Econometrica Supplementary Material

SUPPLEMENT TO "EXCESS IDLE TIME"

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APPENDIX S.A: UNDERSTANDING THE DYNAMICS OF EXIT

LIQUIDITY is an elusive concept with various dimensions. Our focus, in EXcess Idle Time, is on execution costs (i.e., on "tightness" in the language of Kyle (1985)) rather than on price impacts ("depth" or "resiliency"). This section further illustrates the ability of EXIT to operate as an (il)liquidity measure. We compare it to true execution costs. We do so by varying the sampling frequency Δ, for a given threshold, and by varying the threshold ξ, for a given sampling frequency.

Specifically, we conduct simulations in which the data generating process features a *time-varying* modification on the baseline model in Section 4: we allow for changing execution costs (c) across days. The costs evolve according to the AR(1) model:

$$c_t = c_0 + \phi c_{t-1} + \sigma_c \varepsilon_t, \tag{A.1}$$

where the ε_t's are i.i.d. normal. The model parameters are set as being equal to those estimated in Section 7 and reported in Table II: $\delta = 0.010$, $s = 1.905 \cdot 10^{-4}$, $\sigma = 0.927\%$, and $\mathcal{I} = 0.794$. The parameters of the autoregression in Eq. (A.1) are estimated using the daily time series of c estimates ($c = s + f$).

We compute EXIT using 1-minute returns ($\Delta = 1/420$), 5-minute returns ($\Delta = 1/84$), and 10-minute returns ($\Delta = 1/42$). For each frequency, we choose three thresholds ξ, namely $\frac{1}{20}\sigma\Delta^{1/2+1/100}$, $\frac{1}{5}\sigma\Delta^{1/2+1/100}$, and $\frac{1}{2}\sigma\Delta^{1/2+1/100}$. Figure S.A.1 reports results for all nine pairs (Δ, ξ).

For any of the chosen thresholds, the correlation between EXIT and c increases with increases in sampling frequency. Consider the smaller threshold value of $\frac{1}{20}\sigma\Delta^{1/2+1/100}$. The 1- and 5-minute frequencies lead to similar results, with correlations higher than 80%. At 10 minutes, however, the correlation is a lower figure of about 65%. Similar findings apply to alternative threshold choices, with rapid decays in correlation associated with excessively sparse intra-daily sampling.

We note that the empirical frequency (calculated *without* a threshold) of daily zero returns (*zeros*) has been employed successfully in empirical finance work as an (il)liquidity proxy (e.g., Lesmond (2005), Bekaert, Harvey, and Lundblad (2007), Naes, Skjeltorp, and

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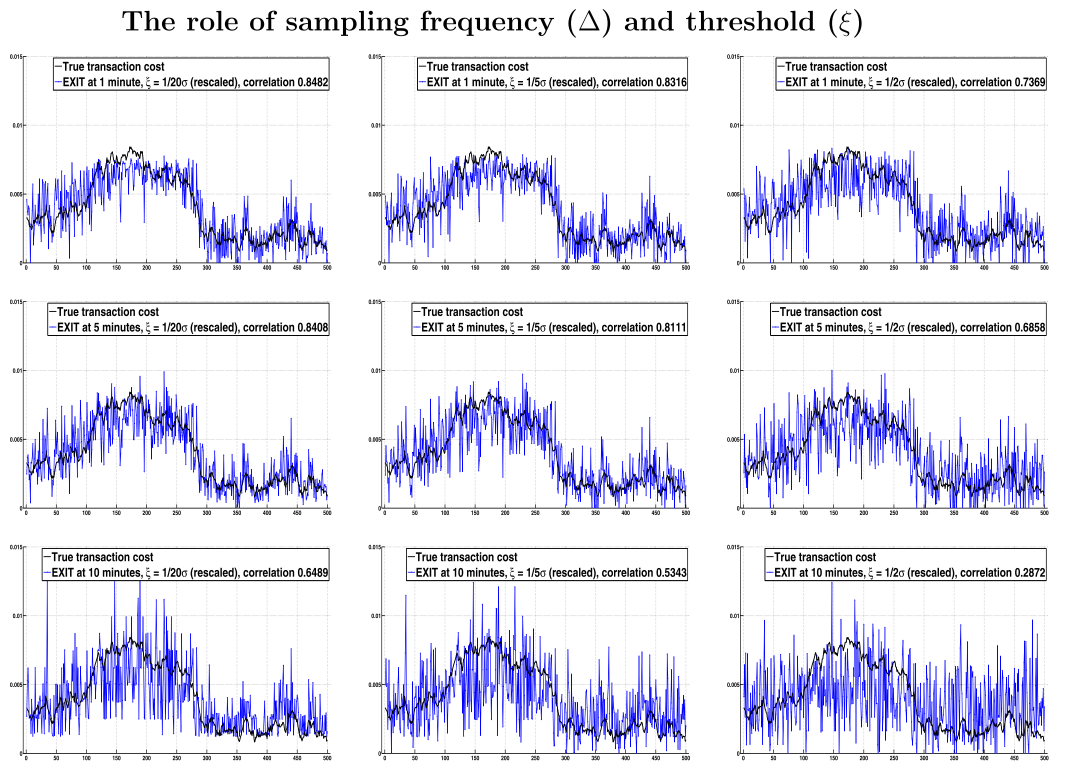

