

SUPPLEMENT TO “RANGE-DEPENDENT ATTRIBUTE WEIGHTING IN CONSUMER CHOICE: AN EXPERIMENTAL TEST”
(*Econometrica*, Vol. 90, No. 2, March 2022, 799–830)

JASON SOMERVILLE
Federal Reserve Bank of New York

APPENDIX A: PROOFS AND ADDITIONAL RESULTS

PROOF OF PROPOSITION 1: THE PROOF OF PROPOSITION 1 follows from the following two results. The first states that the price space $(p_h, p_l) \in \mathbb{R}_+^2$ can be partitioned into five mutually exclusive and collectively exhaustive regions. In three of these regions, the theory predicts only two of the three options are ever chosen: l or o , h or o , and l or h , respectively. I refer to these regions as the h -decoy, l -decoy, and o -decoy regions in the main text, and they are formally defined below by the sets A , B , and C . The second result builds on the first and establishes existence and uniqueness of the l - o , h - o , and l - h indifference boundaries within these three regions.

Result 1 (Pairwise Choice Bounds) Suppose A3 holds. Then, for choice set $\mathcal{C}(p_h, p_l)$, the price space $(p_h, p_l) \in \mathbb{R}_+^2$ can be partitioned into the mutually exclusive and collectively exhaustive regions depicted visually in Figure A.1 and furthermore:

- I. In region $A \equiv \{(p_h, p_l) \in \mathbb{R}_+^2 : p_h \geq q_h \text{ and } q_l/p_l \geq q_h/p_h\}$, l or o is chosen.
- II. In region $B \equiv \{(p_h, p_l) \in \mathbb{R}_+^2 : p_l \geq q_h \text{ and } p_h \leq p_l\}$, h or o is chosen.
- III. In region $C \equiv \{(p_h, p_l) \in \mathbb{R}_+^2 : p_h \leq q_h \text{ and } q_l/p_l \leq q_h/p_h\}$, l or h is chosen.
- IV. In region $D \equiv \{(p_h, p_l) \in \mathbb{R}_+^2 : p_h > q_h \text{ and } q_l/p_l < q_h/p_h \text{ and } p_h > p_l\}$, o is chosen.
- V. In region $E \equiv \{(p_h, p_l) \in \mathbb{R}_+^2 : p_h < q_h \text{ and } q_l/p_l < q_h/p_h \text{ and } p_l < q_h\}$, h is chosen.

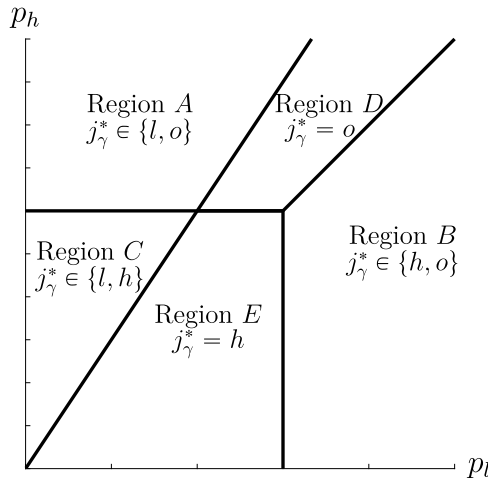


FIGURE A.1.—Preference regions defined in Result 1. The optimal choice for a range-weighting agent is denoted by $j_\gamma^* \equiv \arg \max_j [g(\Delta_q; \gamma)q_j - g(\Delta_p; \gamma)p_j]$.

Jason Somerville: jason.somerville@ny.frb.org

PROOF: *Part I.* Suppose $p_h > p_l$ and $p_h > q_h$, hence $\Delta_q = q_h$ and $\Delta_p = p_h$. By A3(ii), $V_h = g(q_h; \gamma) \cdot q_h - g(p_h; \gamma) \cdot p_h < 0 = V_o$, and therefore h will never be chosen. This condition can be restated as $q_h/p_h < g(p_h; \gamma)/g(q_h; \gamma)$. Furthermore, a necessary condition for l to be preferred to o is

$$\begin{aligned} & g(q_h; \gamma) \cdot q_l - g(p_h; \gamma) \cdot p_l > 0 \\ \Leftrightarrow & \frac{q_l}{p_l} > \frac{g(p_h; \gamma)}{g(q_h; \gamma)} > \frac{q_h}{p_h}. \end{aligned}$$

Part II. If $p_h < p_l$, then $\Delta_q = q_h$ and $\Delta_p = p_l$. By A3(ii), $g(p_l; \gamma) \cdot p_l > g(p_h; \gamma) \cdot p_h$, and hence,

$$\begin{aligned} V_l &= g(q_h; \gamma) \cdot q_l - g(p_l; \gamma) \cdot p_l \\ &< g(q_h; \gamma) \cdot q_h - g(p_l; \gamma) \cdot p_l \\ &< g(q_h; \gamma) \cdot q_h - g(p_h; \gamma) \cdot p_h = V_h \end{aligned}$$

and so l is never chosen. Furthermore, a necessary condition for o to be chosen is $V_o > V_h$, which holds whenever

$$\begin{aligned} 0 &> g(q_h; \gamma) \cdot q_h - g(p_l; \gamma) \cdot p_h \\ \Leftrightarrow 0 &> g(q_h; \gamma) \cdot q_h - g(p_l; \gamma) \cdot p_l \\ \Leftrightarrow g(p_l; \gamma) \cdot p_l &> g(q_h; \gamma) \cdot q_h \\ \Leftrightarrow p_l &> q_h \quad (\text{by A3(ii)}). \end{aligned}$$

Part III. Suppose $p_h > p_l$ and $p_h < q_h$, then $\Delta_q = q_h$ and $\Delta_p = p_h$. By A3(ii), $V_h = g(q_h; \gamma) \cdot q_h - g(p_h; \gamma) \cdot p_h > 0 = V_o$, and therefore o will never be chosen. Rearranging gives $q_h/p_h > g(p_h; \gamma)/g(q_h; \gamma)$ and therefore, a necessary condition for l to be chosen is $V_h < V_l$,

$$\begin{aligned} & g(q_h; \gamma) \cdot q_h - g(p_h; \gamma) \cdot p_h < g(q_h; \gamma) \cdot q_l - g(p_h; \gamma) \cdot p_l \\ \Leftrightarrow & g(q_h; \gamma) \cdot (q_h - q_l) < g(p_h; \gamma) \cdot (p_h - p_l) \\ \Leftrightarrow & \frac{q_h - q_l}{p_h - p_l} < \frac{g(p_h; \gamma)}{g(q_h; \gamma)} < \frac{q_h}{p_h} \\ \Leftrightarrow & \frac{q_l}{p_l} > \frac{q_h}{p_h}. \end{aligned}$$

Part IV. From Part I, $p_h > p_l$ and $p_h > q_h$ implies that h is never chosen and, furthermore, that a necessary condition for l to be chosen is that $q_l/p_l > q_h/p_h$. By definition of D , all prices must satisfy $q_l/p_l < q_h/p_h$, and therefore l will never be chosen. Hence, choice-set agents will always choose o in region D .

Part V. First, consider $p_h \geq p_l$. Then from Part III, o will never be chosen, and furthermore, as the necessary condition for l to be chosen fails to hold, h must be chosen. Second, it was shown in Part II that for $p_h < p_l$, l will never be chosen, and as the necessary condition for choosing o does not hold, h must again be chosen. *Q.E.D.*

Result 2 (Boundary Existence and Uniqueness): Suppose A3 holds and the choice set is $C(p_h, p_l)$ for some $(p_h, p_l) \in \mathbb{R}_+^2$. Then, for any γ , there exists a unique partition of:

- I. A into mutually exclusive and collectively exhaustive subsets $\{A_l, A_o, A_i\}$ such that range-weighting agents are indifferent between l and o for all $(p_h, p_l) \in A_i$, prefer o for all $(p_h, p_l) \in A_o$, and prefer l for all $(p_h, p_l) \in A_l$. Furthermore, any $\gamma' \neq \gamma$ yields a distinct partition $\{A'_l, A'_o, A'_i\} \neq \{A_l, A_o, A_i\}$.
- II. B into mutually exclusive and collectively exhaustive subsets $\{B_h, B_o, B_i\}$ such that range-weighting agents are indifferent between h and o for all $(p_h, p_l) \in B_i$, prefer o for all $(p_h, p_l) \in B_o$, and prefer h for all $(p_h, p_l) \in B_h$. Furthermore, any $\gamma' \neq \gamma$ yields a distinct partition $\{B'_h, B'_o, B'_i\} \neq \{B_h, B_o, B_i\}$.
- III. C into mutually exclusive and collectively exhaustive subsets $\{C_h, C_l, C_i\}$ such that range-weighting agents are indifferent between h and l for all $(p_h, p_l) \in C_i$, prefer l for all $(p_h, p_l) \in C_l$, and prefer h for all $(p_h, p_l) \in C_h$. Furthermore, any $\gamma' \neq \gamma$ yields a distinct partition $\{C'_h, C'_l, C'_i\} \neq \{C_h, C_l, C_i\}$.

