

ONLINE APPENDIX TO “MIXED STRATEGIES IN THE INDEFINITELY
REPEATED PRISONER’S DILEMMA”
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APPENDIX OA–1: ONLINE APPENDIX

TABLE OA1
QUIZ SUMMARY.

Question Order	Question Type	HQ Correct	LQ Correct	Difference
1	quizMatchesAndPeriods1	116/124	26/34	0.17
2	quizMatchesAndPeriods2	111/124	24/34	0.19
3	quiz1	120/124	27/34	0.17
4	quiz2	117/124	22/34	0.3
5	quiz3	123/124	29/34	0.14
6	quiz4	119/124	29/34	0.11
7	quiz5	121/124	31/34	0.06
8	quiz6	124/124	32/33	0.03
9	quizFitHistory1	114/124	21/33	0.28
10	quizFitHistory2	120/124	24/33	0.24
11	quizFitHistory3	118/124	17/33	0.44
12	quizFitHistory4	117/124	21/33	0.31
13	quizWhichRule1	76/124	2/30	0.55
14	quizWhichRule2	118/124	15/30	0.45
15	quizWhichRule3	116/124	9/29	0.63
16	quizWhichRule4	111/124	8/28	0.61
17	quizWhichRule5	110/123	10/26	0.51
18	addRuleQuiz	99/121	1/16	0.76
19	deleteRuleQuiz	121/121	10/12	0.17
20	editRuleQuiz	109/119	5/12	0.5

Note: Note the 5/12 because of subjects that passed the quiz being placed into the low quiz group.

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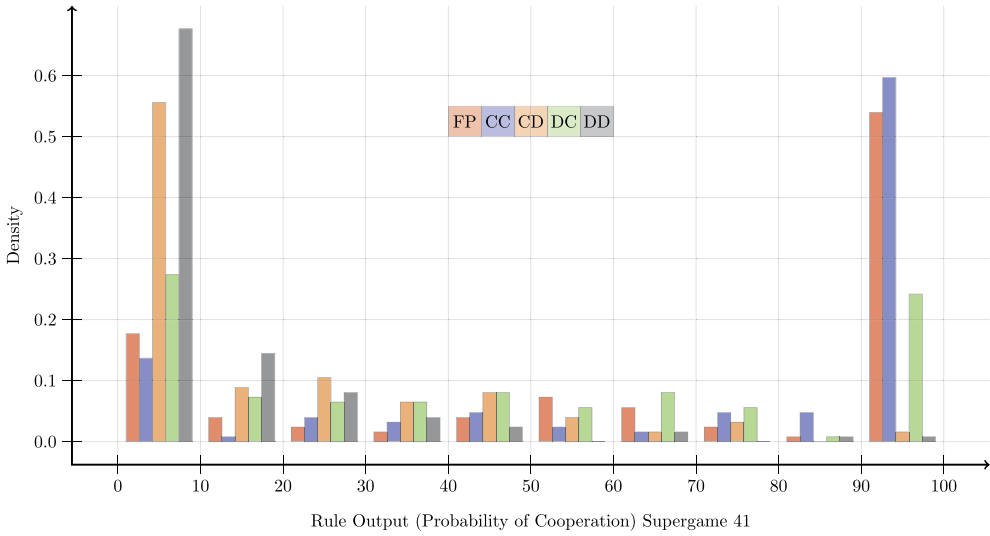


FIGURE OA1.—Frequency of Cooperation After Each Memory-1 History. *Note:* This plot shows the percentage of subjects that have rules with different outputs in the second locked-response stage. For example, the far left bar on this plot shows that about 17% of subjects have First Period Rules (FP) that cooperate with probability 0–10%.