FIGURE S.A.1.—We simulate the model in Section 4 with Eq. (A.1). EXIT is computed using three sampling frequencies—1, 5, and 10 minutes—and three thresholds—$\frac{1}{20}\sigma\Delta^{1/2+1/100}$, $\frac{1}{5}\sigma\Delta^{1/2+1/100}$, and $\frac{1}{2}\sigma\Delta^{1/2+1/100}$. It is then plotted along with true execution costs c. The execution costs are assumed to have the following dynamics across days: $c_t = c_0 + \phi c_{t-1} + \sigma_c \varepsilon_t$, with the ε_ts i.i.d. normal. The autoregressive parameters are estimated on the time series of $c = s + f$ estimates in Section 7. In particular, $c_0 = 3.44 \cdot 10^{0.5}$, $\phi = 0.989$, and $\sigma_c = 2.22 \cdot 10^{-4}$. The constants δ, s, σ, and $\mathcal{I}$ are (mean, daily) estimates from data (see Table 2): $\delta = 0.010$, $s = 1.905 \cdot 10^{-4}$, $\sigma = 0.927\%$, and $\mathcal{I} = 0.794$.

Odegaard (2011)). These experiments indicate that the use of intra-daily data, a defining feature of the measure we propose, leads to estimates that are less noisy and, therefore, more informative about true execution costs than low-frequency (daily) *zeros*. The benefit of using high-frequency information to extract a clearer signal in the measurement of (il)liquidity is intuitive and analogous to what is found in the realized variance literature (see, e.g., Andersen and Benzoni (2009), for a review).

Figure S.A.1 also shows that, for any of the chosen frequencies, a smaller threshold leads to a higher correlation between EXIT and c.

Using a model specification and estimates justified in Section 7, these results are suggestive of the importance of high(er) frequencies, as well as small(er) thresholds, in the computation of daily EXIT estimates. In particular, the choice of 5-minute sampling and a threshold ξ computed as $\frac{1}{20}\sigma\Delta^{1/2+1/100}$ appears, in light of these findings, reasonable.

A thorough evaluation of these conclusions, along with issues of implementation, is contained in Section 5 of the main text.

APPENDIX S.B: ROBUST FINITE SAMPLE IMPLEMENTATION

PROOF OF THEOREM 3: We largely follow Newey and West (1987). Write

$$\begin{aligned}
&\widehat{H}_{T/n} - H_{T/n} \\
&= \sum_{l=-m_n}^{m_n} \left(\frac{m_n - |l|}{m_n}\right)(\widehat{\gamma}_{l,T/n} - \gamma_{l,T/n}) + \sum_{l=-m_n}^{m_n} \left[\left(\frac{m_n - |l|}{m_n}\right) - 1\right]\gamma_{l,T/n} - \sum_{|l|>m_n} \gamma_{l,T/n} \\
&= \sum_{l=-m_n}^{m_n} \left(\frac{m_n - |l|}{m_n}\right)(\widehat{\gamma}_{l,T/n} - \gamma^*_{l,T/n}) + \sum_{l=-m_n}^{m_n} \left(\frac{m_n - |l|}{m_n}\right)(\gamma^*_{l,T/n} - \gamma_{l,T/n}) \\
&\quad + \sum_{l=-m_n}^{m_n} \left[\left(\frac{m_n - |l|}{m_n}\right) - 1\right]\gamma_{l,T/n} - \sum_{|l|>m_n} \gamma_{l,T/n} \\
&= \mathrm{I}_{T/n} + \mathrm{II}_{T/n} + \mathrm{III}_{T/n} + \mathrm{IV}_{T/n},
\end{aligned}$$