PROOF: *Part I.* For any $(p_h, p_l) \in A$, fix p_h . Define $f_A(p_l) \equiv V_l - V_o = g(q_h) \cdot q_l - g(p_h) \cdot p_l$, which is continuous and strictly decreasing in p_l . $f_A(0) > 0$ and hence $j^*(p_h, p_l) = l$. Then,

$$f_A\left(\frac{p_h q_l}{q_h}\right) = g(q_h)q_l - g(p_h)\frac{p_h q_l}{q_h} = g(q_h)\frac{q_l p_h}{q_h} \left[\frac{q_h}{p_h} - \frac{g(p_h)}{g(q_h)} \right] < 0,$$

as the term in brackets is less than zero (see the proof of Result 1 Part I), and hence $j^*(p_h, p_l) = o$. Then, by the intermediate value theorem, there exists a unique $\bar{p} > 0$ such that $f_A(\bar{p}) = 0$. As $f_A(p_l)$ is strictly decreasing, then for all $p_l \leq \bar{p}$, $j^*(p_h, p_l) = l$, and for all $p_l \geq \bar{p}$, $j^*(p_h, p_l) = o$. Define $A_i = \{(p_h, p_l) \in A : p_l = \bar{p}\}$, $A_l = \{(p_h, p_l) \in A : p_l < \bar{p}\}$, and $A_o = \{(p_h, p_l) \in A : p_l > \bar{p}\}$; then $\{A_l, A_o, A_i\}$ is a unique partition of A .

To show that $\{A_l, A_o, A_i\}$ is distinct for any γ , note that $f_A(\bar{p}) = 0$ implies $\bar{p}(\gamma) = q_l g(q_h; \gamma) / g(p_h; \gamma)$, which is continuous and strictly decreasing in γ as $q_h < p_h$. Therefore, for any $\gamma' \neq \gamma$, it follows that $\bar{p}(\gamma') \neq \bar{p}(\gamma)$ and hence $\{A_l, A_o, A_i\} \neq \{A'_l, A'_o, A'_i\}$.

Part II. For any $(p_h, p_l) \in B$, fix p_l . Define $f_B(p_h) \equiv V_h - V_o = g(q_h; \gamma) \cdot q_h - g(p_l; \gamma) \cdot p_h$, which is continuous and strictly decreasing in p_h . $f_B(0) > 0$ and hence $j^*(p_h, p_l) = h$. $f_B(p_l) < 0$ (by A3(ii)) and hence $j^*(p_h, p_l) = o$. Then, by the intermediate value theorem, there exists a unique \hat{p} such that $f_B(\hat{p}) = 0$. As $f_B(p_h)$ is strictly decreasing, then for all $p_h \leq \hat{p}$, $j^*(p_h, p_l) = h$, and for all $p_h \geq \hat{p}$, $j^*(p_h, p_l) = o$. Define $B_i = \{(p_h, p_l) \in B : p_h = \hat{p}\}$, $B_h = \{(p_h, p_l) \in B : p_h < \hat{p}\}$, and $B_o = \{(p_h, p_l) \in B : p_h > \hat{p}\}$; then $\{B_h, B_o, B_i\}$ is a partition of B . Furthermore, $f_B(\hat{p}) = 0$ implies that $\hat{p}(\gamma) = q_l g(q_h; \gamma) / g(p_l; \gamma)$, which is continuous and strictly decreasing in γ as $q_h < p_l$. Therefore, for any $\gamma' \neq \gamma$, it follows that $\hat{p}(\gamma') \neq \hat{p}(\gamma)$ and hence $\{B_h, B_o, B_i\} \neq \{B'_h, B'_o, B'_i\}$.

Part III. For any $(p_h, p_l) \in C$, fix p_l . Define $f_C(p_h) \equiv V_h - V_l = g(q_h; \gamma) \cdot [q_h - q_l] - g(p_h; \gamma) \cdot [p_h - p_l]$, which is continuous and strictly decreasing in p_h (by A3(ii)). $f_C(q_h) = -g(q_h; \gamma)[q_l - p_l] < 0$ and hence $j^*(p_h, p_l) = l$. Recall that for all $(p_h, p_l) \in C$, $q_h/p_h > g(p_h)/g(q_h)$ (see proof of Result 1, Part III) and in particular for $p_h = p_l(q_h/q_l)$, which

implies $q_l/p_l > g(\frac{p_l q_h}{q_l})/g(q_h)$. Therefore,

$$\begin{aligned} f_C\left(\frac{p_l q_h}{q_l}\right) &= g(q_h)(q_h - q_l) - g\left(\frac{p_l q_h}{q_l}\right)\left[\frac{p_l q_h}{q_l} - p_l\right] \\ &= g(q_h)(q_h - q_l)\frac{p_l}{q_l}\left[\frac{q_l}{p_l} - \frac{g\left(\frac{p_l q_h}{q_l}\right)}{g(q_h)}\right] > 0 \end{aligned}$$

and hence $j^*(p_h, p_l) = h$. Then, by the intermediate value theorem, there exists a unique $\tilde{p} > 0$ such that $f_C(\tilde{p}) = 0$. As $f_C(p_h)$ is strictly decreasing, then for all $p_h \leq \tilde{p}$, $j^*(p_h, p_l) = h$, and for all $p_h \geq \tilde{p}$, $j^*(p_h, p_l) = l$. Define $C_i = \{(p_h, p_l) \in C : p_h = \tilde{p}\}$, $C_h = \{(p_h, p_l) \in C : p_h < \tilde{p}\}$, and $C_l = \{(p_h, p_l) \in C : p_h > \tilde{p}\}$; then $\{C_h, C_l, C_i\}$ is a unique partition of C .

It remains to show that $\tilde{p}(\gamma)$ is strictly increasing in γ . Recall that by definition, $f_C(\tilde{p}) = g(q_h; \gamma)(q_h - q_l) - g(\tilde{p}; \gamma)(\tilde{p} - p_l) = 0$. Taking the total derivative with respect to γ gives

$$\begin{aligned} &g_\gamma(q_h; \gamma)(q_h - q_l) \\ &- \left[\left(g_{\tilde{p}}(\tilde{p}(\gamma); \gamma) \frac{\partial \tilde{p}(\gamma)}{\partial \gamma} + g_\gamma(\tilde{p}(\gamma); \gamma) \right) (\tilde{p} - p_l) + \frac{\partial \tilde{p}(\gamma)}{\partial \gamma} g(\tilde{p}(\gamma); \gamma) \right] = 0 \\ \Rightarrow \quad &\frac{\partial \tilde{p}(\gamma)}{\partial \gamma} = \frac{g_\gamma(q_h; \gamma)(q_h - q_l) - g_\gamma(\tilde{p}(\gamma); \gamma)(\tilde{p}(\gamma) - p_l)}{g_{\tilde{p}}(\tilde{p}(\gamma); \gamma)(\tilde{p}(\gamma) - p_l) + g(\tilde{p}(\gamma); \gamma)}. \end{aligned}$$

To sign the denominator, consider two cases. If g is (weakly) increasing in the Δ (i.e., $g_{\tilde{p}}(\tilde{p}(\gamma); \gamma) \geq 0$), then the expression in the denominator is strictly positive. If instead g is (weakly) decreasing in the range (i.e., $g_{\tilde{p}}(\tilde{p}(\gamma); \gamma) \leq 0$), then by A3(ii),

$$\frac{\partial [\tilde{p}(\gamma) \cdot g(\tilde{p}(\gamma); \gamma)]}{\partial \tilde{p}(\gamma)} = g_{\tilde{p}}(\tilde{p}(\gamma); \gamma) \tilde{p}(\gamma) + g(\tilde{p}(\gamma); \gamma) > 0$$

and hence, $g_{\tilde{p}}(\tilde{p}(\gamma); \gamma)(\tilde{p}(\gamma) - p_l) + g(\tilde{p}(\gamma); \gamma) \geq g_{\tilde{p}}(\tilde{p}(\gamma); \gamma) \tilde{p}(\gamma) + g(\tilde{p}(\gamma); \gamma) > 0$. Therefore, the denominator is always strictly positive. To sign the numerator, note that $f_C(\tilde{p}(\gamma)) = 0$

$$\Rightarrow \frac{g(q_h; \gamma)}{g(\tilde{p}(\gamma); \gamma)} = \frac{\tilde{p}(\gamma) - p_l}{q_h - q_l}. \quad (*)$$

Furthermore, as $q_h > \tilde{p}(\gamma)$, $g(q_h; \gamma)/g(\tilde{p}(\gamma); \gamma)$ is strictly increasing in γ , and hence

$$\frac{g_\gamma(q_h; \gamma) \cdot g(\tilde{p}(\gamma); \gamma) - g_\gamma(\tilde{p}(\gamma); \gamma) \cdot g(q_h; \gamma)}{g(\tilde{p}(\gamma); \gamma)^2} > 0.$$

Combining the above with (*) gives

$$\frac{g_\gamma(q_h; \gamma)}{g_\gamma(\tilde{p}(\gamma); \gamma)} > \frac{g(q_h; \gamma)}{g(\tilde{p}(\gamma); \gamma)} = \frac{\tilde{p}(\gamma) - p_l}{q_h - q_l},$$

which can be rewritten as $g(q_h; \gamma)(q_h - q_l) - g_\gamma(\tilde{p}(\gamma); \gamma)(\tilde{p}(\gamma) - p_l) > 0$ and therefore, $\partial \tilde{p}(\gamma) / \partial \gamma > 0$. Hence, for $\gamma' \neq \gamma$, it follows that $\tilde{p}(\gamma') \neq \tilde{p}(\gamma)$ and hence $\{C_h, C_l, C_i\} \neq \{C'_h, C'_l, C'_i\}$. Q.E.D.