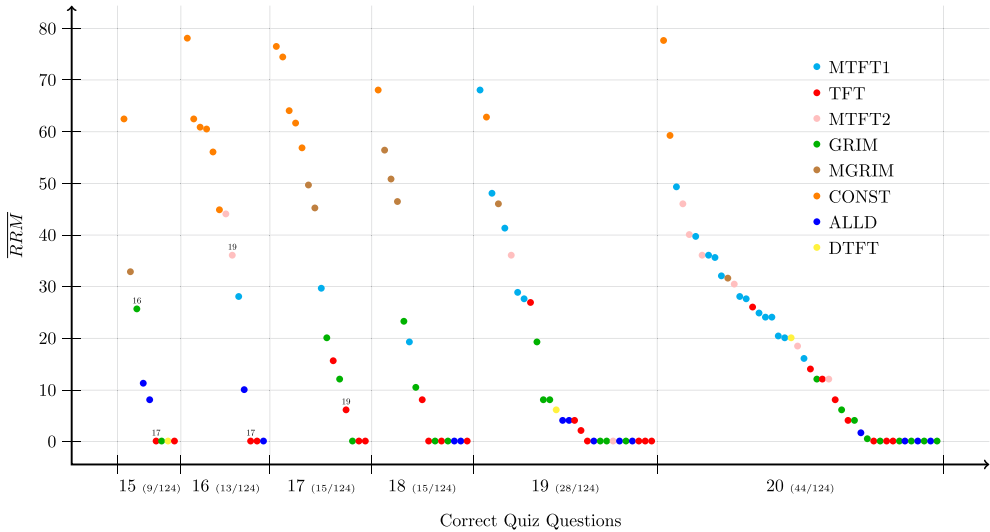


FIGURE OA2.—Quiz Scores vs. Strategy-Randomness Measure. *Note:* The points are colored according to the clustering from supergame 41. A number above a point represents the total number of quiz questions attempted. If there is no number, then that subject attempted all 20 quiz questions.

TABLE OA2
STRATEGY COMPARISON ACROSS TREATMENTS (NO ALLD).

	M1D95	M2+D95	RR2018	M2+D75
Proportion Mixed	0.741 (0.060)	0.560 (0.092)	0.000 (0.000)	0.683 (0.083)
Memory-1 Randomness (M1RM)	23.829 (3.330)	19.092 (2.234)	10.407 (1.980)	18.973 (3.095)
Action Randomness (ARM)	22.878 (3.291)	10.737 (2.209)	0.000 (0.000)	12.674 (3.582)
Strategy Length	0.955 (0.024)	2.920 (0.223)	4.934 (0.640)	3.095 (0.301)
Rules Used	4.786 (0.081)	7.933 (0.514)	8.750 (0.561)	6.683 (0.503)
Action Rule Length (ARL)	0.798 (0.026)	1.020 (0.076)	1.125 (0.057)	0.842 (0.091)

Note: This table replicates Table IV except all subjects using ALLD in the second locked-response stage have been removed.

TABLE OA3
STRATEGY COMPARISON ACROSS TREATMENTS (NONSIMULATED).

	M1D95	M2+D95	M2+D75
Proportion Mixed	0.669 (0.055)	0.525 (0.097)	0.513 (0.092)
Memory-1 Randomness (M1RM)	19.799 (3.045)	16.030 (2.507)	12.660 (2.762)
Action Randomness (ARM)	15.919 (3.113)	6.430 (1.891)	8.226 (3.232)
Strategy Length	0.927 (0.031)	2.525 (0.211)	2.038 (0.252)
Rules Used	3.855 (0.120)	5.900 (0.475)	3.769 (0.397)
Action Rule Length (ARL)	0.779 (0.034)	1.218 (0.104)	0.605 (0.101)

Note: This table replicates Table IV except all terms are calculated based on observed play from the second locked-response stage.

TABLE OA4
STRATEGY COMPARISON ACROSS TREATMENTS (RULE SET ONLY).

