below the participation threshold, implying that the wealth distribution among employed investors shows the characteristic positive skew found in the data and that inequality increases with age.

Finally, the model is used to study the period of negative real 5-year Treasury returns and elevated real stock returns that followed the recession of 2007 to 2009 in the US. In the model, the Treasury return shocks observed between 2009 and 2013, when considered in isolation, lead to a significant rebalancing of household portfolios towards stock holdings. The adjustments are transitory though during the employment stage. This is

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[1]Browning, Crawford, and Knoef (2012) gave a detailed description of the "age-period-cohort" problem. According to them, the problem can be traced back at least to Ryder (1965); see also Ameriks and Zeldes (2004).

[2]The results regarding equity holdings are in line with findings by Fagereng, Gottlieb, and Guiso (2017) and Gomes and Michaelides (2005)

the case, because the negative effect of the debt return shocks on the portfolio return are compensated by the higher stock share so that wealth, and hence the participation rate are nearly unaffected. Consequently, the initial adjustments are undone when the Treasury return rises again. The observed shocks to the returns of both assets jointly cause average wealth to rise and the participation rate to increase. The change in average wealth has persistent effects on the holdings of all three assets and the wealth distribution among agents that are hit by the shocks at an intermediate age is more unequal for the remainder of nearly their entire lives.

A large literature is concerned with portfolio choice over the life cycle. Early contributions by Merton (1969, 1971) and Samuelson (1969) analyze optimal portfolio choice neglecting the asset market participation decision. More recent examples include, but are not limited to, Alan (2006), Bonaparte, Cooper, and Zhu (2012), Campanale, Fugazza, and Gomes (2015), Cocco, Gomes, and Maenhout (2005), Fagereng, Gottlieb, and Guiso (2017), Gomes and Michaelides (2005), and Haliassos and Michaelides (2003).[3] In these papers, households are restricted to holdings of stocks and a riskless asset. I relax this constraint by adding a long-term bond to the portfolio choice problem. Bagliano, Fugazza, and Nicodano (2014) studied a life-cycle model with a safe asset and two risky assets focusing on how the portfolio shares evolve when the stock return is correlated with labor income. They assume that the second risky asset is identical to stocks aside from the mean and variance of its return, not incorporating characteristics of long-term bond returns like a realistic degree of autocorrelation. Campbell and Viceira (2001) and Wachter (2003) considered asset allocation problems with long-term bonds but do not study life-cycle effects. In the model used here, long-term debt is only partially liquid, reflecting the fact that long-term debt like US savings bonds can be sold only at a substantial cost in the first years after they have been issued. As in Campanale, Fugazza, and Gomes (2015), the composition of financial wealth therefore becomes an important state variable in the portfolio choice problem. While they assume that only holdings of the risk-free asset can be transformed costlessly into consumption, the portfolio composition matters here because of a maturity-specific liquidity constraint.

The paper is organized as follows. Section 2 presents the empirical results. It starts by describing the data set, then gives a detailed discussion of the identification strategies employed and finally shows the estimation results. Section 3 contains a description of the model, its calibration, and the resulting policy functions. In Section 4, model simulations are confronted with the data and the model is applied to the 2009–2013 period in the US. Section 5 concludes.

2. Stylized facts

This section presents stylized facts on long-term government bond and stock holdings of US households which inform and provide a benchmark for the model-based analysis that follows. With the purely descriptive approach many times adopted in the literature, adjustments of conditional asset shares cannot be reliably isolated from changes at the

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[3]The model follows this literature in abstracting from informational frictions or incentive problems that may arise if households delegate the portfolio-choice decision to a portfolio manager.

extensive margin and life-cycle effects cannot be reliably isolated from sampling period and cohort effects.[4] The section therefore contains a careful discussion of the models estimated and the strategies used to achieve parameter identification.

2.1 *Data*

The data set employed is constructed from the seven consecutive waves of the SCF collected between 1989 and 2007.[5] Since the data consist of repeated cross-sections rather than a panel, I am not able to track *individuals* over time. However, due to the large amount of households included in each survey wave, one can track *cohorts* of individuals defined by their birth year over the sample period.[6]

It should be noted that there is an intentional oversampling of wealthy households in the SCF relative to the US population. This is done to allow for more precise estimates of financial asset holdings, which are highly concentrated among households in the upper tail of the wealth distribution, and to correct for the fact that the nonresponse rate is positively correlated with wealth (Kennickell (2008)). The benefit of much improved estimation precision comes at the cost of being able to make inferences for wealthier households only. Nonetheless, I believe that uncovering the life-cycle patterns in asset holdings among those that are the likely holders of the assets in question is of considerable interest. Descriptive statistics of the sample are shown in Table 1.

The SCF contains information on a large variety of assets held by households, which I divide into three categories. These categories are long-term government debt, stocks, and a residual category that mainly includes cash/liquidity, short-term sovereign debt, and corporate bonds. Most assets that appear in household balance sheets can be fully attributed to one of these three categories. When this is not the case, a careful partial assignment is done based on additional information about the institutions that issue the asset in question.

According to the "Monthly Statement of the Public Debt of the United States" from December 2007, 22.1% of total marketable debt held by the public took the form of Bills (maturity of 1 year or less), while the remaining 77.9% were issued in the form of Notes, Bonds, and TIPS (maturity of 2 years or more). Assuming that agents are homogeneous in regards to the maturity composition of government debt in their portfolios, 77.9% of the marketable US government debt held by a household is assigned to long-term government debt and the remaining part to the residual category. Savings bonds and tax-exempt bonds, for example, are fully assigned to the long-term debt category, since they typically have a maturity of several years.

Funds held in individual retirement accounts (IRAs) are also divided into more than one category. An IRA is a tax-advantaged retirement savings plan. Funds transferred

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[4]A notable exception is Fagereng, Gottlieb, and Guiso (2017) who equally account for the participation decision by estimating a latent variable model and use a set of strategies to address the age-period-cohort problem which partially overlaps with that employed here but focus solely on the stock market.

[5]Data from later years are not used here to avoid bias introduced by crisis-specific effects. Data from before 1989 are not used due to changes in the availability of a subset of variables.

[6]This fact was pointed out by Deaton and Paxson (1994) and Deaton (1997).

TABLE 1. Descriptive statistics.

Variable	Mean	Variance	Min	Max
Household (h.h.) composition				
Age of h.h. head	50.32	10.04	26	76
Marital status of h.h. head	0.74	0.44	0	1
Number of children living in h.h.	0.98	1.20	0	10
Family income (gross, in thousands)				
Wages	210.04	1143.30	−86.92	88,630.81
Interest, dividends	112.17	981.75	0	51,550.12
Sales of stocks, bonds, real estate	149.97	1795.72	−1759.41	114,890.5
Retirement income, pensions, annuities	7.60	75.46	0	6550.77
Other, transfers	6.05	232.04	−1450.44	57,408.45
Highest education of h.h. head				
None	0.07	0.26	0	1
School diploma	0.24	0.42	0	1
Some college education	0.16	0.36	0	1
College diploma	0.53	0.50	0	1
Ethnic background of h.h. head				
White and non-Hispanic	0.84	0.36	0	1
Black	0.08	0.27	0	1
Hispanic	0.04	0.20	0	1
Other	0.04	0.19	0	1
Occupational category of h.h. head				
Managerial and professional	0.45	0.50	0	1
Technical, sales, and services	0.21	0.41	0	1
Other	0.18	0.39	0	1
Not working	0.15	0.36	0	1

Note: Nominal variables in 2010 US dollars.

into an IRA can be requested to be allocated to a large variety of financial assets. The Employee Benefit Research Institute (EBRI) collects data on the allocation of assets in IRAs.[7] Based on these data, I attribute 45.8% of the funds held in an IRA to stocks, 18.4% to long-term government debt, and the remainder to the last category. The assignment of all assets into the three broad categories is described in detail in Section A in the Appendix and summarized in Table A.1.

Figures 1–3 illustrate the average shares of the portfolio categories constructed in this way. Each line represents the average portfolio share of a given birth-year cohort at a particular age. Since data points are available only every 3 years, both respondent age and cohort (birth year) are divided into 3-year intervals. For example, the earliest data available are from 1989. The youngest age group considered includes households with a "household head" aged 26–28.[8] Individuals that are 26–28 of age in 1989 belong to the

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[7] See EBRI Note "IRA Asset Allocation and Characteristics of the CDHP Population, 2005–2010" from May 2011, available at www.ebri.org/publications/notes.

[8] In the SCF, the term "household" refers to a "primary economic unit," which consists of a core couple or economically-dominant individual and other individuals that are financially interdependent with that
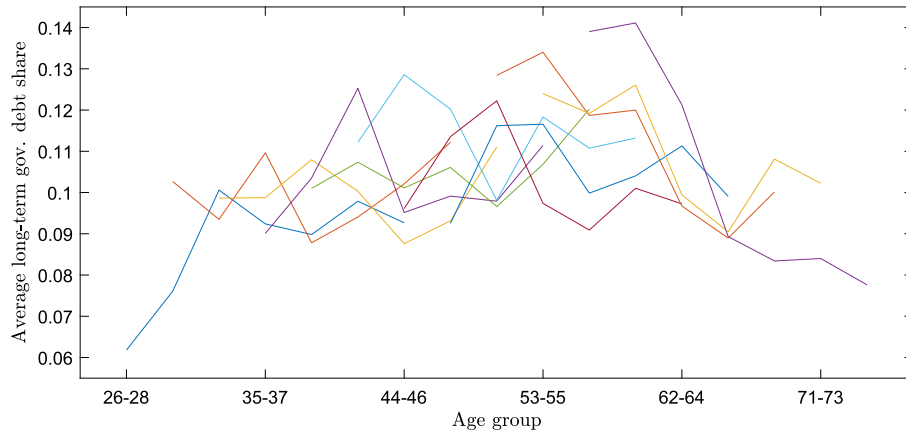
Figure 1. Average portfolio share of long-term government debt.
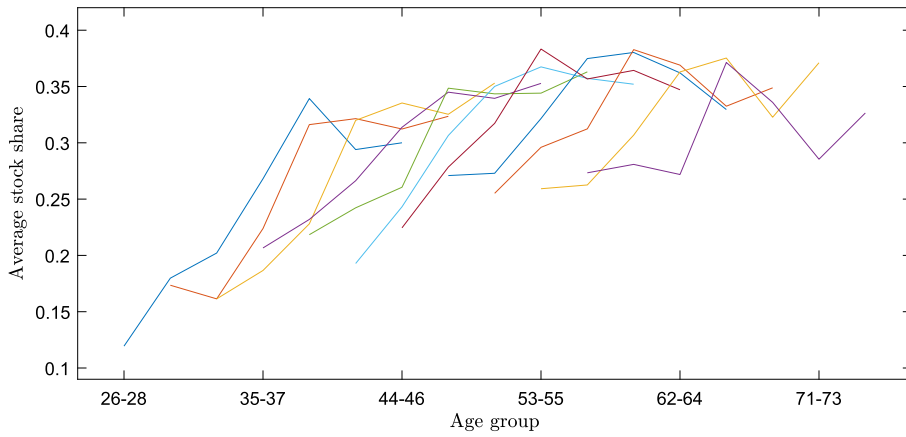

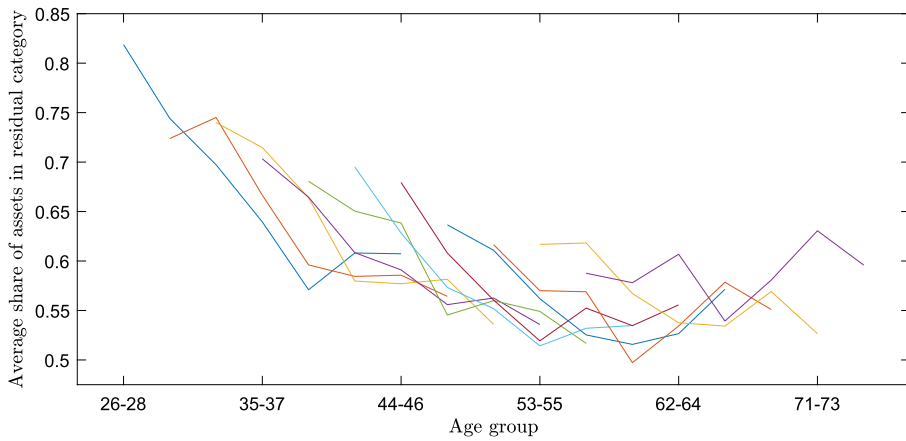Figure 2. Average portfolio share of equity.

Figure 3. Average portfolio share of cash/liquidity and other financial assets.

birth-year cohort 1961–63. The 1961–63 cohort is sampled seven times between 1989 and 2007. Its members are aged 29–31 in 1992, 32–34 in 1995 and so on. In Figures 1–3, the lines most to the left represent the average portfolio shares held by the 1961–63 cohort. Similarly, the lines starting with age group 29–31 represent the average portfolio shares of the 1958–60 cohort. Altogether, the sample contains the eleven cohorts born between 1931–33 and 1961–63, each observed at seven consecutive age groups between the ages of 26–28 and 74–76.[9]

The average portfolio share of stocks is increasing in household portfolios with a peak in the late fifties or early sixties of the household head. Average long-term government debt holdings behave in a somewhat similar way, although the pattern is less well pronounced. The average portfolio share of the residual category follows a pattern that is markedly distinct from that of the average long-term government bond share. However, it is a well-known fact the averages computed in Figures 1–3 provide a biased picture of the composition of household portfolios. The reasons are twofold.

First, the adjustments visible in the figures can be due to changes at the intensive or extensive margin. A number of papers report that the rate of participation in the stock market first increases and later decreases significantly over the life cycle.[10] This suggests that the inverse U-shape in Figure 2 largely results from households entering and exiting the stock market rather than adjustments at the intensive margin. An important question explored below is whether or not the same is true for holdings of long-term government debt. Figures A.1 to A.3, which plot the participation rates in the data, provide first suggestive evidence for the importance of adjustments at the extensive margin.

Second, as discussed in Ameriks and Zeldes (2004), it is not possible from the figures to disentangle the effects of age, observation period, and cohort. Age effects are related to education, family formation, and retirement. Period effects result from events that occur at the time of data collection. For example, the dot-com-bubble and its bursting is reflected in the survey waves from 2001 and 2004. Cohort effects include cohort-specific experiences like growing up during war time as was the case for the oldest cohorts in the sample. Even if, for example, we were to observe a figure of the same kind as Figures 1–3 in which all lines were perfectly aligned such that they formed one single upward-sloping line, we could not say whether this was due to pure age effects or a combination of time and cohort effects.[11]

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couple or individual. The "household head" is defined as the single economically-dominant individual in a household without a core couple, the male in a household with a mixed-sex core couple and the older individual in the case of a same-sex core couple.