It follows from Result 2 that for any $\gamma \neq \gamma'$, the choice correspondence $j^*(p_h, p_l; \gamma) \neq j^*(p_h, p_l; \gamma')$ for all $(p_h, p_l) \in \mathbb{R}_+^2$. Therefore, there exists either no γ such that $j^*(p_h, p_l; \gamma) = j(p_h, p_l)$ for all $(p_h, p_l) \in \mathbb{R}_+^2$, or a unique γ such that $j^*(p_h, p_l; \gamma) = j(p_h, p_l)$ for all $(p_h, p_l) \in \mathbb{R}_+^2$. Q.E.D.

PROOF OF PROPOSITION 2: Let f denote the joint distribution of error difference terms. The probability of choosing h or l is

$$\begin{aligned} P(h \text{ or } l) &= 1 - P(o) \\ &= 1 - \Pr(\widehat{\varepsilon}_{ol} > U_l - U_o) \Pr(\widehat{\varepsilon}_{oh} > U_h - U_o) \\ &= 1 - \int_{\widehat{\varepsilon}_{ol}=U_l-U_o}^{\infty} \int_{\widehat{\varepsilon}_{oh}=U_h-U_o}^{\infty} f(\widehat{\varepsilon}_{ol}, \widehat{\varepsilon}_{oh}) d\widehat{\varepsilon}_{oh} d\widehat{\varepsilon}_{ol}. \end{aligned}$$

As $f(\widehat{\varepsilon}_{lo}, \widehat{\varepsilon}_{lh}) > 0$ for all $(\widehat{\varepsilon}_{lo}, \widehat{\varepsilon}_{lh})$, the probability of choosing h or l is strictly increasing (decreasing) if and only if the bounds of both integrals contract (expand). First, consider how an increase in p_h affects the bounds of the inner integral. By A1, U_l and U_o are independent of p_h , while U_h is strictly decreasing in p_h . Therefore, the bound $U_h - U_o$ is strictly decreasing in p_h , which leads to a strict increase in $P(o)$ and hence a strict decrease in $P(h \text{ or } l)$. Finally, consider how an increase in p_l affects the bounds of the outer integral. By A1, U_h and U_o are independent of p_l , while U_l is strictly decreasing in p_l . Therefore, the bound $U_l - U_o$ is strictly decreasing in p_l , which leads to a strict increase in $P(o)$ and hence a strict decrease in $P(h \text{ or } l)$. Q.E.D.

PROOF OF PROPOSITION 3: Let $j, k, k' \in \{h, l, o\}$ denote the three distinct options in choice set $\mathcal{C}(p_h, p_l)$. The probability of choosing j is

$$\begin{aligned} P(j) &= \Pr(\widehat{\varepsilon}_{jk} > V_k - V_j, \widehat{\varepsilon}_{jk'} > V_{k'} - V_j) \\ &= \int_{\widehat{\varepsilon}_{jk}=V_k-V_j}^{\infty} \int_{\widehat{\varepsilon}_{jk'}=V_{k'}-V_j}^{\infty} f(\widehat{\varepsilon}_{jk}, \widehat{\varepsilon}_{jk'}) d\widehat{\varepsilon}_{jk} d\widehat{\varepsilon}_{jk'}. \end{aligned} \quad (\text{A.4})$$

As $f(\widehat{\varepsilon}_{jk}, \widehat{\varepsilon}_{jk'}) > 0$ for all $(\widehat{\varepsilon}_{jk}, \widehat{\varepsilon}_{jk'})$, the probability of choosing j is strictly increasing (decreasing) if and only if the bounds of both integrals expand (contract).

Part (a): h-decoy region. To see how the integral bounds respond to changes in p_h in the h -decoy region, note that for any prices $(p_h, p_l) \in A$, $V_o - V_l = g(p_h; \gamma)p_l - g(q_h; \gamma)q_l$, $V_h - V_o = g(q_h; \gamma)q_h - g(p_h; \gamma)p_h$, and $V_h - V_l = g(q_h; \gamma)(q_h - q_l) - g(p_h; \gamma)(p_h - p_l)$. By the properties of g , it follows that $V_h - V_l$ and $V_h - V_o$ are strictly decreasing in p_h for all γ , while $V_o - V_l$ is strictly increasing in p_h if $\gamma > 0$, strictly decreasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Given these properties and the choice probability formula (A.4):

- (i) The first integral bound for $P(h)$ is $V_l - V_h$, which is strictly increasing in p_h for all γ . The second is $V_o - V_h$, which is also strictly increasing in p_h for all γ . Thus, $P(h)$ is strictly decreasing in p_h for all γ .

- (ii) The first integral bound for $P(l)$ is $V_h - V_l$, which is strictly decreasing in p_h for all γ . The second is $V_o - V_l$, which is strictly increasing in p_h if $\gamma > 0$, strictly decreasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Thus, $P(l)$ is strictly increasing in p_h if $\gamma \leq 0$ and ambiguous if $\gamma > 0$.
- (iii) The first integral bound for $P(o)$ is $V_h - V_o$, which is strictly decreasing in p_h for all γ . The second is $V_l - V_o$, which is strictly decreasing in p_h if $\gamma > 0$, strictly increasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Thus, $P(o)$ is strictly increasing in p_h if $\gamma \geq 0$ and ambiguous if $\gamma < 0$.

Part (b): l-decoy region. For any prices $(p_h, p_l) \in B$, $V_l - V_o = g(q_h; \gamma)q_l - g(p_l; \gamma)p_l$, $V_l - V_h = g(p_l; \gamma)(p_h - p_l) - g(q_h; \gamma)(q_h - q_l)$, and $V_o - V_h = g(p_l; \gamma)p_h - g(q_h; \gamma)q_h$. By the properties of g , it follows that $V_l - V_o$ and $V_l - V_h$ are strictly decreasing in p_l for all γ , while $V_o - V_h$ is strictly increasing in p_l if $\gamma > 0$, strictly decreasing in p_l if $\gamma < 0$, and unaffected by p_l if $\gamma = 0$. Given these properties and the choice probability formula (A.4):

- (i) The first integral bound for $P(h)$ is $V_l - V_h$, which is strictly decreasing in p_l for all γ . The second is $V_o - V_h$, which is strictly increasing in p_l if $\gamma > 0$, strictly decreasing in p_l if $\gamma < 0$, and unaffected by p_l if $\gamma = 0$. Thus, $P(h)$ is strictly increasing in p_l if $\gamma \leq 0$ and ambiguous if $\gamma > 0$.
- (ii) The first integral bound for $P(l)$ is $V_h - V_l$, which is strictly increasing in p_l for all γ . The second is $V_o - V_l$, which is also strictly increasing in p_l if $\gamma > 0$. Thus, $P(l)$ is strictly decreasing in p_l for all γ .
- (iii) The first integral bound for $P(o)$ is $V_l - V_o$, which is strictly decreasing in p_l for all γ . The second is $V_h - V_o$, which is strictly decreasing in p_l if $\gamma > 0$, strictly increasing in p_l if $\gamma < 0$, and unaffected by p_l if $\gamma = 0$. Thus, $P(o)$ is strictly increasing if $\gamma \geq 0$ and ambiguous if $\gamma < 0$.

Part (c): o-decoy region. Fix the price difference $p_h - p_l \equiv c > 0$. Then for any prices $(p_h, p_l) \in C$, $V_l - V_o = g(q_h; \gamma)q_l - g(p_h; \gamma)p_l$, $V_h - V_o = g(q_h; \gamma)q_h - g(p_h; \gamma)p_h$, and $V_h - V_l = g(q_h; \gamma)(q_h - q_l) - g(p_h; \gamma)c$. By the properties of g , it follows that $V_l - V_o$ and $V_h - V_o$ are strictly decreasing in p_h for all γ , while $V_h - V_l$ is strictly decreasing in p_h if $\gamma > 0$, strictly increasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Given these properties and the choice probability formula (A.4):

- (i) The first integral bound for $P(h)$ is $V_o - V_h$, which is strictly increasing in p_h for all γ . The second is $V_l - V_h$, which is strictly increasing in p_h if $\gamma > 0$, strictly decreasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Thus, $P(h)$ is strictly decreasing in p_h if $\gamma \geq 0$ and ambiguous if $\gamma < 0$.
- (ii) The first integral bound for $P(l)$ is $V_o - V_l$, which is strictly increasing in p_h for all γ . The second is $V_h - V_l$, which is strictly decreasing in p_h if $\gamma > 0$, strictly increasing in p_h if $\gamma < 0$, and unaffected by p_h if $\gamma = 0$. Thus, $P(l)$ is strictly decreasing if $\gamma \leq 0$ and ambiguous if $\gamma > 0$.
- (iii) The first integral bound for $P(o)$ is $V_h - V_o$, which is strictly decreasing in p_h for all γ . The second is $V_l - V_o$, which is also strictly decreasing in p_h for all γ . Thus, $P(o)$ is strictly increasing in p_h for all γ . Q.E.D.