	M1D95	M2+D95	RR2018	M2+D75
Proportion Mixed	0.702 (0.053)	0.600 (0.091)	0.000 (0.000)	0.641 (0.093)
Memory-1 Randomness (M1RM)	21.700 (3.025)	12.835 (2.762)	0.000 (0.000)	11.579 (3.167)
Strategy Length	0.935 (0.028)	3.100 (0.213)	5.366 (0.616)	3.038 (0.332)
Rules in Set	5.282 (0.124)	8.775 (0.608)	11.037 (0.957)	7.115 (0.608)

Note: This table replicates Table IV except all statistics in table are calculated using rule sets from the second locked-response stage only. Proportion mixed is the number of subjects that specify at least one rule with mixed output. M1RM is calculated by ignoring all memory-2+ rules in the rule set. Strategy length is the longest rule in the set. Rules in set is the number of rules in the set.

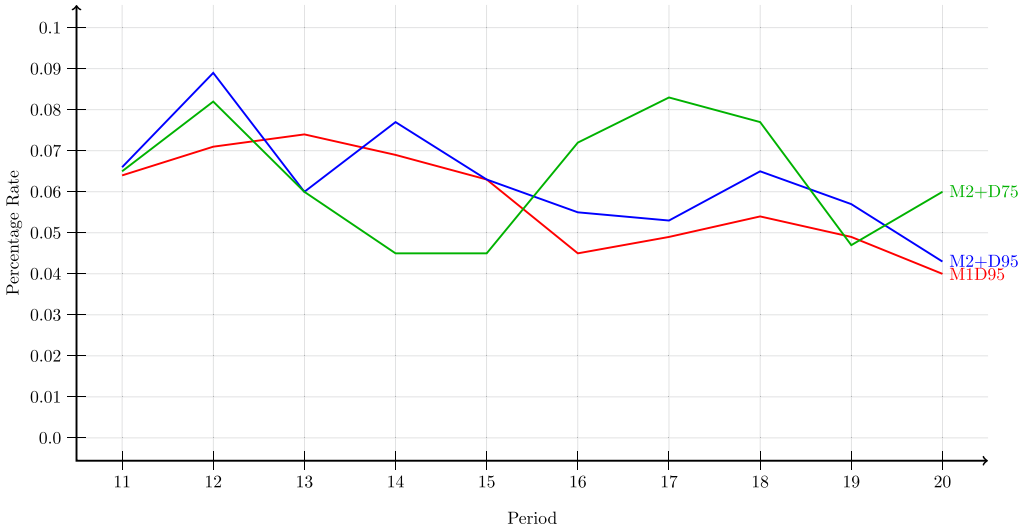


FIGURE OA3.—Rate of Deviations From Rule Set During Nonbinding Stage. *Note:* A deviation from the rule set occurred when a subject’s rule set suggested one action in a given period, and the subject decided to choose the other action in that period. The displayed lines show the average deviation rate for all subjects over the nonbinding supergames.

TABLE OA5
COOPERATION COMPARISON BETWEEN TREATMENTS.