[9]Only cohorts that fall inside this age interval at all seven survey waves are considered. Younger and older age groups are not examined due to a lack of sufficient data. Table A.2 in the Appendix shows the number of observations for each cohort-age pair. The data set contains multiple imputations as is explained in more detail in the table notes.

[10]See Ameriks and Zeldes (2004), Fagereng, Gottlieb, and Guiso (2017), and Guiso, Haliassos, and Jappelli (2003). Haliassos (2008) contained a more general summary of the literature on limited participation in asset markets.

[11]Time effects could cause each individual line to be sloped upwards and cohort effects of increasing size could result in all lines aligning precisely in the way previously mentioned.

2.2 *Identification*

As outlined above, identification problems arise from sample selection and perfect multicollinearity of a respondent's cohort, the age at which they are sampled and the year in which the survey is conducted (birth year + age = observation period). An unresolved issue in the literature on equity holdings over the life cycle is that estimation results are somewhat dependent on the underlying identifying assumptions.[12] I therefore present the results obtained under three different identification strategies. Two are borrowed from the literature and one is novel. A number of robust findings emerge. To be able to motivate the strategies employed below, the nature of the identification problem is laid out before in detail.

2.2.1 *Sample selection*   The self-selection of agents into participants and nonparticipants in the market for a given financial asset results in a sample selection problem. If agents enter and exit systematically over the course of the life cycle, the age effects on the conditional portfolio share are estimated with bias. To address this issue, I employ a standard latent variable model with a Probit selection equation. Formally, the model is given by

$$s_i = x'_{2,i}\beta_2 + \sigma_{12}\lambda\big(x'_{1,i}\hat{\beta}_1\big) + e_{2,i} \tag{1}$$

$\forall i$ where $s_i > 0$ is i's portfolio share of the asset in question, $\lambda \equiv \phi(x'_{1,i}\hat{\beta}_1)/\Phi(x'_{1,i}\hat{\beta}_1)$ is the Inverse Mills Ratio and $\hat{\beta}_1$ is obtained from estimating the first-stage Probit model

$$\Pr(P_i = 1 | x_{1,i}) = \Pr\big(x'_{1,i}\beta_1 + e_{1,i} > 0\big)$$
$$= \Phi\big(x'_{1,i}\beta_1\big) \tag{2}$$

$P_i = 1$ if i is a participant in the market for the asset considered and $P_i = 0$ otherwise. The error terms are normal, $e_{1,i} \sim N(0, 1)$ and $e_{2,i} \sim N(0, \sigma^2)$, with $\mathrm{Cov}(e_{1,i}, e_{2,i}) = \sigma_{12}$.

2.2.2 *The "age-period-cohort problem"*   Due to the multicollinearity described above, a simple linear model that aims to separate age, period, and cohort effects is underidentified. To see this, consider the following example.[13] Let a_i denote the age of respondents, t_i the time period in which they are sampled and c_i their year of birth. Suppose that observations are available for two consecutive time periods, $t_i \in \{t^1, t^2\}$, and that three consecutive cohorts are sampled in both periods, $c_i \in \{c^1, c^2, c^3\}$. Age can then take on four distinct values, $a_i \in \{a^1, a^2, a^3, a^4\}$.[14]

A projection of some variable of interest y_i on age, period and cohort indicators is

$$y_i = \alpha_1 a_i^1 + \alpha_2 a_i^2 + \alpha_3 a_i^3 + \alpha_4 a_i^4$$
$$+ \theta_2 t_i^2$$
$$+ \gamma_2 c_i^2 + \gamma_3 c_i^3 + e_i, \tag{3}$$

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[12]See Ameriks and Zeldes (2004) and Gomes and Michaelides (2005) for detailed discussions.
[13]See also Browning, Crawford, and Knoef (2012).
[14]For example, if $t_i \in \{2000, 2001\}$ and $c_i \in \{1950, 1951, 1952\}$ then $a_i \in \{48, 49, 50, 51\}$.

where $x_i^n = 1$ if $x_i = x^n$ and $x_i^n = 0$ otherwise for $x \in \{a, t, c\}$ and $n \in \{1, 2, 3, 4\}$. Note that t_i^1, c_i^1 and a constant have been omitted to prevent each set of binary variables from summing to the constant. However, the fact that there exists a linear relationship between the age, observation period and cohort of each respondent implies that the data matrix pertaining to equation (3) is not invertible and that parameter estimates cannot be computed using standard methods. More precisely, the linear relationship between age, period, and cohort implies[15]

$$2a_i^1 + a_i^2 - a_i^4 + t_i^2 = c_i^2 + 2c_i^3. \tag{4}$$

Inserting (4) into (3) yields

$$
\begin{aligned}
y_i = {} & \tilde{\alpha}_1 a_i^1 + \tilde{\alpha}_2 a_i^2 + \tilde{\alpha}_3 a_i^3 + \tilde{\alpha}_4 a_i^4 \\
& + \tilde{\gamma}_2 c_i^2 + \tilde{\gamma}_3 c_i^3 + e_i,
\end{aligned} \tag{5}
$$

where

$$
\begin{pmatrix} \tilde{\alpha}_1 \\ \tilde{\alpha}_2 \\ \tilde{\alpha}_3 \\ \tilde{\alpha}_4 \\ \tilde{\gamma}_2 \\ \tilde{\gamma}_3 \end{pmatrix} =
\begin{bmatrix}
1 & 0 & 0 & 0 & -2 & 0 & 0 \\
0 & 1 & 0 & 0 & -1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 1 & 1 & 0 \\
0 & 0 & 0 & 0 & 2 & 0 & 1
\end{bmatrix}
\begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \alpha_3 \\ \alpha_4 \\ \theta_2 \\ \gamma_2 \\ \gamma_3 \end{pmatrix}. \tag{6}
$$

Using equation (5), one can estimate the six reduced form parameters $\tilde{\alpha}_1$, $\tilde{\alpha}_2$, $\tilde{\alpha}_3$, $\tilde{\alpha}_4$, $\tilde{\gamma}_2$, $\tilde{\gamma}_3$. From (6), it is clear though that it is not possible to solve for the seven structural parameters $\alpha_1, \alpha_2, \alpha_3, \alpha_4, \theta_2, \gamma_2, \gamma_3$ knowing the reduced form parameters. The structural parameters are underidentified, unless at least *one* parameter restriction is imposed. It can be easily shown that this result generalizes to scenarios with more observation periods and cohorts.

To be able to judge the robustness of the estimation results, I pursue three distinct identification strategies. The first and most standard is to impose an equality restriction on neighboring cohort effects, that is, to impose

$$\gamma_n = \gamma_{n+1} \tag{7}$$

for some n. This restriction formally reduces the generality of the model, yet the bias it introduces should be expected to be small if two neighboring cohorts can be identified that have a sufficiently similar history.

The second strategy was suggested by Deaton and Paxson (1994), and more recently used by Fagereng, Gottlieb, and Guiso (2017) among others. The idea is to attribute cyclical fluctuations to time effects and trends to age and cohort effects. This is achieved by

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[15]Continuing the previous example, a person that is born say in 1952 and surveyed in 2001 is aged 49 when surveyed; thus $t_i^2 = c_i^3 = a_i^2 = 1$ and $a_i^1 = a_i^4 = c_i^2 = 0$. It is straightforward to verify that (4) holds for all six such combinations of binary-variable values for which the birth year and the age sum to the observation period.

requiring time effects to sum to zero and to be orthogonal to a linear time trend, that is,

$$g'\theta = 0, \tag{8}$$

where $g = (0, 1, \ldots, T-1)'$ is the trend, θ is the vector of coefficients on the time dummies and T is the number of observation periods. This set of restrictions correctly identifies all effects if indeed only age and cohort effects are trending. In the context here, one cannot be sure however that there is no trend in time effects. In particular, in the time period examined (1989–2007), stocks became a more widely-used mode of saving. Imposing (8) when a trend in time effects is present in the data could cause the coefficients on the age and cohort variables to jointly pick up this trend and, therefore, to be biased. In the second-stage regression, I therefore follow Fagereng, Gottlieb, and Guiso (2017) in detrending the dependent variable, the portfolio share of a given asset, by subtracting its cross-sectional average at each time period. Since this is not feasible in a binary dependent variable model, I add a linear time trend as an explanatory variable at the first stage. This implies that one additional dummy has to be excluded from the Probit model.

Under the final identification strategy, the time dummies are replaced with the first p principal components of a large set of stationary macroeconomic time series covering the entire sample period. This resolves the linear dependence of the independent variables. As before, the asset share is detrended and a trend is added to the selection equation. To the extent that the principal components contain the effects otherwise picked up by the time dummies, this modification allows controlling for age, period, and cohort effects without parameter restrictions. In particular, institutional and regulatory changes concerning the usage of different savings instruments can be expected to be reflected in asset prices and interest rates. Note that to provide a meaningful addition to the previous identification approach, p should not be chosen too large.[16]

2.3 Estimation and results

Beginning with the second-stage regression, the equations estimated in case of the first identification strategy (parameter restriction on cohort effects) are

$$s_i = a_i'\alpha_2 + t_i'\theta_2 + c_i'\gamma_2 + s\lambda_i + z_{2,i}'\delta_2 + e_{2,i}, \tag{9}$$

$$\Pr(P_i = 1 | x_{1,i}) = \Phi\big(a_i'\alpha_1 + t_i'\theta_1 + c_i'\gamma_1 + z_{1,i}'\delta_1\big) \tag{10}$$

$a_i = (a_i^{26-28}, a_i^{29-31}, \ldots, a_i^{74-76})'$ is a complete set of age dummies for seventeen age groups, $t_i = (t_i^{1992}, t_i^{1995}, \ldots, t_i^{2007})'$ is a vector of 6 year dummies and $c_i = (c_i^{1934-36},$

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[16]Suppose a model with Deaton–Paxson restrictions contains T' time dummies, which, together with the two constraints that the time effects be orthogonal to a linear trend and sum to zero, can be summarized by $T' - 2$ variables constructed in an appropriate way. Then, if the principal components included under the final identification strategy are also approximately orthogonal to a linear trend and mean zero, a model with $p \geq T' - 2$ principal components spans the same space as the one with time dummies and Deaton–Paxson restrictions. To avoid this case, it is ensured in the estimation below that $p < T' - 2$.
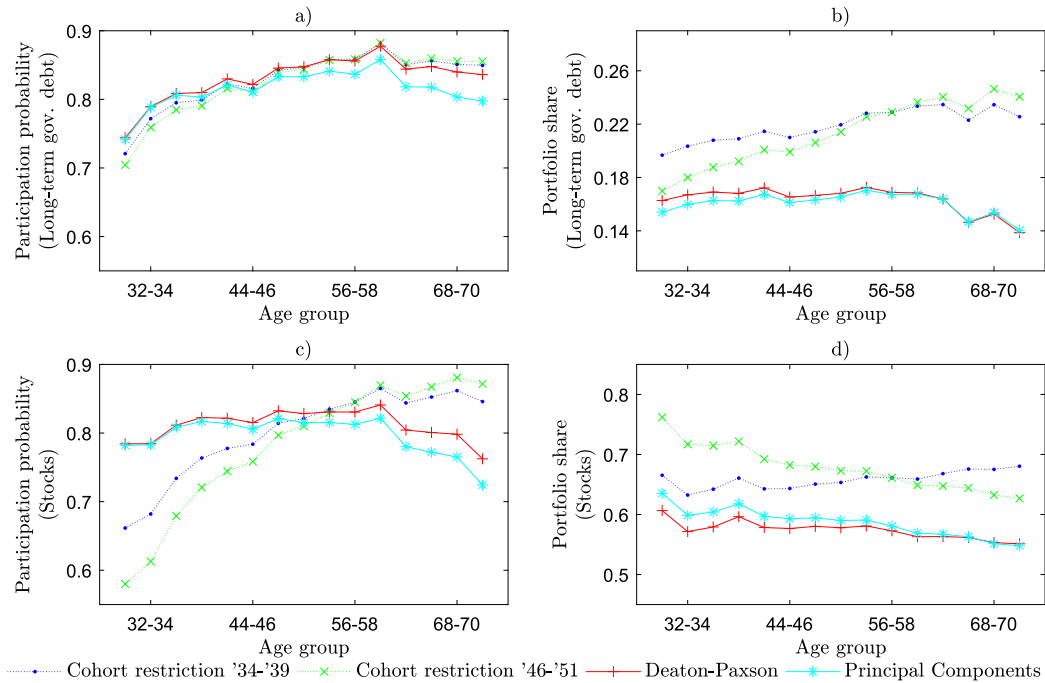
FIGURE 4. Predicted participation probabilities and conditional asset shares.

$c_i^{1937-39}, \ldots, c_i^{1961-63})'$ contains a dummy for each of ten cohorts. $z_{2,i}$ and $z_{1,i}$ are additional household-specific controls and $x_{1,i} \equiv (a_i, t_i, c_i, z_{1,i})'$. In the case of the other two strategies, the equations are modified as explained in the previous section. Information on the controls used in the estimation and a detailed discussion of the exclusion restrictions imposed in the second step of the selection model are contained in Section D of the Appendix.

Figure 4 plots the estimation results for all three identification strategies outlined before including separate sets of results for two different cohort restrictions. The first cohort restriction equates the effects of the two oldest cohorts in the sample, 1934–36 and 1937–39, the second one those from the first two post-war cohorts, 1946–48 and 1949–51. The cohort effects of the oldest respondents are equated, since it seems likely for any differences between them to wash out over the years until the sampling period and the second restriction may appear reasonable from a historical perspective. In the model in which the time dummies are dropped, the first $p = 3$ principal components of a large set of macroeconomic aggregates from the US are used.[17] Panels (a) and (c) show the marginal values, the average predicted probabilities, of being a stockholder and a long-term government debt holder, respectively, for each age group. Panels (b) and (d) graph the corresponding average predicted portfolio shares conditional on participation in the respective asset market. Since the dependent variable in (b) and (d) is detrended

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[17]Details about the macroeconomic time series employed and the principal components are given in the Online Appendix found in the Supplementary Material of this paper (Tischbirek (2019)) of this paper.

when the Deaton–Paxson restrictions are imposed and when the principal components are used to capture time effects, the mean asset share conditional on participation is added to the average predicted values in these two instances to produce the estimates shown.