PROPOSITION 4—Equivalence for Binary Choice: *Suppose the choice set is $\{(q_h, p_h), (q_l, p_l)\}$, where q_h and q_l are fixed such that $q_h > q_l$ and that A3 holds. Let $j_0^* \equiv \arg \max_{j \in \{h, l\}} U_j$ and $j_\gamma^* \equiv \arg \max_{j \in \{h, l\}} V_j$. Then, $j_\gamma^* = j_0^*$ for all $(p_h, p_l) \in \mathbb{R}_+^2$ and for all γ .*

PROOF: Given choice set $\mathcal{C} = \{(q_h, p_h), (q_l, p_l)\}$, the quality and price ranges are $\Delta_q = q_h - q_l$ and $\Delta_p = |p_h - p_l|$. First, consider the case where $p_l > p_h$. Both $V_h - V_l = g(q_h -$

$q_l; \gamma) \cdot (q_h - q_l) + g(p_l - p_h; \gamma) \cdot (p_l - p_h) > 0$ and $U_h - U_l = (q_h - q_l) + (p_l - p_h) > 0$, and therefore $j_\gamma^* = j_0^* = h$ for all γ . For $p_h \geq p_l$, h will be preferred whenever $V_h = g(q_h - q_l; \gamma) \cdot q_h - q_l > g(p_h - p_l; \gamma) \cdot (p_h - p_l) = V_l$. This holds if and only if $g(\Delta_q; \gamma) \cdot \Delta_q > g(\Delta_p; \gamma) \cdot \Delta_p$. By A3(ii), this holds if and only if $\Delta_q > \Delta_p$, which is equivalent to $U_h > U_l$. Likewise, $V_h \leq V_l \Leftrightarrow U_h \leq U_l$, and therefore for all γ , $j_\gamma^* = j_0^* = h$ if $U_h > U_l$ and $j_\gamma^* = j_0^* = l$ if $U_h < U_l$, with indifference at $U_h = U_l$. Thus, $j_\gamma^* = j_0^*$ for all $(p_h, p_l) \in \mathbb{R}_+^2$ and γ . *Q.E.D.*

APPENDIX B: ADDITIONAL EXPERIMENTAL DESIGN DETAILS

B.1. Products

TABLE B.I
PRODUCTS OFFERED IN THE EXPERIMENT.

	<i>high variety</i>	<i>low variety</i>
<i>Quantity</i>		
Lindor Chocolate Truffles	300 pieces	100 pieces
Cinema Card	10 passes	5 passes
Starbucks Coffee	40 cups	20 cups
Gillette Razors	16 blades	8 blades
Insignia TV	24 inch	19 inch
Uber Credit	\$80	\$40
<i>Duration</i>		
Blue Apron Subscription	2 weeks	1 week
Yoga Subscription	2 month	1 month
Wine of the Month Subscription	2 month	1 month
Cheese of the Month Subscription	2 month	1 month
Beer of the Month Subscription	2 month	1 month
Flowers of the Month Subscription	2 month	1 month
<i>Functionality</i>		
Roku Streaming Device	Roku Ultra TV	Roku Stick
Amazon Tablet	Fire 8	Fire 7
<i>Brand</i>		
Bluetooth Speaker	Bose Soundlink	AmazonBasics
Water Purifier + 6 Filters	Brita	AmazonBasics
Hardcase Luggage	Samsonite	AmazonBasics
Rechargeable Batteries + Charger	Energizer	AmazonBasics
Laptop Backpack	Northface	AmazonBasics
Sunglasses	Ray-Ban	Sungait

B.2. Screenshots of the Experimental Tasks

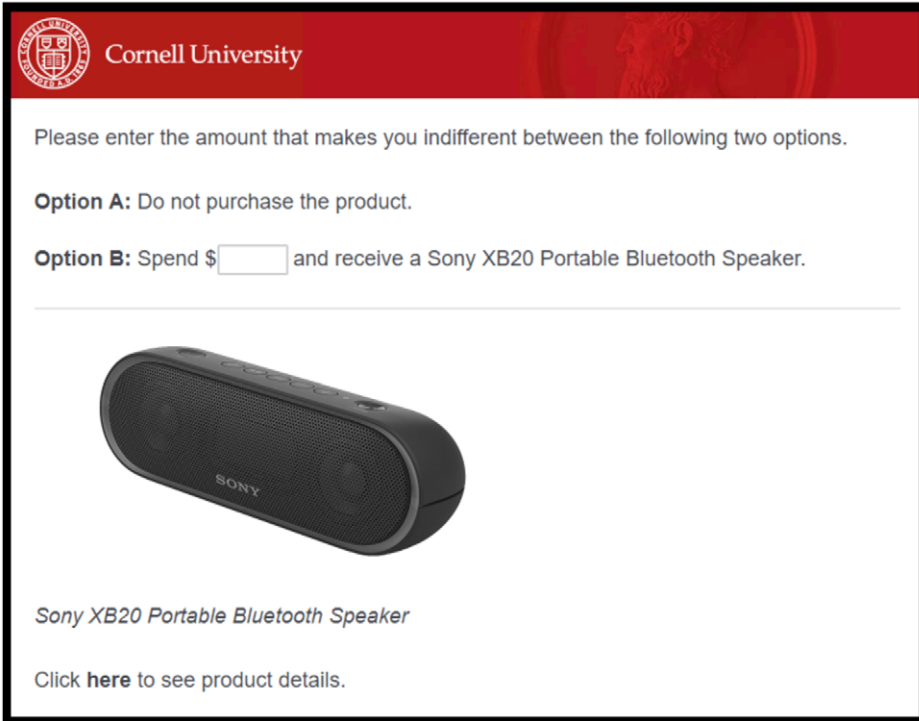


FIGURE B.1.—Example screenshot of the first-stage valuation task. Participants could enter any positive amount and could reveal product details by clicking on the bold text “here”.



FIGURE B.2.—Example screenshot of the second-stage choice task. Participants decided whether to purchase a given product by responding “yes” or “no”. Prices start at \$0 and increase in increments of \$5 up to a maximum of \$100.

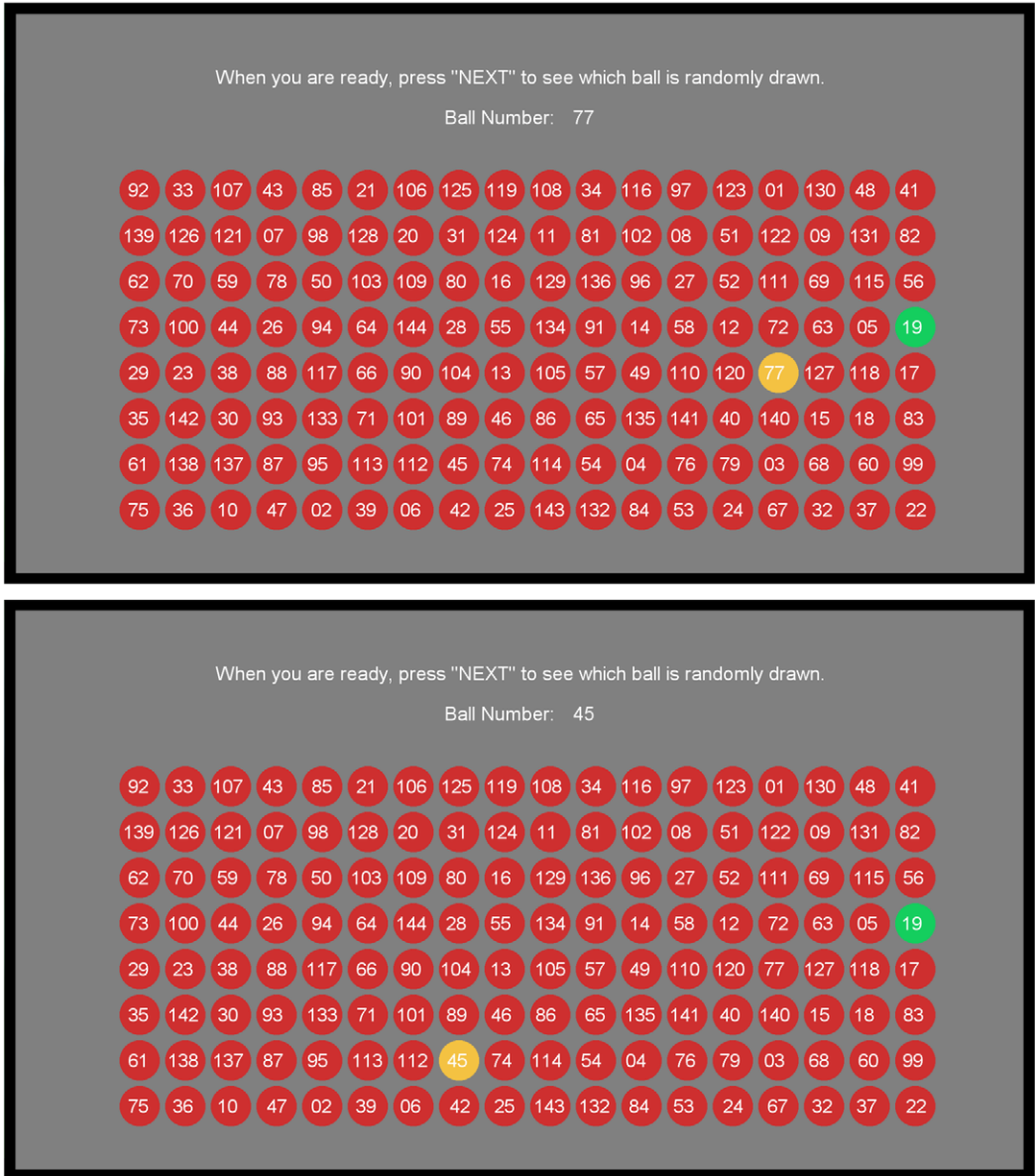


FIGURE B.3.—Example screenshots of the incentive animation. The ball highlighted in green is the number chosen by the participant. In the second-stage choice task, participants can choose one number (i.e., ball to turn green). During the second-stage valuation task incentive, nine numbers can be chosen. The yellow ball is the randomly chosen number. A new randomly chosen number is drawn every 0.01 seconds and highlighted in yellow. The animation stops when the participant hits the space key (labeled “NEXT”). If the animation stops on the participant’s chosen number, (19 in this example), the participant wins a bonus cash prize of \$80 and their previous choice is implemented for real.