Experiment	M1D95	M2D95	M2D75	M1D95	M2D95	M2D75	M1D95	M2D95	M2D75	M1D95	M2D95	M2D75
Groups	14	8	8	14	8	8	14	8	8	14	8	8
Subjects	124	80	78	124	80	78	124	80	78	124	80	78
Supergames	1–10	1–10	1–10	11–20	11–20	11–20	21–40	21–40	21–40	41–60	41–60	41–60
Type	DR	DR	DR	NB	NB	NB	LR	LR	LR	LR	LR	LR
First Periods	0.56 (0.05)	0.62 (0.07)	0.44 (0.07)	0.66 (0.06)	0.63 (0.07)	0.42 (0.09)	0.65 (0.04)	0.63 (0.06)	0.43 (0.1)	0.68 (0.05)	0.63 (0.07)	0.36 (0.1)
First 4 Periods	0.53 (0.04)	0.62 (0.06)	0.39 (0.07)	0.61 (0.04)	0.58 (0.05)	0.39 (0.1)	0.54 (0.04)	0.55 (0.04)	0.35 (0.09)	0.53 (0.05)	0.53 (0.05)	0.3 (0.09)
Last 4 Periods	0.47 (0.04)	0.58 (0.05)	0.36 (0.07)	0.55 (0.04)	0.5 (0.06)	0.37 (0.08)	0.44 (0.04)	0.44 (0.05)	0.32 (0.07)	0.42 (0.04)	0.45 (0.06)	0.26 (0.07)
All Periods	0.49 (0.04)	0.59 (0.05)	0.38 (0.07)	0.57 (0.04)	0.53 (0.05)	0.38 (0.09)	0.47 (0.04)	0.47 (0.04)	0.34 (0.08)	0.46 (0.04)	0.47 (0.06)	0.28 (0.07)
CC	0.86 (0.02)	0.91 (0.03)	0.78 (0.06)	0.84 (0.03)	0.84 (0.03)	0.9 (0.05)	0.78 (0.04)	0.78 (0.05)	0.67 (0.07)	0.79 (0.03)	0.81 (0.05)	0.75 (0.09)
CD	0.35 (0.03)	0.39 (0.05)	0.2 (0.05)	0.27 (0.03)	0.32 (0.04)	0.19 (0.05)	0.17 (0.02)	0.26 (0.05)	0.14 (0.05)	0.12 (0.02)	0.21 (0.04)	0.12 (0.05)
DC	0.45 (0.03)	0.44 (0.07)	0.37 (0.06)	0.54 (0.04)	0.52 (0.07)	0.33 (0.06)	0.46 (0.04)	0.49 (0.09)	0.34 (0.06)	0.45 (0.04)	0.52 (0.07)	0.26 (0.05)
DD	0.13 (0.02)	0.14 (0.03)	0.15 (0.03)	0.16 (0.03)	0.19 (0.06)	0.15 (0.05)	0.14 (0.02)	0.15 (0.04)	0.13 (0.04)	0.11 (0.02)	0.12 (0.05)	0.1 (0.04)

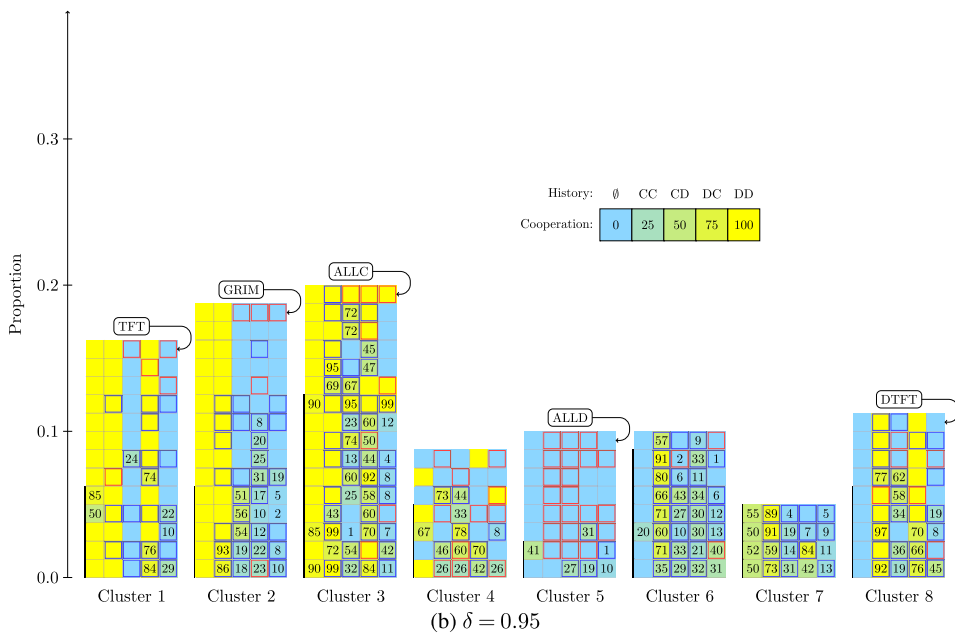
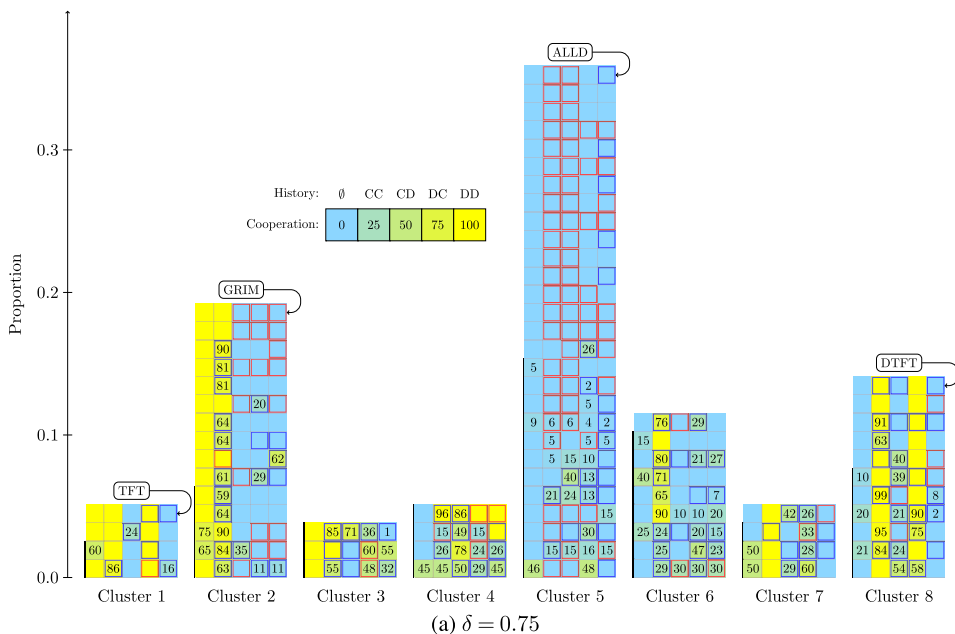