From the figure, it becomes obvious that imposing different ex ante plausible cohort restrictions does not yield robust estimates. While all estimates for long-term debt market participation are of similar shape, the estimates obtained when cohort restrictions are employed deviate significantly from each other and from the results obtained under the remaining two identification schemes in the panels (b) to (d). Experimenting with different cohort restrictions showed that the discrepancies are even more severe for other pairs of economically plausible restrictions, likely because trends in the cohort effects not accounted for by the model are forced into the estimates of the age effects. However, the results obtained using Deaton–Paxson restrictions and principal components nearly coincide despite of their distinct way of accounting for time influences and are consistent with previous findings about equity holdings from the literature.[18]

Several stylized facts emerge from the estimations that make use of Deaton–Paxson restrictions or principal components. The profile of participation in the market for long-term government debt shows a pronounced hump shape. Participation rates rise over the course of nearly the entire working life and then begin to decline at the age of 62–64 as household members retire. The age effects on the conditional portfolio share of long-term government debt are mildly increasing at first and roughly constant from the mid-forties until retirement. A significant decline is not observable until after the age of 65. Overall, the results suggest that there is a clear inverse U-shape in participation rates and that the conditional portfolio share is nondecreasing until retirement, but falls thereafter. Stock market participation takes an almost identical shape to participation in the market for long-term government debt. The conditional stock share is monotone declining from 39–41 onwards.

The life-cycle dynamics of stock-market participation and the conditional share of stocks have been a topic of debate in the literature. In summarizing the existing empirical evidence, Gomes and Michaelides (2005) stated that (1) stock-market participation increases over the working life, (2) there is some evidence which suggests that participation rates decline after retirement, and (3) there is "no clear pattern of equity holdings over the life cycle." I interpret the results presented here as support for (1) and (2). In recent work, Fagereng, Gottlieb, and Guiso (2017) found evidence for the conditional stock share to decline over the life cycle using administrative panel data from the Norwegian Tax Registry.[19] Regarding (3), my estimates are more in line with their findings.[20]

Table 2 provides more detailed information about the estimations for the long-term debt share. The results from the models with cohort restrictions are included for com-

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[18]The estimated cohort and time effects are shown in the Online Appendix.

[19]Considering cross-sectional data only, other studies conclude that the conditional equity share may be mildly increasing or also mildly hump-shaped; see Campanale, Fugazza, and Gomes (2015) for a short discussion.

[20]In the Online Appendix, it is shown that the stylized facts are robust to reassigning corporate bond holdings to the long-term debt category.

TABLE 2. Estimation results for long-term government debt share.

	Deaton–Paxson		Principal Comp.		Cohort Restr. 1		Cohort Restr. 2	
	1st st.	2nd st.	1st st.	2nd st.	1st st.	2nd st.	1st st.	2nd st.
λ		0.065		0.066		0.065		0.065
		(0.021)		(0.021)		(0.021)		(0.021)
ρ		0.445		0.449		0.445		0.445
$\min(N_{\text{imp}})$	17,202	12,673	17,202	12,673	17,202	12,673	17,202	12,673
Total N	86,030	63,437	86,030	63,437	86,030	63,437	86,030	63,437
Sign. tests								
Age eff's	0.000	0.066	0.000	0.090	0.000	0.000	0.000	0.000
(d.o.f.)	(16)	(17)	(16)	(17)	(17)	(17)	(17)	(17)
Time eff's	0.029	0.325			0.051	0.000	0.051	0.000
(d.o.f.)	(5)	(5)			(6)	(6)	(6)	(6)
Cohort eff's	0.319	0.007	0.318	0.094	0.256	0.114	0.256	0.114
(d.o.f.)	(10)	(10)	(10)	(10)	(9)	(9)	(9)	(9)
Pri. comp's			0.027	0.153				
(d.o.f.)			(3)	(3)				

Note: Results of first and second stage estimation shown for four models—Deaton–Paxson restrictions, principal components of macroeconomic variables replacing time dummies, cohort effects equated for '34–'36 and '37–'39 (Cohort Restr. 1), cohort effects equated for '46–'48 and '49–'51 (Cohort Restr. 2). Models estimated using two-step estimator (Heckit). Data are multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors (in parentheses) are adjusted in an appropriate way. N_{imp} is number of obs. for imputation imp $\in \{1, 2, \ldots, 5\}$. For joint significant tests, avrg. p-value shown (each test stat. $\sim \chi^2$), degrees of freedom in parentheses.

parison purposes. The coefficient on the inverse Mills ratio λ is significant and the correlation of first-stage and second-stage residuals ρ is estimated to be positive, confirming the need to address the selection problem. SCF data are multiply imputed. The size of the smallest imputation group $\min(N_{\text{imp}})$ therefore gives a more accurate picture of the number of respondents in the sample than the total amount of observations N. With Deaton–Paxson restrictions and principal components, all age effects are significant. The significance level is slightly higher in the selection equations than in the conditional portfolio share equations, in line with the estimated size of the slope coefficients. Time effects play an important role at the first stage but cease to do so at the second stage. Thus, the participation decision is strongly influenced by time effects even after controlling for a linear trend. Demeaning the conditional long-term debt share successfully eliminates time effects at the second stage. The estimated cohort effects are jointly significant only for the conditional asset share.

To uncover the interdependence between the different portfolio components, I additionally estimate the models for the stock share with (financial and nonfinancial) wealth and either the long-term bond share or the share of the residual category "cash" as independent variables. The results are shown in Table 3. Conditional on wealth, a higher long-term debt share is correlated with a higher probability of being a stockholder, while the opposite is true for the portfolio share of cash, as one would expect. The estimates from the second stage suggest that the elasticities of substitution between long-term debt and equity and between cash and equity differ, reflected in coefficients of −0.52

TABLE 3. Substitution of long-term government debt and cash with equity.

	Deaton–Paxson		Deaton–Paxson		Principal Comp.		Principal Comp.	
	1st st.	2nd st.	1st st.	2nd st.	1st st.	2nd st.	1st st.	2nd st.
Long-t. debt	3.305	−0.516			3.306	−0.516		
	(0.164)	(0.022)			(0.164)	(0.022)		
Cash			−5.241	−0.884			−5.239	−0.884
			(0.119)	(0.008)			(0.119)	(0.008)
Wealth	0.0066	0.0004	0.0041	0.0000	0.0066	0.0004	0.0041	0.0000
(in millions)	(0.0013)	(0.0001)	(0.0013)	(0.0000)	(0.0013)	(0.0001)	(0.0013)	(0.0000)
$\min(N_{\mathrm{imp}})$	17,202	12,750	17,202	12,750	17,202	12,750	17,202	12,750
Total N	86,030	63,799	86,030	63,799	86,030	63,799	86,030	63,799
Sign. tests								
Age eff's	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.001
(d.o.f.)	(16)	(17)	(16)	(17)	(16)	(17)	(16)	(17)
Time eff's	0.000	0.096	0.000	0.033				
(d.o.f.)	(5)	(5)	(5)	(5)				
Cohort eff's	0.332	0.000	0.231	0.041	0.234	0.000	0.465	0.241
(d.o.f.)	(10)	(10)	(10)	(10)	(10)	(10)	(10)	(10)
Pri. comp's					0.000	0.027	0.000	0.019
(d.o.f.)					(3)	(3)	(3)	(3)

Note: Results of first and second stage shown for four models estimated using two-step estimator (Heckit). Data are multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors (in parentheses) are adjusted in an appropriate way. N_{imp} is number of obs. for imputation imp $\in \{1, 2, \ldots, 5\}$. For joint significant tests, avrg. p-value shown (each test stat. $\sim \chi^2$), degrees of freedom in parentheses.

and −0.88, respectively.[21] Additional cash holdings are associated with a larger reduction in the stock share than additional long-term debt holdings. In addition to differing age profiles, this suggests that long-term debt plays a significant and distinctive role in the dynamic rebalancing of household portfolios. The model outlined in the following section allows studying these relationships in more detail.

3. Model

There is a large number of agents who are faced with an asset market participation decision and, conditional on participation, an asset allocation problem in each period of their lives. I refer to model agents interchangeably as households or investors below.[22] Investors are born employed. They retire and subsequently die, providing them with a motive to save for retirement and to deplete their asset stock once retired. Asset market participation is costly, but allows an investor to hold stocks *and* long-term government debt. A non-participant is able to save only through a riskless and low-interest bearing

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[21]Significantly differing values also emerge from a naive OLS regression among stock and long-term debt holders. See Table A.3 in the Appendix.

[22]A model agent can be viewed as a household that either is in direct control of the consumption-savings and the portfolio-choice decision or delegates the latter decision to a portfolio manager that is informed about the risks faced by the household and its preferences toward them.

asset that is comparable to short-term bonds or cash. Thus, agents who choose to invest in only one of the two risky assets have to incur the entire asset market participation cost.[23] Investors are subject to uninsurable idiosyncratic and aggregate risk. Idiosyncratic risk arises from nonfinancial income and aggregate risk results from the returns on stocks and long-term government debt.

3.1 *Life-cycle stages*

Each investor $i \in I$ lives for T periods and goes through an employment and a retirement stage. Investors are born employed at the beginning of period $t = 1$, retire in period $T_{\text{ret}} > 1$ and die at the end of period $T > T_{\text{ret}}$. Note that the model describes the decisions of a large number of agents belonging to the *same* generation and that, as a result, there is no interaction between different, potentially overlapping, generations. Investors supply labor inelastically as long as they remain employed, which entitles them to an exogenous income stream given by

$$Y_{i,t} = P_{i,t}U_{i,t}, \quad \ln U_{i,t} \sim N\left(-0.5\sigma_u^2, \sigma_u^2\right), \tag{11}$$

$$P_{i,t} = GP_{i,t-1}N_{i,t}, \quad \ln N_{i,t} \sim N\left(-0.5\sigma_n^2, \sigma_n^2\right). \tag{12}$$

Labor income has a transitory component $U_{i,t}$ and a persistent component $P_{i,t}$.[24] The logarithm of $P_{i,t}$ follows a random walk with drift. The expectation of the shock to the persistent component of income $N_{i,t}$ and the expectation of the transitory shock $U_{i,t}$ equal one, so that, in expectation, the labor income of all agents grows at the common rate $G - 1$.[25] Retired investors receive a pension $\Omega_{i,t} = \omega P_{i,T_{\text{ret}}}$ which is a fraction of the persistent income that they obtained in the last period in which they were employed as in the model of Gomes and Michaelides (2005) among others. This specification captures the empirical fact that differences in income that develop over the course of the working life persist among retired investors.

3.2 *Investment opportunities*

There are three types of assets available to the investors: a one-period bond, stocks, and long-term government debt. Long-term government debt has a maturity of δ periods. A strategy frequently adopted in the literature is to assume that long-term bonds are entirely illiquid, or more precisely, that they have to be held until maturity. Aside from understating the liquidity of long-term government debt, this assumption leads to a big inflation of the state space as δ becomes large, causing exact solutions to portfolio choice

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[23]The model does not include separate participation costs for the long-term government debt market and the stock market to reduce the dimensionality of the portfolio choice problem. Participation in both markets is highly correlated in the sample with a coefficient of 0.80 and Figure 4 suggests that this simplification yields a good approximation of observed household behavior. Figure A.8 shows that reestimating the empirical models with a joint asset-market participation decision does not alter the stylized facts described in the previous section.

[24]This income process is frequently used in the literature and originally due to Carroll, Hall, and Zeldes (1992). They refer to P somewhat ambiguously as "permanent labor income."

[25]In general, if $\ln x \sim N(\mu, \sigma)$, then $\mathrm{E}x = \exp(\mu + 0.5\sigma^2)$, therefore, $\mathrm{E}U_{i,t} = \mathrm{E}N_{i,t} = 1$.

problems to be computationally burdensome. A specification is proposed here that, in accordance with the US long-term bond market, allows investors to access some of the funds held in the form of long-term debt in each period and that makes the portfolio optimization computationally feasible.

An investor that has purchased long-term government bonds in period $t - 1$ at the amount of $Q_{i,t-1}$ receives a Calvo-type signal for each infinitesimal unit of $r_{q,t}Q_{i,t-1}$ in period t indicating whether it can be sold or not. $r_{q,t}$ is the annual gross return on the long-term government bond. A positive signal is received with probability δ^{-1}, implying that each infinitesimal unit has to be held on average for δ periods. Thus, portfolios are chosen in all periods subject to the constraint

$$Q_{i,t} \geq (1 - \delta^{-1}) r_{q,t} Q_{i,t-1}. \tag{13}$$

Comparable to the case in which long-term bonds have to be held until maturity, the minimum expected holding period of the entire stock is equal to its maturity, but a fraction of this stock can be accessed in each period. Since the probability of being able to sell a given unit of long-term debt is time-constant, all long-term debt held by i can be summarized by one single state variable. Modeling long-term government bonds as a perpetuity as in Woodford (2001) would equally permit all long-term debt to be represented by a single state variable. However, the specification chosen here emphasizes the imperfect liquidity of long-term government debt, which is an important characteristic of assets such as US savings bonds and tax-exempt bonds.[26]

The one-period bond yields the riskless gross return r_b. Following Bonaparte, Cooper, and Zhu (2012), the gross stock return $r_{s,t}$ evolves according to a two-state Markov process, $r_{s,t} \in \{r_s^l, r_s^h\}$, with mean r_s and standard deviation σ_{rs}. Accounting for capital gains and dividends, Bonaparte, Cooper, and Zhu cannot reject that the annual stock return in the US is serially uncorrelated. $r_{s,t}$ is therefore assumed to be i.i.d. across periods with probabilities of a half for both return states. The return on long-term government debt equally follows a two-state Markov process. The mean, the standard deviation, and the transition matrix are given by r_q, σ_{rq}, and Γ_{rq}, respectively. No restrictions are placed on Γ_{rq}, allowing for persistence in the government bond return process.

Holdings of the short-term bond $B_{i,t}$ are costless. Investments in stocks $S_{i,t}$ and long-term government debt $Q_{i,t}$ are associated with a cost of size $\Psi_{i,t} = \psi P_{i,t}$ if i is employed and $\Psi_{i,t} = \psi P_{i,T_{\text{ret}}}$ if i is retired that has to be paid in each period of active participation in the markets for stocks or long-term government debt. $\Psi_{i,t}$ represents, for example, costs associated with the acquisition of information about financial markets and is scaled to the persistent component of income in order to capture the opportunity cost of time.[27] Investors are not considered active participants in financial markets if they hold no stocks and allow potential previously-acquired holdings of long-term bonds to

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[26]The two specifications are similar though. For a perpetuity, the pay-off stream from a one-dollar investment is $\rho, \rho^2, \rho^3, \ldots$ for some $\rho \in [0, \beta^{-1})$. Here, if government debt is run down at the fastest possible rate, this stream is $(1 - \delta^{-1})r_{q,t+1}, (1 - \delta^{-1})^2 r_{q,t+1}r_{q,t+2}, (1 - \delta^{-1})^3 r_{q,t+1}r_{q,t+2}r_{q,t+3}, \ldots$ with $(1 - \delta^{-1}) \in [0, 1)$.

[27]In Alan (2006), the cost of stock-market participation is equally made dependent on the persistent component of labor income, however, it is incurred only the first time an agent enters the market and not, for example, at a later reentry.

mature at the fastest possible rate. Thus, if the investor chooses not to pay $\Psi_{i,t}$, $S_{i,t} = 0$ and (13) holds with equality.[28] In addition, stock holdings are subject to a variable cost $\psi_s S_{i,t}$, reflecting the monetary costs of maintaining a stock portfolio. The role played by the two types of costs is revisited below in more detail.