B.3. Additional Price Generation Details

This section provides the precise method used for drawing prices in the experiment. In the h -decoy region (defined by the set A in Section A), prices were drawn from zone

$m = 1, 2, 3, 4, 5, 6$ by first drawing some $p_h > q_h$ (subject to the restrictions outlined in the main text), and then p_l was drawn at random from the interval bounded by the indifference boundaries for $\bar{\gamma}_m$ and $\bar{\gamma}_{m+1}$; more precisely, zone m in h -decoy region was defined as $z_m^A = \{(p_h, p_l) \in A : p_l \in (q_l(q_h/p_h)^{\bar{\gamma}_m}, q_l(q_h/p_h)^{\bar{\gamma}_{m+1}})\}$. In the l -decoy region (defined by the set B in Section A), prices from zone m were obtained by first drawing some $p_l > q_l$, and then randomly drawing p_h from the interval generated by indifference boundaries $\bar{\gamma}_m$ and $\bar{\gamma}_{m+1}$; specifically, zone m in the l -decoy region was defined as $z_m^B = \{(p_h, p_l) \in B : p_h \in (q_h(q_h/p_l)^{\bar{\gamma}_m}, q_h(q_h/p_l)^{\bar{\gamma}_{m+1}})\}$. In the o -decoy region (defined by the set C in Section A), prices were drawn from zone m by first drawing some $p_l < q_l$, and then the price of h was drawn such that $p_h > \underline{p}_h^*$ and $p_h < \bar{p}_h^*$, where \underline{p}_h^* is the solution to $(q_h/\underline{p}_h^*)^{\bar{\gamma}_m} = (\underline{p}_h^* - p_l)/(q_h - q_l)$ and \bar{p}_h^* is the solution to $(q_h/\bar{p}_h^*)^{\bar{\gamma}_{m+1}} = (\bar{p}_h^* - p_l)/(q_h - q_l)$; hence, $z_m^C = \{(p_h, p_l) \in C : p_h \in (\underline{p}_h^*, \bar{p}_h^*)\}$.

B.4. Instructions to Participants

Stage 1: Valuation Task Instructions. The next few screens will provide detailed instructions about what this survey entails. It is very important that you understand how it works, so please read all instructions carefully. Not only will this allow us to collect more precise data, it will also help you to maximize your payment for participating.

In this survey you will view many goods, one at a time. Your task is to enter the amount (in \$) that makes you indifferent between purchasing the good and not purchasing the good. There is no deception or “tricks” in this study; we are interested in your preferences for a variety of goods and your payment will be maximized when you respond truthfully and accurately. At your in-person session at LEEDR, you will have the opportunity to win a bonus prize of \$80, in addition to your fixed payment of \$20. On average, one participant in every five will win this prize and if you are a winner, one of your decisions from this survey may be drawn at random to be implemented for real (around a 20% chance). When a trial is implemented you may end up purchasing a product based on the value you entered. If your value is above the randomly drawn market price, you will receive the product and have that market price deducted from your total payment. Since you don’t know which decision will be selected, you should treat each one as if it was real. The next few screens will explain to you why it is always in your best interest to enter the true maximum value that you are willing to pay for each product.

Why is it in your interest to enter your true maximum value for the product? You might think that your best strategy is to enter an amount less than the item is worth to you. This is INCORRECT. The price you pay (i.e., the market price) is determined by a random number generator, and NOT by the price you enter. Thus, if you enter a price less than your true value, you would not be able to affect the price that you pay, but might end up losing the opportunity to buy the item at a “good” price.

As an example, suppose the “product” is simply a \$10 bill. If you enter your true maximum willingness-to-pay (i.e., \$10), you will receive the good whenever the randomly drawn market price is less than \$10, and pay that price. For example, if the randomly drawn market price is \$4, you will buy a \$10 bill for only \$4. If the randomly drawn market price is above \$10, you will not buy the \$10 bill. It follows that by bidding your maximum buying price, you make a “profit” since you always end up paying less than the item is worth to you. You should never enter more than your maximum willingness-to-pay, as you may end up paying more than \$10 for a \$10 bill. What happens if you enter a price less than your true value? For example, suppose you say you are only willing to pay \$1 for a \$10 bill and the randomly drawn market price is \$5. As the amount you entered is

less than the randomly drawn market price, you would miss out on the opportunity to purchase the \$10 bill for only \$5! Your maximum buying price for the products that you will see next will not always be as obvious as it is for a \$10 bill. However, the logic is the exact same—you will maximize your payout for participating when you enter your true valuation for the product as precisely as possible.

Example 1: Let's go through a hypothetical example. Suppose that you stated that your maximum buying price for a golden egg was \$50. If the randomly drawn market price is \$14, then what outcome would occur?

- \$0 will be deducted from your payment and you will receive no golden eggs.
- \$14 will be deducted from your balance and you will receive a golden egg.
- \$64 will be deducted from your balance and you will receive a golden egg.
- \$50 will be deducted from your balance and you will receive a golden egg.
- \$14 will be deducted from your balance and you will receive no golden eggs.

That's correct! As the randomly drawn price of the golden egg is below your maximum buying price, you will purchase it for the market price of \$14.

Example 2: Let's go through another hypothetical example. Suppose that you stated that your maximum buying price for a silver egg was \$25. If the randomly drawn price was \$32, then what outcome would occur?

- \$0 will be deducted from your payment and you will receive no silver eggs.
- \$32 will be deducted from your balance and you will receive a silver egg.
- \$25 will be deducted from your balance and you will receive no silver eggs.
- \$25 will be deducted from your balance and you will receive a silver egg.
- \$32 will be deducted from your balance and you will receive no silver eggs.

That's correct! As the randomly drawn price of the silver egg is above your maximum buying price, you will not purchase it and \$0 will be deducted from your payment. Thank you for your attention so far. When you are ready to begin the survey, click "Next".

Stage 2: Choice Task Instructions. Welcome to the LEEDR Lab and thank you for agreeing to take part in this study. Your base payment for participating is \$20 and you will have several opportunities to win an additional \$80. Around one in every five participants will win this bonus payment. There are two parts to today's session—phase 1 and phase 2—and each should take around 20 minutes to complete. There is no deception or "tricks" involved; we are interested in your preferences and your payment will be maximized when you respond truthfully and accurately. The next few screens will provide detailed instructions about phase 1. It is very important that you understand how it works and how your payment will be calculated, so please read all instructions carefully. Not only will this allow us to collect more precise data, it will also help to maximize your payment for participating. If anything is unclear at any point, raise your hand and a lab assistant will be right over to help clarify.

In phase 1, three options will be displayed onscreen and your job is simply to choose the one you like the most. Each screen will feature two kinds of the same product, with prices displayed on an attached tag, along with the option of not buying either. You will see many choices like this, one after another, and each time you will be asked to choose your preferred option using the keys labeled "A", "B", and "C" on the keyboard in front of you. In total, you will make 18 choices in phase 1, with each differing in product assortment and prices.

Each choice that you make today might be implemented for real, so it is important that you treat each decision as if you are actually purchasing one of the options. After each of the 18 choices that you make, you will be asked to choose a number between 1 and 144.

You will then draw a number at random and if the number you chose comes up, then the following will occur:

- 1) You will win a bonus payment of \$80.
- 2) You will purchase whichever option you just chose at the posted price.

If you do purchase a product, the buying price will be deducted from your final payment (i.e., \$20 plus bonuses). Purchased products will be delivered electronically where possible or shipped to a location of your choosing. Note that the price you see will never exceed your payment, so you will always leave with a positive amount of money. All purchases are final, so please consider each decision carefully. Finally, it is important to remember that each decision is independent; your choice and the outcome of one decision have no bearing on future options or outcomes. Everything “resets” after each decision, so try not to think about your previous decisions when you are faced with a new choice.

Thank you for your attention so far. To summarize:

- (i) Respond each time as if your choice will be implemented for real.
- (ii) Treat each decision as if it were the only choice you make today.
- (iii) You will leave with a positive amount of money (and possibly a product).

When you are ready to begin, press “NEXT” to proceed.

Stage 2: Valuation Task Instructions. In phase 2 you will view several goods, one at a time. Each time your task is simply to decide whether or not you would like to purchase the displayed good at the posted price. Prices will start at zero (i.e., free!) and begin to increase. Once the price gets sufficiently high, you can switch your preference from “buy” to “don’t buy”. Remember, there is no deception or “tricks” in this study; we are interested in your preferences for a variety of goods and your payment will be maximized when you respond truthfully and accurately. If anything is unclear, raise your hand and a lab assistant will be right over to help clarify.

You may end up purchasing a product based on the decisions that you’re about to make, so it is important that you treat each one as if it were real. At the end of phase 2, you will have another opportunity to win a bonus prize of \$80. You will be able to select 9 numbers (i.e., green balls) between 1 and 144, and if any of your numbers are drawn, the following will occur:

- 1) You will win a bonus payment of \$80.
- 2) A decision from phase 2 or your online survey will drawn at random and implemented for real.

