Econometrica Supplementary Material

SUPPLEMENT TO "PRECAUTIONARY SAVINGS, ILLIQUID ASSETS, AND THE AGGREGATE CONSEQUENCES OF SHOCKS TO HOUSEHOLD INCOME RISK: APPENDICES"

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CHRISTIAN BAYER
Dept. of Economics, Universität Bonn

RALPH LUETTICKE
Dept. of Economics, University College London

LIEN PHAM-DAO
Research Data and Service Centre, Deutsche Bundesbank

VOLKER TJADEN
Blue Yonder GmbH

APPENDIX A: PROOF OF PROPOSITION 1

IN THE FOLLOWING APPENDIX, we prove Proposition 1. For this purpose, we first establish the following two lemmas.

LEMMA 1: *In the high income state in period* 2, *the borrowing constraint never binds if* $R^k < (1 + R^{k\frac{\xi-1}{\xi}})^\xi$.

PROOF: Suppose the opposite. Then also in the low income state the constraint must bind, thus $c_3^{H,L} = R^k k_1$ and $c_2^H = y + b_1 + \sigma$. Optimality of $b_2 = 0$ requires $c_2^H < R^k k_1$. At the same time, optimality in period 1 requires $u'(c_1) = R^k u'(c_3)$ (there is no uncertainty regarding period 3 consumption) and thus $c_3 = R^{k^{1/\xi}} c_1$, implying $c_1 = \frac{y-b_1}{1+R^{k\frac{1-\xi}{\xi}}}$ and $c_3 = (y - b_1)\frac{R^{k^{1/\xi}}}{1+R^{k\frac{1-\xi}{\xi}}}$. However, then $c_2^H > c_3$, as $R^{k^{1/\xi}} < 1 + (R^k)^{\frac{\xi-1}{\xi}}$ by assumption. This contradicts optimality in period 2. *Q.E.D.*

Christian Bayer: christian.bayer@uni-bonn.de
Ralph Luetticke: r.luetticke@ucl.ac.uk
Lien Pham-Dao: lien.pham-dao@bundesbank.de
Volker Tjaden: volker.tjaden@mailbox.org

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LEMMA 2: *If the household holds positive amounts of both assets in period* 1 *and* $R^k \neq 1$, *then the borrowing constraint binds in period* 2 *in the low income state*.

PROOF: As the household holds positive amounts of both assets, it is not borrowing constrained in period 1, and hence $u'(c_1) = Eu'(c_2)$ and $u'(c_1) = R^k Eu'(c_3)$ from the optimality in period 1. Optimality in period 2 in the absence of binding borrowing constraints implies $c_2^{H,L} = c_3^{H,L}$, such that $Eu'(c_3) = R^k Eu'(c_3)$, which contradicts $R^k \neq 1$. *Q.E.D.*

We can use these lemmas to prove the actual proposition, which is repeated for the reader's convenience.

PROPOSITION 1: *Define* $b_1^*(\sigma)$ *and* $k_1^*(\sigma)$, *the optimal liquid and illiquid asset holdings*. *Define* $\tilde{b}_1(\sigma)$, *the liquid asset holdings of a household that does not have the option to invest in an illiquid asset*. *Now suppose income uncertainty is large enough such that* $b_1^*(\sigma) > 0$ *and the returns on the illiquid investment are larger than* 1, $R^k > 1$, *but not too large*, *that is*, $1 < R^k < (1 + R^{k\frac{\xi-1}{\xi}})^{\xi}$. *Then* $\frac{\partial b_1^*}{\partial \sigma} > \frac{\partial \tilde{b}_1}{\partial \sigma} > 0 > \frac{\partial k_1^*}{\partial \sigma}$, *that is*, *liquid asset holdings increase in* σ *and they increase more than in a model where all assets are liquid*, *while illiquid asset holdings decrease*.

PROOF: The optimal liquid and illiquid asset holdings are determined by the two Euler equations in period 1. From Lemmas 1 and 2, we know that in period 2, the household will not be borrowing constrained in the high income state, but it will be constrained in the low income state. Moreover, we also know that then $k_1 > 0$ follows as marginal utility diverges to infinity for $c \to 0$. Therefore, the two Euler equations for b_1 and k_1 read

$$u'(y - b_1^* - k_1^*) - \frac{1}{2}\left[u'\left(\frac{y + b_1^* + \sigma + R^k k_1^*}{2}\right) + u'(y + b_1^* - \sigma)\right] = 0, \tag{S1}$$

$$u'(y - b_1^* - k_1^*) - R^k \frac{1}{2}\left[u'\left(\frac{y + b_1^* + \sigma + R^k k_1^*}{2}\right) + u'(R^k k_1^*)\right] = 0. \tag{S2}$$

These Euler equations define $k_1^*(\sigma)$ and $b_1^*(\sigma)$ as implicit functions of σ.

Removing the option to invest in the illiquid asset, the household is never borrowing constrained in period 2 and the demand for liquid assets $\tilde{b}_1(\sigma)$ is given by the Euler equation

$$u'(y - \tilde{b}_1) - \frac{1}{2}\left[u'\left(\frac{y + \tilde{b}_1 + \sigma}{2}\right) + u'\left(\frac{y + \tilde{b}_1 - \sigma}{2}\right)\right] = 0. \tag{S3}$$

We can now use the implicit function theorem to calculate how asset demand changes in σ. This yields

$$\begin{pmatrix} \dfrac{\partial b_1^*}{\partial \sigma} \\ \dfrac{\partial k_1^*}{\partial \sigma} \end{pmatrix} = \frac{1}{4}\begin{pmatrix} A_1 & A_2 \\ A_2 & A_3 \end{pmatrix}^{-1} \begin{pmatrix} 2u''_{2L} - u''_{2H} \\ -R^k u''_{2H} \end{pmatrix}$$

with

$$A_1 := u''_1 + 1/4(u''_{2H} + 2u''_{2L}) < 0,$$

$$A_2 := u_1'' + 1/4R^k u_{2H}'' < 0,$$
$$A_3 := u_1'' + 1/4R^{k2}(u_{2H}'' + u_{3L}'') < 0,$$

and

$$u_{2L}'' := u''(c_2^L) < u_{3L}'' := u''(c_3^L) < u_1'' := u''(c_1) < u_{2H}'' := u''(c_2^H) < 0,$$

and thus

$$\begin{pmatrix} \frac{\partial b_1^*}{\partial \sigma} \\ \frac{\partial k_1^*}{\partial \sigma} \end{pmatrix} = \frac{1}{4(A_1A_3 - A_2^2)} \begin{pmatrix} A_3 & -A_2 \\ -A_2 & A_1 \end{pmatrix} \begin{pmatrix} 2u_{2L}'' - u_{2H}'' \\ -R^k u_{2H}'' \end{pmatrix}$$
$$= \frac{1}{4(A_1A_3 - A_2^2)} \begin{pmatrix} A_3(2u_{2L}'' - u_{2H}'') + R^k A_2 u_{2H}'' \\ -A_2(2u_{2L}'' - u_{2H}'') - R^k A_1 u_{2H}'' \end{pmatrix}.$$

In particular, making use of $u_{2H}'' < u_{3L}'' < u_{2L}'' < 0$, we obtain that the numerator

$$A_1A_3 - A_2^2 > (u_1'' + 3/4u_{2H}'')(u_1'' + 1/2R_k^2 u_{2H}'') - (u_1'' + 1/4R_k u_{2H}'')$$
$$= \frac{5}{16}R_k^2(u_{2H}'')^2 + \frac{3+2R_k^2}{4}u_1''(u_{2H}'') - \frac{2}{4}R_k u_1''(u_{2H}'') > 0$$

is positive. This implies, as $A_{1,2,3}$, $2u_{2L}'' - u_{2H}''$, and u_{2H}'' are all negative,

$$\frac{\partial b_1^*}{\partial \sigma} > 0 > \frac{\partial k_1^*}{\partial \sigma}.$$

Moreover, we can estimate a lower bound on $\frac{\partial b_1^*}{\partial \sigma}$ as

$$\frac{\partial b_1^*}{\partial \sigma} = \frac{1}{4}\frac{A_3}{A_1A_3 - A_2^2}(2u_{2L}'' - u_{2H}'') + \frac{1}{4}\frac{A_2}{A_1A_3 - A_2^2}R^k u_{2H}''$$
$$> \frac{1}{4}\frac{A_3}{A_1A_3}(2u_{2L}'' - u_{2H}'') + \frac{1}{4}\frac{A_2}{A_1A_3 - A_2^2}R^k u_{2H}''$$
$$> \frac{\frac{1}{4}(2u_{2L}'' - u_{2H}'')}{u_1'' + \frac{1}{4}(u_{2H}'' + 2u_{2L}'')}.$$

This term, the lower bound on $\frac{\partial b_1^*}{\partial \sigma}$, has a form similar to the derivative of liquid asset demand to income risk when the household can only hold liquid assets. In that case, we obtain

$$\frac{\partial \tilde{b}_1}{\partial \sigma} = \frac{\frac{1}{4}(\tilde{u}_{2L}'' - \tilde{u}_{2H}'')}{\tilde{u}_1'' + \frac{1}{4}(\tilde{u}_{2H}'' + \tilde{u}_{2L}'')}.$$

Now define a function $U(u'(c)) = u''(c)$. This function $U = -\xi u'^{\frac{\xi+1}{\xi}}$ is negative, decreasing, and convex in marginal utility. Therefore, the Euler equation implies $0 >$

$1/2(u''_{2H} + u''_{2L}) > u''_1$ due to convexity and $u''_1 > u''_{2L}$ because U is decreasing. Analogous formulas apply for the case without illiquid assets. We can use these estimates to obtain an upper bound on $\frac{\partial \tilde{b}_1}{\partial \sigma}$ and to simplify the lower bound on $\frac{\partial b_1^*}{\partial \sigma}$:

$$
\begin{aligned}
\frac{\partial \tilde{b}_1}{\partial \sigma} &= \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{4\tilde{u}''_1 + (\tilde{u}''_{2H} + \tilde{u}''_{2L})} \le \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{3(\tilde{u}''_{2H} + \tilde{u}''_{2L})} = \frac{\tilde{u}''_{2L} - \tilde{u}''_{2H}}{6\tilde{u}''_{2H} + 3(\tilde{u}''_{2L} - \tilde{u}''_{2H})} \\
&= \frac{1}{3 + 6\dfrac{\tilde{u}''_{2H}}{\tilde{u}''_{2L} - \tilde{u}''_{2H}}}, \\
\frac{\partial b_1^*}{\partial \sigma} &> \frac{2u''_{2L} - u''_{2H}}{4u''_1 + (u''_{2H} + 2u''_{2L})} \ge \frac{2u''_{2L} - u''_{2H}}{6u''_{2L} + u''_{2H}} = \frac{2u''_{2L} - u''_{2H}}{3(2u''_{2L} - u''_{2H}) + 4u''_{2H}} \\
&= \frac{1}{3 + 4\dfrac{u''_{2H}}{2u''_{2L} - u''_{2H}}},
\end{aligned}
$$

which implies $\frac{\partial \tilde{b}_1}{\partial \sigma} < \frac{\partial b_1^*}{\partial \sigma}$ because $4\frac{u''_{2H}}{2u''_{2L} - u''_{2H}} < 6\frac{\tilde{u}''_{2H}}{\tilde{u}''_{2L} - \tilde{u}''_{2H}}$ as $0 > \tilde{u}''_{2L} > u''_{2L}$ and $0 > u''_{2H} > \tilde{u}''_{2H}$. This completes the proof. *Q.E.D.*

APPENDIX B: DYNAMIC PLANNING PROBLEM WITH TWO ASSETS AND FIXED ADJUSTMENT PROBABILITIES

The dynamic planning problem of a household in the model is characterized by two Bellman equations: V_a in the case where the household can adjust its capital holdings and V_n otherwise. We will first go through the problem with exogenous adjustment probabilities, as the first-order conditions of the model with adjustment decisions that describe portfolio and consumption choices turn out to be of the same structure as under given adjustment probabilities.

With fixed adjustment probabilities, the value functions are given by

$$
\begin{aligned}
&V_a(b, k, h; \Theta, R^b, s) \\
&\quad = \max_{k', b'_a \in \Gamma_a} u[x(b, b'_a, k, k', h)] \\
&\qquad + \beta[\nu E V_a(b'_a, k', h'; \Theta', R'_b, s') + (1-\nu) E V_n(b'_a, k', h'; \Theta', R'_b, s')], \\
&V_n(b, k, h; \Theta, R^b, s) \\
&\quad = \max_{b'_n \in \Gamma_n} u[x(b, b'_n, k, k, h)] \\
&\qquad + \beta[\nu E V_a(b'_n, k, h'; \Theta', R'_b, s') + (1-\nu) E V_n(b'_n, k, h'; \Theta', R'_b, s')],
\end{aligned}
\tag{S4}
$$

where the budget sets are given by

$$
\begin{aligned}
\Gamma_a(b, k, h; \Theta, R^b, s) = \Bigg\{ & k' \ge 0, b' \ge \underline{B} \mid q(k' - k) + b' \le rk + \frac{R^b}{\pi} b \\
& + (1-\tau)\left(\frac{\gamma}{1+\gamma} whN + \mathbb{I}_{h=0}\Pi\right)\Bigg\},
\end{aligned}
$$

$$\Gamma_n\big(b, k, h; \Theta, R^b, s\big) = \bigg\{ b' \geq \underline{B} \mid b' \leq rk + \frac{R^b}{\pi} b + (1-\tau)\left(\frac{\gamma}{1+\gamma} whN + \mathbb{I}_{h=0}\Pi\right)\bigg\},$$

$$x\big(b, b', k, k', h; \Theta, R^b, s\big) = \frac{\gamma}{1+\gamma} whN + rk + \frac{R^b}{\pi} b - q\big(k'-k\big) - b',$$

where q, π, and Π are functions of (Θ, R^b, s).

To save on notation, let Ω be the set of idiosyncratic state variables controlled by the household, let Z be the set of states outside the household's control, let $\Gamma_i : \Omega \to \Omega$ be the correspondence describing the feasibility constraints, and let $A_i(z) = \{(\omega, y) \in \Omega \times \Omega : y \in \Gamma_i(\omega, z)\}$ be the graph of Γ_i. Hence the states and controls of the household problem can be defined as

$$\Omega = \big\{\omega = (b, k) \in \mathbb{R}^2 : \underline{B} \leq b < \infty, 0 \leq k < \infty\big\},$$
$$z = \big\{h, \Theta, R^b, s\big\},$$

and the return function $F : A \to R$ reads as

$$F\big(\Gamma_i(\omega, z), \omega; z\big) = \frac{x_i^{1-\gamma}}{1-\gamma}.$$

Define the value before the adjustment/non-adjustment shock is realized as

$$v(\omega, z) := \nu V_a(\omega, z) + (1-\nu) V_n(\omega, z).$$

Now we can rewrite the optimization problem of the household in terms of the definitions above in a compact form:

$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega, z)} \big[F(\omega, y; z) + \beta E v\big(y, z'\big)\big], \tag{S5}$$

$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega, z)} \big[F(\omega, y; z) + \beta E v\big(y, z'\big)\big]. \tag{S6}$$

Finally we define the mapping $T : C(\Omega) \to C(\Omega)$, where $C(\Omega)$ is the space of bounded, continuous and weakly concave functions:

$$(Tv)(\omega, z) = \nu V_a(\omega, z) + (1-\nu) V_n(\omega, z),$$
$$V_a(\omega, z) = \max_{y \in \Gamma_a(\omega, z)} \big[F(\omega, y; z) + \beta E v\big(y, z'\big)\big],$$
$$V_n(\omega, z) = \max_{y \in \Gamma_n(\omega, z)} \big[F(\omega, y; z) + \beta E v\big(y, z'\big)\big].$$

B.1. *Properties of Primitives*

Abstracting from the noncontinuity in R at $b = 0$, the following properties of the primitives of the problem obviously hold.