3.3 *Optimization problem*

The optimal plan of investor $i \in I$ solves the problem described in this section in each period $t = 1, 2, \ldots, T$. The indices i and t are suppressed below for notational clarity.

3.3.1 *Financial-market participants*　The budget constraint of an investor that participates in financial markets is given by

$$C + S(1 + \psi_s) + B + Q + \Psi = r_s S_{-1} + r_b B_{-1} + r_q Q_{-1} + \Theta, \tag{14}$$

where nonfinancial income $\Theta \in \{Y, \Omega\}$ equals Y if the investor is employed and Ω otherwise. The sum of expenditures on consumption, stocks, short-term bonds, long-term government debt, and all costs incurred must be equal to income, which is given by the gross return on last period's investments and nonfinancial income.

Defining "cash on hand" as

$$X \equiv r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + \Theta \tag{15}$$

and illiquid assets as

$$Z \equiv \left(1 - \delta^{-1}\right) r_q Q_{-1}, \tag{16}$$

one can express the budget constraint as

$$C + S(1 + \psi_s) + B + Q + \Psi = X + Z. \tag{17}$$

In equation (17), income is divided into liquid funds X that can be freely allocated toward all types of expenditures and illiquid funds Z which are tied to a reinvestment in long-term government debt. Using this notation, the liquidity constraint on long-term government (13) debt becomes

$$Q \geq Z \tag{18}$$

requiring investors to carry an amount of long-term debt forward into the next period that is at least as large as the amount of illiquid assets brought into the period.

In the event of participation in the current period, the optimal portfolio choice satisfies

$$v_p(X, Z, r_q, P, t) = \max_{C,S,B,Q} u(C) + \beta E_{U',P',r_s',r_q'|P,r_q} v(X', Z', r_q', P', t+1) \tag{19}$$

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[28]If nonparticipants were able to reduce long-term bond holdings at a faster rate, there would be liquidity gains associated with not acquiring information about financial markets. If they were able to reduce long-term bond holdings at a slower rate, the expected average holding period of long-term bonds would be larger than the maturity of the bond. Therefore, the assumption that (13) must hold with equality for nonparticipants seems most plausible.

together with (15)–(18) and the regularity conditions $(S, B, Q) \geq (\underline{S}, \underline{B}, \underline{Q})$. Here, $u :$ $\mathbb{R}^+ \to \mathbb{R}$ is the period utility function with $u'(C) > 0$ and $u''(C) < 0$ for all $C \in \mathbb{R}^+$, $v_p : \mathbb{R}^2 \times (\mathbb{R}^+)^2 \times \mathbb{N} \to \mathbb{R}$ is the indirect utility function conditional on financial-market participation in the current period and v is unconditional indirect utility derived below. Since retirement income is deterministic, the expectation has to be taken over U' and P' only if the investor is employed.

3.3.2 *Financial-market nonparticipants* The budget constraint of a household that does not participate in financial markets is

$$C + B + Q = r_s S_{-1} + r_b B_{-1} + r_q Q_{-1} + \Theta$$
$$= X + Z. \tag{20}$$

As discussed before, for a nonparticipant

$$Q = Z \tag{21}$$

which implies that the budget constraint can be written as

$$C + B = X. \tag{22}$$

The equation above is independent of Q, reflecting the fact that the only choice that a nonparticipant faces is how to allocate cash on hand toward consumption and savings at the risk-free rate.

In this case, the solution must satisfy

$$v_n(X, Z, r_q, P, t) = \max_{C,B} u(C) + \beta \mathrm{E}_{U',P',r_q'|P,r_q} v\big(X', Z', r_q', P', t+1\big) \tag{23}$$

as well as (15), (16), (20), (21), and $B \geq \underline{B}$, where $v_n : \mathbb{R}^2 \times (\mathbb{R}^+)^2 \times \mathbb{N} \to \mathbb{R}$ gives indirect utility if the household does not participate and retired agents face no risk from nonfinancial income as explained above.

3.3.3 *Participation decision* In each period, an investor has to decide whether or not to participate in financial markets having solved the consumption-savings problem and the portfolio choice problem in the case of participation. The value of the problem of an investor is given by

$$v(X, Z, r_q, P, t) = \max\big\{v_p(X, Z, r_q, P, t), v_n(X, Z, r_q, P, t)\big\}. \tag{24}$$

At each point in the state space, the investor decides to participate in financial markets if the value from participating is higher than that from not participating.

3.4 *Computation*

The model has to be solved numerically. The fact that agents are able to invest in a third financial asset with a persistent stochastic return increases the dimensionality of the problem significantly in comparison to other recent models of portfolio choice over the

life cycle.[29] There are three continuous state variables in addition to the prevailing level of the return on long-term debt and the investors' age. The optimization involves four continuous controls. One state variable can be eliminated from the problem by dividing all endogenous variables by the persistent component of labor income P. Below, lower case letters denote normalised variables, for example, $x \equiv X/P$. Period utility is assumed to be of CRRA form with a coefficient of relative risk aversion γ. This implies that all value functions introduced above are homogeneous of degree $1 - \gamma$. For example, the indirect utility of an employed investor that participates in financial markets in the current period can be expressed as

$$v_p(x, z, r_q, t) = \max_{c,s,b,q} u(c) + \beta \mathrm{E}_{U',N',r'_q,r'_s|r_q} (GN')^{1-\gamma} v(x', z', r'_q, t+1). \qquad (25)$$

The stochastic growth rate of the persistent component of income $P'/P = GN'$ raised to $1 - \gamma$ now premultiplies v in the expected value accounting for uncertainty resulting from persistent income shocks. All normalized model equations together with more details on their derivation are listed in Section E of the Appendix.

3.5 *Calibration*

Table 4 shows the calibration used for the simulations presented in the sections that follow. Note that the model is written entirely in real terms. A period corresponds to a year. Since the age group 26–28 is the youngest contained in the estimations described in Section 2, it is assumed that agents are born at the age of 27. T_{ret} and T are selected such that agents retire just before turning 63 years of age and die at the age of 80, respectively, consistent with data from the US Census Bureau. A value of five for δ implies that the long-term bond approximates 5-year government debt. This is in line with the average maturity of all outstanding marketable securities issued by the US Treasury which averaged 59.7 months between January 2000 and December 2007.[30]

As in Bonaparte, Cooper, and Zhu (2012) and Campanale, Fugazza, and Gomes (2015), the risk-free rate is set to 2%, a value that is commonly used in the literature. Bonaparte, Cooper, and Zhu further estimated the average net return of stocks in the US, inclusive of dividend payments, to be 6.33% with a a standard deviation of 0.155 and no serial correlation, which I also adopt here. The mean excess return of 5-year government debt over the risk-free rate is set to 0.6%, the standard deviation of 5-year US debt over the sampling period was about 0.011. The 5-year government debt return is modeled using a two-state Markov process whose transition matrix is found by first fitting an AR(1) process to the data and then finding the transition probabilities that best describe the estimated AR(1) process as proposed in Tauchen and Hussey (1991). r_q remains in state $k \in \{l, h\}$ with probability 0.663 and switches with the converse probability.

The standard deviations of the shocks to the transitory and the persistent component of income are chosen based on the estimates reported in the seminal contribution by Carroll, Hall, and Zeldes (1992). Several papers draw on these results, including

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[29]Examples of models with two assets include Bonaparte, Cooper, and Zhu (2012), Campanale, Fugazza, and Gomes (2015), and Cocco, Gomes, and Maenhout (2005).

[30]Between January 2000 and March 2016, the average debt maturity was 60.7 months.

TABLE 4. Calibration.

Parameter	Value	Description	Target/Source
T_{ret}	36	Retirement period	Avrg. US retirement age
T	54	Death period	US life expectancy
δ	5	Maturity of government debt	Avrg. maturity of marketable US debt
β	0.96	Discount factor	Gomes and Michaelides (2005)
γ	5	Coefficient of relative risk aversion	Gomes and Michaelides (2005)
σ_u	0.16	SD of log transitory income shock	Carroll, Hall, and Zeldes (1992)
σ_n	0.12	SD of log persistent income shock	Carroll, Hall, and Zeldes (1992)
G	1.03	Mean income growth	Haliassos and Michaelides (2003), Viceira (2001)
ω	0.6	Replacement ratio	Campanale, Fugazza, and Gomes (2015), Munnell and Soto (2005)
r_b	1.02	Gross return of riskless bond	Bonaparte, Cooper, and Zhu (2012)
r_s	1.0633	Mean gross stock return	Bonaparte, Cooper, and Zhu (2012)
σ_{rs}	0.155	SD of stock return	Bonaparte, Cooper, and Zhu (2012)
r_q	1.026	Mean gross return of δ-year bond	5-year Treasury Notes return
σ_{rq}	0.011	SD of δ-year bond return	5-year Treasury Notes return
$\Gamma_{rq,kk}$	0.663	$\Pr(r_q^k \vert r_q^k)$ for $k \in \{l, h\}$	5-year Treasury Notes return
ψ_s	0.015	Variable cost of stock holdings	Avrg. expense ratio of equity funds
ψ	0.035	Participation cost parameter	Campanale, Fugazza, and Gomes (2015), Gomes and Michaelides (2005)
$\underline{B}$, $\underline{Q}$, $\underline{S}$	0	Borrowing limits	Campanale, Fugazza, and Gomes (2015)

Gomes and Michaelides (2005) and Haliassos and Michaelides (2003). Carroll, Hall, and Zeldes (1992) set income growth to 2%, subsequent papers use a slightly higher value of 3%, which I follow here (Haliassos and Michaelides (2003), Viceira (2001)). The replacement ratio employed is 0.6 in line with Campanale, Fugazza, and Gomes (2015) and Munnell and Soto (2005).[31]

A number of authors that examine stock holdings over the life-cycle report estimates of the discount factor and the coefficient of relative risk aversion. Although the results vary considerably, generally a relatively high degree of discounting and risk aversion is required to match the data. The estimates of Bonaparte, Cooper, and Zhu (2012) are $\beta = 0.69$ and $\gamma = 7.24$. Alan (2006) estimated $\beta = 0.92$ and $\gamma = 1.6$. Cagetti (2003) showed that both variables strongly depend on education with estimates for groups of different education levels ranging from 0.78 to 1.14 for β and from 2.40 to 8.13 for γ. Campanale, Fugazza, and Gomes (2015) employed $\beta = 0.94$ and $\gamma = 5$. Following Gomes and Michaelides (2005), I use standard values of $\beta = 0.96$ and $\gamma = 5$.[32]

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[31]Munnell and Soto (2005) found that, in the US, the replacement rate ranges from about 0.6 to 0.8 for households covered by a pension plan and from about 0.45 to 0.6 for those without pension coverage depending on the precise definitions used.

[32]As in Alan (2006) and Bonaparte, Cooper, and Zhu (2012), expected utility is discounted at a constant rate. Time-variation in the discount factor could be introduced through age-dependent conditional death probabilities which according to data from the National Center for Health Statistics (NCHS) are small until old age though.

The high structural estimates for the discount rate and the coefficient of relative risk aversion are related to a puzzle that poses a challenge to the literature on portfolio choice over the life cycle. According to standard models, it is optimal for households to invest a bigger share into high-risk and high-return assets like stocks than they do according to survey data. A number of strategies have been employed to align model-based predictions more closely with the empirical evidence. Calibrations with high, occasionally extreme levels of discounting and risk aversion respectively lower the benefits from the high average return and increase the sensitivity of households toward the high volatility of stocks. Transaction costs which imply that investors cannot costlessly convert stock holdings into consumption equally reduce the value of stock holdings for households (Campanale, Fugazza, and Gomes (2015)). Disastrous labor income shocks occurring with a small probability make total income, ceteris paribus, more risky and, therefore, lead households to reduce portfolio risk by lowering the stock share. Finally, an isolated decrease in the elasticity of intertemporal substitution, implemented by generalizing CRRA utility to the Epstein–Zin–Weil recursive form, causes households to increase savings, and thus financial income which also implies a lower optimal stock share.[33]

The focus of this paper lies on the dynamic patterns according to which liquidity, long-term debt, and stocks are substituted as wealth is accumulated and deaccumulated with age rather than the effect that the addition of long-term debt to the portfolio choice problem of households has on the mean level of stock holdings. To keep the analysis as clean as possible, therefore merely two types of realistically-calibrated costs whose effects are easily understood are included in the model. The fixed cost of asset-market participation ψ gives rise to a meaningful participation decision and the variable cost of stock holdings ψ_s to an interior solution to the conditional asset allocation problem. Gomes and Michaelides (2005) calibrated stock-market entry costs to 2.5% of the persistent component of labor income, Alan (2006) estimates a value of about 2.1%. In the model of Bonaparte, Cooper, and Zhu (2012), agents incur a transaction cost in each period in which stock holdings are adjusted. Their estimate for these costs is 1.2% of total labor income. Campanale, Fugazza, and Gomes (2015) consider transaction costs that are comparable to those in Bonaparte, Cooper, and Zhu (2012) which they calibrate to 4%–7%, depending on the level of education, in a their preferred scenario. In accordance with these values, ψ is set to 3.5% here.

The Investment Company Institute (ICI) publishes data on the fees associated with investments in equity funds, which I use as an approximation of the marginal cost of holding a stock portfolio. The average expense ratio, the ratio of annual fees to the total size of the investment, fell from 1.6% in 2000 to 1.5% in 2007 and 1.3% in 2015.[34] Based on these figures, a value of 1.5% is chosen for ψ_s. Following the literature, the possibility of borrowing and short selling is excluded in the model (Campanale, Fugazza, and Gomes (2015)).[35]

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[33]See Cocco, Gomes, and Maenhout (2005) for more details on the effects of disastrous labor income shocks and changes in the elasticity of intertemporal substitution.

[34]See www.icifactbook.org. Asset-weighted averages are slightly smaller, however, the expense ratio does not include costs like portfolio transaction fees, brokerage costs, or sales charges.

[35]Cocco, Gomes, and Maenhout (2005) provided a detailed discussion of this assumption.