where $\gamma^*_{l,T/n} = \frac{n-|l|}{n}\gamma_{l,T/n}$. Now,

$$|\mathrm{IV}_{T/n}| \le \sum_{|l|>m_n} |\gamma_{l,T/n}| = \sum_{l=-\infty}^{\infty} |\gamma_{l,T/n}| - \sum_{l=-m_n}^{m_n} |\gamma_{l,T/n}| \to 0$$

as $n \to \infty$ and $m_n \to \infty$, given Assumption 1. Next, write

$$|\mathrm{III}_{T/n}| \le \sum_{l=-m_n(l\neq 0)}^{m_n} \left|\left(\frac{m_n - |l|}{m_n}\right) - 1\right| |\gamma_{l,T/n}| = \sum_{l=-m_n(l\neq 0)}^{m_n} \frac{|l|}{m_n}|\gamma_{l,T/n}|.$$

Because of the absolute convergence of the autocovariances (Assumption 1), we have $\gamma_{l,T/n} = O(1/|l|^{1+\epsilon})$, with $\epsilon > 0$ arbitrarily small. This, however, implies that

$$|\mathrm{III}_{T/n}| \le \sum_{l=-m_n(l\neq 0)}^{m_n} \frac{|l|}{m_n}|\gamma_{l,T/n}| \sim \frac{1}{m_n} \sum_{l=-m_n(l\neq 0)}^{m_n} O\left(\frac{1}{|l|^{\epsilon}}\right) \to 0,$$

where convergence to zero is guaranteed by the Cesàro mean theorem (given $\frac{1}{|l|^{\epsilon}} \to 0$ as $|l| \to \infty$). Similarly,

$$\begin{aligned}
|\mathrm{II}_{T/n}| &\le \sum_{l=-m_n}^{m_n} \left|\frac{m_n - |l|}{m_n}\right| \left|\gamma^*_{l,T/n} - \gamma_{l,T/n}\right| \le \sum_{l=-m_n}^{m_n} \left|\frac{m_n - |l|}{m_n}\right| \frac{|l|}{n}|\gamma_{l,T/n}| \\
&\le \frac{m_n}{n} \sum_{l=-m_n(l\neq 0)}^{m_n} \frac{|l|}{m_n}|\gamma_{l,T/n}| + \sum_{l=-m_n(l\neq 0)}^{m_n} \frac{|l|}{m_n}\frac{|l|}{n}|\gamma_{l,T/n}| \\
&\big(\text{since } |l|/m_n \le 1\big) \le 2\frac{m_n}{n} \sum_{l=-m_n(l\neq 0)}^{m_n} \frac{|l|}{m_n}|\gamma_{l,T/n}|.
\end{aligned}$$

This bound, however, converges to zero faster than the bound on $\mathrm{III}_{T/n}$ since $\frac{m_n}{n} \to 0$ by assumption. Next, we notice that

$$\begin{aligned}
\mathrm{I}_{T/n} &= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\left(\widehat{\gamma}_{l,T/n}-\gamma^*_{l,T/n}\right)\\
&= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\left(\frac{1}{n}\sum_{j=1}^{n-|l|}(e_{j+|l|,T/n,\xi_{T/n}}-\mathrm{EXIT}_{T/n})(e_{j,T/n,\xi_{T/n}}-\mathrm{EXIT}_{T/n})-\gamma^*_{l,T/n}\right)\\
&= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\\
&\quad\times\left(\frac{1}{n}\sum_{j=1}^{n-|l|}\big(e_{j+|l|,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)\big(e_{j,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)-\gamma^*_{l,T/n}\right)\\
&\quad+\big(\mathbb{E}(\mathrm{EXIT}_{T/n})-\mathrm{EXIT}_{T/n}\big)\sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\frac{1}{n}\sum_{j=1}^{n-|l|}\big(e_{j+|l|,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)\\
&\quad+\big(\mathbb{E}(\mathrm{EXIT}_{T/n})-\mathrm{EXIT}_{T/n}\big)\sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\frac{1}{n}\sum_{j=1}^{n-|l|}\big(e_{j,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)\\
&\quad+\big(\mathrm{EXIT}_{T/n}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)^2\sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\left(\frac{n-|l|}{n}\right).
\end{aligned}$$