FIGURE OA4.—Robustness Treatment Clusters Relative to M1D95 Exemplars. *Note:* This figure replicates Figure 7(b) except that the cluster exemplars are taken from Figure 2.

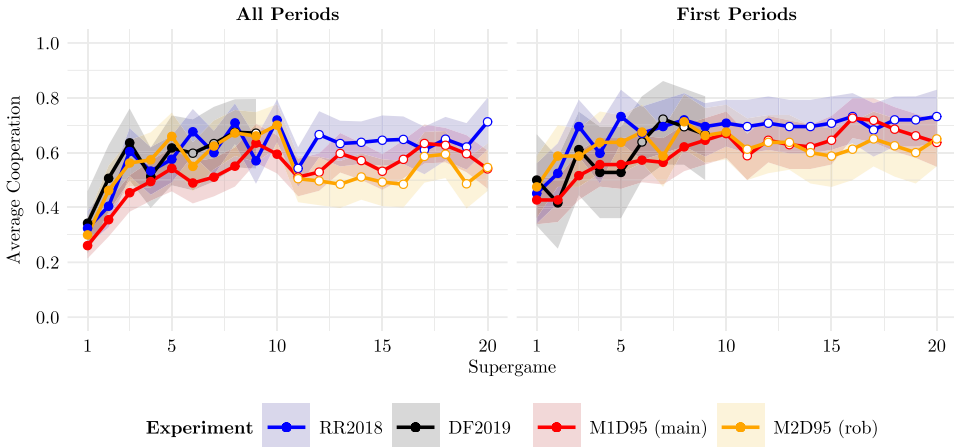


FIGURE OA5.—Cooperation Comparison for $\delta = 0.95$. *Note:* Solid points represent direct-response stage. White points represent nonbinding stage. 95% bootstrapped confidence intervals are superimposed. Dal Bó and Fréchette (2019) run two sessions with different numbers of supergames in each stage. Gray points for (supergames 6 and 7) represent one session being in the direct-response stage while the other being in the nonbinding stage. To test whether our instructions influence behavior in the first period of supergame 1, we run nonparametric permutation tests using an individual subject as a unit of observation. When doing the pairwise comparisons, the lowest p-value is 0.45, indicating no significant difference between any of the four treatments.