P1: *Properties of sets* Ω, $\Gamma_a(\omega, z)$, *and* $\Gamma_n(\omega, z)$:

(i) *The set Ω is a convex subset of* $\mathbb{R}^3$.
(ii) *The correspondence* $\Gamma_i(\cdot, z) : \Omega \to \Omega$ *is non-empty, compact-valued, continuous, monotone and convex for all* z.

P2: *Properties of return function F. The return function F is bounded, continuous, strongly concave, $\mathbb{C}^2$ differentiable on the interior of A, and strictly increasing in each of its first two arguments.*

B.2. *Properties of the Value and Policy Functions*

LEMMA 3: *The mapping T defined by the Bellman equation for v fulfills Blackwell's sufficient conditions for a contraction on the set of bounded, continuous, and weakly concave functions $\mathbb{C}(\Omega)$.*
(a) *It satisfies discounting.*
(b) *It is monotonic.*
(c) *It preserves boundedness (assuming an arbitrary maximum consumption level).*
(d) *It preserves strict concavity.*
Hence, the solution to the Bellman equation is strictly concave. The policy is a single-valued function in (b, k), and so is optimal consumption.

PROOF: The proof proceeds item by item and closely follows Stokey and Lucas (1989), taking into account that the household problem in the extended model consists of two Bellman equations.
(a) Discounting. Let $a \in R_+$ and let the rest be defined as above. Then it holds that

$$\begin{aligned}\big(T(v+a)\big)(\omega, z) &= \nu \max_{y\in\Gamma_a(\omega,z)}\big[F(\omega, y, z) + \beta E v\big(y, z'\big) + a\big] \\ &\quad + (1-\nu) \max_{y\in\Gamma_n(\omega,z)}\big[F(\omega, y, z) + \beta E v\big(y, z'\big) + a\big] \\ &= (Tv)(\omega, z) + \beta a.\end{aligned}$$

Accordingly, T fulfills discounting.
(b) Monotonicity. Let $g : \Omega \times Z \to R^2$, $f : \Omega \times Z \to R^2$, and $g(\omega, z) \geq f(\omega, z)\ \forall \omega, z \in \Omega \times Z$. Then it follows that

$$\begin{aligned}(Tg)(\omega, z) &= \nu \max_{y\in\Gamma_a(\omega,z)}\big[F(\omega, y, z) + \beta E g\big(y, z'\big)\big] \\ &\quad + (1-\nu) \max_{y\in\Gamma_n(\omega,z)}\big[F(\omega, y, z) + \beta E g\big(y, z'\big)\big] \\ &\geq \nu \max_{y\in\Gamma_a(\omega,z)}\big[F(\omega, y, z) + \beta E f\big(y, z'\big)\big] \\ &\quad + (1-\nu) \max_{y\in\Gamma_n(\omega,z)}\big[F(\omega, y, z) + \beta E f\big(y, z'\big)\big] \\ &= Tf(\omega, z).\end{aligned}$$

The objective function for which Tg is the maximized value is uniformly higher than the function for which Tf is the maximized value. Therefore, T preserves monotonicity.
(c) Boundedness. From properties P1, it follows that the mapping T defines a maximization problem over the continuous and bounded function $[F(\omega, y) + \beta E v(y, z'))]$ over the compact sets $\Gamma_i(\omega, z)$ for $i = \{a, n\}$. Hence the maximum is attained. Since F and v are bounded, Tv is also bounded.

(d) Strict Concavity. Let $f \in C''(\Omega)$, where C'' is the set of bounded, continuous, strictly concave functions on Ω. Since the convex combination of two strictly concave functions is strictly concave, it is sufficient to show that $T_i[C''(\Omega)] \subseteq C''(\Omega)$, where T_i is defined by

$$T_i v = \max_{y \in \Gamma_i(\omega, z)} \big[F(\omega, y, z) + \beta E v(y, z')\big], \quad i \in \{a, n\}.$$

Let $\omega_0 \neq \omega_1$, $\theta \in (0, 1)$, and $\omega_\theta = \theta\omega_0 + (1-\theta)\omega_1$. Let $y_j \in \Gamma_i(\omega_j, z)$ be the maximizer of $(T_i f)(\omega_j)$ for $j = \{0, 1\}$ and $i = \{a, n\}$, $y_\theta = \theta y_0 + (1-\theta) y_1$:

$$\begin{aligned}(T_i f)(\omega_\theta, z) &\geq \big[F(\omega_\theta, y_\theta, z) + \beta E f(y_\theta, z')\big] \\ &> \theta\big[F(\omega_0, y_0, z) + \beta E f(y_0, z')\big] + (1-\theta)\big[F(\omega_1, y_1, z) + \beta E f(y_1, z')\big] \\ &= \theta(Tf)(\omega_0, z) + (1-\theta)(Tf)(\omega_1, z).\end{aligned}$$

The first inequality follows from y_θ being feasible because of convex budget sets. The second inequality follows from the strict concavity of f. Since ω_0 and ω_1 are arbitrary, it follows that $T_i f$ is strictly concave, and since f is arbitrary, it follows that $T[C''(\Omega)] \subseteq C''(\Omega)$.

Q.E.D.

LEMMA 4: *The value function is $\mathbb{C}^2$ and the policy function $\mathbb{C}^1$ differentiable.*

PROOF: The properties of the choice set P1, of the return function P2, and the properties of the value function proven in (3) fulfill the assumptions of Santos's (1991) theorem on the differentiability of the policy function. According to the theorem, the value function is C^2 and the policy function C^1 is differentiable. Note that strong concavity of the return function holds for CRRA utility, because of the arbitrary maximum we set for consumption. *Q.E.D.*

LEMMA 5: *The total savings $S_i^* := b_i^*(\omega, z) + q(z)k_i^*(\omega, z)$ and consumption c_i^*, $i \in \{a, n\}$, are increasing in ω if $r(z)$ is positive. In the adjustment case, total savings and consumption are increasing in total resources $\mathcal{R}_a(z) = [q(z) + r(z)]k + b\frac{R(b,z)}{\pi(z)}$ for any $r(z)$.*

PROOF: Define $\tilde{v}(S, z) := \max_{\{b,k|b+q(z)k\leq S\}} Ev(b, k; z')$ and define resources in the case of no adjustment as $\mathcal{R}_n = r(z)k + b\frac{R(b,z)}{\pi(z)}$. Since v is strictly concave and increasing, so is $\tilde{v}$ by the line of the proof of Lemma 3(d). Denote $\varphi(z) = (1-\tau)(\frac{\gamma}{1+\gamma} w(z)hN + \mathbb{I}_{h=0}\Pi(z))$. Now we can (re)write the planning problem as

$$V_a(b, k; z) = \max_{S \leq \varphi(z) + \mathcal{R}_a} \left[u\left(\varphi(z) + \big[q(z) + r(z)\big]k + b\frac{R(b, z)}{\pi(z)} - S\right) + \beta\tilde{v}(S, z)\right],$$

$$V_n(b, k; z) = \max_{b' \leq \varphi(z) + \mathcal{R}_n} \left[u\left(\varphi(z) + r(z)k + b\frac{R(b, z)}{\pi(z)} - b'\right) + \beta E v(b', k; z')\right].$$

Due to differentiability, we obtain the (sufficient) first-order conditions

$$\begin{aligned}\frac{\partial u(\varphi(z) + \mathcal{R}_a - S)}{\partial c} &= \beta\frac{\partial\tilde{v}(S, z)}{\partial S}, \\ \frac{\partial u(\varphi(z) + \mathcal{R}_n - b')}{\partial c} &= \beta\frac{\partial v(b', k; z)}{\partial b'}.\end{aligned} \tag{S7}$$

Since the left-hand sides are decreasing in $\omega = (b, k)$ and increasing in S (respectively, b'), and the right-hand side is decreasing in S (respectively, b'),

$$S_i^* = \begin{cases} qk' + b' & \text{if } i = a, \\ qk + b', & \text{if } i = n, \end{cases}$$

must be increasing in ω.

Since the right-hand side of (S7) is hence decreasing in ω, so must be the left-hand side of (S7). Hence consumption must be increasing in ω. The last statement follows directly from the same proof. *Q.E.D.*

B.3. *Euler Equations*

Denote the optimal policies for consumption, for bond holdings, and for capital as x_i^*, b_i^*, and k^*, $i \in \{a, n\}$, respectively. The first-order conditions for an inner solution in the (non-) adjustment case read

$$k^*: \frac{\partial u(x_a^*)}{\partial x} q = \beta E\left[\nu \frac{\partial V_a(b_a^*, k^*; z')}{\partial k} + (1-\nu) \frac{\partial V_n(b_a^*, k^*; z')}{\partial k}\right], \tag{S8}$$

$$b_a^*: \frac{\partial u(x_a^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(b_a^*, k^*; z')}{\partial b} + (1-\nu) \frac{\partial V_n(b_a^*, k^*; z')}{\partial b}\right], \tag{S9}$$

$$b_n^*: \frac{\partial u(x_n^*)}{\partial x} = \beta E\left[\nu \frac{\partial V_a(b_n^*, k; z')}{\partial b} + (1-\nu) \frac{\partial V_n(b_n^*, k; z')}{\partial b}\right]. \tag{S10}$$

Note the subtle difference between (S9) and (S10), which lies in the different capital stocks k' versus k in the right-hand side expressions.

Differentiating the value functions with respect to k and m, we obtain

$$\begin{aligned}
\frac{\partial V_a(b, k; z)}{\partial k} &= \frac{\partial u[x_a^*(b, k; z)]}{\partial x}\big(q(z) + r(z)\big), \\
\frac{\partial V_a(b, k; z)}{\partial b} &= \frac{\partial u[x_a^*(b, k; z)]}{\partial x} \frac{R(b, z)}{\pi(z)}, \\
\frac{\partial V_n(b, k; z)}{\partial b} &= \frac{\partial u[x_n^*(b, k; z)]}{\partial x} \frac{R(b, z)}{\pi(z)}, \\
\frac{\partial V_n(b, k; z)}{\partial k} & \\
&= r(z) \frac{\partial u[x_n^*(b, k; z)]}{\partial x} \\
&\quad + \beta E\left[\nu \frac{\partial V_a[b_n^*(b, k; z), k; z']}{\partial k} + (1-\nu) \frac{\partial V_n[b_n^*(b, k; z), k; z']}{\partial k}\right] \\
&= r(z) \frac{\partial u[x_n^*(b, k; z)]}{\partial x}
\end{aligned} \tag{S11}$$

$$+\beta\nu E\frac{\partial u\{x_a^*[b_n^*(b,k;z),k;z],k;z'\}}{\partial x}\big(q(z')+r(z')\big)$$
$$+\beta(1-\nu)E\frac{\partial V_n\{[b_n^*(b,k;z),k;z],k;z'\}}{\partial k}$$

such that the marginal value of capital in non-adjustment is defined recursively.

Now we can plug the second set of equations into the first set of equations and obtain the Euler equations (in slightly shortened notation)

$$\frac{\partial u[x_a^*(b,k;z)]}{\partial x}q(z) = \beta E\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;z')]}{\partial x}\big[q(z')+r(z')\big]+(1-\nu)\frac{\partial V_n(b_a^*,k^*;z')}{\partial k}\right], \tag{S12}$$

$$\frac{\partial u[x_a^*(b,k;z)]}{\partial x} = \beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u[x_a^*(b_a^*,k^*;z')]}{\partial x}+(1-\nu)\frac{\partial u[x_n^*(b_a^*,k^*;z')]}{\partial x}\right], \tag{S13}$$

$$\frac{\partial u[x_n^*(b,k;z)]}{\partial x} = \beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u[x_a^*(b_n^*,k;z')]}{\partial x}+(1-\nu)\frac{\partial u[x_n^*(b_n^*,k;z')]}{\partial x}\right]. \tag{S14}$$

In words, when deciding between the liquid and the illiquid asset, the household compares the one-period return difference between the two assets $E\frac{R(b^*,z')}{\pi(z')}-E\frac{r(z')+q(z')}{q(z)}$, weighted with the marginal utility under adjustment and the probability of adjustment, and the difference between the return in the no adjustment case, $E\frac{R(b^*,z')}{\pi(z')}\frac{\partial u[x_n^*(b_a^*,k^*;z')]}{\partial x}$, and the marginal value of illiquid assets when not adjusting $\frac{\partial V_n(b_a^*,k^*;z')}{\partial k}$. The latter reflects both the utility derived from the dividend stream and the utility from occasionally selling the asset. (We abstract from the nondifferentiability at $b=0$ in this.)

B.4. *Algorithm*

The algorithm we use to solve for optimal policies is a version of the Hintermaier and Koeniger's (2010) extension of the endogenous grid method, originally developed by Carroll (2006).

It works iteratively until convergence of policies as follows. Start with some guess for the policy functions x_a^* and x_n^* on a given grid $(b,k)\in B\times K$. Define the shadow value of capital:

$$\beta^{-1}\psi(b,k;z):=\nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}\big[q(z')+r(z')\big]\right\}$$
$$+(1-\nu)E\frac{\partial V_n[b_n^*(b,k,z),k;z']}{\partial k}$$

$$
\begin{aligned}
&= \nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}[q(z')+r(z')]\right\} \\
&\quad + (1-\nu)E\left\{\frac{\partial u\{x_n^*[b_n^*(b,k,z),k;z']\}}{\partial x}r(z')\right\} \\
&\quad + (1-\nu)E\{\psi[b_n^*(b,k,z),k;z']\}.
\end{aligned}
$$

Guess initially $\psi = 0$. Then iterate over the following steps:

Step 1. Solve for an update of x_n^* by standard endogenous grid methods using equation (S14), and denote $b_n^*(b,k;z)$ as the optimal bond holdings without capital adjustment.

Step 2. For every k' on-grid, find some (off-grid) value of $\tilde{b}_a^*(k';z)$ such that combining (S13) and (S12) yields

$$
\begin{aligned}
0 &= \nu E\left\{\frac{\partial u[x_a^*(\tilde{b}_a^*(k',z),k';z')]}{\partial x}\left[\frac{q(z')+r(z')}{q(z)} - \frac{R(b',z')}{\pi(z')}\right]\right\} \\
&\quad + (1-\nu)E\left\{\frac{\partial u[x_n^*(\tilde{b}_a^*(k',z),k';z')]}{\partial x}\left[\frac{r(z')}{q(z)} - \frac{R(b',z')}{\pi(z')}\right]\right\} \\
&\quad + (1-\nu)E\left[\frac{\psi(\tilde{b}_a^*(k',z),k';z')}{q(z)}\right].
\end{aligned}
$$

Note well that $E\psi$ takes the stochastic transitions in h' into account and does not replace the expectations operator in the definition of ψ. If no solution exists, set $\tilde{b}_a^* = \underline{B}$. Uniqueness (conditional on existence) of $\tilde{b}_a^*$ follows from the strict concavity of v.

Step 3. Solve for total initial resources by solving the Euler equation (S13) for $\tilde{x}^*(k',z)$, such that

$$
\begin{aligned}
&\tilde{x}^*(k',z) \\
&\quad = \frac{\partial u}{\partial x}^{-1}\left\{\beta E\frac{R(b^*,z')}{\pi(z')}\left[\nu\frac{\partial u\{x_a^*[b_a^*(k',z),k';z']\}}{\partial x} + (1-\nu)\frac{\partial u\{x_n^*[b_a^*(k',z),k';z']\}}{\partial x}\right]\right\},
\end{aligned}
$$

where the right-hand side expressions are obtained by interpolating $x_a^*(b_a^*(k',z),k',z')$ from the on-grid guesses $x_a^*(b,k;z)$ and taking expected values with respect to z'.

This way we obtain total nonhuman resources $\tilde{\mathcal{R}}_a(k',z)$ that are compatible with plans $(b^*(k'),k')$ and a consumption policy $\tilde{\tilde{x}}_a^*(\tilde{\mathcal{R}}_a(k',z),z)$ in total resources.