If a decision from phase 2 is chosen to be implemented, the price will be determined by a random draw. Crucially, since you don’t know what price will be selected, treat each buying decision for a given good as if it were the only one you make. The prices that you will see next have been pre-selected and, therefore, your answers will have no impact on the prices you will see. Put differently, it is always in your best interest to respond truthfully because it will maximize your payment for participating. If you end up purchasing a product, the buying price will be deducted from your final payment (i.e., \$20 plus bonuses). All products will be delivered electronically where possible or shipped to a location of your choosing. Note that the price you see will never exceed your payment, so you will always leave with a positive amount of money. All purchases are final, so please consider each decision carefully.

Thank you for your attention so far. To summarize:

- (i) Respond each time as if your choice will be implemented for real.
- (ii) Treat each decision as if it were the only choice you make today.
- (iii) You will leave with a positive amount of money (and possibly a product).

When you are ready to begin, press “NEXT” to proceed.

APPENDIX C: ADDITIONAL ANALYSIS AND ROBUSTNESS

C.1. *Heterogeneous-Agent Structural Estimates*

This section takes a parametric approach to estimating individual heterogeneity in the range-weighting parameter γ . With 18 observations per participant, it is possible to estimate the model separately for each individual in the sample—thus estimating a (γ_i, λ_i) for each i .¹ The gray bars in Figure C.1 plot the distribution of the point estimates for both γ_i and λ_i . The majority of the γ_i point estimates are consistent with relative thinking: 75.3% of point estimates are negative. However, the results also reveal substantial heterogeneity in the magnitude of both γ and λ .

Individual-level estimates based on small samples are inherently noisy, complicating inference. To obtain more precise estimates of the distributions of γ and λ , I develop a parametric model of heterogeneity. Figure C.1 suggests that both parameters follow an approximately log-normal distribution. Therefore, let

$$\log(\gamma_i + \kappa) = \mathbf{x}_i \boldsymbol{\beta}_\gamma + \epsilon_\gamma, \quad (\text{C.1})$$

$$\log(\lambda_i) = \mathbf{x}_i \boldsymbol{\beta}_\lambda + \epsilon_\lambda, \quad (\text{C.2})$$

where $(\epsilon_\lambda, \epsilon_\gamma)$ are independent normally distributed errors with mean zero, \mathbf{x}_i is a vector of individual-specific demographic controls, and κ is a shift parameter that allows γ_i to take negative values while preserving the log-normal structure. Gender, age, age-squared, educational attainment, income bracket, and marital status are all included in \mathbf{x}_i . Rather than estimating two parameters per individual (300 in total), this approach estimates individual-specific means as a function of demographics, $\mu_{\gamma,i} \equiv \mathbf{x}_i \boldsymbol{\beta}_\gamma$ and $\mu_{\lambda,i} \equiv \mathbf{x}_i \boldsymbol{\beta}_\lambda$, along with variances indexed by σ_γ and σ_λ . I estimate the distribution parameters, $\theta = (\boldsymbol{\beta}_\gamma, \boldsymbol{\beta}_\lambda, \sigma_\gamma, \sigma_\lambda, \kappa)$, using simulated maximum likelihood (Train (2009)).

Table C.I presents the heterogeneous-agent model results. The main statistics of interest are the expected value and standard deviation of γ_i and λ_i . Column (1) presents the

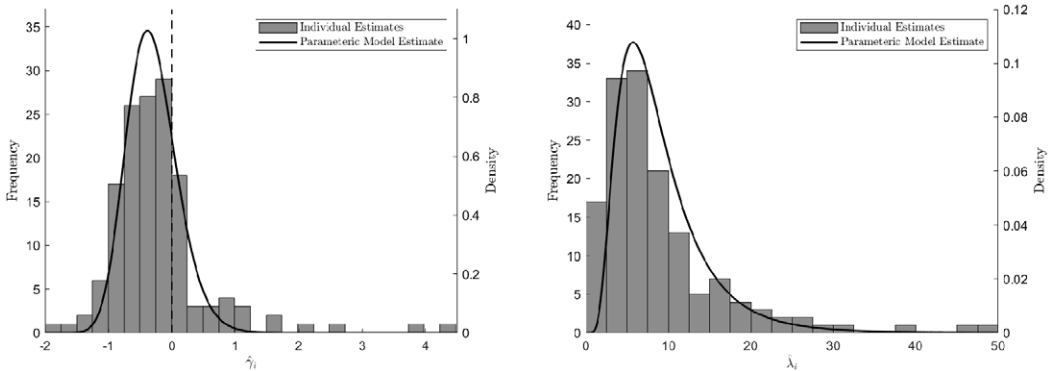


FIGURE C.1.—Distribution of individual-level estimates. Each (γ_i, λ_i) is estimated from equation (1) using 18 choice observations per person. The attribute weights are normalized to sum to 2. The solid black lines depict the implied distributions of γ_i and λ_i based on the parametric model estimates in column (3) of Table C.I.

¹The estimation was unable to produce estimates for four participants. Choices for these individuals could not be rationalized by any γ values, which resulted in λ estimates indicating random choice.

TABLE C.I
HETEROGENEOUS MODEL ESTIMATES.

	(1)	(2)	(3)	(4)
<i>Range Weighting: γ</i>				
Mean Expected Value	0	0	-0.31	-0.31
	-	-	[-0.39, -0.22]	[-0.40, -0.21]
Mean Standard Deviation	0	0	0.40	0.37
	-	-	[0.30, 0.51]	[0.26, 0.47]
<i>Scale: λ</i>				
Mean Expected Value	12.98	13.03	9.07	9.05
	[11.31, 15.09]	[11.21, 15.30]	[7.90, 10.31]	[7.95, 11.68]
Mean Standard Deviation	8.18	7.49	5.46	4.69
	[5.46, 11.50]	[4.40, 10.01]	[3.67, 7.56]	[2.75, 8.87]
Ranging Weighting	No	No	Yes	Yes
Demographics	No	Yes	No	Yes
# Obs.	2700	2700	2700	2700
# Ind.	150	150	150	150
$SLL(\hat{\theta})$	-2175	-2168	-2070	-2057
Comparison Model	-	(1)	(1)	(3)
LR Statistic	-	13.86	209.9	26.8
p-value	-	0.127	<0.001	0.061

Note: Individual-cluster-robust bootstrapped 95% confidence intervals in brackets. The attribute-weighting function is defined by $g(\Delta x; \gamma_i) = (\Delta x)^{\gamma_i}$ for attribute $x \in \{q, p\}$. λ_i is the scale parameter of the type-I extreme-value error for individual i . Heterogeneity is modeled by $\log(\gamma_i + \kappa) \sim N(\mathbf{x}_i \boldsymbol{\beta}_\gamma, \sigma_\gamma)$ and $\log(\lambda_i) \sim LN(\mathbf{x}_i \boldsymbol{\beta}_\lambda, \sigma_\lambda)$. Demographic variables included in \mathbf{x}_i are gender, age, age-squared, educational attainment, income bracket, and marital status.

estimates of the surplus-maximizing benchmark, $\gamma_i = 0$ for all i . The scale-parameter estimates imply large and variable choice noise relative to the average surpluses on offer. Column (2) includes demographics as mean shifters as described by equation (C.2). The parameter estimates are quantitatively similar and there is no significant improvement in model fit based on a log-likelihood ratio test.

Column (3) allows for range-dependent attribute weighting as indexed by γ_i , but restricts the model to a constant mean: $\mu_{\gamma,i} = \mu_\gamma$ and $\mu_{\lambda,i} = \mu_\lambda$ for all i . The estimated expected value of γ is -0.31, which is close to the homogeneous model estimate. The standard deviation estimate of 0.40 indicates substantial heterogeneity relative to the mean. For example, the estimates imply that surplus maximization lies 0.8 standard deviations above the mean and the limiting case of relative thinking ($\gamma = -1$) is 1.7 standard deviations below the mean. Allowing γ_i and λ_i to follow a mixture distribution does not yield a significant improvement in fit ($p > 0.05$). This supports individual-level results showing there is a single, relative-thinking type that differs in the intensity but not the direction of range weighting.

Column (4) adds the full set of demographic variables to the estimation, hence estimating equations (C.1) and (C.2). Including observable characteristics does not alter the expected-value estimates, but does yield smaller standard deviations. There is a modest improvement in model fit based on a likelihood-ratio test, though it is not statistically significant at the conventional level. The solid black lines in Figure C.1 illustrate the implied population distributions of γ and λ using the preferred specification in column (3) of Table C.I. Overall, the estimated distributions closely mirror the individual estimates.

C.2. Additional Tests of Price Signaling

In this section, I describe additional analyses to assess whether price signaling can explain the experimental results. First, I consider whether the results in Section 5.1 are sensitive to the definition of the price-signal index, s_{ig} . Rather than defining an index, I restrict the analysis to situations in which participants observed opposing price signals between their first and second encounters with a product. For example, do participants who first observe low (*o*-decoy) prices make choices consistent with relative thinking when they later experience higher (*h*-decoy) prices? This approach does not rely on any assumptions about how prices signaled quality but comes at the expense of statistical power.