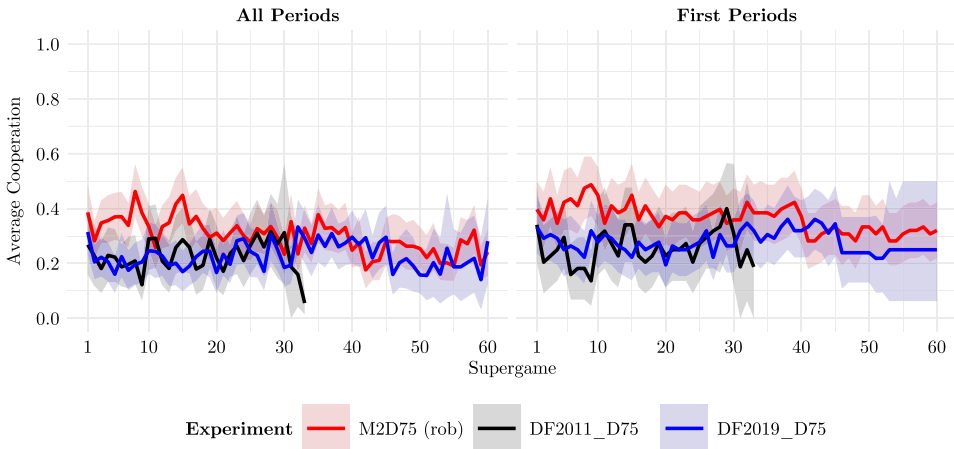


FIGURE OA6.—Cooperation Comparison for $\delta = 0.75$. *Note:* 95% bootstrapped confidence intervals are superimposed (unit of observation is a subject). Dal Bó and Fréchette (2011) run three sessions via direct response with different numbers of supergames. Dal Bó and Fréchette (2019) run three sessions with different numbers of supergames in each stage. To test whether our instructions influence behavior in the first period of supergame 1, we run nonparametric permutation tests using an individual subject as a unit of observation. When doing the pairwise comparisons, the lowest p-value is 0.49, indicating no significant difference between any of the three treatments.

TABLE OA6
COOPERATION COMPARISON WITH PRIOR STUDIES WITH $\delta = 0.75$ AND $R = 32$.

Experiment	M2+D75	DF2019	M2+D75	DF2019	M2+D75	DF2019	M2+D75	DF2019
Subjects	78	44	78	44	78	72	78	72
Supergames	01–10	01–10	11–20	11–20	01–10	01–10	11–20	11–20
Type	DR	DR	NB	DR	DR	DR	NB	DR/NB
First Periods	0.42 (0.07)	$\sim^{0.11}$ 0.23 (0.04)	0.38 (0.09)	$\sim^{0.45}$ 0.27 (0.05)	0.42 (0.07)	$\sim^{0.32}$ 0.28 (0.07)	0.38 (0.09)	$\sim^{0.48}$ 0.26 (0.11)
All Periods	0.36 (0.06)	$\sim^{0.14}$ 0.21 (0.03)	0.35 (0.08)	$\sim^{0.41}$ 0.24 (0.04)	0.36 (0.06)	$\sim^{0.14}$ 0.22 (0.03)	0.35 (0.09)	$\sim^{0.39}$ 0.21 (0.08)

TABLE OA7
COOPERATION COMPARISON WITH PRIOR STUDIES WITH $\delta = 0.95$ AND $R = 32$.