Step 4. Since (consumption) policies are increasing in resources, we can obtain consumption policy updates as follows. Calculate total resources for each (b,k) pair $\mathcal{R}_a(b,k) = (q+r)k + b\frac{R(b)}{\pi}$ and use the consumption policy obtained before to update $x_a^*(b,k,z)$ by interpolating at $\mathcal{R}_a(b,k)$ from the set $\{(\tilde{\tilde{x}}_a^*(\tilde{\mathcal{R}}_a(k',z),z),\mathcal{R}_a(k',z))|k' \in K\}$.[1]

[1]If a boundary solution $\tilde{b}^*(\underline{B}) > \underline{B}$ is found, we use the "n" problem to obtain consumption policies for resources below $\tilde{b}^*(\underline{B})$.

Step 5. Update ψ: Calculate a new value of ψ using (S11), such that

$$\psi^{new}(b,k,z) = \beta\nu E\left\{\frac{\partial u\{x_a^*[b_n^*(b,k,z),k;z']\}}{\partial x}\left[q(z')+r(z')\right]\right\}$$
$$+\beta(1-\nu)E\left\{\frac{\partial u\{x_n^*(b_n^*(b,k,z),k;z')]}{\partial x}r(z')\right\}$$
$$+\beta(1-\nu)E\{\psi^{\text{old}}[b_n^*(b,k,z),k;z']\},$$

making use of the *updated* consumption policies.

Note that we wrote the algorithm in a general form that covers both Krusell–Smith equilibria, steady states, and first-order perturbations in aggregate dynamics. The difference lies in specifying the prices $r(z)$, $q(z)$, and $R(b,z)$, $\pi(z)$. In a Krusell–Smith equilibrium these are given by the forecasting rules; in the steady state, prices are fixed; and in an approximation of the aggregate dynamics by first-order perturbation following Reiter (2002, 2009, 2010), current and future prices get perturbed and obtained as solution to a system of linearized difference equations.

APPENDIX C: Dynamic Planning Problem With Two Assets and Logistic Distribution of Adjustment Costs

With logistically distributed adjustment costs, concavity of the value function is no longer guaranteed, because ν will depend on $(\omega;z)$. If the function EV in equation (13) is convex, then the policy functions will still be continuously differentiable and the value function will be twice differentiable because the prerequisites of Lemmas 4 and 5 are still fulfilled.

Let $f(\chi)$ be the density function of the adjustment costs. Since $V_a \geq V_n$, we can write

$$EV(\omega;z) = V_n(\omega;z) + \int_0^{V_a(\omega;z)-V_n(\omega;z)} \big(V_a(\omega;z)-V_n(\omega;z)-\chi\big)f(\chi)\,d\chi.$$

In turn, if $f(\chi)>0$ for all $\chi>0$ (the adjustment cost distribution has unbounded support on $\mathbb{R}_+$), the derivative of EV w.r.t. ω takes the form

$$\frac{\partial EV}{\partial\omega} = \frac{\partial V_n}{\partial\omega} + \nu^*(\omega;z)\left[\frac{\partial V_a}{\partial\omega}-\frac{\partial V_n}{\partial\omega}\right].$$

In words, first-order conditions of a model with fixed adjustment probabilities and a model with state-dependent adjustment probabilities are the same. We make use of this fact and simply replace the state-independent adjustment probability by a guess for an adjustment probability function in the algorithm described in Appendix B.4. We then update the adjustment probabilities by making use of the closed-form solution to the expected adjustment costs under the logistic distribution assumption for χ when calculating the value functions in iteration (n):

$$V_a^{(n)} = u\big(x_a^{*(n)}\big) + \beta EV_{(n)}\big(b_a^{*(n)},k^{*(n)};z'\big),$$
$$V_n^{(n)} = u\big(x_n^{*(n)}\big) + \beta EV_{(n)}\big(b_n^{*(n)},k;z'\big),$$

where

$$EV^{(n)} = \nu^{*(n)}V_a^{(n)} + \big(1-\nu^{*(n)}\big)V_n^{(n)} - AC\big(\nu^{*(n)};\mu_\chi,\sigma_\chi\big),$$

where μ_χ and σ_χ are the mean and the scale of the logistic distribution

$$F(\chi) = \frac{1}{1 + \exp\left\{-\frac{\chi - \mu_\chi}{\sigma_\chi}\right\}}.$$

The adjustment probability can be updated after the two value functions have been calculated for a given $\nu^*(\omega, z)$ as

$$\nu^{*(n+1)}(\omega, z) = F\left[V_a^{(n)}(\omega, z) - V_n^{(n)}(\omega, z)\right].$$

Given the new adjustment probabilities, consumption and savings policies can be determined again using the endogenous grid method. The expected conditional adjustment cost is given by

$$\begin{aligned} AC(\nu; \mu_\chi, \sigma_\chi) &= \int_0^{F^{-1}(\nu)} \chi dF(\chi) = \int_0^{\nu} F^{-1}(p)\, dp \\ &= \int_0^{\nu} \mu_\chi + \sigma_\chi\left[\log p - \log(1-p)\right] dp \\ &= \mu_\chi \nu + \sigma_\chi\left[\nu \log \nu + (1-\nu)\log(1-\nu)\right]. \end{aligned}$$

Given that concavity of the value functions is not guaranteed, we check for monotonicity of the derivatives of the value function and for uniqueness of the optimal portfolio solution in the algorithm, implementing thereby a version of Fella's (2014) algorithm, and find that the solution turns out to be globally concave.

APPENDIX D: PRICE-LEVEL DETERMINACY

This appendix sketches in a more technical way than the main text the question of price-level determinacy in our non-Ricardian setup. Making the argument in full is beyond the scope of this paper, so we restrict ourselves to a setup with only bonds, no aggregate shocks, and flexible prices. With flexible prices there is no output-inflation feedback, so we can drop output as a determinant of government rules.

The equation that then determines the price level is the bond-market clearing condition (see Woodford (1995)), which simplifies to the well known series of consumption Euler equations under complete markets and a representative agent. Given that total output is fixed under flexible prices, this market clearing condition, when substituted in for the supply of bonds, simplifies to

$$\left(\frac{B_t R_t^b/\pi_t}{\bar{B}\bar{R}^b/\bar{\pi}}\right)^{\rho_B} \left(\frac{\pi_t}{\bar{\pi}}\right)^{-\gamma_\pi} = \frac{B^d\left(\Theta_t; R_t^b; \pi_t\right)}{\bar{B}}. \tag{S15}$$

Since in our model tax policy does not adjust, there is no direct feedback from government policy to bond demand through household budgets, but only through goods/bonds markets. Of course, the demand for bonds depends on the entire path of future prices and wealth distributions (and insofar as our notation is sloppy). We can, however, use the logic of a local approximation in aggregates (cf. Reiter (2002), a variant of which we use to solve the model), and write the first-order expansion of (S15) as

$$\rho_B\left(\hat{B}_t + \hat{R}_t^b - \hat{\pi}_t\right) - \gamma_\pi \hat{\pi}_t = \zeta_B(\hat{B}_t - \hat{\pi}_t) + \zeta_R\left(\hat{R}_{t+1}^b - E_t\hat{\pi}_{t+1}\right),$$

where the $\hat{x}$ is a log deviation in a variable x from its steady-state value, and $\zeta_{B,R}$ are the *aggregate* wealth and interest elasticities of savings. We approximate the true dynamics assuming approximate aggregation, that is, that all changes in aggregate government debt affect debt demand as if they were proportionally distributed according to the steady-state distribution. This implies that any change in end-of-period $t-1$ government debt, B_t, has the same impact on total demand of liquid assets as has a change in beginning-of-period real debt, $\hat{B}_t - \hat{\pi}_t$, through inflation and we do not need to model the dynamics of Θ_t explicitly. Note that this is a simplifying assumption that we make only here to obtain analytical results, but not in the actual solution of our model.

Inserting the Taylor rule and plugging in the laws of motion for bonds and nominal rates, we obtain a system of three equations:

$$\begin{aligned}
E_t\hat{\pi}_{t+1} &= \left[(1-\rho_R)\theta_\pi + \frac{\gamma_\pi}{\zeta_R}\right]\hat{\pi}_t + \frac{\zeta_B - \rho_B}{\zeta_R}\big(\hat{B}_t - \hat{\pi}_t + \hat{R}^b_t\big) + \frac{\rho_R\zeta_R - \zeta_B}{\zeta_R}\hat{R}^b_t,\\
\hat{B}_{t+1} &= \rho_B\big(\hat{B}_t - \hat{\pi}_t + \hat{R}^b_t\big) - \gamma_\pi\hat{\pi}_t,\\
\hat{R}^b_{t+1} &= \rho_R\hat{R}^b_t + (1-\rho_R)\theta_\pi\pi_t,
\end{aligned}$$

which we can write in terms of total real outstanding government obligations $\hat{O}_t = \hat{B}_t - \hat{\pi}_t + \hat{R}^b_t$ as

$$\begin{aligned}
E_t\hat{\pi}_{t+1} &= \left[(1-\rho_R)\theta_\pi + \frac{\gamma_\pi}{\zeta_R}\right]\hat{\pi}_t + \frac{\zeta_B - \rho_B}{\zeta_R}\hat{O}_t + \frac{\rho_R\zeta_R - \zeta_B}{\zeta_R}\hat{R}^b_t,\\
E_t\hat{O}_{t+1} &= -\left(\frac{\zeta_R + 1}{\zeta_R}\gamma_\pi\right)\hat{\pi}_t + \left(\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B\right)\hat{O}_t + \frac{\zeta_B}{\zeta_R}\hat{R}^b_t,\\
\hat{R}^b_{t+1} &= \rho_R\hat{R}^b_t + (1-\rho_R)\theta_\pi\pi_t.
\end{aligned}$$

For government obligations, we can invoke a transversality condition to rule out exploding paths. This means that if $\hat{O}$ explodes for $\hat{O} \neq 0$, $\hat{O} = 0$ is the only solution. This directly implies that $\hat{\pi} = 0$ because real beginning-of-period bonds and interest rates are predetermined.

For the special case of an interest rate peg, $\theta_\pi = \rho_R = 0$, and no active fiscal stabilization, $\gamma_\pi = 0$, this implies

$$E_t(\hat{O}_{t+1}) = \left(\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B\right)\hat{O}_t,$$

such that the government obligations are stable whenever $-1 < \frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B < 1$, which implies local indeterminacy of the price level. If by contrast $\frac{\rho_B - \zeta_B}{\zeta_R} + \rho_B > 1$, then any deviation from steady-state inflation leads to an explosive path of log real debt, and $\hat{B}_t - \hat{\pi}_t = 0$ is the only solution to the system.

In a representative agent model, we have $\zeta_B = 1$ and $\zeta_R \approx \frac{1}{\xi}\frac{\bar{C}}{\bar{B}}$, which gives rise to fiscal theories of the price level (see Leeper (1991)), when households assume that primary surpluses do not adjust when the real value of debt changes, $\rho_B = 1$ ("fiscal dominance"). With incomplete markets, the elasticity of savings to wealth is strictly smaller than one, $\zeta_B < 1$, such that the critical value for ρ_B is also strictly less than 1. Therefore, we have that even when the government does make sure that future surpluses repay any level of government debt, the price level is still determinate, because households are not indifferent about the paths of government debt. Yet, if government debt is "too" stable, in the sense that debt reverts relatively fast to its steady-state level, $\rho_B < \zeta_B$, indeterminacy still arises.

APPENDIX E: SOLVING THE MODEL WITH AGGREGATE SHOCKS

E.1. *Local Approximation*

Our model has a three-dimensional idiosyncratic state space with two endogenous states. We experimented with the grid size for liquid and illiquid asset holdings as well as for the process of productivity. Given that we focus on second moment changes, we require $n_h = 26$ productivity states and find that with a log-spaced grid for assets, results are no longer affected by grid size beyond $n_b = 80$, $n_k = 80$ points. This means that a tensor grid contains $n_b \times n_k \times n_h = 166{,}400$ points. This renders solving the model by perturbing the histogram and the value functions on a tensor grid infeasible such that we cannot apply a perturbation method without state-space reduction, as in Reiter (2002).

Instead, we develop a variant of Reiter's (2009) method to solve heterogeneous agent models with aggregate risk. We represent the dynamic system as a set of nonlinear difference equations, for which

$$E_t F(X_t, X_{t+1}, Y_t, Y_{t+1}) = 0$$

holds, where the set of control variables is $Y_t = (V_t, \frac{\partial V_t}{\partial b}, \frac{\partial V_t}{\partial k}, \tilde{Y}_t)$, that is, value functions and their marginals with respect to k, b as well as some aggregate controls $\tilde{Y}_t$ such as dividends, wages, and so forth. The set of state variables $X_t = (\Theta_t, R_t^b, s_t)$ is given by the histogram Θ_t of the distribution over (b, k, h) and the aggregate states R_t^b, s_t. In principle, we can solve this system with Schmitt-Grohé and Uribe's (2004) method as argued in Reiter (2002), but in practice the state space is too rich and the solution becomes numerically infeasible and unstable.

Hence, we need to reduce the dimensionality of the system. We therefore first approximate value functions and their derivatives at all grid points around their value in the stationary equilibrium without aggregate risk, $V^{SS}(b, k, h)$, by a sparse polynomial $P(b, k, h)$ with parameters $\Omega_t = \Omega(\Theta_t, R_t^b, s_t)$. For example, we write the value function as

$$V\big(b, k, h; \Theta_t, R_t^b, s_t\big)/V^{SS}(b, k, h) \approx P(b, k, h)\Omega_t.$$

Note the difference from a global approximation of the functions for finding the stationary equilibrium without aggregate risk. Here, we only use the sparse polynomial to capture *deviations* from the stationary equilibrium values, compared to Ahn, Kaplan, Moll, Winberry, and Wolf (2017) and different from Winberry (2016) and Reiter (2009). We define the polynomial basis functions in such a way that the grid points of the tensor grid coincide with the Chebyshev nodes for this basis.

In the system F, we then use the Bellman equation to obtain V_t from V_{t+1} on a tensor grid and then calculate the difference of Ω_t to the regression coefficients for the polynomial that fits $V_t(b, k, h)/V^{SS}(b, k, h)$.

This reduces the number of variables in the difference equation substantially, but leaves us still with too many state variables from the histogram at the tensor grid. Reiter (2010) and Ahn et al. (2017) suggest using state-space reduction techniques to deal with this issue. In continuous time, the state-space reduction can be done based on a Taylor expansion in time derivatives. In discrete time, there is no obvious basis for the state-space reduction and the Jacobi matrices involved are substantially less sparse.

Yet, we use Sklar's theorem and write the distribution function in its copula form such that $\Theta_t = \mathcal{C}_t(F_t^b, F_t^k, F_t^h)$ with the copula $\mathcal{C}_t$ and the marginal distributions for liquid and illiquid assets and productivity $F_t^{b,k,h}$. Now fixing $\mathcal{C}_t = \mathcal{C}$ can break the curse of dimensionality, reducing the number of state variables from $n_b \times n_k \times n_h = 166,400$ to $n_b + n_k + n_h = 186$, as we now only need to perturb the marginal distributions.

Fixing the copula $\mathcal{C}$ to the copula from the stationary distribution, the approximation does not impose any restriction on the stationary distribution when aggregate shocks are absent, such that the approximation then becomes exact. Therefore, it is less restrictive for the stationary state than assuming a parametric form for the distribution function. The copula itself is obtained by fitting a cubic spline to the stationary distribution of ranks in b, k, and h.

The idea behind this approach is that given the economic structure of the model, prices only depend on aggregate asset demand and supply, as in Krusell and Smith (1998), and not directly on higher moments of the *joint* distributions Θ_t and Θ_{t+1}. Our approach imposes no restriction on how the marginal distributions change, that is, how many more or less liquid assets the portfolios of the xth percentile have. It only restricts the change in the likelihood of a household being among the $x\%$ percent richest in liquid assets to be among the $y\%$ percent richest in illiquid assets. We check whether the time-constant copula assumption creates substantial numerical errors and find none by comparing it to the Krusell and Smith (1998) solution. See Figure S1 for a comparison of the impulse response functions (IRFs) for our baseline calibration.