Table C.II presents the results. In panel (a), I restrict the sample to participants who first experienced *o*-decoy prices and then made a choice at *h*-decoy prices. Consistent with relative thinking, higher prices of the decoy option led to an increase in the probability of choosing *l* and a decrease in the probability of buying *o*. Panel (b) restricts the analysis to participants who saw *o*-decoy prices followed by *l*-decoy prices for a good. The estimates are again consistent with relative thinking: higher decoy prices increased the probability of buying *h* and decreased the probability of not buying. Finally, panel (c) analyzes the choice behavior of participants who first saw *h*- or *l*-decoy prices and then made a choice at *o*-decoy prices. The results confirm that, despite observing conflicting price signals, parallel price increases caused demand to shift from *l* to *h*.

Next, I explore whether the structural estimates are robust to controlling for price signaling quality. As described in Section 5.1, relative thinking and price signaling make different predictions when participants make more than one choice per good. The panel structure of the data therefore allows me to separately identify these two forces. Formally, I estimate equations (2) and (3). In the estimation, I set initial quality equal to the first-stage reservation values. Column (1) of Table C.III reproduces the baseline estimates for $\alpha_h = \alpha_l = 0$ —see column (2) of Table C.VI. Column (2) allows price to signal quality, but restricts the model to no range weighting ($\gamma = 0$). The estimates are consistent with

TABLE C.II
CHOICE PROBABILITY ESTIMATES WITH CONFLICTING PRICE SIGNALS.

Dependent Variable:	(1) Buy <i>high</i>	(2) Buy <i>low</i>	(3) Don't Buy
	Panel (a): <i>h</i> -decoy region <i>Previous price region: o-decoy</i>		
Price of <i>h</i>	-0.19 (0.11)	0.55 (0.18)	-0.38 (0.21)
	Panel (b): <i>l</i> -decoy region <i>Previous price region: o-decoy</i>		
Price of <i>l</i>	0.40 (0.23)	0.13 (0.09)	-0.56 (0.23)
	Panel (c): <i>o</i> -decoy region <i>Previous price region: h- or l-decoy</i>		
Price of <i>h</i>	0.96 (0.30)	-1.21 (0.34)	0.20 (0.16)

Note: Average marginal effects derived from logistic regression estimates. The regressions either control for the price of *l* (panel a), the price of *h* (panel b), or the price difference between *h* and *l* (panel c). All numbers reported in percentages. Panel (a) contains 153 observations across 103 participants, panel (b) contains 185 observations across 113 participants, and panel (c) contains 316 observations from 138 participants. Individual-cluster-robust standard errors in parentheses.

TABLE C.III
PRICE AS A SIGNAL OF QUALITY ESTIMATES.

	(1)	(2)	(3)
$\hat{\gamma}$	-0.31 [-0.40, -0.21]		-0.24 [-0.36, -0.11]
$\hat{\alpha}_h$		0.21 [0.14, 0.29]	0.09 [-0.04, 0.19]
$\hat{\alpha}_l$		0.12 [0.03, 0.21]	0.06 [-0.04, 0.15]
$\hat{\lambda}$	11.78 [10.52, 13.33]	12.89 [11.65, 14.37]	11.89 [10.58, 13.29]
BIC	4748.3	4771.2	4761.0
Individuals	150	150	150
Observations	2700	2700	2700

Note: Individual-cluster-robust bootstrapped 95% confidence intervals in brackets. Estimation allows for price signaling quality and range-dependent attribute weighting as defined in equations (2) and (3). Column (1) restricts the estimation to $\alpha_h = \alpha_l = 0$ and column (2) restricts the estimation to $\gamma = 0$.

prices signaling quality, with a larger impact for the high-quality option relative to the low-quality option. Column (3) allows for both range weighting and price signaling quality. The results show that positive price signals had a small, statistically insignificant impact on perceived quality. Furthermore, the attribute-weighting parameter γ is robust to the inclusion of price signaling. The range-weighting specification in column (1) provides the best fit to the data as measured by BIC. A likelihood-ratio test confirms that adding price signaling yields no improvement in the model fit relative to the range-weighting-only specification ($p = 0.21$).

C.3. Modeling Lagged Price Dependency

To model the impact of the previous price ranges, I define the effective price range as

$$\tilde{\Delta}_{p,it} = \sum_{k=0}^K \phi_{ik} \Delta_{p,it-k}, \quad (\text{C.3})$$

where $\phi_{i0} = 1 - \sum_{k=0}^K \phi_{ik}$ and K is the number of lags included in the specification. Modeling price range spillovers amounts to simply replacing $\Delta_{p,t}$ by $\tilde{\Delta}_{p,t}$ in determining the attribute weights. In terms of estimation, more structure is required to estimate the parameters $\boldsymbol{\phi}_i = (\phi_{i1}, \dots, \phi_{iK})$. To model heterogeneity across individuals, let

$$\begin{bmatrix} \log(\gamma_i + \kappa) \\ \log(\lambda_i) \\ \boldsymbol{\phi}_i \end{bmatrix} \sim N \left(\begin{bmatrix} \mu_\gamma \\ \mu_\lambda \\ \boldsymbol{\mu}_\phi \end{bmatrix}, \begin{bmatrix} \sigma_\gamma & \cdot & \cdot \\ 0 & \sigma_\lambda & \cdot \\ 0 & 0 & \boldsymbol{\Sigma}_\phi \end{bmatrix} \right),$$

where $\boldsymbol{\mu}_\phi = (\mu_1, \dots, \mu_K)$ and $\boldsymbol{\Sigma}_\phi = \text{diag}(\sigma_1, \dots, \sigma_K)$. Note that $K = 0$ is equivalent to the baseline model with no lagged price dependency. Given this specification, the parameters $\theta = (\mu_\gamma, \mu_\lambda, \boldsymbol{\mu}_\phi, \sigma_\gamma, \sigma_\lambda, \boldsymbol{\Sigma}_\phi, \kappa)$ can be estimated using simulated maximum likelihood. Table C.IV reports the results for $K \in \{0, 1, 2, 3\}$.

TABLE C.IV
LAGGED-PRICE-RANGE ESTIMATES.

	(1) $K = 0$	(2) $K = 1$	(3) $K = 2$	(4) $K = 3$
Mean γ_i	-0.31 [-0.39, -0.22]	-0.32 [-0.41, -0.23]	-0.31 [-0.39, -0.10]	-0.28 [-0.38, -0.08]
SD γ_i	0.40 [0.30, 0.51]	0.44 [0.34, 0.57]	0.46 [0.33, 0.73]	0.50 [0.36, 0.71]
Mean λ_i	9.07 [7.90, 10.31]	9.20 [7.87, 10.68]	9.21 [7.84, 10.77]	9.07 [7.74, 10.78]
SD λ_i	5.46 [3.67, 7.56]	5.95 [4.07, 8.19]	6.13 [4.09, 9.01]	5.95 [3.91, 8.86]
Mean ϕ_{1i}		0.07 [0.01, 0.18]	0.06 [0.01, 0.21]	0.05 [-0.01, 0.20]
SD ϕ_{1i}		0.10 [0.00, 0.22]	0.05 [0.00, 0.27]	0.07 [0.00, 0.26]
Mean ϕ_{2i}			0.02 [-0.04, 0.17]	0.02 [-0.03, 0.14]
SD ϕ_{2i}			0.07 [0.00, 0.24]	0.00 [0.00, 0.24]
Mean ϕ_{3i}				0.03 [-0.03, 0.10]
SD ϕ_{3i}				0.01 [0.00, 0.06]
$SLL(\hat{\theta})$	-2175	-1954	-1830	-1706
Individuals	150	150	150	150
Observations	2700	2550	2400	2250

Note: Individual-cluster-robust bootstrapped 95% confidence intervals in brackets. The attribute-weighting function is $g(\Delta x; \gamma_i) = (\Delta x)^{\gamma_i}$ for attribute $x \in \{q, p\}$. The effective price range is $\tilde{\Delta}_{p,it} = \sum_{k=0}^K \phi_{ik} \Delta_{p,it-k}$, where $\phi_{i0} = 1 - \sum_{k=0}^K \phi_{ik}$ and K is the number of lags included in the specification. Attribute weights are normalized to sum to 2. λ_i is the scale parameter of an additive type-I extreme-value error for individual i . Heterogeneity is modeled by $\log(\gamma_i + \kappa) \sim N(\mu_\gamma, \sigma_\gamma)$, $\log(\lambda_i) \sim LN(\mu_\lambda, \sigma_\lambda)$, and $\phi_i \sim N(\mu_\phi, \Sigma_\phi)$.

C.4. Reservation Values

The experimental design elicited two reservation value proxies of quality for both the high- and low-quality variants of the nine products that each participant saw during the choice task. In Section 4, I used the average of these reservation values as a proxy for quality in the estimation. In this section, I explore whether there are systematic differences between these two values and estimate the optimal weighting of these two measures.

Table C.V presents the average first- and second-stage reservation values by product. The values are systematically higher in the first stage relative to the second stage. This difference could be driven by differences in methodology. The first elicitation used an open-ended free response with no upper bound. To the extent that participants may have perceived a relatively high upper bound on prices, this could have decreased price sensitivity and therefore led to inflated reservation values relative to the underlying quality. It is also possible that participants misunderstood the incentive mechanism, which could cause them to report inflated reservation values (Cason and Plott (2014)). Conversely, I elicited the second-stage reservation values using an ascending price. Research has shown that whether prices are increasing or decreasing leads to framing effects (Andersen et al. (2006)). In this case, participants might have stated lower reservation values due to the ascending format.

TABLE C.V
FIRST- VERSUS SECOND-STAGE QUALITY ELICITIONS (IN \$).