Experiment	M1D95	DF2019	M1D95	DF2019	M2+D95	DF2019	M2+D95	DF2019
Subjects	124	36	124	36	80	36	80	36
Supergames	01–05	01–05	06–10	06–10	01–05	01–05	06–10	06–10
Type	DR	DR	DR	DR/NB	DR	DR	DR	DR/NB
First Periods	0.5 (0.05)	$\sim^{0.81}$ 0.52 (0.07)	0.61 (0.06)	$\sim^{0.51}$ 0.69 (0.1)	0.58 (0.06)	$\sim^{0.47}$ 0.52 (0.07)	0.66 (0.06)	$\sim^{0.83}$ 0.69 (0.1)
All Periods	0.42 (0.04)	$\sim^{0.32}$ 0.52 (0.05)	0.56 (0.04)	$\sim^{0.32}$ 0.65 (0.06)	0.51 (0.05)	$\sim^{0.83}$ 0.52 (0.05)	0.64 (0.04)	$\sim^{0.95}$ 0.65 (0.07)

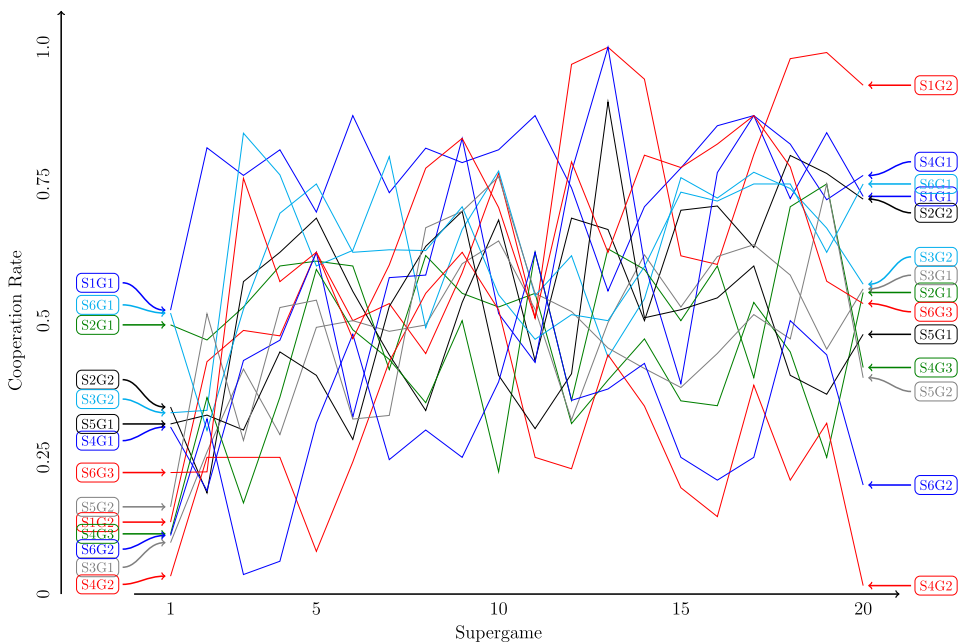
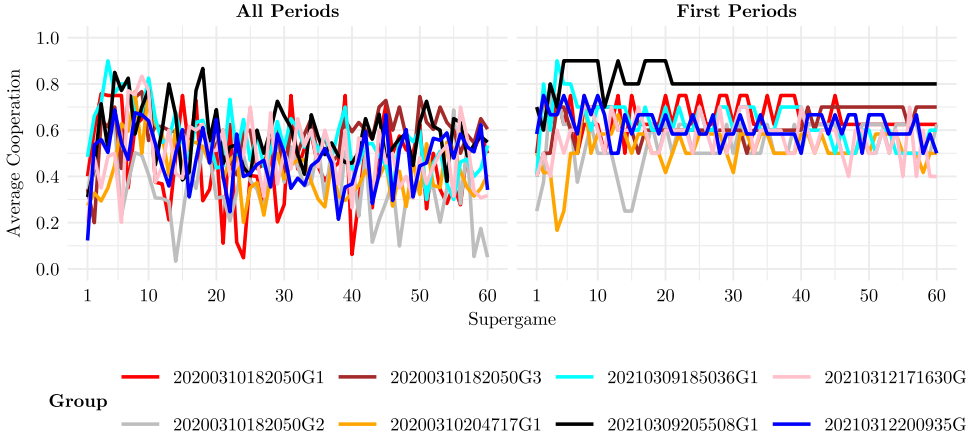
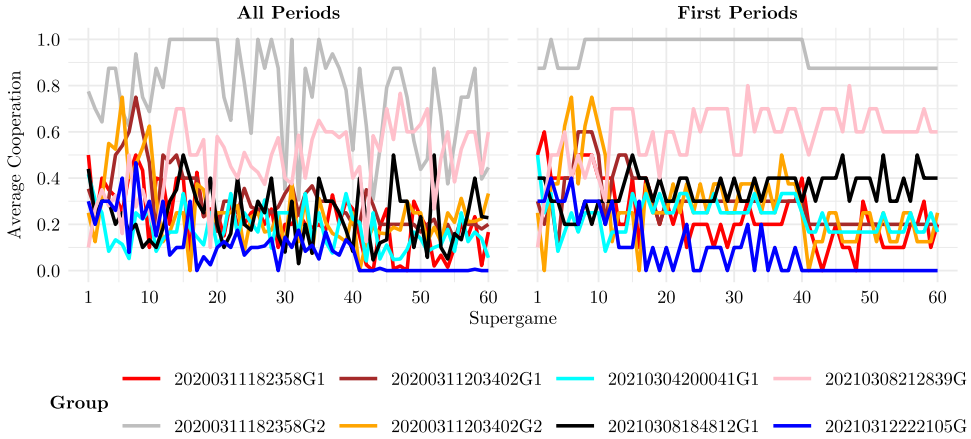