In addition, we calculate the $\mathbf{R}^2$ statistics for the estimate $\mathcal{C}(F_{t+1}^b, F_{t+1}^k, F_{t+1}^h)$ of distribution Θ_{t+1},

$$\mathbf{R}^2 = 1 - \frac{\int \left[d\mathcal{C}\left(F_{t+1}^b, F_{t+1}^k, F_{t+1}^h\right) - d\Theta_{t+1}\right]^2}{\int [d\Theta_{t+1}]^2},$$

plugging in for $F_{t+1}^{b,k,h}$ the linearized solutions $H(F_t^{b,k,h}, R_t^B, s_t)$ and for $d\Theta_{t+1}$ the solution from iterating the histogram forward given the policy functions. This yields a measure of fit for our approximation of the distribution function by a fixed copula. Absent aggregate shocks, the measure is 100% by construction. Given the solution technique, the appropriateness of the fixed copula assumption is captured by the derivative $\frac{\partial \mathbf{R}^2}{\partial x_t}$ of the $\mathbf{R}^2$ statistics with respect to state variable x_t. We find that this derivative is roughly 0.00019% with respect to uncertainty, such that, extrapolating linearly, the $\mathbf{R}^2$ at 99.9999% remains extremely high after a 1 standard deviation increase in uncertainty (a shock of size 0.54).

Finally, we check the quality of the linearized solution (in aggregate shocks) by solving the household planning problem given the implied expected continuation values from our solution technique but solving for the actual intratemporal equilibrium, as suggested by Den Haan (2010). We simulate the economy over $T = 1000$ periods and calculate the differences between our linearized solution and the nonlinear one. The maximum difference is 0.45% for the capital stock and 1.66% for bonds, while the mean absolute errors are substantially smaller; see Table SI.

E.2. *Krusell–Smith Equilibrium*

As an alternative to the solution method laid out above, we assume that households use forecasting rules to predict future prices on the basis of a restricted set of moments, as in Krusell and Smith (1997, 1998).

TABLE SI

DEN HAAN (2010) STATISTIC[a]

	Absolute Error (in %) for			
	Price of Capital q_t	Capital K_t	Inflation π_t	Real Bonds B_t
Mean	0.0310	0.1071	0.0359	0.4434
Max	0.1030	0.4543	0.2119	1.6634

[a] Differences in percent between the simulation of the linearized solution of the model and a simulation in which we solve for the actual intratemporal equilibrium prices in every period for $t = \{1, \dots, 1000\}$; see Den Haan (2010).

Specifically, these rules "nowcast" inflation, π_t, and capital price, q_t, and forecast the term $[\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}]$ in the Phillips curve. These rules are used when calculating the continuation values in the Bellman equation. We assume these functions to be log linear in government debt, B_t, last period's nominal interest rate, R_t^b, the aggregate stock of capital, K_t, average h_{it} (denoted H_t below), and the uncertainty state, s_t (and s_{t+1} for the forecasting term).

We formulate the problem in terms of relative price nowcasts and inflation forecasts such that we have a description of the conditional distributions of all future prices households expect. Note also that it is sufficient to write the problem in terms of *price* nowcasts and the Phillips curve forecast, because given these, households can back out future state variables describing aggregate *quantities*, $\{K_{t+s}, B_{t+s}\}$, from the government's budget constraint and the capital supply function, and future nominal rates R_{t+s}^b from the Taylor rule.

In detail, this means that when households know K_t, B_t, R_t^b, s_t, and H_t, they can back out markups from the Phillips curve (16) using the stipulated rules for inflation in t and the conditional inflation forecasts for $t+1$. Given this, they can calculate real wages and total output. In turn, they know future government debt, B_{t+1}, from the government's budget constraint (18). The future nominal interest rate, R_{t+1}^b, is pinned down by the Taylor rule (17). Finally, from the nowcast for capital prices (20), households can determine the next period's capital stock K_{t+1}. Using these model-implied forecasts for K_{t+1}, B_{t+1}, R_{t+1}^b, and H_{t+1}, households can then forecast the next period's inflation, capital prices and so forth, conditional on shock realizations ad infinitum. The law of motion for average productivity is given analytically by

$$\begin{aligned}\log H_{t+1} &:= \log \int h_{it+1} = \frac{1}{2}\operatorname{var}(\log h_{it+1}) \\ &= \rho_h^2 \frac{1}{2}\operatorname{var}(\log h_{it}) + \frac{1}{2}\bar{\sigma}_h^2 \exp(s_t) = \rho_h^2 \log H_t + \frac{1}{2}\bar{\sigma}_\varepsilon^2 \exp(s_t).\end{aligned}$$

The functional forms we use in the nowcasts/ forecasts of prices, letting the coefficients depend on the uncertainty state (a caret denotes deviations from steady state), are

$$\begin{aligned}\log \pi_t = {}& \beta_\pi^1(s_t) + \beta_\pi^2(s_t)\log \hat{B}_t + \beta_\pi^3(s_t)\log \hat{K}_t \\ &+ \beta_\pi^4(s_t)\hat{R}_t^b + \beta_\pi^5 H_t^{-1},\end{aligned} \tag{S16}$$

$$\begin{aligned}\log q_t = {}& \beta_q^1(s_t) + \beta_q^2(s_t)\log \hat{B}_t + \beta_q^3(s_t)\log \hat{K}_t \\ &+ \beta_q^4(s_t)\hat{R}_t^b + \beta_q^5 H_t^{-1},\end{aligned} \tag{S17}$$

$$\left[\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}\right] = \beta^1_{E\pi}(s_t) + \beta^2_{E\pi}(s_t)\log\hat{B}_t + \beta^3_{E\pi}(s_t)\log\hat{K}_t + \beta^4_{E\pi}(s_t)\hat{R}^b_t + \beta^5_{E\pi}H^{-1}_t + \beta^6_{E\pi}(s_{t+1}). \tag{S18}$$

Whether these rules yield good nowcasts of prices depends on the asset-demand functions, $b^*_{a,n}$ and k^*. If these are sufficiently close to linear in human capital, h, and in nonhuman wealth, b and k, at the mass of Θ_t, B_t and K_t will suffice and we can expect approximate aggregation to hold. For our exercise, the four endogenous aggregate states, R^b_t, H_t, B_t, and K_t, and the aggregate stochastic state s_t are sufficient to describe the evolution of the aggregate economy.

Technically, finding the equilibrium is the same as in Krusell and Smith (1997), as we need to find market clearing prices within each period. Concretely, this means that the posited rules, (S16)–(S18), are used to solve for households' policy functions. Having solved for the policy functions conditional on the forecasting rules, we then simulate n independent sequences of economies for $t = 1, \dots, T$ periods, keeping track of the actual distribution Θ_t. In each simulation, the sequence of distributions starts from the stationary distribution implied by our model without aggregate risk. We then calculate in each period t the optimal policies for market clearing inflation rates and capital prices assuming that households resort to the policy functions derived under rules (S16)–(S18) from period $t + 1$ onward. Having determined the market clearing prices, we obtain the next period's distribution Θ_{t+1}. In doing so, we obtain n sequences of equilibria. The first 250 observations of each simulation are discarded to minimize the impact of the initial distribution. We next re-estimate the parameters of (S16)–(S18) from the simulated data and update the parameters accordingly. By using $n = 20$ and $T = 750$, it is possible to make use of parallel computing resources and obtain 10,000 equilibrium observations. Subsequently, we recalculate policy functions and iterate until convergence in the forecasting rules.

The posited rules, (S16)–(S18), approximate the aggregate behavior of the economy fairly well. The equilibrium values for the parameters of the rules are given in Table SII. The minimal within sample R^2 is above 99%. The forecast performance is not perfect because we need to force households to effectively approximate the process for $\log\int h$ by a three-state Markov chain. This variable moves slowly and leads to small but persistent low frequency errors.

E.3. *Comparison of Results*

Figure S1 compares the impulse response functions obtained from the Reiter method solution to the nonlinear Krusell–Smith solution. The Krusell–Smith impulse response functions are generated by linearly interpolating the policy functions, setting the uncertainty state to exactly its expected path after a 1 standard deviation shock, that is, they are obtained without simulation.

The impulse responses look qualitatively similar across the two methods. One should take the results and hence the differences, however, with a grain of salt, as we need to approximate the continuous aggregate states in the Krusell and Smith algorithm very coarsely with three grid points each for K_t, B_t, R^b_t, and H_t, and five grid points for s_t. In addition, we need to decrease the points on the idiosyncratic assets grids to 40 each, as the total number of nodes with $n_b \times n_k \times n_h \times n_s \times n_R \times n_B \times n_K \times n_H \approx 16E(+6)$ is already very large. This leads to an underestimation of the persistence of the uncertainty shock and the slow moving average idiosyncratic productivity, which decreases the aggregate effects on impact, but makes them somewhat more persistent.

TABLE SII

LAWS OF MOTION FOR KRUSELL AND SMITH[a]

	s_1	s_2	s_3	s_4	s_5
Price of Capital $\log q_t$ (R^2: 99.38)					
β_q^1	−1.65	−1.74	−1.84	−1.95	−2.16
β_q^2	−0.02	−0.02	−0.02	−0.02	−0.01
β_q^3	−0.28	−0.28	−0.28	−0.28	−0.28
β_q^4	−0.03	−0.03	−0.03	−0.03	−0.02
β_q^5	1.90	1.90	1.90	1.90	1.90
Inflation $\log \pi_t$ (R^2: 99.95)					
β_π^1	−9.65	−9.76	−9.85	−9.95	−10.09
β_π^2	−0.07	−0.06	−0.05	−0.04	−0.03
β_π^3	−0.05	−0.06	−0.06	−0.07	−0.08
β_π^4	−0.01	−0.01	−0.01	−0.01	−0.01
β_π^5	9.96	9.96	9.96	9.96	9.96
Expectation Term $[\log(1+\pi_{t+1})\frac{Y_{t+1}}{Y_t}]$ (R^2: 99.68)					
$\beta_{E\pi}^1$	−8.54	−8.62	−8.69	−8.77	−8.88
$\beta_{E\pi}^2$	−0.06	−0.05	−0.05	−0.04	−0.04
$\beta_{E\pi}^3$	−0.04	−0.04	−0.05	−0.05	−0.06
$\beta_{E\pi}^4$	−0.00	−0.01	−0.01	−0.01	−0.01
$\beta_{E\pi}^5$	8.76	8.76	8.76	8.76	8.76

[a]For readability, all values are multiplied by 100.

APPENDIX F: ESTIMATION OF THE STOCHASTIC VOLATILITY PROCESS FOR HOUSEHOLD INCOME

F.1. *Data*

We estimate the income process based on the Survey of Income and Program Participation (SIPP) panels 1984–1987, 1990–1993, 1996, 2001, 2004, and 2008. We do not use the 1988 and 1989 surveys because of their known deficiencies due to the survey design and small sample size that resulted from budgetary constraints.

The SIPP panels provide monthly individual income data for up to 3 years (more in the 2008 survey) for each household member for each wave. The waves we use span the period 1983Q4–2013Q1. We constrain the sample to households with two married adults whose head is between 30 and 55 years of age, and calculate for each household the labor income after taxes and transfers using NBER TAXSIM. We aggregate income to quarterly frequency and restrict the sample to households that supply at least 260 hours of work (both spouses together) per quarter (50% of full-time work).

We then estimate the predictable part of log household income, based on age and education dummies, and a linear quadratic term in age for each education level. Furthermore, we control for time effects, ethnicity, and the number of dependent children. The residuals from this regression form the basis of our subsequent analysis. We eliminate the top and bottom 0.5% of the residuals from each age–quarter cell to remove outliers.

Then we construct a sequence of quarterly panels containing, for each household in the panel, current residual income and two lags thereof. We use these data to calculate for each quarter and age (expressed in quarters) the variance and the first two autocovariances of residual income. We estimate the sampling variance–covariance of the empirical variance–autocovariance estimates for each quarter and age cell by bootstrapping, where we stratify by age and quarter.

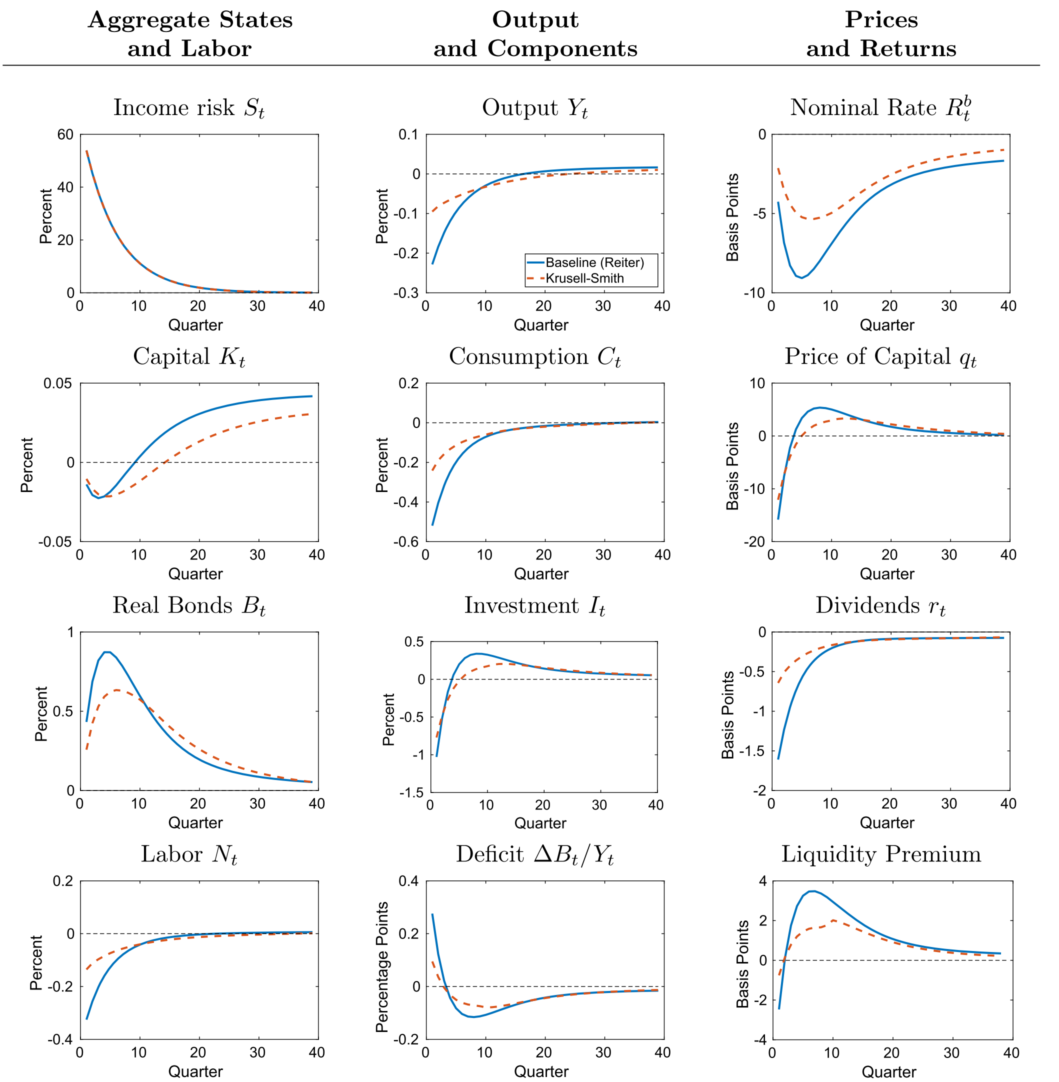

FIGURE S1.—Comparison of the Krusell–Smith versus the Reiter method. *Notes*: The liquidity premium is $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

F.2. *Estimation*

Our estimation strategy uses the theoretical (autoco-) variances, $\omega_{0,j}^2(c,t)$ for $j = 0, 1, 2$, as described in equation (4) in Section 2 and their sample counterparts, $\mathbf{ac}_{0,j}^2(c,t)$, $j = 0, 1, 2$, to construct a quasi maximum likelihood (QML) estimator. It is only a *quasi* maximum likelihood estimator, as we treat sampling error for the variance terms as if they are normally distributed although they might not be. Let ψ denote the sampling error. Then we have

$$\psi_j(c,t) = \omega_{0,j}^2(c,t) - \mathbf{ac}_{0,j}^2(c,t). \tag{S19}$$

We estimate the covariance matrix of ψ, $\Sigma_\psi(c,t)$, by bootstrapping age–quarter strata. With these terms in hand, we can specify the log pseudo-likelihood as

$$-2\log L = \sum_{(c,t)\in S} \boldsymbol{\psi}'(c,t)\Sigma_\psi(c,t)^{-1}\boldsymbol{\psi}(c,t) + \sum_{t\in T} \left(\varepsilon^s_t\right)^2/\sigma^2_s + \#T\log\sigma^2_s, \tag{S20}$$

where S is the set of all cohort–quarter pairs we observe, that is, the cohorts 1959Q1–2013Q1 (denoted by the quarter they turn 30) between 1983Q4 and 2013Q1, and T is the set of quarters for which we estimate shocks, that is, 1976Q1–2013Q1. We force $\sum_{t\in T}\varepsilon^s_t = 0$.