3.6 *Portfolio rebalancing*

The policy functions associated with the optimisation problem laid out above illustrate how investors rebalance their portfolios as financial wealth increases. In Figure 5, the
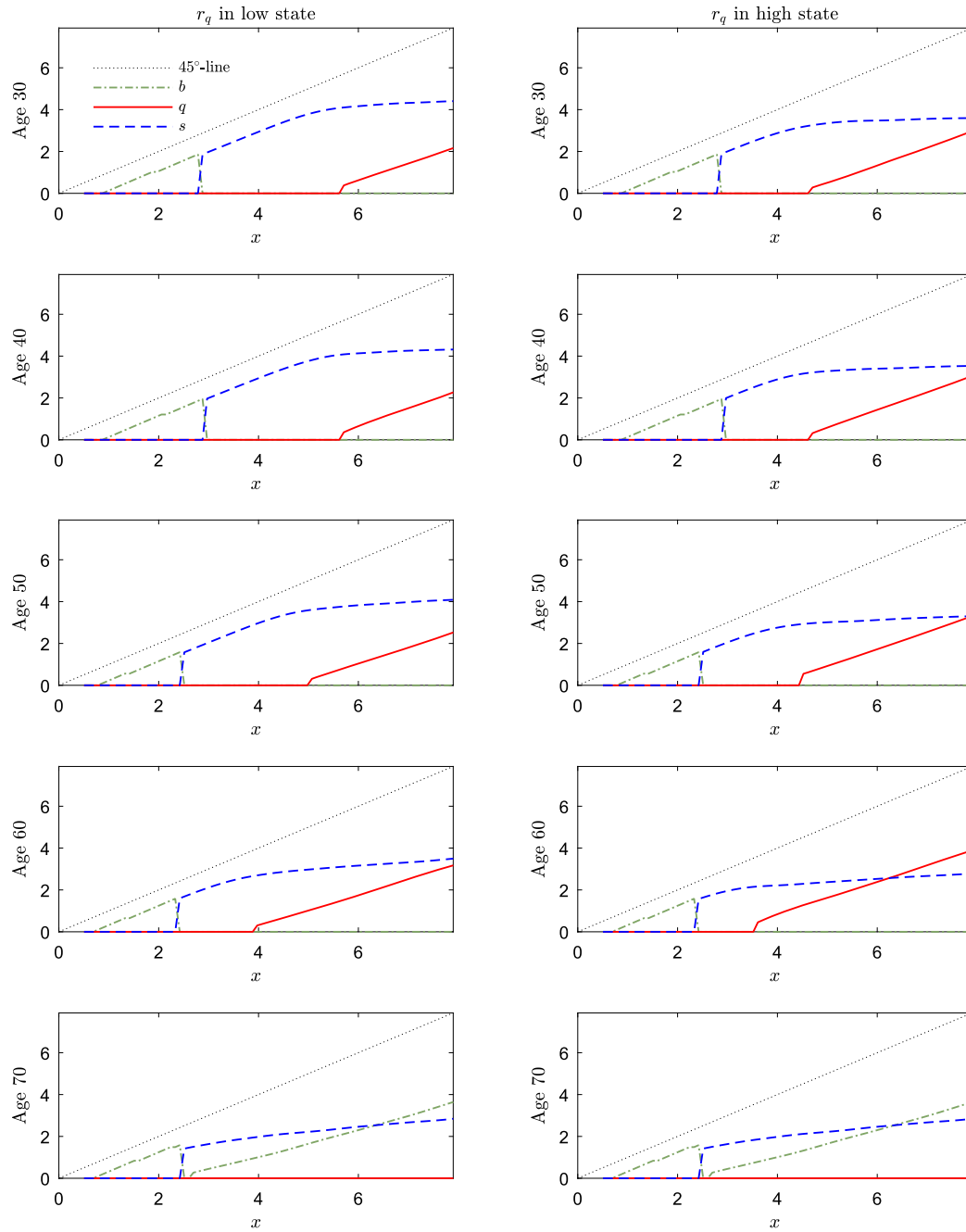

FIGURE 5. Policy functions (for $z = 0$).

optimal choice of financial assets is plotted against cash on hand at different ages. Potential illiquid asset holdings brought into the period are set to zero for now. The long-term interest rate is in its low state in the panels on the left and in its high state in the panels on the right. All variables are normalized by the trending persistent component of income as described before.

The solution follows a similar pattern at all ages shown with the exception of the last. It is highly nonlinear. At very low levels of cash on hand, investors do not participate in the markets for stocks and long-term debt. All savings are done using the risk-free asset since the higher expected returns obtainable when they participate would not sufficiently compensate investors for the additional risks born and the participation cost as well as the proportional costs incurred. As cash on hand increases, the benefits from participating in financial markets outweigh the costs. Holdings of the riskless asset as a savings device then play a role only for older households and younger households with significantly larger levels of cash on hand than are shown in the figure.[36] Note that consumption, given by the difference between the 45-degree line and riskless bond holdings for nonparticipants, initially remains nearly constant as investors aim to accumulate wealth.

Just above the participation threshold, agents allocate their investments toward stocks. As cash on hand increases, investors reduce the risk that they are exposed to *through their financial portfolio* by rebalancing their portfolios away from stocks and toward government debt. Because the expected return on long-term debt is higher when it is currently in the high state, investors start to substitute long-term debt for stocks at lower levels of cash on hand in this case.

To understand why portfolios are adjusted in this way, suppose first that labor and retirement income were nonstochastic. At low levels of cash on hand, labor and retirement income make up a large part of overall income. Thus, investments in financial assets could be comparably risky without giving rise to much risk exposure on aggregate. At high levels of cash on hand, labor and retirement income make up only a small fraction of overall income. Thus, investors would be exposed to more risk on aggregate if the same share of investments were made in the form of stocks. As a result, given that relative risk aversion is assumed to be constant, an equal or even increasing portfolio share of stocks across cash-on-hand levels could not be part of the solution to the investors' portfolio choice problem.[37]

Viceira (2001) showed that investors with stochastic nonfinancial income behave in an analogous way. Provided that the nonfinancial income is uncorrelated with the asset returns, agents choose their portfolios as if this income were generated by an investment in a safe asset of a size below the expected discounted value of its future payment

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[36]There is no intrinsic value of holding the riskless asset in the model. A more significant portfolio share would be obtained for financial market participants, for example, if this asset were interpreted as "money" and a cash-in-advance constraint were introduced.

[37]The initial increase in the risk exposure of asset market participants at cash on hand levels beyond the "kink" in holdings of the riskless asset is a result of the lower bound on government debt and riskless bond holdings. Without borrowing constraints, investors would hold negative amounts of at least one of the two assets initially.

stream.[38] Consequently, they also reduce the stock share in their financial portfolio as wealth is accumulated. A comparable effect can be observed in models in which investors have the choice between stock and a safe asset only.[39] In this class of models, it is the share of the riskless asset that increases with cash on hand as the stock share is reduced. At high levels of wealth, these models therefore assign a role to money holdings as a savings instrument that the model discussed here partly assigns to long-term debt holdings.

At the retirement stage, here represented by the policy functions of an investor at the age of 70, stock holdings are smaller for all levels of cash on hand than at younger age. Lower nonfinancial earnings imply that households reduce the risk incurred through their portfolio. Because long-term debt is partially illiquid, stock holdings are substituted with holdings of the safe asset toward the end of the life cycle.

The policy functions depend on the model parameters in an intuitive way. An increased excess return of long-term debt induces employed investors to rebalance stocks towards debt at lower levels of cash on hand. Higher participation costs defer asset market participation, particularly during the retirement phase. Less risk aversion, a smaller discount factor and a higher retirement age lead investors to enter asset markets at lower wealth levels. The corresponding figures for investors at the age of 40 and 70 are contained in the Online Appendix.

Figure 6 shows how the investment decisions are influenced by existing long-term bond positions. It plots the optimal level of investment in all three financial assets against cash on hand for increasing values of illiquid wealth z. Results are only shown for an investor aged 60, that is, an employed investor who is likely to have accumulated illiquid assets, and $r_q = r_q^l$ for conciseness. The higher the illiquid wealth of households, the less they invest in the riskless asset conditional on not participating in government-debt and equity markets and the more liquid wealth is required to make full asset market participation worthwhile. Intuitively, the more funds are currently locked up in previous investments but will become available in the future, the less willing investors are to pay the participation fee to make new investments in stock or long-term bonds. The optimal amount of new long-term bonds purchased is constant as long as the liquidity
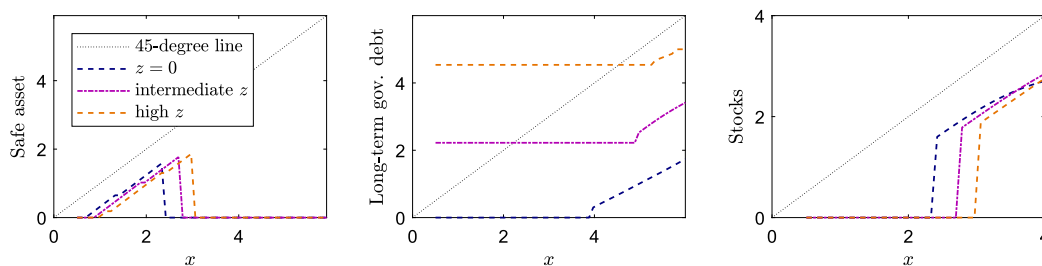

FIGURE 6. Optimal holdings of the riskless asset (left), long-term debt (middle), and stocks (right) as a function of liquid and illiquid financial wealth (age 60, r_q in low state).

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[38]This is shown formally in Proposition 3 of Viceira (2001).

[39]See, for example, Alan (2006) and Fagereng, Gottlieb, and Guiso (2017).

constraint is binding. The more illiquid wealth investors possess, the higher is the level of liquid assets at which they wish to hold long-term bonds in excess of the required amount.

4. Simulations

This section reports results from simulations of the baseline model outlined above and an application to the period of historically low US Treasury returns between 2009 and 2013.

4.1 *Baseline results*

I simulate the model for $T = 80 - 26 = 54$ years, for I individuals with idiosyncratic shocks $\{U_{i,t}, N_{i,t}\}_{t=1}^{T}$, and for J aggregate shock sequences $\{r_{s,t}, r_{q,t}\}_{t=1}^{T}$. I and J are chosen sufficiently large to guarantee full convergence of all model moments presented below. Households have an initial endowment of liquid financial assets amounting to 50% of the permanent component of labor income and half of the households additionally have illiquid initial wealth at the value of 25% of permanent labor income, which is approximately in line with the data from the SCF. Figure 7 plots the participation rate and the asset shares conditional on participation implied by the model against the empirical estimates from Section 2. The estimates shown use the Deaton–Paxson identifying assumptions and principal components to capture time effects, respectively. The behavior of households in the SCF coincides with the predictions of the life-cycle model in several ways.
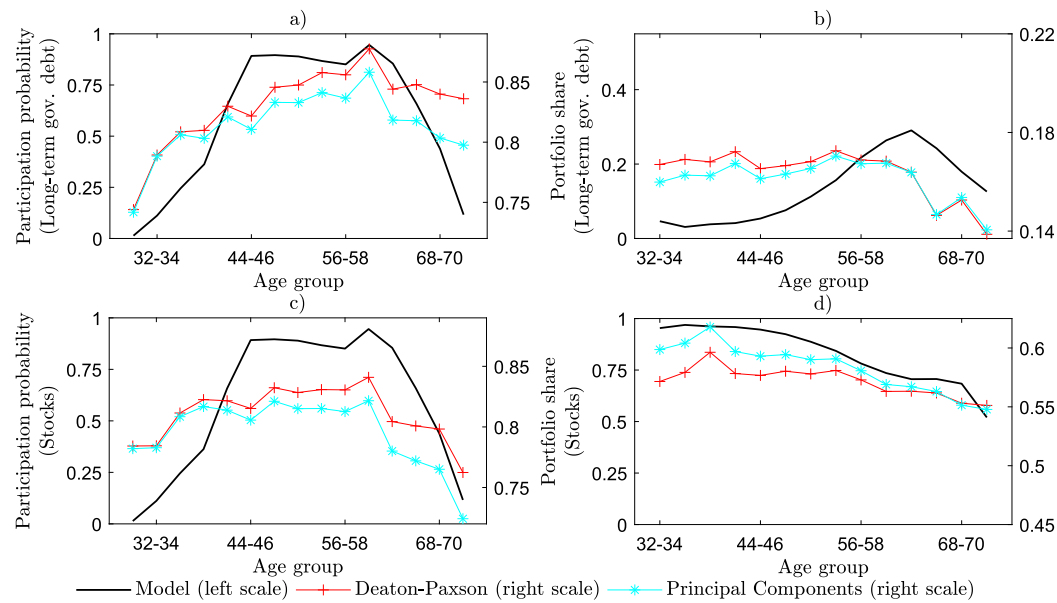
Figure 7. Financial-market participation and conditional portfolio shares.

The dynamics of asset market participation in the model mirror the patterns of participation in the markets for equity and long-term debt estimated from the SCF data. Participation in financial markets is low at first in the model, since initial wealth and hence desired savings are too low for it to be profitable to pay the participation cost and then increases sharply as agents accumulate wealth. The participation rate first flattens and then peaks at the age of 59–61. It declines again as agents retire and fall back to wealth levels at which paying the participation cost to invest in the two risky assets ceases to be beneficial.

The panels (b) and (d) show the average simulated conditional asset shares together with the corresponding empirical estimates. The conditional long-term government debt share in the model increases between the ages of 35–37 and 62–64 and then declines. A comparable pattern is visible but less well pronounced in the data. The long-term debt share equally increases between 32–34 and 41–43 as well as between 44–46 and 53–55 but at a smaller scale and the estimated peak occurs at a slightly younger age. Because cash on hand on average increases with age during the employment stage, households rebalance their portfolios from stocks toward long-term bonds to reduce the risk they are facing through their financial portfolio in the model. Accordingly, the model predicts a declining conditional stock share over the course of the working life which is confirmed by the SCF data. When agents retire, nonfinancial income falls which implies that they wish to decrease the risk of their financial investments. The fact that agents also save less partially offsets this effect. On aggregate the stock share continues to decline. The lack of liquidity of long-term debt makes it increasingly less attractive as a replacement for stock holdings during the retirement phase. Since both the stock share and the long-term debt share fall, the share of the residual category increases. The empirical estimates suggest that the same is true in the US data.

While the stylized facts about the life-cycle dynamics of asset market participation, the conditional stock share and the conditional long-term debt share uncovered in Section 2 can be rationalized using the model, the predicted dynamics are more muted in the SCF estimates compared to the model. This may suggest that a standard life-cycle model somewhat overstates the responsiveness of US households to life-cycle influences but may equally be a result of noise in the survey data or the fact that a part of the asset holdings have to be imputed. The investment decisions of the model agents are crucially driven by wealth accumulation. Therefore, it is instructive to investigate the evolution of wealth in more detail to further gauge the congruence of observed household decisions with optimal model behavior.

Figure 8 compares the distribution of wealth normalized by nonfinancial income for six intermediate age groups of employed investors in the model and in the SCF.[40] In both cases, kernel density estimates based on a Gaussian kernel are reported.[41] The

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[40]In the model, wealth is the sum of liquid and illiquid funds normalised by the permanent component of labor income $(x + z)$. In the SCF data, I construct the corresponding measure by summing financial assets (financial asset holdings and income from interest and dividends) with nonfinancial income (wage income, retirement income, and transfers) and dividing by the latter.