FIGURE OA7.—Cooperation Rate By Groups (Main M1 $\delta = 0.95$).

FIGURE OA8.—Cooperation Rate By Groups (Robustness M2 + $\delta = 0.95$).FIGURE OA9.—Cooperation Rate By Groups (Robustness M2 + $\delta = 0.75$).

APPENDIX OA-2: DETAILS OF THE SIMULATION

We use the method from [Romero and Rosokha \(2018\)](#), which involves creating simulated opponents and seeing how each subject's rule set plays against the simulated opponents. [Romero and Rosokha \(2018\)](#) contains a detailed description of the benefits of this method and why it is chosen over alternative methods. A simulated opponent's play is generated using a two-state Markov transition matrix with parameters $a, b \in [0, 1]$,

$$P = \begin{matrix} C \\ D \end{matrix} \begin{bmatrix} a & 1-a \\ 1-b & b \end{bmatrix}.$$

The parameter a represents the probability that the sequence plays C after playing C in the previous period, and the parameter b represents the probability that the sequence plays D after playing D in the previous period. To generate the play of a single simulated opponent, we do the following process:

1. Randomly select parameters a and b from a uniform distribution over the unit interval ($U[0, 1]$), which gives a realization of the Markov transition matrix \hat{P} .

2. Start in a random state of the Markov Chain based on the stationary distribution of \hat{P} .
3. Generate a sequence of actions C or D using \hat{P} . The length of this sequence is based on the continuation probability $\delta = 0.95$.¹

We use this procedure to generate sequences of play for 500 simulated opponents. Unlike [Romero and Rosokha \(2018\)](#), strategies in this experiment could randomize. To keep the outcome of randomization as similar as possible, for each sequence of play, we generate a corresponding sequence of action random numbers containing integers between 1 and 100 (inclusive). The combination of these 500 sequences of play and 500 sequences of action random numbers are referred to as *simulated play*. This simulated play can be fed into a subject's rule set to generate *simulated output* for that subject. The simulated output contains both the action prescribed by the corresponding subject's rule set as well as the probability used to choose that action. Note that in expectation, the simulated play has $500 \times 20 = 10,000$ periods of choices and, therefore, the simulated output also has 10,000 periods of choices in expectation. We use these simulated outputs to calculate a number of statistics that help us understand the complex rule sets in the robustness treatments.

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¹We still use $\delta = 0.95$ sequences when analyzing $\delta = 0.75$ treatments for comparability reasons. If we used different length sequences for different treatments, then identical strategies would yield different statistics.