We directly estimate the shock series, ε_t, together with the parameters for the persistent income shocks (ρ_h, ρ_s, $\bar{\sigma}_p$, σ_s), the transitory and permanent part, (σ_τ, σ_μ, ρ_τ), and the time trend (θ_1, θ_2). However, since the data contain only limited information on shocks far before the sample starts, we set all shocks 8 years before the first sample year (i.e., before 1976Q1) to their unconditional mean, that is, to zero, and exclude them from the calculation of the likelihood. Eight years correspond roughly to the half-life of income shocks, $-\frac{\log 1/2}{\log\rho_h} \approx 34$ quarters, and thus twice the half-life of deviations in income variances.

F.3. *Bootstrapped Standard Errors*

Since asymptotic standard errors might be misleading, we bootstrap the standard errors for our estimates. Yet, bootstrapping the estimator is not entirely trivial. The errors $\psi(c,t)$ are heteroscedastic. Cells with more information and more income inequality will have higher sampling variation in the (autoco-) variances of income. What is more, bootstrapping the microdata to capture sampling error alone also does not suffice, since the sampling uncertainty also regards the time period we sampled, not only the individuals in the sample.

Therefore, we proceed as follows to obtain bootstrapped standard errors for Table 1. We draw $b = 1,\dots,B$ bootstrap samples of shocks $\{\varepsilon^s_{t,b}{}^*\}^{b=1\dots B}_{t\in T}$ from the estimated shock series $\{\hat{\varepsilon}^s_t\}_{t\in T}$. We then feed the shocks through the model under the estimated parameters to obtain bootstrapped theoretical autocovariances $\omega_{0,j}(c,t)^*_b$ for $b = 1,\dots,B$. We then use a wild bootstrap using a Rademacher distribution (see Davidson and Flachaire (2008)) for ν^* to draw from the estimated sampling errors $\hat{\psi}_j(c,t)$ generating the bootstrapped sampling errors

$$\psi_{j_b}^*(c,t) = \nu^*_b(c,t)\hat{\psi}_j(c,t),$$

that is, we draw the entire vector of measurement error for all three autocovariances for a cohort–year cell. We then generate the resampled data to re-estimate the parameters and shocks from

$$\mathbf{ac}^{2\ \ *}_{0,j_b}(c,t) = \omega_{0,j_b^*}(c,t) + \psi_{j_b}^*(c,t).$$

This leaves us with B samples from which to estimate parameters and shocks. To calculate the standard deviation for each individual income risk shock ε^s_t, we subtract the actual value of the shocks ε^{s*}_{tb} in each single bootstrap replication from the estimated shock value for that bootstrap.

TABLE SIII
PARAMETER ESTIMATES[a]

ρ_h	ρ_s	$\bar{\sigma}$	σ_s	
0.979	0.839	0.059	0.539	
(0.060)	(0.065)	(0.029)	(0.097)	
ρ_τ	σ_τ	σ_μ	θ_1	θ_2
0.339	0.115	0.271	3.385	−4.401
(0.009)	(0.002)	(0.005)	(0.999)	(1.280)

[a] Bootstrapped standard errors are given in parentheses. Time trend parameters are estimated coding the time 1959Q1–2013Q1 as $-1, \ldots, 1$. The estimate for the average uncertainty $\bar{\sigma}$ includes the average time–trend effect for 1983–2013.

F.4. *Results*

Table SIII summarizes the estimated parameter values. The estimated income risk and income risk shocks have been displayed in Section 2. There is a positive but decreasing trend in income risk. We take this trend into account in our model by including the average trend term in the baseline uncertainty.

APPENDIX G: WEALTH DISTRIBUTION, ASSET CLASSES, AND OTHER AGGREGATE VARIABLES

G.1. *Data From the Flow of Funds*

We can map our definition of liquid assets to the quarterly Flow of Funds (FoF), Table Z1. The financial accounts report the aggregate balance sheet of the U.S. household sector (including nonprofit organizations serving households) and are used in our analysis to quantify changes in the aggregate ratio of net liquid to net illiquid assets on a quarterly basis. Net liquid assets are defined as total currency and deposits, money market fund shares, various types of debt securities (Treasury, agency backed, government sponsored enterprise (GSE) backed, municipal, corporate, and foreign), loans (as assets), and total miscellaneous assets net of consumer credit, depository institution loans not elsewhere classified, and other loans and advances.

Net illiquid wealth is composed of real estate at market value, life insurance reserves, pension entitlements, equipment and nonresidential intellectual property products of nonprofit organizations, proprietors' equity in noncorporate business, corporate equities, and mutual fund shares subtracting home mortgages as well as commercial mortgages. The Flow of Funds computes proprietors' equity in noncorporate business as the sum of all capital expenditures and financial assets of that business minus its liabilities. Therefore, and this is in line with our assumption of nontradable pure profits, it does not contain any goodwill.

G.2. *Data From the Survey of Consumer Finances*

We use 11 waves of the Survey of Consumer Finances (SCF, 1983–2013) to calibrate our model and to compare the cross-sectional implications of our model with the data.

Net liquid assets are classified as all households' savings and checking accounts, call and money market accounts (incl. money market funds), certificates of deposit, all types of bonds (such as savings bonds, U.S. government bonds, Treasury bills, mortgage-backed bonds, municipal bonds, corporate bonds, and foreign and other tax-free bonds), and private loans net of credit card debt.

TABLE SIV

HOUSEHOLD PORTFOLIO COMPOSITION[a]

Moments	Model	Data
Fraction with $b < 0$	0.16	0.16
Fraction with $k > 0$	0.88	0.89
Fraction with $b \leq 0$ and $k > 0$	0.15	0.14
Gini liquid wealth	0.84	0.86
Gini illiquid wealth	0.79	0.79
Gini total wealth	0.78	0.78

[a] Averages over the 1983–2013 SCFs using the respective cross-sectional sampling weights. Households whose liquid asset holdings fall below minus half quarterly average income are dropped from the sample. Ratios of liquid to illiquid wealth are estimated by first estimating local linear functions that map the percentile of the wealth distribution into average liquid and average illiquid asset holdings for each year, then averaging over years and finally calculating the ratios. Survey of consumer finances 1983–2013: Married households with head between 30 and 55 years of age.

All other assets are considered to be illiquid. Most households hold their illiquid wealth in real estate and pension wealth from retirement accounts and life insurance policies. Furthermore, we identify business assets, other nonfinancial and managed assets, and corporate equity in the form of directly held mutual funds and stocks as illiquid, because a large share of equities owned by private households is not publicly traded or widely circulated (see Kaplan, Moll, and Violante (2017)). From gross illiquid asset holdings, we subtract all debt except for credit card debt.

We exclude cars and car debt from the analysis altogether. What is more, we exclude from the analysis households that hold massive amounts of credit card debt such that their net liquid assets are below -1 month of average household income, which is the debt limit we use in our model. Moreover, we exclude all households whose equity in illiquid assets is below the negative of 1 average annual income. This excludes roughly 5% of U.S. households on average from our analysis and amounts to a debt limit on unsecured debt of \$9273 U.S. in 2013, for example. Table SIV displays some key statistics of the distribution of liquid and illiquid assets in the population.

We estimate the asset holdings at each percentile of the net worth distribution by running a local linear regression that maps the percentile rank in net worth into the net liquid and net illiquid asset holdings. In detail, let LI_{it} and IL_{it} be the value of liquid and illiquid assets of household i in the SCF of year t, respectively. Let ω_{it} be its sample weight. Then we first sort the households by total wealth (LI + IL) and calculate the percentile rank of a household i as $\mathrm{prc}_{it} = \sum_{j<i} \omega_{jt} / \sum_j \omega_{jt}$. We then run, for each percentile, $\mathrm{prc} = 0.01, 0.02, \dots, 1$, a local linear regression. For this regression, we calculate the weight of household i as $w_{it} = \sqrt{\phi(\frac{\mathrm{prc}_{it}-\mathrm{prc}}{h})\omega_{it}}$, where ϕ is the probability density function of a standard normal and $h = 0.05$ is the bandwidth. We then estimate the liquid and illiquid asset holdings at percentile prc at time t as the intercepts $\lambda^{\mathrm{LI,IL}}(\mathrm{prc}, t)$ obtained from the weighted regressions for year t:

$$w_{it}\mathrm{LI}_{it} = \lambda^{\mathrm{LI}}(\mathrm{prc}, t)w_{it} + \beta^{\mathrm{LI}}(\mathrm{prc}, t)(\mathrm{prc}_{it} - \mathrm{prc})w_{it} + \zeta_{it}^{\mathrm{LI}}, \tag{S21}$$

$$w_{it}\mathrm{IL}_{it} = \lambda^{\mathrm{IL}}(\mathrm{prc}, t)w_{it} + \beta^{\mathrm{IL}}(\mathrm{prc}, t)(\mathrm{prc}_{it} - \mathrm{prc})w_{it} + \zeta_{it}^{\mathrm{IL}}, \tag{S22}$$

where ζ is an error term.

We can use these estimates, for example, to calculate average portfolio liquidity at time t as $\sum_{\mathrm{prc}} \lambda^{\mathrm{LI}}(\mathrm{prc}, t) / \sum_{\mathrm{prc}} \lambda^{\mathrm{IL}}(\mathrm{prc}, t)$. Figure S2 compares the percentage deviations of

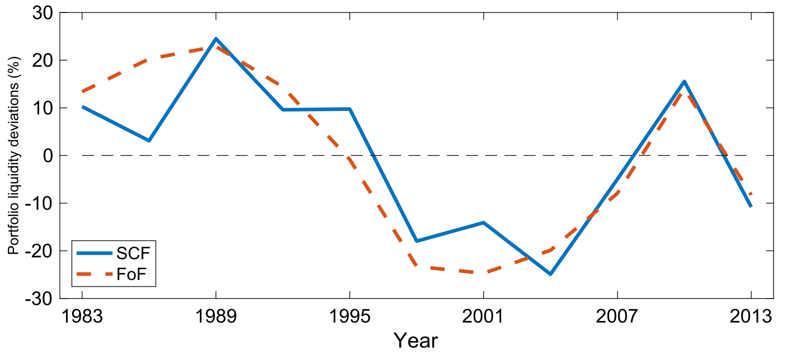

FIGURE S2.—Percentage deviation of portfolio liquidity from mean in SCF and FoF.

these average portfolio liquidity measures from their long-run mean and to those obtained from the FoF data for the years 1983–2013. The figure reveals that both data sources capture the very similar changes in the liquidity ratio over time.

However, it is important to note that the SCF, like many comparable surveys on wealth, systematically underestimates gross financial assets, and, consequently, the average liquid-to-illiquid assets ratio in the FoF is roughly 20%, about twice as high as in the SCF. This is because households are more likely to underreport their financial wealth, and especially deposits and bonds, due to a larger number of potential asset items. In contrast, they tend to overestimate the value of their real estate and equity (compare also Table C.1 in Kaplan, Moll, and Violante (2017)).

G.3. *Other Aggregate Data*

In Section2, we depicted the impulse response functions of the log of real GDP, real personal consumption, real private investment, real wages, and the real government deficit. These variables are taken from the national accounts data provided by the Federal Reserve Bank of St. Louis (series GDPMC1, PCECC96, GPDIC1, GCEC1, AHETPI) and data on government deficits from the NIPA tables for the United States (Table 3.1, BEA).

Data on house prices, Treasury bill returns, and the liquidity premium stem from the same source. We use the secondary market rate of the 3-month Treasury bill (DTB3) as a measure for the short-term nominal interest rate. House prices are captured by the Case–Shiller S&P U.S. National Home Price Index (CSUSHPINSA) divided by the all-items CPI (CPIAUCSL). The liquidity premium we construct from nominal house prices, the CPI for rents, and the rate on 3-month Treasuries. We measure the liquidity premium as the excess realized return on housing. This is composed of the rent–price ratio in t, $\frac{r_{h,t}}{q_t^{\text{house}}}$ plus the quarterly growth rate of house prices, $\frac{q_{t+1}^{\text{house}}}{q_t^{\text{house}}}$ in $t+1$, over the nominal return on riskless 3-month Treasury bills R_t^b (converted to a quarterly rate):

$$LP_t = \frac{r_{h,t}}{q_t^{\text{house}}} + \frac{q_{t+1}^{\text{house}}}{q_t^{\text{house}}} - \left(R_t^b\right)^{\frac{1}{4}}. \tag{S23}$$

Rents are imputed on the basis of the CPI for rents on primary residences paid by all urban consumers (CUSR0000SEHA) fixing the rent–price ratio in 1981Q1 to 4%.

The Solow residual series we use is taken from the latest version (date of retrieval 2017-11-01) of Fernald's raw TFP series (Fernald (2012)). We construct an index from the reported growth rates and use the log of this index.

APPENDIX H: DETAILS ON THE EMPIRICAL ESTIMATES OF THE RESPONSE TO SHOCKS TO HOUSEHOLD INCOME RISK

H.1. *Local Projection Method*

In Figure 3 of Section 2, we presented impulse response functions based on local projections (see Jordà (2005)). This method does not require the specification and estimation of a vector autoregressive model for the true data-generating process. Instead, in the spirit of multi-step direct forecasting, the impulse responses of the endogenous variables X at time $t+j$ to uncertainty shocks, ε_t^s, at time t are estimated using horizon-specific single regressions, in which the endogenous variable shifted ahead is regressed on the current normalized uncertainty shock ε_t^s, a time trend, lagged income risk s_{t-1}, and controls $\mathbf{X}_{t-1}$. These controls are specified as the return on T-bills, R_{t-1}^b, and the log of GDP, Y_{t-1}, of consumption C_{t-1}, of investment I_{t-1}, of TFP, A_{t-1}, and of real wages, w_{t-1}, as well as the GDP share of the government deficit $\Delta B_{t-1}/Y_{t-1}$:

$$X_{t+j} = \beta_{j,0} + \beta_{j,\varepsilon}\varepsilon_t^s/\sigma_s + \beta_{j,t}t + \beta_{j,X}\mathbf{X}_{t-1} + \beta_{j,s}s_{t-1} + \nu_{t+j}, \quad j = 0, \dots, 12. \tag{S24}$$

Hence, the impulse response function $\beta_{j,\varepsilon}$ is estimated just as a sequence of projections of X_{t+j} in response to the standardized shock ε_t^s/σ_s, local to each forecast horizon $j = 0, \dots, 12$ quarters. We focus on the post-Volcker disinflation era and use aggregate time series data from 1983Q1 to 2016Q2.

H.2. *Alternative Identification Schemes*

An important assumption made for employing the local projection method, which directly regresses the shocks on the endogenous variable of interest, is that the identified uncertainty shocks ε_t^s obtained from SIPP data are purely exogenous and orthogonal to all other structural shocks ν_{t+j} in the economy.