[41]The distributions from the data are obtained by pooling the survey waves contained in the sample for each age group.
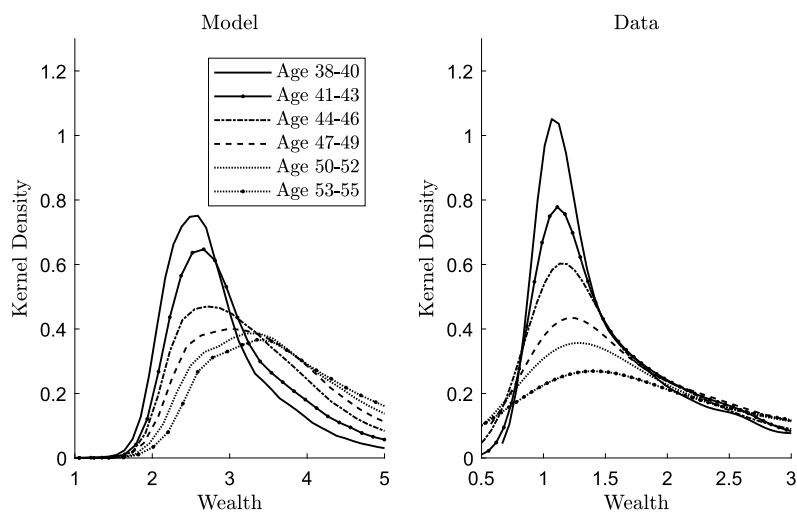
FIGURE 8. Wealth distribution by age.

model successfully generates the characteristic positively-skewed distribution of financial wealth found in the data. The mass of wealth bunched at low levels declines with age, giving rise to an increasingly fatter right tail, and thus an increase in average wealth but also in inequality. In the model, average wealth rises in response to the accumulation of income and the associated rise in equity and debt market participation. The distribution becomes more skewed, because agents that escape from the nonparticipation threshold at a young age accumulate wealth at a faster rate than those that have to exit asset markets occasionally and those that begin to participate persistently at a later stage, despite of rebalancing their portfolios towards safer assets as financial wealth increases. Interestingly, the model therefore provides an intuitive explanation for both the disproportionately high share of wealth held by financial market participants highlighted in Guvenen (2009) and the skew of the wealth distribution discussed in Benhabib and Bisin (2018). The mode of the wealth distribution shifts slightly faster in the model than in the data, however, the evolution of the mean is almost precisely captured as can be seen from Figure 9.

4.2 *Application—historically low US Treasury yields 2009–2013*

The years following the recession of 2007 to 2009 saw historically low levels of government bond returns and very high, although not unprecedented, levels of stock returns. Figure 10 shows the 5-year Treasury constant maturity rate in real terms between 2000 and 2014 in the left panel and the real return of the S&P 500 in the right panel.[42] In 2009, the real 5-year Treasury return became negative, recovered somewhat in the following year, and turned negative again for the following three years. At the same time, the real

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[42]The data on real US stock returns are taken from Robert Shiller's online database, which is freely accessible at www.econ.yale.edu/~shiller/data.htm.
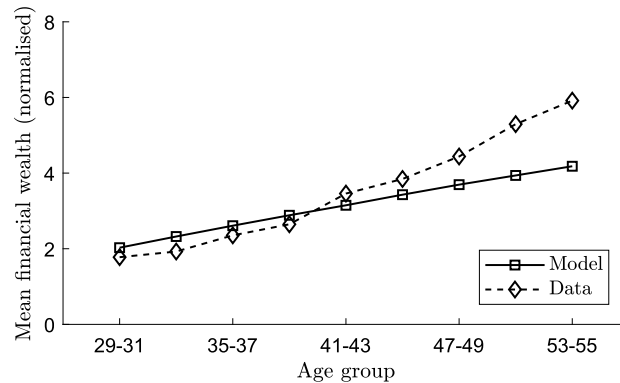
FIGURE 9. Mean wealth by age.

return on US equity, initially driven by stock price gains then also by dividend payments, reached double digits in 4 out of 5 years between 2009 and 2013, breaking the 20% mark in the first and the last year of the interval.

To examine the effects of these conditions on household portfolios through the lens of the model, I conduct the following experiment. The model is simulated letting the stock and the long-term debt return follow the same stochastic processes as outlined above in all years except for a 5-year window, in which the returns fed into the model are matched precisely to those observed in the data between 2009 and 2013 (depicted by stars in Figure 10). Results are shown below for the case that the observed return shocks occur at the age of 45 to 49, that is, for a relatively young employed cohort with already high predicted financial market participation. A set of corresponding results for an older cohort affected just after retirement, at the ages of 63 to 67 is relegated to Appendix F.

The returns of 2009–2013 are treated as exceptional draws from the same distributions as those that govern the asset returns before and after this window, since the period is too short to reliably estimate a separate set of return processes. As a result, assump-
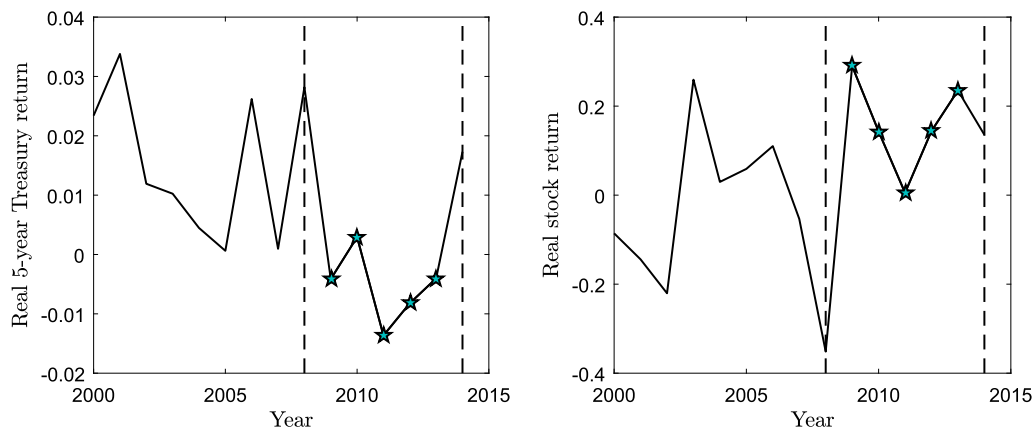
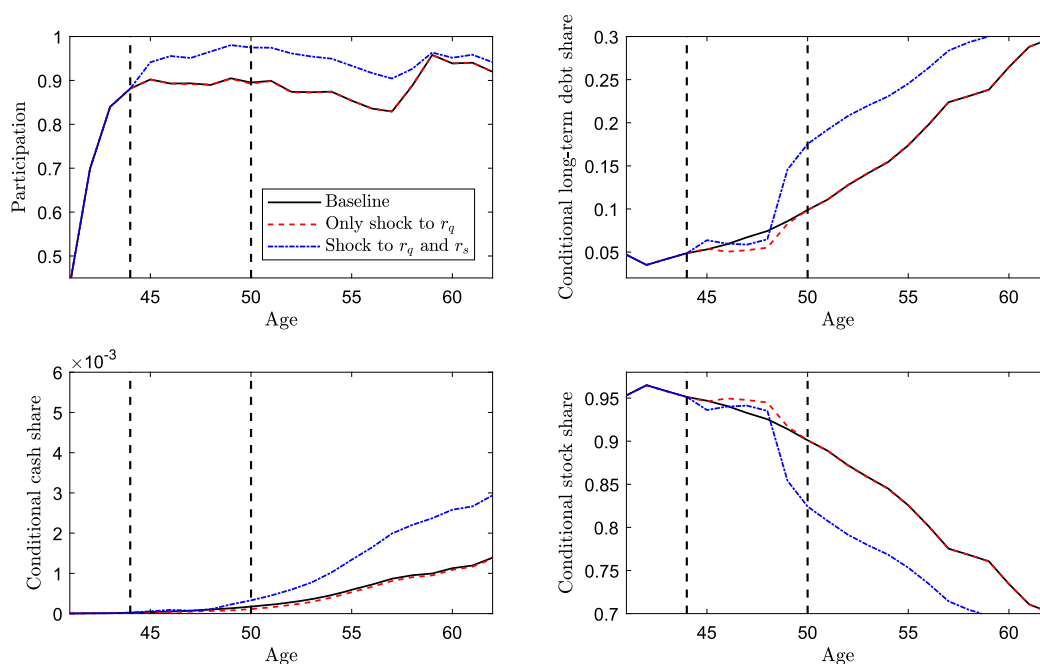FIGURE 10. Real return on 5-year Treasuries and the US stock market (S&P 500).

Figure 11. Participation and conditional asset shares in response to the return shocks of 2009–13 impacting at age 45–49.

tions have to be made about the underlying state of the long-term debt return process. In the first and the last year of the time period examined, the population of model agents is split in half between those forming expectations about the 5-year bond return in the following period based on the high and the low state, so that on aggregate both states are seen as equally likely, reflecting uncertainty about the persistence of the observed shocks. During the intermediate years, agents form expectations based on the conditional distribution pertaining to the low return state. To be able to distinguish the effects caused by the return shocks to each asset, I also report results from the same experiments when the long-term debt return is matched to the 2009–2013 period as described above but the stock return evolves as in the baseline model.

The aggregate dynamics of financial-market participation and the conditional asset shares in response to the historical return shocks are plotted in Figure 11. Dashed vertical lines indicate the last and the first period in which all asset returns follow the baseline specification. Consider the case that only the long-term bond yields are matched to the 5-year period in the data depicted by the dashed lines first. On average, agents in the model react to the decline in bond yields by rebalancing their portfolios toward stock holdings following the initial period of adjustments in expectations. In the three intermediate shock years, the average conditional long-term bond share is lowered by 0.9–1.9 percentage points (pps) and the conditional stock share is increased by about the same amount compared to the baseline outcome. Nearly all adjustments take place between the two risky assets, leaving the share of the safe asset largely unaffected. The decline in the Treasury return is compensated by the increased stock share, implying that the

resulting change in the overall portfolio return and wealth is not large enough to alter the participation rate in a significant way. Consequently, while quantitatively significant on impact, the effects on the composition of household portfolios are of temporary nature. In the year after the final negative government debt return shock has occurred, all adjustments are nearly undone.[43]

The dash-dotted lines show the results for the scenario in which both historical shock sequences are fed into the model. In this case, the long-term debt share is elevated and the stock share is reduced in all periods following the initial return shocks in comparison to the case with only historical Treasury return shocks. In the first shock year, households receive a large return on their investments in equity from the previous year. This income shock immediately pushes participation up by 3.9 pps and induces investors to rebalance some of their stock holdings toward long-term debt holdings. The long-term debt share is increased and the stock share is decreased by 1.0 pps, respectively, compared to the case with only bond return shocks. For the three intermediate shock years, the difference between the dash-dotted and the dashed curve remains roughly constant as the equity returns are less extreme then. A particularly large stock return in the last shock period causes income to increase significantly which in combination with the nonlinear nature of the policy functions leads to an additional long-term debt share of 6.3 pps and a reduction in the stock share of similar size. The safe asset share then also increases although it remains quantitatively less important at this stage. The opposing effect of both types of return shocks implies that the portfolio composition initially remains close to that in the baseline model. However, average wealth is significantly increased towards the end of the period of extreme returns, which leads to persistent deviations of the participation rate and the portfolio shares from their respective baseline paths over the following years. Average participation is 7.9 pps above the baseline at the age of 55 and 1.3 pps at the age of 60. Similarly, long-term debt and stock holdings deviate from the baseline by 7.2 pps and -7.2 pps at the age of 55, and 5.6 pps and -5.8 pps at the age of 60, respectively.

Figure A.10 in the Appendix contains an analogous set of results for households that are confronted with the historical sequence of return shocks at the age of 63 to 67. During the retirement phase, households rebalance their portfolios toward the more liquid safe asset rather than long-term debt. In this case, the stock return shocks therefore lead to a more persistent increase in the stock share.

The historical return shocks have a significant effect on the distribution of wealth in the model. Figure 12 shows the predicted evolution of the wealth distribution among employed investors affected at the age of 45 to 49 as above. The shocks to the stock return raise average wealth by increasing the mass in the right tail of the distribution. Investors that are able to afford significant stock holdings when the shocks first hit experience a large increase in their wealth levels, while the others are "left behind." Deviations from the baseline still exist when agents retire, suggesting that booms in the stock market

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[43]Figure A.9 in Appendix F shows that these results are robust to different assumptions about the long-term return state used to form expectations in the two transitional periods.
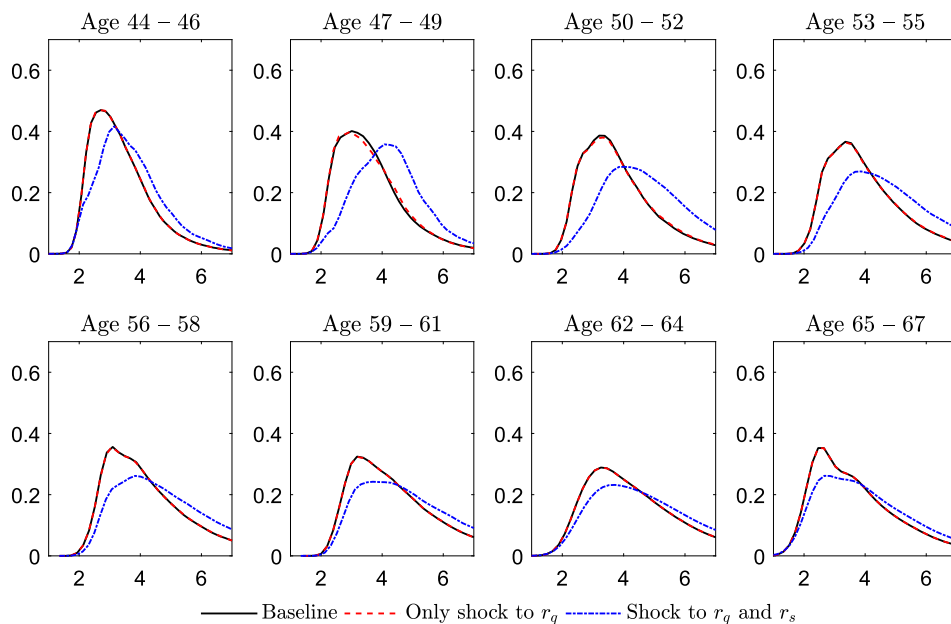
FIGURE 12. Evolution of the wealth distribution in response to the return shocks of 2009–13 impacting at age 45–49.

lead to persistent increases in inequality. For generations that experience the shocks at an older age, the wealth distribution is strongly affected during the retirement stage, as can be seen from Figure A.11.