<i>Variant</i>	First Stage				Second Stage				RRP*	
	<i>high</i>	(s.d.)	<i>low</i>	(s.d.)	<i>high</i>	(s.d.)	<i>low</i>	(s.d.)	<i>high</i>	<i>low</i>
<i>Quantity</i>										
Chocolate Truffles	43.3	(22.0)	18.1	(8.9)	27.5	(18.6)	15.4	(8.0)	75.0	35.0
Cinema Passes	56.6	(23.4)	30.6	(12)	45.4	(25.5)	25.7	(14.4)	95.0	50.0
Starbucks Coffee	53.8	(22.6)	28.8	(11.6)	36.1	(21.6)	22.5	(14.2)	100.0	50.0
Gillette Razors	24.1	(14.2)	13.4	(7.7)	20.3	(9.7)	11.8	(5.8)	52.9	21.0
Insignia TV	57.9	(22.3)	39.9	(19.1)	39.7	(26.7)	28.7	(18.5)	80.0	60.0
Uber Credit	61.4	(17.3)	30.0	(8.2)	50.1	(22.6)	26.9	(11)	80.0	40.0
<i>Duration</i>										
Meal Kit	49.3	(22.3)	28.3	(12.7)	38.2	(20.3)	23.8	(14.8)	120.0	60.0
Yoga Subscription	52.4	(23.6)	29.1	(13.1)	37.5	(20.4)	22.3	(10.7)	100.0	50.0
Wine of the Month	42.9	(20.9)	23.7	(11.1)	35.8	(20.2)	21.8	(10.9)	79.9	40.0
Cheese of the Month	33.6	(18.5)	18.0	(9.5)	28.8	(18.2)	17.5	(9.8)	97.9	49.0
Beer of the Month	38.8	(16.8)	21.5	(9.0)	31.3	(15.6)	19.7	(10.7)	85.9	43.0
Flowers of the Month	42.9	(19.4)	24.4	(10.6)	36.0	(19.4)	21.2	(10.8)	97.9	49.0
<i>Functionality</i>										
Roku	50.6	(22.1)	26.6	(13.7)	34.0	(19.3)	19.2	(11.3)	85.3	50.0
Amazon Tablet	56.2	(19.4)	38.7	(16.0)	44.6	(19.5)	32.2	(16.7)	80.0	50.0
<i>Brand</i>										
Bluetooth Speaker	40.4	(19.8)	21.3	(9.6)	27.7	(18.5)	15.9	(9.8)	80.0	20.0
Water Purifier	33.9	(15.7)	23.9	(12.8)	27.9	(11.2)	18.7	(9.1)	60.0	48.0
Hardcase Luggage	59.4	(19.5)	42.0	(17.1)	41.1	(20.8)	27.6	(14.5)	75.0	50.0
Rechargeable Batteries	20.9	(8.3)	15.2	(6.3)	17.6	(9.8)	13.4	(6.9)	40.3	37.0
Laptop Backpack	47.2	(22.5)	25.1	(14.4)	35.8	(17.6)	20.9	(11.4)	78.9	30.0
Sunglasses	44.3	(22.6)	18.1	(11.3)	29.3	(16.1)	15.0	(7.4)	95.0	15.0

*RRP = Recommended Retail Price.

While the quality estimates are subject to noise and systematic differences, they both contain information about participants' quality perceptions. Building on the intuition in (Gillen, Snowberg, and Yariv (2019)), I estimate the optimal combination of these measures to obtain a more precise estimate of quality. I define quality as $q_{ijt} = \tau q_{ijt}^I + (1 - \tau)q_{ijt}^II$, where q_{ijt}^I is the first reservation value and q_{ijt}^II is the second. Column (1) of Table C.VI reproduces the baseline structural estimates ($\tau = 0.5$). Columns (2) and (3) report the estimates using the first reservation value ($\tau = 1$) or the second reservation value ($\tau = 0$). The γ estimate based on the first reservation values is -0.31 , which is similar in magnitude to the baseline model. Estimated range weighting is more pronounced when the second reservation values are used as proxies for quality, $\hat{\gamma} = -0.47$. Column (4) estimates an optimal τ of 0.25, which implies a 3-to-1 weight on the second reservation value relative to the first. This shift in weight towards the second reservation value leads to more pronounced range-based attribute weighting: the γ estimate of -0.42 lies close to the upper bound of the baseline estimate.

Finally, I assess whether using the first or second reservation values changes the overall pattern of results. Table C.VII presents the premiums implied by the range-weighting estimates using the first or the second reservation values. Overall, while there is some difference in the magnitude of the implied premiums, I find economically meaningful premiums ranging from 11% to 36% across the three regions.

TABLE C.VI
ALTERNATIVE QUALITY SPECIFICATION ESTIMATES.

<i>Reservation Values:</i>	(1) Average	(2) First	(3) Second	(4) Optimal
$\hat{\gamma}$	-0.34 [-0.42, -0.26]	-0.31 [-0.39, -0.21]	-0.47 [-0.54, -0.40]	-0.42 [-0.49, -0.34]
$\hat{\lambda}$	9.19 [8.18, 10.27]	11.78 [10.45, 13.21]	9.61 [8.48, 10.86]	9.19 [8.19, 10.34]
τ	0.5 -	1 -	0 -	0.25 [0.15, 0.37]

Note: Individual-cluster-robust bootstrapped 95% confidence intervals in brackets. The attribute-weighting function is defined by $g(\Delta_x; \gamma) = (\Delta_x)^\gamma$ for attribute $x \in \{q, p\}$. λ is the scale parameter of the type-I extreme-value error. Quality is defined as $q_{ijt} = \tau q_{ijt}^I + (1 - \tau)q_{ijt}^H$. Columns (1), (2), and (3) provide the estimates based on fixed values of τ . Column (4) estimates the optimal τ . Observations: 2,700. Individuals: 150.

TABLE C.VII
QUANTIFYING RANGE-DEPENDENT ATTRIBUTE WEIGHTING: FIRST-STAGE RESERVATION VALUES.

	(1) Surplus Max	(2) Lower Bound	(3) Estimate	(4) Upper Bound
Panel A: First-Stage Reservation Values:				
Range weighting: $\gamma =$	0	-0.39	-0.31	-0.21
Quality: $(q_h, q_l) = (\\$46, \\$26)$				
<i>h</i> -decoy region ($p_h = \$66$)				
Willingness-to-pay for <i>l</i>	\$26.0	\$29.9	\$29.1	\$28.0
% premium vs. surplus max	-	15.2%	11.8%	7.9%
<i>l</i> -decoy region ($p_l = \$65$)				
Willingness-to-pay for <i>h</i>	\$46.0	\$52.6	\$51.2	\$49.5
% premium vs. surplus max	-	14.4%	11.3%	7.5%
<i>o</i> -decoy region ($p_h = \$30$)				
Willingness-to-pay for <i>l</i>	\$10.0	\$13.1	\$12.5	\$11.7
% premium vs. surplus max	-	30.7%	24.8%	17.2%
Panel B: Second-Stage Reservation Values:				
Range weighting: $\gamma =$	0	-0.54	-0.47	-0.40
Quality: $(q_h, q_l) = (\\$35, \\$21)$				
<i>h</i> -decoy region ($p_h = \$58$)				
Willingness-to-pay for <i>l</i>	\$21.0	\$27.6	\$26.6	\$25.7
% premium vs. surplus max	-	31.4%	26.8%	22.4%
<i>l</i> -decoy region ($p_l = \$67$)				
Willingness-to-pay for <i>h</i>	\$35.0	\$49.7	\$47.5	\$45.2
% premium vs. surplus max	-	21.6%	35.7%	29.7%
<i>o</i> -decoy region ($p_h = \$26$)				
Willingness-to-pay for <i>l</i>	\$12.0	\$14.1	\$13.8	\$13.6
% premium vs. surplus max	-	17.3%	15.2%	13.1%

Note: The price in each region is set to the average realized prices in each decoy region, and each region is defined using the specified reservation values. The reported premiums are measured in percentage terms relative to the surplus maximization amount in column (1). The γ values are taken from columns (2) and (3) of Table C.VI. Willingness-to-pay is (i) $WTP(\gamma) = q_l(q_h/p_h)^\gamma$ in the *h*-decoy region, (ii) $WTP(\gamma) = q_h(q_h/p_l)^\gamma$ in the *l*-decoy region, and (iii) defined implicitly by $(q_h/p_h)^\gamma = (p_h - WTP(\gamma))/(q_h - q_l)$ in the *o*-decoy region.

REFERENCES

- ANDERSEN, S., G. W. HARRISON, M. I. LAU, AND E. E. RUTSTRÖM (2006): “Elicitation Using Multiple Price List Formats,” *Experimental Economics*, 9 (4), 383–405. [17]
- CASON, T. N., AND C. R. PLOTT (2014): “Misconceptions and Game Form Recognition: Challenges to Theories of Revealed Preference and Framing,” *Journal of Political Economy*, 122 (6), 1235–1270. [17]
- GILLEN, B., E. SNOWBERG, AND L. YARIV (2019): “Experimenting With Measurement Error: Techniques With Applications to the Caltech Cohort Study,” *Journal of Political Economy*, 127 (4), 1826–1863. [18]
- TRAIN, K. E. (2009): *Discrete Choice Methods With Simulation*. Cambridge University Press. [13]

Co-editor Alessandro Lizzeri handled this manuscript.

Manuscript received 30 April, 2020; final version accepted 2 August, 2021; available online 5 August, 2021.