While this method allows for an identification that is fully consistent with our model, where all uncertainty fluctuations are exogenous, this identification strategy is arguably not very conservative. Therefore, we present additional evidence based on two alternative identification schemes.

Our baseline scheme can be understood as ordering income risk first in a Cholesky-identified structural vector autoregression (SVAR). Our first robustness check therefore takes the opposite extreme assumption and assumes that none of the variables in Figure 3 except for income risk itself reacts to an income risk shock, that is, we estimate

$$\begin{aligned} X_{t+j} = {} & \beta_{j,0} + \beta_{j,\varepsilon}\varepsilon_t^s/\sigma_s + \beta_{j,t}t + \beta_{j,X_t}\mathbf{X}_t + \beta_{j,X_{t-1}}\mathbf{X}_{t-1} + \beta_{j,s}s_{t-1} + \nu_{t+j}, \\ & j = 0, \dots, 12. \end{aligned} \tag{S25}$$

Results can be found in Figures S3 and S4. The estimated output response is slightly smaller and all responses are somewhat more delayed as, by construction, the immediate impact is zero for output and its components, for measured productivity, real wages, and the government's policy variables. Still we find that the liquidity of household portfolios

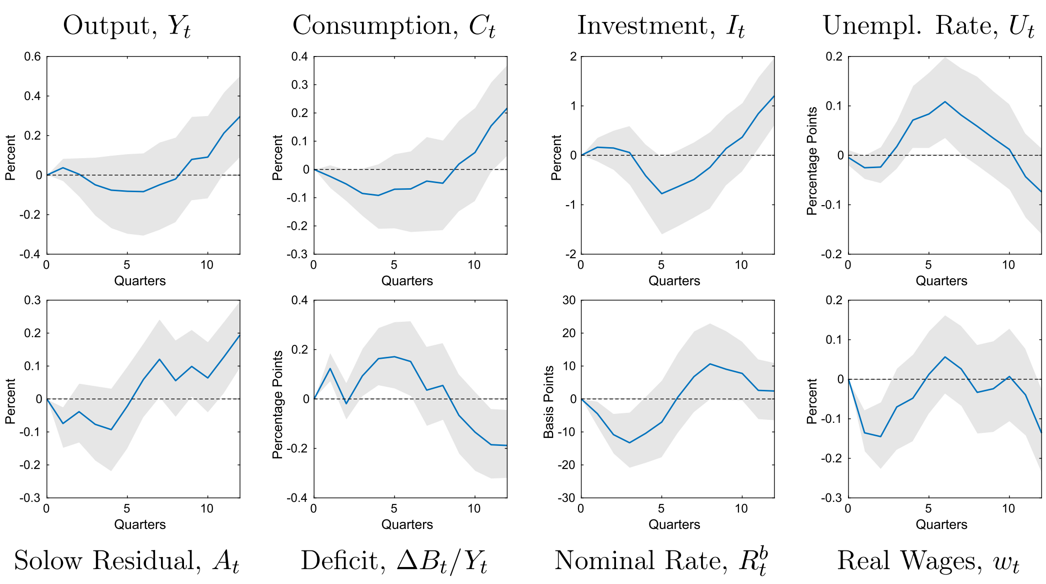

FIGURE S3.—Empirical response to household income risk shock: alternative identification. Estimated response of $\mathbf{X}_{t+j}$, $j = 0, \dots, 12$, where $\mathbf{X}_t = [Y_t, C_t, I_t, A_t, \Delta B_t/Y_t, w_t, R_t^b]$, to the estimated shocks to household income risk, ε_t^s. The regressions control for the current and lagged state of the economy, $\mathbf{X}_t$ and $\mathbf{X}_{t-1}$, respectively, and lagged levels of income risk s_{t-1}. The nominal rate is the 3-month T-bill rate. Bootstrapped 66% confidence bounds are shown in gray (block bootstrap).

increases on impact. The results for house prices and the liquidity premium are slightly more mixed.

Our second alternative identification scheme is somewhat in-between the baseline and the first alternative scheme. In line with the practice to estimate various small SVARs, we estimate the local projection, controlling for all lagged variables and only for current output and the current value of the variable of interest. This is a more parsimonious speci-

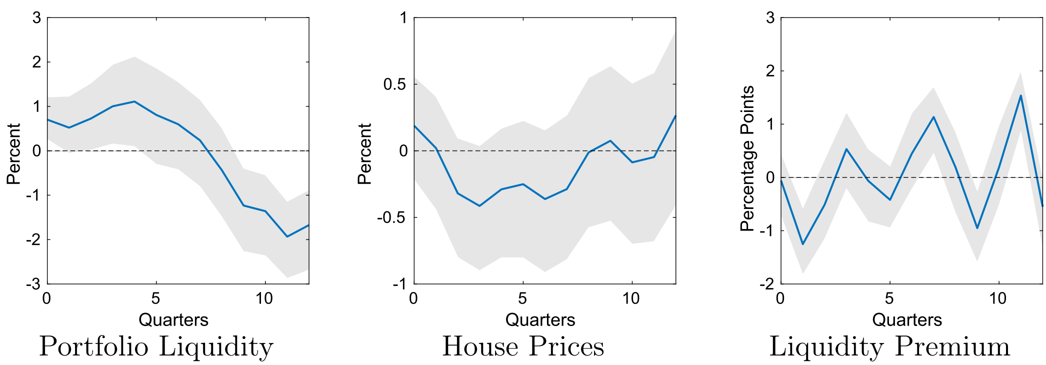

FIGURE S4.—Response of household portfolios, house prices, and the liquidity premium to household income risk shock: alternative identification. Estimated response of the liquidity of household portfolios, the price of houses (Case–Shiller S&P Index), and the difference between the return on housing and the nominal rate (liquidity premium) to income risk using local projections. The set of control variables is as in Figure S3. Bootstrapped 66% confidence bounds are shown in gray (block bootstrap).

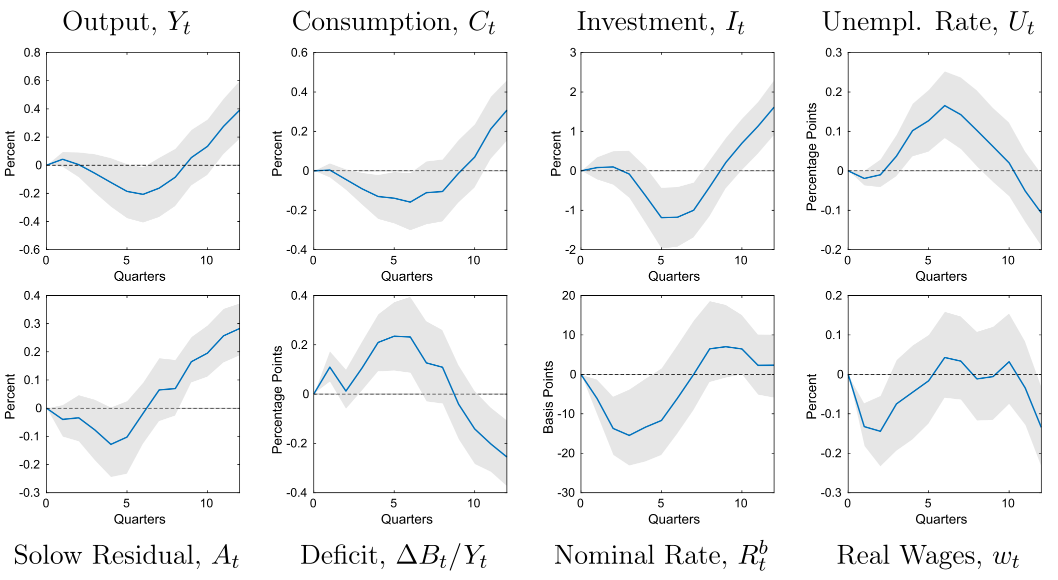

FIGURE S5.—Empirical response to household income risk shock: alternative identification. Estimated response of $\mathbf{X}_{t+j}$, $j = 0, \dots, 12$, where $\mathbf{X}_t = [Y_t, C_t, I_t, A_t, \Delta B_t/Y_t, w_t, R_t^b]$, to the estimated shocks to household income risk, ε_t^s. The regressions control for current output, the current value of the variable of interest, and the lagged state of the economy, X_t, Y_t, and $\mathbf{X}_{t-1}$, respectively, and lagged levels of income risk s_{t-1}. The nominal rate is the 3-month T-bill rate. Bootstrapped 66% confidence bounds are shown in gray (block bootstrap).

fication, but it comes at the cost of identifying a slightly different shock in each regression. Results can be found in Figures S5 and S6. Also here results are very much in line with our baseline treatment of the data.

FIGURE S6.—Response of household portfolios, house prices, and the liquidity premium to household income risk shock: alternative identification. Estimated response of the liquidity of household portfolios, the price of houses (Case–Shiller S&P Index), and the difference between the return on housing and the nominal rate (liquidity premium) to income risk using local projections. The set of control variables is as in Figure S5. Bootstrapped 66% confidence bounds are shown in gray (block bootstrap).

TABLE SV

BUSINESS CYCLE STATISTICS DATA/MODEL[a]

	GDP	C	I	Deficit
Time Series Standard Deviation of ... (%)				
Data	1.38	0.98	6.28	1.33
Model TFP	1.38	0.75	6.28	1.33
Model uncertainty	0.29	0.63	1.33	0.52
Correlation With GDP				
Data	1.00	0.92	0.92	−0.76
Model TFP	1.00	0.87	0.96	−0.86
Model uncertainty	1.00	1.00	0.74	0.08

[a] Real GDP, consumption (C), and investment (I) are in logs. Net government savings (deficit) is given as a fraction of GDP. All data are Hodrick–Prescott (HP) filtered with $\lambda = 1600$. The model refers to the baseline model with TFP or income risk shocks only.

APPENDIX I: UNCONDITIONAL BUSINESS CYCLE STATISTICS

Table SV reports unconditional business cycle statistics for the quarterly U.S. data we use in the empirical sections and for our model.

APPENDIX J: MODEL EXTENSIONS

J.1. *Importance of Illiquid Assets*

Even when all assets are liquid, households will decrease their consumption demand for precautionary motives when income uncertainty rises. We have seen in Section 3 that the presence of illiquid assets introduces a portfolio adjustment in response to the uncertainty shock, which augments the increase in demand for liquid assets.

To show the importance of the portfolio adjustment channel also in our full model, we solve a version of the model where all assets are liquid. In this case, the household portfolio position between the two assets is indeterminate in the steady state as long as the expected returns of both assets are equal,

$$E_t\left[r_{t+1} + \frac{q_{t+1}}{q_t}\right] = E_t\left[\frac{R^b_{t+1}}{\pi_{t+1}}\right], \tag{S26}$$

and in equilibrium they must be equal for households to be willing to hold a positive amount of both assets.

Since our solution method linearizes the problem in the presence of aggregate shocks, the portfolio problem remains indeterminate. Therefore, we assume that all households hold the same bond-to-capital ratio, which is in the aggregate determined by (S26) and by the supply of government bonds.

We recalibrate the discount factor to match again the capital-to-output and bond-to-capital ratio as our baseline model. We also recalibrate the aggregate capital adjustment cost parameter to match again the relative investment volatility in response to TFP shocks but keep all other household, policy, and technological parameters as in our baseline.

The response to an income risk shock changes drastically as Figure S7 shows. Output falls much less than in our baseline, but most important, we get an investment boom. The level of price stickiness is not sufficiently strong to drive down the capital return such that, as central bank interest rates fall, the capital stock and hence investment must go up in equilibrium to equate expected capital and bond returns.

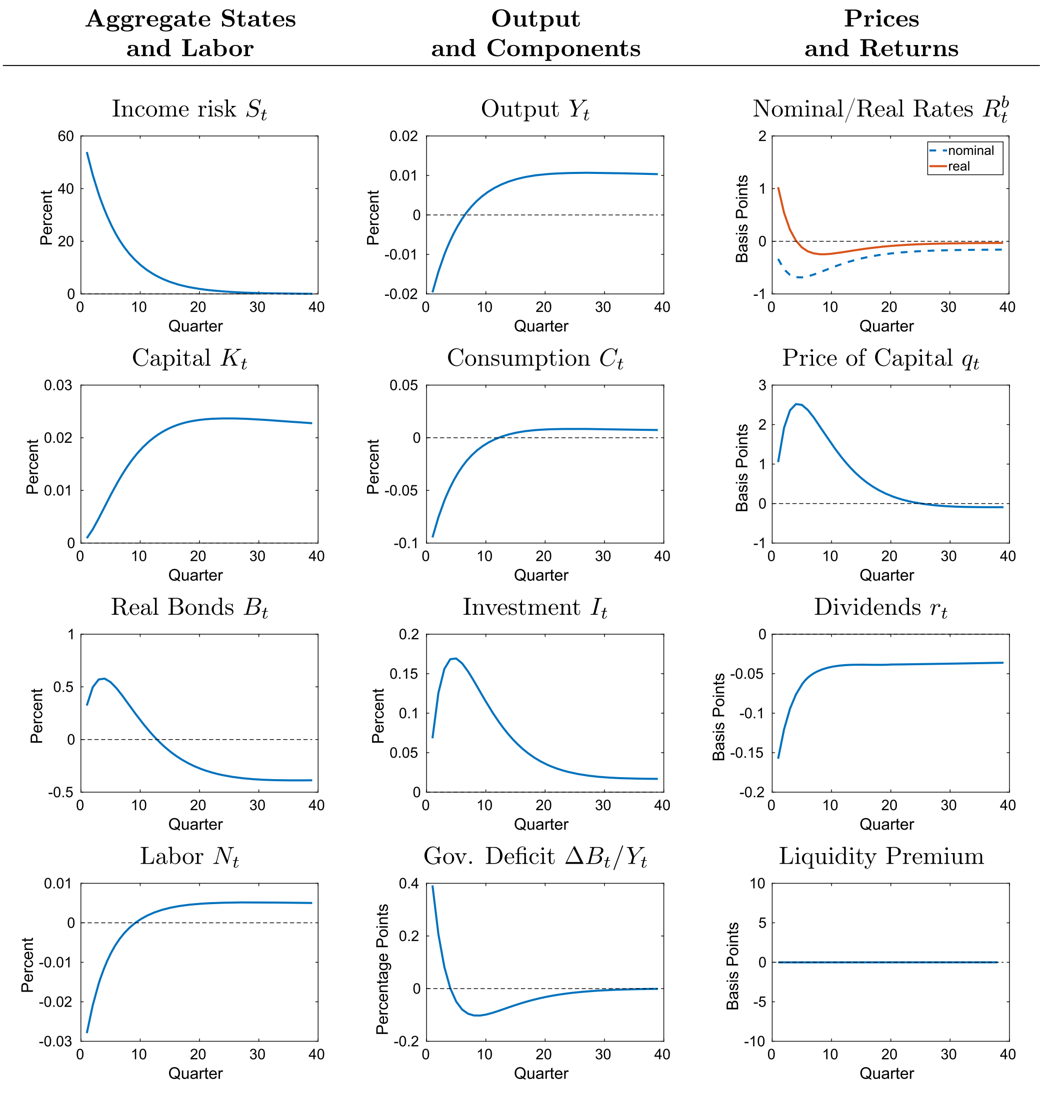

FIGURE S7.—Aggregate response to household income risk with liquid capital. *Notes*: The liquidity premium is $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t\pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

J.2. *The Role of Price Stickiness*

Next we assess the importance of price stickiness for our results. For this purpose, we set the price adjustment costs to (virtually) zero. We find that the output effects of income risk shocks are negligible in this specification; see Figure S8. The income risk shock in our model creates a slump in private demand, but without any price stickiness, this is undone by price-level movements. Inflation falls and, given the Taylor rule, the real interest rate on liquid assets falls too. Households then shift their portfolios back to the illiquid asset that has a higher relative return now. In summary, price stickiness is essential for the negative output movement that we find.