5. Conclusion

Households systematically rebalance their financial portfolios as they accumulate wealth. Nearly riskless assets like cash or bank deposits, long-term bonds like Treasury notes or Treasury bonds, and equity play distinctive roles in this process. At low levels of liquid wealth, the expected benefits from participating in the markets for long-term debt and equity do not outweigh the costs, implying that households save through low-cost assets with a low return. As they accumulate more cash on hand and save more, entering the stock market becomes profitable. Cash holdings due to a pure savings motive markedly decline at this point. At levels of liquid wealth just above the participation threshold, investment income is small compared to labor income. Agents can therefore afford to invest a large share of their financial portfolio in equity without being exposed to an excessive amount of risk on aggregate. The additional accumulation of liquid wealth implies that investment income as a fraction of total income increases. As a result, the average asset-market participant reduces the equity share and increases the share of safer assets to limit their risk exposure. My model suggests that long-term debt rather than cash is used by households to readjust the risk properties of their portfolios in this way.

In accordance with debt instruments like US savings bonds, which can be cashed by households only at a significant cost in the first 5 years after they have been issued, government debt is assumed to be partially illiquid in the model. Consequently, the portfolio-choice problem of households in a given period is dependent on past investments in long-term debt. The higher the illiquid wealth of a household before retirement, the less it invests in the riskless asset conditional on not acquiring new government debt and stocks, the more liquid wealth is required for full asset-market participation to be beneficial and the higher the level of cash on hand at which new government bonds are purchased.

The microlevel behavior in the model gives rise to aggregate dynamics that are in line with data from the SCF. The asset-market participation rate is inverse U-shaped, the average stock share monotonically decreases and the average long-term government-bond share mildly increases until the age of 62 to 64 and then declines. The wealth distribution is increasingly skewed to the right, because households that begin to invest in equity consistently at a young age accumulate wealth faster than those that join later and those that are forced to exit temporarily.

Cocco (2004) investigates the role that investment in housing plays for household portfolios. He shows that for a fixed participation cost the rate of equity market participation and, at a young age in particular, the equity share, are lower if model agents are permitted to invest in housing. The interactions of housing with long-term debt investments and their implications for equity holdings are left for future work.

A model-based decomposition of the effects resulting from the fluctuations in asset returns observed in the US between 2009 and 2013 shows that the observed decline in long-term bond yields has sizeable effects on the composition of household portfolios. Since the average participation rate and wealth remain nearly unaffected, these effects are of temporary nature. The observed stock-return shocks give households an additional motive to rebalance their portfolios in the short-run, lead the asset-market participation rate to increase and shift additional mass into the right tail of the wealth distribution, which leads to persistent changes in the conditional asset shares. Naturally, the decomposition used here is hypothetical. An important question relevant, for example, for evaluating the macroeconomic effects of large-scale purchases of long-term bonds by central banks is to what degree lowered bond yields exert upward pressure on stock prices. In the presence of a strong link between the two, the model suggests that unconventional monetary policy in the form of government debt purchases raises average consumption but causes inequality to increase for several decades.

Appendix A: Asset categorization

This section shows how the set of financial assets sampled in the SCF is divided into the three broad categories stocks, long-term government debt, and residual assets. I begin by discussing selected assignments. The categorization of all assets is shown in Table A.1.

TABLE A.1. Asset categorization.

Asset Type	Asset Sub-type	Stocks	Long-Term Gov't Debt	Residual
Stocks		1	0	0
Bonds				
	Savings bonds	0	1	0
	Tax-exempt bonds	0	1	0
	US gov't (agency) debt	0	0.779	0.221
	Corporate and foreign bonds	0	0	1
	Mortgage-backed bonds	0	0	1
Transaction accounts				
	Checking accounts	0	0	1
	Saving accounts	0	0	1
	Money market deposit accounts	0	0	1
	Money market mutual funds	0	0	1
	Call accounts at brokerages	0	0	1
Certificates of deposit		0	0	1
Pooled investment funds				
	Stock funds	1	0	0
	Tax-free bond funds	0	1	0
	Government bond funds	0	0.779	0.221
	Other bond funds	0	0	1
	Hybrid funds	0.5	0.389	0.111
	Other funds	0	0	1
Retirement accounts				
	IRAs and Keogh plans	0.458	0.184	0.358
	Thrift plans	0.458	0.184	0.358
	Current pensions	0	0	1
Life insurance, annuities		0	0	1
Other managed assets		1	0	0

Savings bonds and tax-exempt bonds are issued by the Department of the Treasury and municipal, county, or state governments, respectively. They typically have a maturity of at least 5 years and are therefore treated as long-term government debt.

US government (agency) debt contains Bills, Notes, and Bonds. As explained in the text, Bills are classified as short-term and Notes/Bonds as long-term. According to the "Monthly Statement of the Public Debt of the United States" from December 2007, 22.1% of total marketable debt held by the public were Bills and the remainder Notes/Bonds. Households are assumed to hold long-term and short-term debt at this proportion.

Certificates of deposit are a fixed-term savings product issued by banks, credit unions and thrift institutions; they are allocated to the residual category.

Government bond investment funds are allocated in the same way as direct holdings of US government debt.

Hybrid investment funds are allocated equally to stocks and government debt. Government debt is again split into long-term and short-term debt as described above.

Individual Retirement Accounts (IRAs) and Keogh Plans can include a variety of different financial assets. The Employee Benefit Research Institute (EBRI) estimates that,

in 2008, a total of 45.8% of funds in IRAs were held in the form of equity (38.5% in equity mutual funds, directly held individual stocks, and other 100%-equity investment vehicles; they add to this 60% of balanced, lifestyle/life cycle, target-date funds, and any other funds that have a partial investment in equities and bonds, which make up 12.1% of all funds in IRAs).[44] I assign 18.4% to long-term government debt (the remaining 40% of hybrid equity-bond funds and all bond investments). Money and other assets make up 35.8% and are classified as "residual" for the purposes of this paper. Keogh Plans are similar to IRAs but designed for self-employed individuals. I assume that funds held in Keogh Plans are invested as in IRAs.

Life insurance and annuities are assigned to the residual category, since life insurances and annuities are claims to a safe future payoff (stream).

Other managed assets include trusts and managed investment accounts which are counted toward stocks here.

Appendix B: Additional descriptive statistics

Table A.2. Distribution of observations across age groups and cohorts.

Age Group	Cohort (Birth Year)										
	'31–'33	'34–'36	'37–'39	'40–'42	'43–'45	'46–'48	'49–'51	'52–'54	'55–'57	'58–'60	'61–'63
26–28	0	0	0	0	0	0	0	0	0	0	495
29–31	0	0	0	0	0	0	0	0	0	510	858
32–34	0	0	0	0	0	0	0	0	822	1009	1047
35–37	0	0	0	0	0	0	0	850	1102	1200	1123
38–40	0	0	0	0	0	0	906	1155	1227	1305	1296
41–43	0	0	0	0	0	1125	1125	1328	1447	1601	1362
44–46	0	0	0	0	890	1229	1462	1386	1517	1365	1230
47–49	0	0	0	810	1162	1265	1411	1690	1608	1511	0
50–52	0	0	834	1035	1286	1519	1327	1725	1535	0	0
53–55	0	916	1000	1086	1206	1480	1321	1566	0	0	0
56–58	825	800	937	1117	1180	1595	1295	0	0	0	0
59–61	855	945	1010	1016	1292	1437	0	0	0	0	0
62–64	943	782	905	1097	1235	0	0	0	0	0	0
65–67	739	756	867	1020	0	0	0	0	0	0	0
68–70	733	711	760	0	0	0	0	0	0	0	0
71–73	690	705	0	0	0	0	0	0	0	0	0
74–76	518	0	0	0	0	0	0	0	0	0	0
Σ	5303	5615	6313	7181	8251	9650	8847	9700	9258	8501	7411

Note: $N = 86{,}030$. SCF data are multiply imputed. For each respondent, there are five observations in the data. To approximate the number of respondents in each field in the table, the number shown therefore has to be divided by five. Age groups and cohorts are chosen such that there is a sufficient number of responses in each age-cohort pairing. Figures are not multiples of five if asset shares are missing for some but not all imputations.

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[44]See "IRA Asset Allocation and Characteristics of the CDHP Population, 2005–2010," May 2011, available at www.ebri.org/publications/notes.
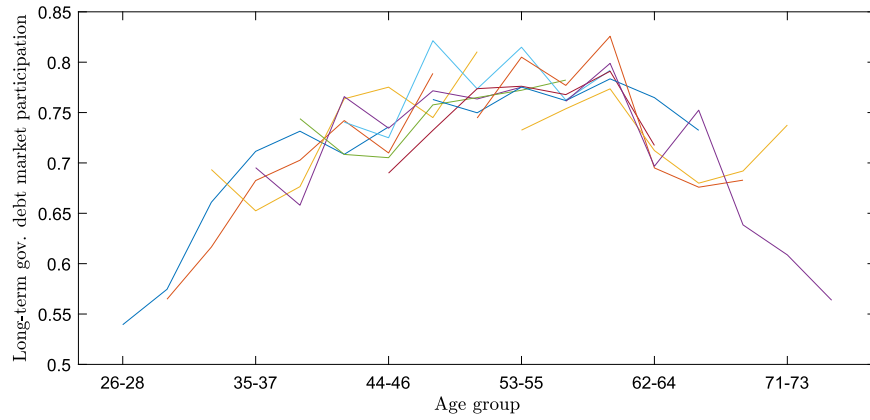
Appendix C: Participation data

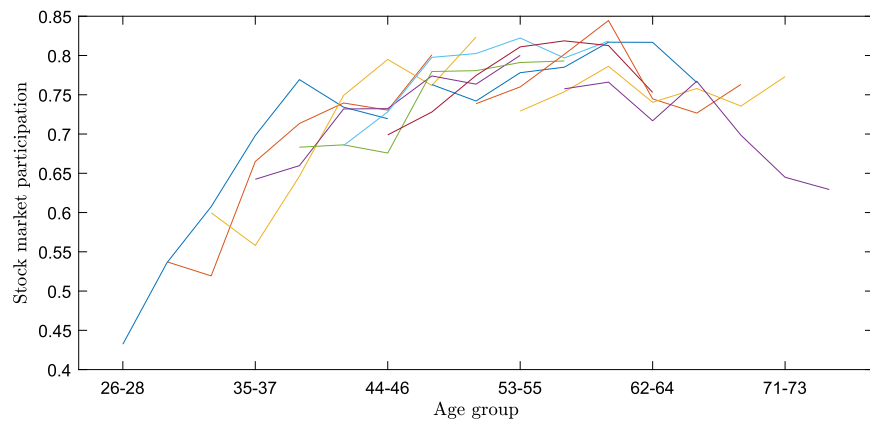Figure A.1. Participation in long-term government debt market.

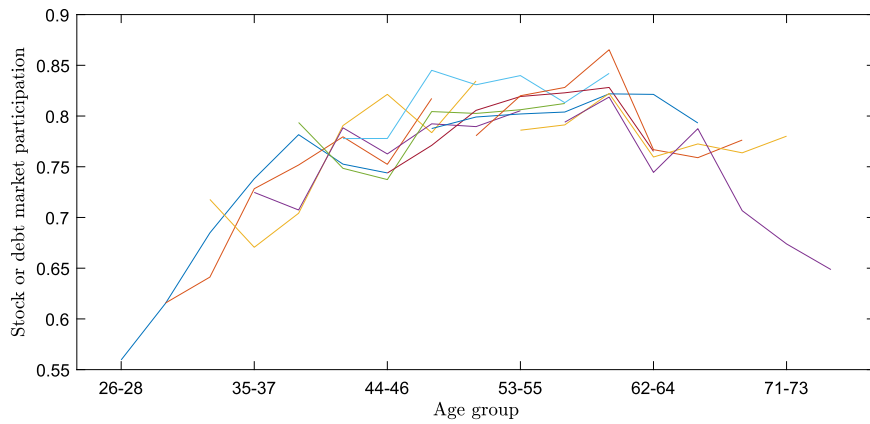Figure A.2. Participation in stock market.

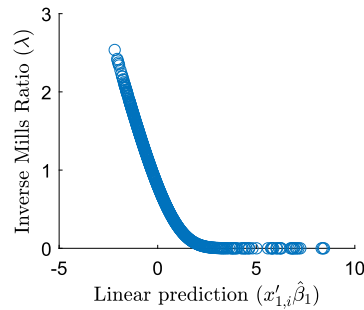Figure A.3. Participation in long-term gov. debt or stock market.

Figure A.4. Inverse Mills ratio (Stock market participation).

Appendix D: Estimation

The controls included in the estimations are binary variables indicating the marital status, education, race, type of employment as well as information on household debt and pension plans. The exclusion restrictions imposed in the estimations are discussed below.

In the model of the stock share, the assumption maintained is that, at low levels of education, a marginal amount of additional schooling increases the likelihood of stock market participation but not the portfolio share conditional on participation. Compared to some years of high-school attendance, a completed high-school education may provide individuals with knowledge about investment opportunities and lower the costs of access to the stock market, but should have no direct effect on the stock share conditional on participation when other characteristics such as the occupation of the individual are controlled for. In the Probit selection equation, having earned a high-school diploma and visited college for some time without graduating has a significant effect on the participation probability (at the 1% level) and is insignificant in a regression of the second-step residuals on the explanatory variables (at the 5% level).

Without an exclusion restriction, multicollinearity issues arise if the inverse Mills ratio (IMR) is close to being linear in the linear predictions of the Probit equation. Figure A.4 which plots the IMR against the linear predictions shows that this is not a concern here. Figure A.5 depicts the predicted stock share for the case that the exclusion restriction is imposed and the case that it is not imposed and the identification relies on functional form assumptions. Deaton–Paxson restrictions are used to address the age-
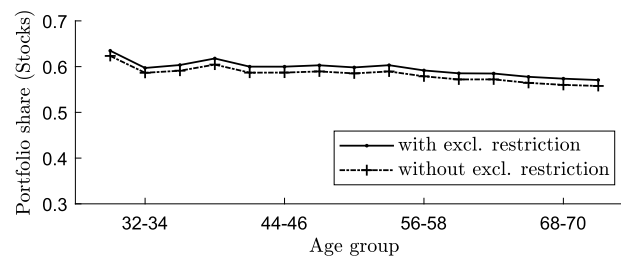

Figure A.5. Estimated age profile of stock share.

period-cohort problem. The estimates are almost identical despite of the fact that the excluded variables are strongly significant in the selection equation. Excluding the variables in the outcome equation has a small "direct" effect resulting in an insignificant parallel shift of the estimated age profile. However, the shape of the estimated age profile is unaffected by the exclusion restriction. Following the view in the literature that the Heckman sample selection model is the correct model to separate stock market participation from the conditional portfolio share (see, e.g., Fagereng, Gottlieb, and Guiso (2017)), we can therefore be confident that the age dynamics of the conditional stock share are captured accurately.