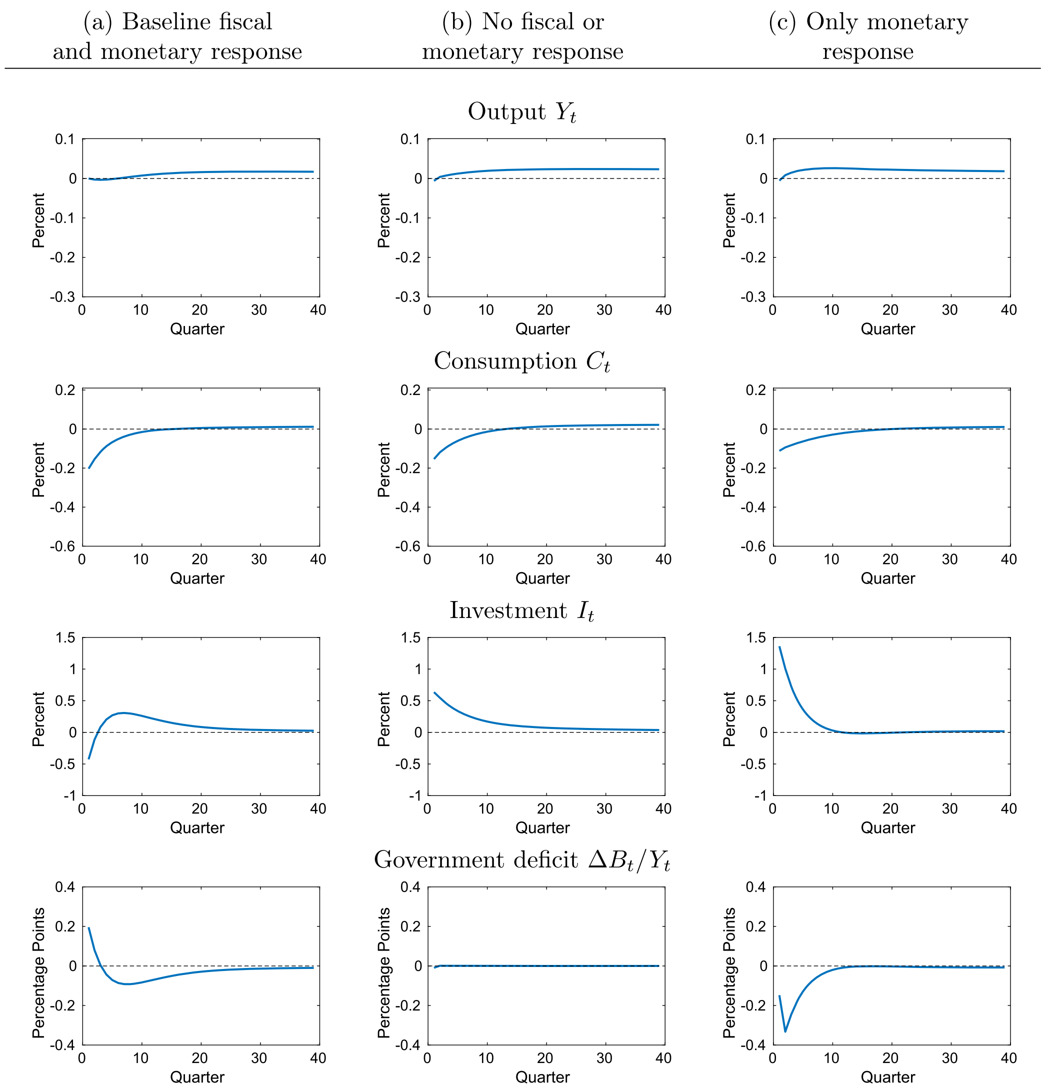

FIGURE S8.—Aggregate response to household income risk shock with flexible prices. *Notes*: The liquidity premium is $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R^b_t}{E_t\pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized. Columns (a), $\gamma_\pi = 1.5$, $\gamma_T = 0.5075$, $\rho_B = 0.86$, $\theta_\pi = 1.25$, $\rho_R = 0.8$; columns (b), $\gamma_\pi = 0$, $\gamma_T = 0$, $\rho_B = 1$, $\theta_\pi = 0$, $\rho_R = 1$; columns (c), $\gamma_\pi = 0$, $\gamma_T = 0$, $\rho_B = 0.86$, $\theta_\pi = 1.25$, $\rho_R = 0.8$.

The exact path of aggregates depends on the fiscal and monetary rules in place. With no fiscal response to inflation, the path of aggregates is very similar to that under perfect monetary stabilization; see Figure S8, columns (b) and (c). When government spending increases in response to falling inflation, the effect on and through the real amount of government debt is different; see Figure S8, column (a). Outstanding government debt increases more persistently and crowds out private consumption and investment on impact.

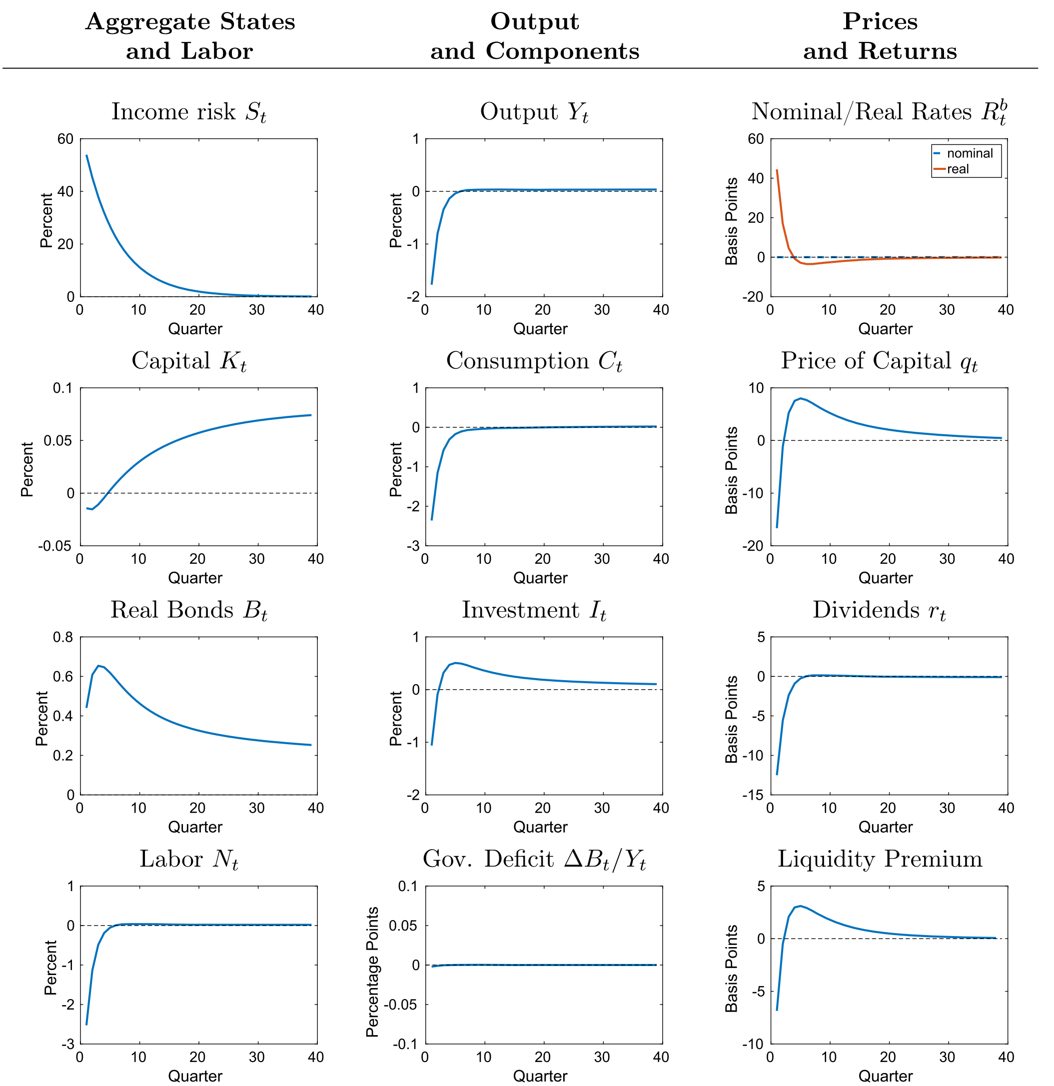

FIGURE S9.—Aggregate response to household income risk shock without stabilization. *Notes*: The liquidity premium is $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R^b_t}{E_t\pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized. All stabilization policy parameters are set to zero; $\theta_\pi = 0$, $\gamma_\pi = 0$, $\gamma_\mathcal{T} = 0$, and $\rho_B = 1$.

J.3. *Response of the Model Without Stabilization*

To capture what happens if governments do not stabilize (e.g., at the zero lower bound (ZLB)), we produce the impulse responses for our baseline calibration with an interest rate peg and no fiscal stabilization; see Figure S9. Output, consumption, and investment fall drastically.

J.4. *Introducing Asset-Backed Securities*

A potential limitation of our model is that it does not include mortgages—an important way for the banking sector to create liquid assets out of illiquid investments. To capture the effect of mortgages, we assume that any newly created capital good is partly credit financed, that is, any investment creates an illiquid asset and a liquid asset-backed security at the same time. Let ζ be the number of bonds issued as mortgages per unit of capital. We assume that the fraction ζ cannot be adjusted by the household sector. However, households can buy back the mortgages originated from "their" illiquid assets.

For this purpose, we model a wedge $\underline{R}$ between the borrowing and the lending rate on mortgages due to the costs of intermediation. The rate paid to lenders is the central bank's interest rate R_t^b, and the rate borrowers pay is $R_t^b + \underline{R}$. This means that, net of the payments on the asset-backed securities, the dividend stream from illiquid assets decreases to $r_t = F_K(K, N) - \delta - \zeta(R_t^b + \underline{R})$. However, the price of illiquid assets also decreases, which now is $q_t = 1 - \zeta + \phi \frac{K_{t+1} - K_t}{K_t}$ in equilibrium, because the producer of each unit of illiquid assets can sell ζ units of asset-backed securities for each unit of the illiquid capital good in addition to that good itself.

To allow households to adjust the extent to which they effectively draw their mortgages, we assume that the borrowing wedge does not apply to securities backed by assets the household itself owns. This means that the household now faces three marginal interest rates on liquid assets, such that the total payout on liquid assets is

$$\text{repayment} = \begin{cases} R_t^b(b_{it} - \zeta k_{it}) + \zeta k_{it}\big(R_t^b + \underline{R}\big) & \text{if } b_{it} \geq \zeta k_{it}, \\ \big(R_t^b + \underline{R}\big) b_{it} & \text{if } 0 < b_{it} < \zeta k_{it}, \\ \big(R_t^b + \overline{R}\big) b_{it} & \text{if } b_{it} < 0. \end{cases}$$

The highest interest rate applies to unsecured borrowing $b < 0$. An intermediate interest rate applies if the household buys back securities originated from the illiquid asset it owns, $0 < b < \zeta k$, that is, that the household saves by paying back a mortgage. The lowest interest rate applies when the household accumulates liquid assets beyond those it has originated.

The bond market equilibrium condition then reads

$$\begin{aligned} &\zeta K_{t+1} + B_{t+1} \\ &\quad = \int \int \int_{b \geq \underline{B}} \big[\nu^* b_a^*\big(b, k, h; q_t, \pi_t, R_{t+1}^b\big) \\ &\qquad + \big(1 - \nu^*\big) b_n^*\big(b, k, h; q_t, \pi_t, R_{t+1}^b\big)\big]\, d\Theta_t(b, k, h), \end{aligned} \tag{S27}$$

where ζK_{t+1} is the amount of asset-backed securities in circulation. The market clearing condition for illiquid assets remains unchanged.

We have recalibrated the amount of government debt to keep the average portfolio liquidity unchanged when not counting securities held by the issuer. The ratio of mortgage liabilities of households to their net worth in the Flow of Funds (Table Z1-B.101) is roughly 10%. We set $\zeta = 10\%$ and calibrate $\underline{R} = 1\%$ p.a.

Figure S10 shows the impulse responses for our baseline calibration with asset-backed securities (ABS). Compared to our baseline scenario, the recessionary impact of uncertainty is larger, because the rebalancing of portfolios implies a decline in the supply of liquid assets as households reduce the stock of capital.

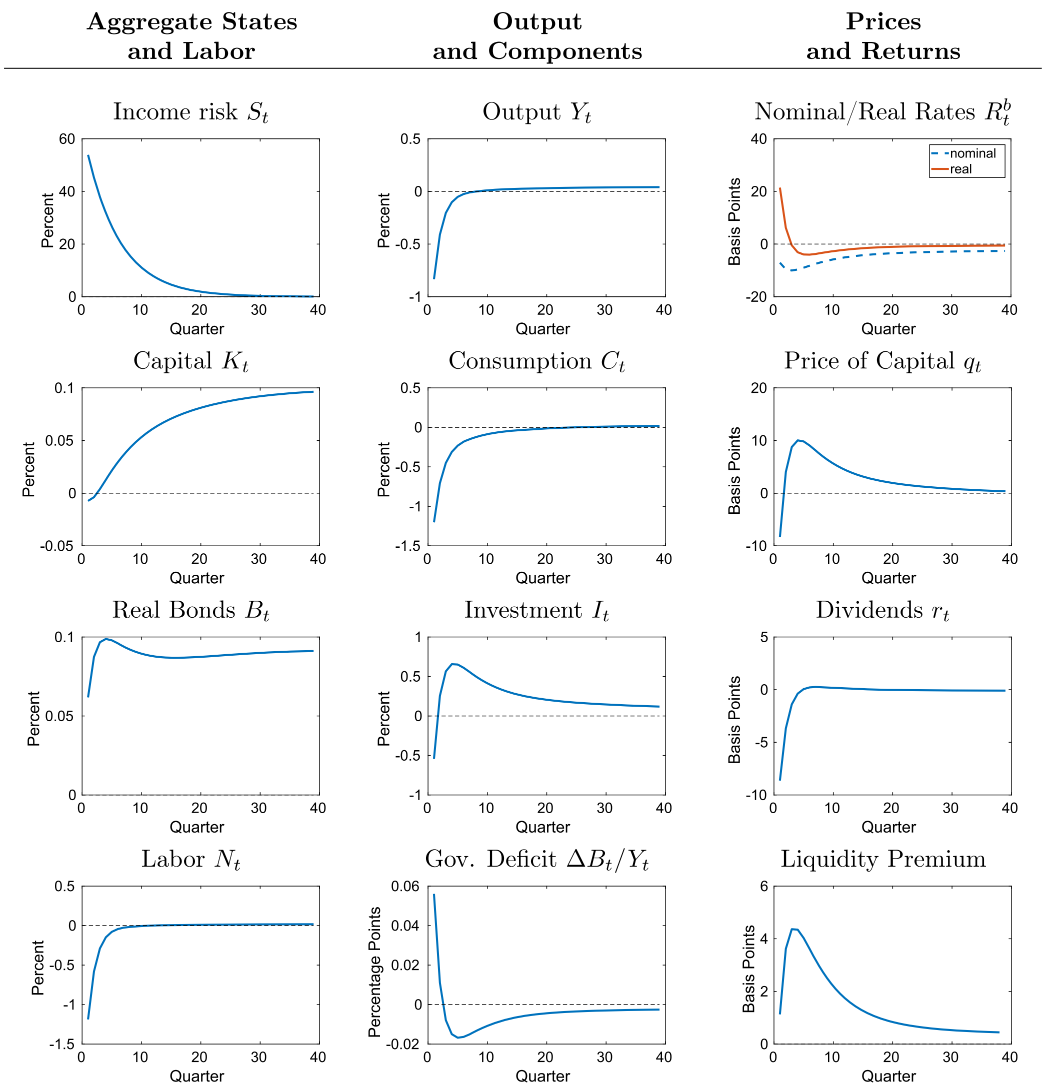

FIGURE S10.—Aggregate response to household income risk with asset-backed securities. *Notes*: The liquidity premium is $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t\pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

J.5. *Response of the Model to TFP Shocks*

Given our solution technique, it is straightforward to extend the model by other shocks. For our calibration, we use an extension with time-varying total factor productivity in production, such that $Y_t = A_t F(K_t, L_t)$, where A_t is total factor productivity and follows an autoregression (AR(1)) process in logs with a persistence of 0.95 and a standard deviation of 0.00965. We use this model variant to calibrate capital adjustment costs. Figure S11 shows the IRFs to a TFP shock.

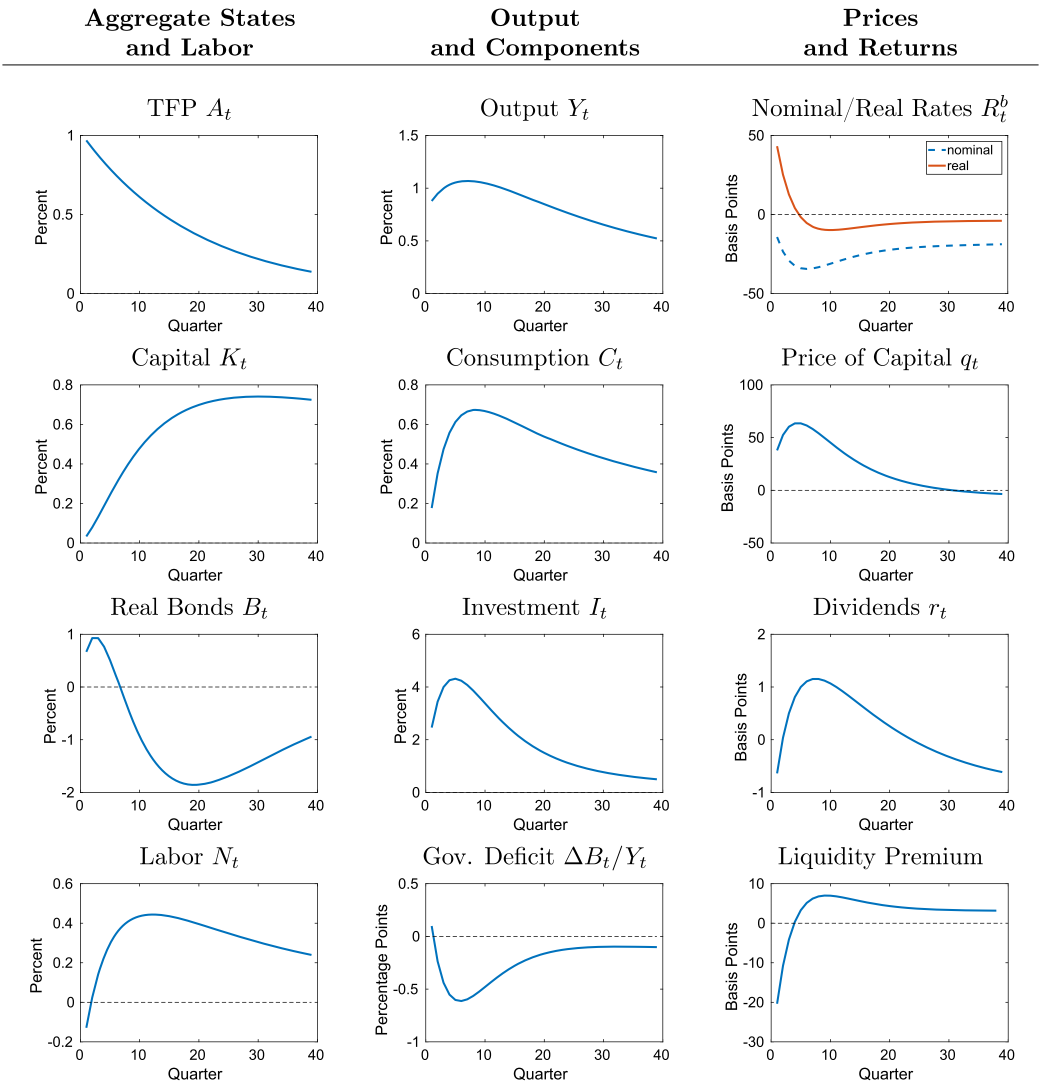

FIGURE S11.—Aggregate response to a TFP shock. *Notes*: The liquidity premium is $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in TFP. All rates (dividends, interest, liquidity premium) are *not* annualized.

APPENDIX K: INDIVIDUAL CONSUMPTION RESPONSES TO PERSISTENT AND TRANSITORY INCOME SHOCKS

For the model to provide a useful framework for welfare analysis, it is important that the model replicates the empirical evidence on consumption responses to persistent and transitory income shocks (in partial equilibrium). For this purpose, we consider the average consumption elasticity to a persistent increase in income and an increase in liquid assets proportional to income (transitory income shock). These two elasticities are key to understanding the consumption smoothing behavior of an incomplete markets model; see Kaplan and Violante (2010) and Blundell, Pistaferri, and Preston (2008). Table SVI provides these statistics for our model.

TABLE SVI

CONSUMPTION SMOOTHING IN MODEL AND DATA[a]

	Elasticity of Consumption to Transitory and Persistent Income Shocks	
	Data	Model
Transitory income change	0.05	0.05
Persistent income change	0.43	0.44

[a]Data correspond to Kaplan and Violante (2010).

The model replicates the fact that transitory income shocks are well insured, while persistent income shocks are much less insured. Given the below unit-root autocorrelation of persistent income, our model predicts persistent income shocks to be somewhat better insured in comparison to assuming permanent shocks.

APPENDIX L: ROBUSTNESS CHECKS

For the risk aversion parameter and the Frisch elasticity of labor supply, we take standard values from the literature as there is no direct counterpart in the data. To account for this calibration strategy, we check the robustness of our findings with respect to the assumed parameter values. We do so by varying one parameter at a time while recalibrating to match the moments of Table 3 by adjusting the discount factor, the mean and variance of the distribution of adjustment costs, and the borrowing penalty.

We find our results are qualitatively robust to all the considered parameter variations. The impulse response functions for output, consumption, investment, and the liquidity premium are displayed in Figure S12. When we reduce the risk aversion, households do not decrease investment demand as much as in the baseline and, conversely, the liquidity premium increases less. The illiquidity of capital is less important to households. An increase in the inverse Frisch elasticity is very similar to an increase in risk aversion. As can be seen from the household budget constraint, when labor supply is maximized out, the lower is the inverse Frisch elasticity, the less do the resources the household has for composite consumption fluctuate with productivity h. The recalibration of the illiquidity of capital only partially offsets this, because the return movements through central bank policy become relatively more important when households are effectively less affected by changes in income risk (either because they are less risk averse or better insured through the labor market).

Furthermore, we set the discount factor used in the firms' maximization problem to the median stochastic discount factor for entrepreneurs, taking into account that an entrepreneur household becomes a worker household with a certain probability. The resulting discount factor is roughly 77% quarterly. Such an extreme discounting has some impact on results, making the recessionary effects of income risk larger because future expected deflation does help less to stabilize output, hence rendering monetary policy with interest rate smoothing less effective.

As a second robustness check, we vary the utility costs of portfolio adjustment. First, we make the adjustment probability more reactive to the value gained from adjustment by lowering the variance of the logistic distribution from which households draw the adjustment cost. Second, we consider a case of almost fixed adjustment probabilities by increasing the variance of the logistic distribution drastically. Third, we lower the mean of the logistic distribution such that the average adjustment probability goes up to 20% (and

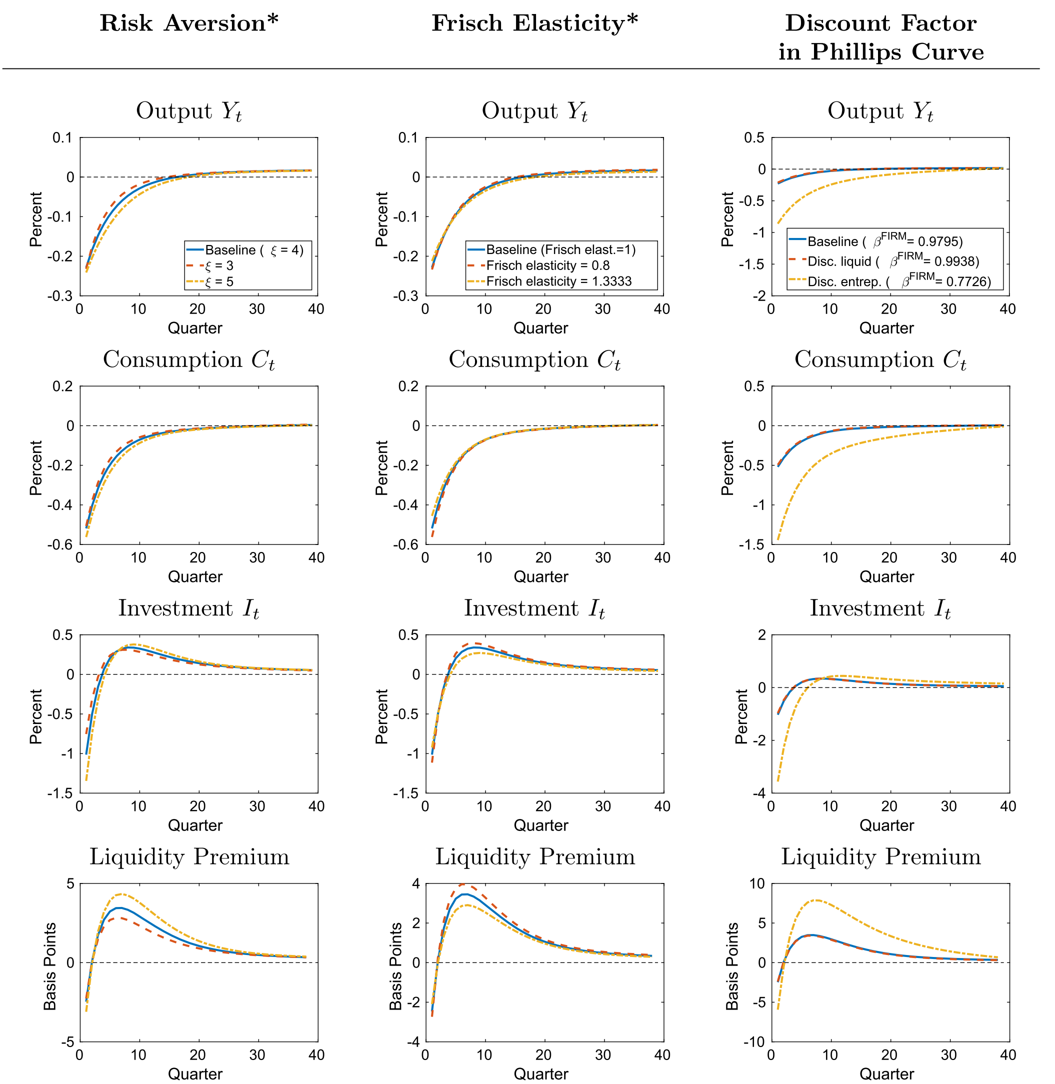

FIGURE S12.—Robustness A: Aggregate response to household income risk shock. *Notes*: The liquidity premium is $\frac{Eq_{t+1}+r_t}{q_t} - \frac{R_t^b}{\pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized. The asterisk (*) indicates recalibration to match the moments of Table 3 by adjusting the discount factor, the mean and variance of the distribution of adjustment costs, and the borrowing penalty.

the average portfolio liquidity falls). All three cases show results qualitatively similar to our baseline; see Figure S13.

Making adjustment more state dependent yields quantitatively very similar results: the investment response is only slightly muted. When adjustment probabilities are fully exogenous, the investment response is slightly larger. When the illiquid asset is more liquid, the effect of a shock to income risk becomes stronger in the short run, but also shorter lived. The economic intuition seems to be as follows. When the illiquid asset is very liquid, the demand for liquid assets becomes smaller, but also less elastic to the return differences

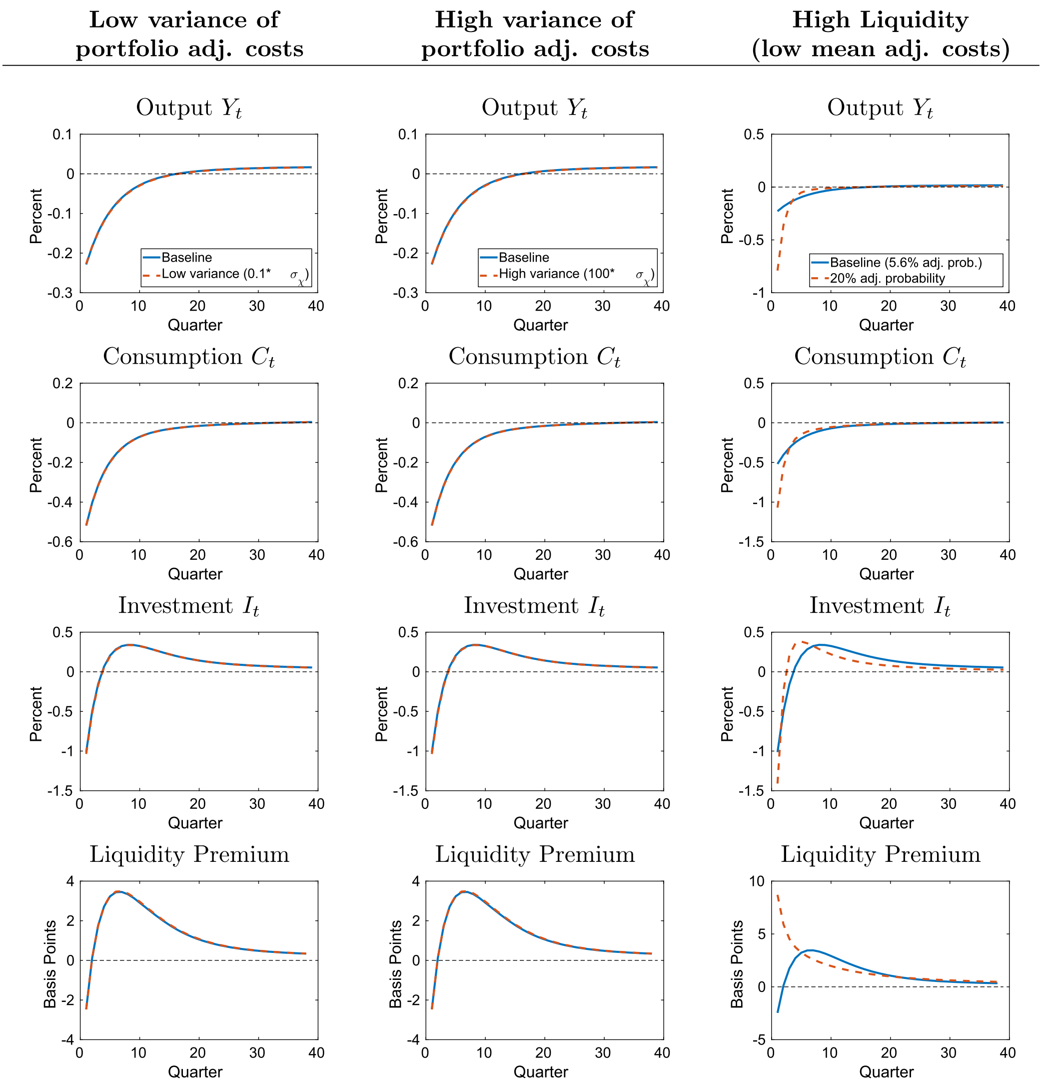

FIGURE S13.—Robustness B: Aggregate response to household income risk shock. *Notes*: The liquidity premium is $\frac{E_t q_{t+1}+r_t}{q_t} - \frac{R_t^b}{E_t \pi_{t+1}}$. Impulse responses to a 1 standard deviation increase in the variance of income shocks. All rates (dividends, interest, liquidity premium) are *not* annualized.

between the assets. Therefore, the central bank's intervention that cuts rates can stabilize less. The supply of liquid assets itself becomes more important, but with a small stock of outside liquidity, the same relative growth in government bonds stabilizes aggregate demand less. However, as soon as the stock of liquid funds has increased sufficiently, households start to invest in illiquid assets again.

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