The variable excluded from the second stage of the model for long-term government debt is a binary variable indicating whether the household head is "not doing any work for pay at the present time." As we would expect, it has a highly significant negative effect on the participation decision. The hypothesis is that it does not directly affect the portfolio share of long-term government debt among those who hold the asset. The average age of those responding positively is significantly larger than that of individuals responding negatively (57.9 compared to 48.9) suggesting that positive responses come, at a disproportionate amount, from retired individuals. Age effects are controlled for in the second stage regression so that the hypothesis holds true for this group unless there are effects from being retired that the age dummies do not pick up. Positive responses also come from individuals that have not yet reached retirement age. These cases are unproblematic if there are other household members that provide the family with labor income, since the source of family income should not affect portfolio shares. To address cases where labor income is diminished as a result of one or more household members being involuntarily unemployed and where the family participates in the market for long-term debt nevertheless, a set of educational and occupational variables are used to control for potential common drivers of unemployment and the long-term government debt share.

In the Probit selection equation, the excluded variable is highly significant (at the 1% level) and is insignificant again in a regression of the residuals from the observation equation on the explanatory variables (at the % 10 level). Figures A.6 and A.7 show that the IMR visibly deviates from linearity although less than in the case of stocks and the exclusion restriction does not have a strong effect on the shape of the estimated age profile as above.
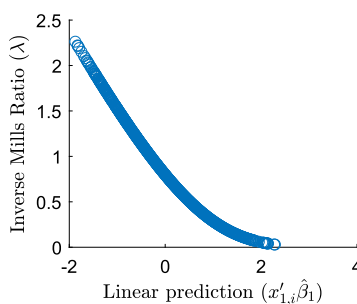

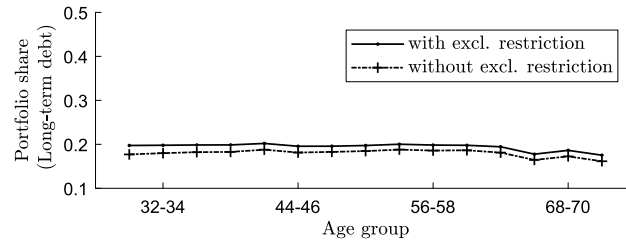FIGURE A.6. Inverse Mills ratio (long-term gov. debt market participation).

FIGURE A.7. Estimated age profile of long-term gov. debt share.

APPENDIX E: NORMALIZATION

Consider the amount of funds brought into the period first. Cash on hand of an employed investor can be normalized as follows:

$$X = r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + Y, \tag{26}$$

$$\frac{X}{P} = r_s \frac{S_{-1} P_{-1}}{P_{-1} P} + r_b \frac{B_{-1} P_{-1}}{P_{-1} P} + \delta^{-1} r_q \frac{Q_{-1} P_{-1}}{P_{-1} P} + \frac{PU}{P}, \tag{27}$$

$$x = \frac{r_s s_{-1} + r_b b_{-1} + \delta^{-1} r_q q_{-1}}{GN} + U \tag{28}$$

and illiquid assets as

$$z = \frac{(1 - \delta^{-1}) r_q q_{-1}}{GN}. \tag{29}$$

TABLE A.3. Conditional correlations of long-term debt and cash share with stock share.

	OLS	
	(1)	(2)
Long-term debt	−0.638	
	(0.018)	
Cash		−0.754
		(0.012)
Wealth	0.0006	−0.0001
(in millions)	(0.0001)	(0.0001)
$\min(N_{\mathrm{imp}})$	4299	4299
Total N	21,531	21,531
Sign. tests		
Time Eff's	0.000	0.000
(d.o.f.)	(6)	(6)

Note: Dependent variable: Stock share. Data are multiply imputed. For each respondent, there are five observations in the data. Point estimates are averages over five separate estimations. Strd. errors (in parentheses) are adjusted in an appropriate way. N_{imp} is number of obs. for imputation imp $\in \{1, 2, \ldots, 5\}$. For joint significant tests, avrg. p-value shown (each test stat. $\sim F$), degrees of freedom in parentheses.
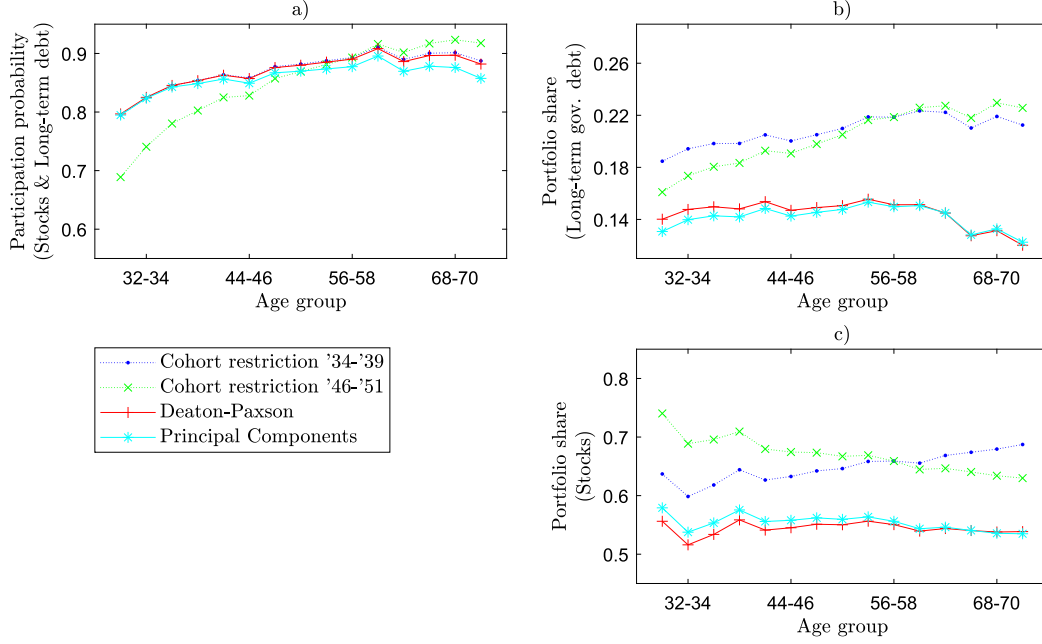
FIGURE A.8. Predicted joint participation profile and conditional asset shares.

For a retired investor, one obtains

$$X = r_s S_{-1} + r_b B_{-1} + \delta^{-1} r_q Q_{-1} + \Omega, \tag{30}$$

$$\frac{X}{P_{T_{\text{ret}}}} = r_s \frac{S_{-1}}{P_{T_{\text{ret}}}} + r_b \frac{B_{-1}}{P_{T_{\text{ret}}}} + \delta^{-1} r_q \frac{Q_{-1}}{P_{T_{\text{ret}}}} + \frac{\omega P_{T_{\text{ret}}}}{P_{T_{\text{ret}}}}, \tag{31}$$

$$x = r_s s_{-1} + r_b b_{-1} + \delta^{-1} r_q q_{-1} + \omega \tag{32}$$

and

$$z = \left(1 - \delta^{-1}\right) r_q q_{-1}. \tag{33}$$

The budget constraint of a financial-market participant and a nonparticipant then become, respectively,

$$c + s(1 + \psi_s) + b + q + \psi = x + z, \tag{34}$$

$$c + b + q = x + z. \tag{35}$$

Indirect utility can be transformed in a similar way. Starting with an employed participant, we have

$$v_p(X, Z, r_q, P, t) = \max_{C,S,B,Q} u(C) + \beta \mathrm{E}_{U',P',r_s',r_q'|P,r_q} v\left(X', Z', r_q', P', t+1\right), \tag{36}$$

$$\frac{v_p(X, Z, r_q, P, t)}{P^{1-\gamma}} = \max_{C,S,B,Q} \frac{u(C)}{P^{1-\gamma}} + \beta \mathrm{E}_{U',P',r_s',r_q'|P,r_q} \frac{v\left(X', Z', r_q', P', t+1\right)}{P^{1-\gamma}}, \tag{37}$$

$$v_p\left(\frac{X}{P}, \frac{Z}{P}, r_q, 1, t\right) = \max_{C,S,B,Q} u\left(\frac{C}{P}\right) + \beta \mathrm{E}_{U',P',r_s',r_q'|P,r_q} v\left(\frac{X'}{P}, \frac{Z'}{P}, r_q', \frac{P'}{P}, t+1\right), \tag{38}$$

$$v_p(x, z, r_q, t) = \max_{c,s,b,q} u(c) + \beta E_{U',P',r_s',r_q'|P,r_q} v\left(x'\frac{P'}{P}, z'\frac{P'}{P}, r_q', \frac{P'}{P}, t+1\right), \quad (39)$$

$$v_p(x, z, r_q, t) = \max_{c,s,b,q} u(c) + \beta E_{U',N',r_s',r_q'|r_q} (GN')^{1-\gamma} v(x', z', r_q', t+1). \quad (40)$$

Indirect utility of an employed nonparticipant is

$$v_n(x, z, r_q, t) = \max_{c,b} u(c) + \beta E_{U',N',r_q'|r_q} (GN')^{1-\gamma} v(x', z', r_q', t+1) \quad (41)$$

and

$$v(x, z, r_q, t) = \max\{v_p(x, z, r_q, t), v_n(x, z, r_q, t)\}. \quad (42)$$

The corresponding equations for a retired investor are

$$v_p(x, z, r_q, t) = \max_{c,s,b,q} u(c) + \beta E_{r_s',r_q'|r_q} v(x', z', r_q', t+1), \quad (43)$$

$$v_n(x, z, r_q, t) = \max_{c,b} u(c) + \beta E_{r_q'|r_q} v(x', z', r_q', t+1). \quad (44)$$

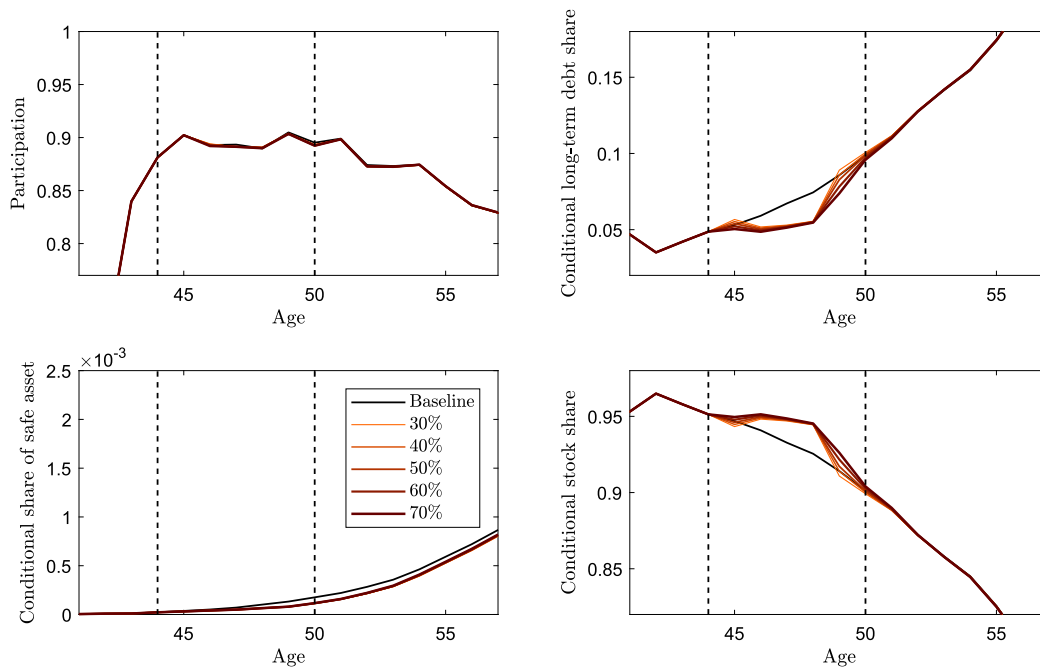## Appendix F: Additional simulations

Figure A.9. Participation and conditional asset shares in response to the 5-year Treasury return shocks of 2009–13 impacting at age 45–49.
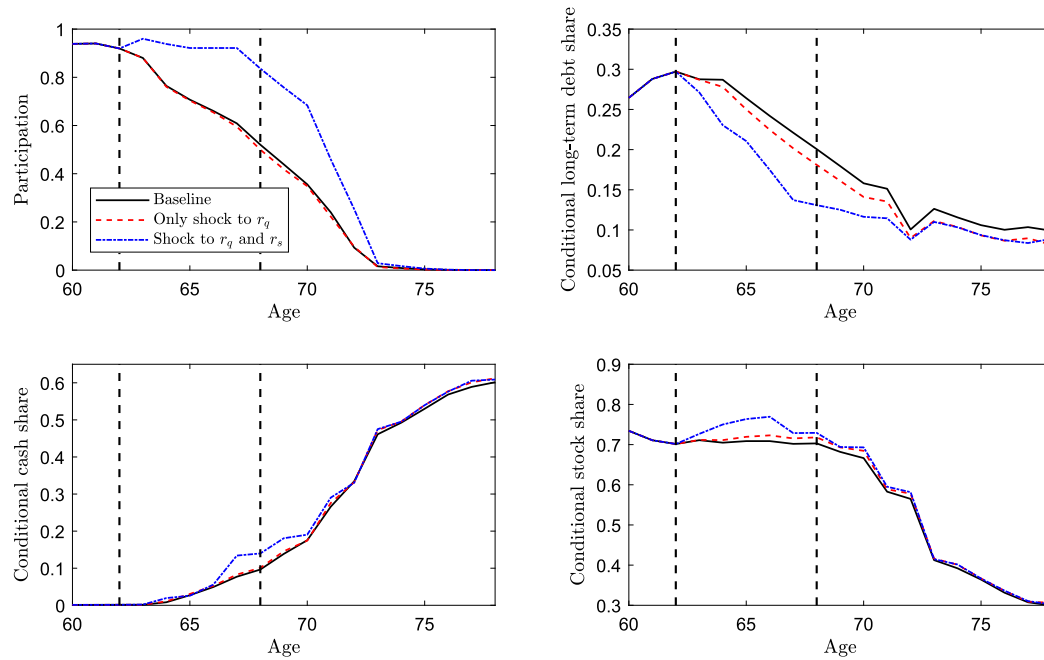
FIGURE A.10. Participation and conditional asset shares in response to the return shocks of 2009–13 impacting at age 63–67.
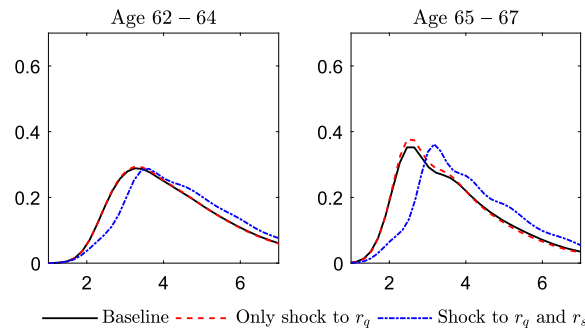

FIGURE A.11. Wealth distributions with return shocks impacting at age 63–67.

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