Write the first term as

$$\begin{aligned}
\mathrm{I}^1_{T/n} &= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\left(\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n}\right)\\
&= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\\
&\quad\times\left(\frac{1}{n}\sum_{j=1}^{n-|l|}\big(e_{j+|l|,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)\big(e_{j,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)-\gamma^*_{l,T/n}\right)\\
&= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\frac{1}{n}\sum_{j=1}^{n-|l|}\Big(\big(e_{j+|l|,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)\\
&\quad\times\big(e_{j,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n})\big)-\gamma_{l,T/n}\Big)\\
&= \sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)\frac{1}{n}\sum_{j=1}^{n-|l|}Z_{j,|l|,T/n},
\end{aligned}$$

where $Z_{j,|l|,T/n}=(e_{j+|l|,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n}))(e_{j,T/n,\xi_{T/n}}-\mathbb{E}(\mathrm{EXIT}_{T/n}))-\gamma_{l,T/n}$.

We have

$$\begin{aligned}
\mathbb{P}\{|\mathrm{I}^1_{T/n}| > \varepsilon\} &= \mathbb{P}\left\{\left|\sum_{l=-m_n}^{m_n}\left(\frac{m_n-|l|}{m_n}\right)(\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n})\right| > \varepsilon\right\} \\
\left(\text{since } \left|\frac{m_n-|l|}{m_n}\right| \le 1\right) &\le \mathbb{P}\left\{\sum_{l=-m_n}^{m_n}|\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n}| > \varepsilon\right\} \\
&\le \mathbb{P}\left\{\exists l : |\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n}| > \frac{\varepsilon}{2m_n+1}\right\} \\
&\le \sum_{l=-m_n}^{m_n}\mathbb{P}\left\{|\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n}| > \frac{\varepsilon}{2m_n+1}\right\} \\
&\le \frac{(2m_n+1)^2}{\varepsilon^2}\sum_{l=-m_n}^{m_n}\mathbb{E}(\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n})^2.
\end{aligned}$$

By an application of Cauchy–Schwarz inequality, Assumption 3 implies $\mathbb{E}(|Z_{j,|l|,T/n}|^{2(r+\delta)}) < C_8$ for all j and l. Given Assumptions 2 and 3, we obtain

$$\mathbb{E}(\widetilde{\gamma}_{l,T/n}-\gamma^*_{l,T/n})^2 = \mathbb{E}\left(\frac{1}{n}\sum_{j=1}^{n-|l|}Z_{j,|l|,T/n}\right)^2 \le \frac{C_9 n(|l|+1)}{n^2},$$

using Lemma 6.19 in White (1984) (see, also, Eq. (10) in Newey and West (1987)). Hence,

$$\begin{aligned}
\mathbb{P}\{|\mathrm{I}^1_{T/n}| > \varepsilon\} &\le \frac{(2m_n+1)^2}{\varepsilon^2}\sum_{l=-m_n}^{m_n}\frac{C_9(|l|+1)}{n} \\
&\le C_9\frac{(2m_n+1)^2}{n\varepsilon^2}\left[\sum_{l=-m_n}^{m_n}1+\left(\sum_{l=0}^{m_n}l+\sum_{l=-m_n}^{0}-l\right)\right] \\
&= C_9\frac{(2m_n+1)^2}{n\varepsilon^2}(2m_n+1)+C_9\frac{(2m_n+1)^2}{n\varepsilon^2}\left(2\left(\frac{m_n^2+m_n}{2}\right)\right).
\end{aligned}$$

Thus, $\mathrm{I}^1_{T/n} \xrightarrow{p} 0$, as $n \to \infty$, provided $\frac{m_n^4}{n} \to 0$. The remaining terms in $\mathrm{I}_{T/n}$ can be treated similarly and have a faster (than $\mathrm{I}^1_{T/n}$) vanishing rate. *Q.E.D.*

APPENDIX S.C: ADDITIONAL DETAILS ON IMPLEMENTATION

Our implementation of EXIT and its confidence bands relies on the results in Theorem 2 (the case with microstructure noise). While it is important to take microstructure noise into account explicitly for economic reasons discussed in the main text, we reduce its impact on volatility estimation by following the procedure illustrated below.

Even though the paper's theory allows for unevenly spaced observations, we use (both with data and in simulation) evenly sampled returns over a day to compute EXIT. Thus, $H'_{3/2} = 1$ and $H_2(T) = T$.

Denote by $p_1, \dots, p_n$ the logarithmic prices observed over one day, so that n is the total number of transactions at the highest observation frequency. Denote, also, by $\tilde{r}_1, \dots, \tilde{r}_M$ the logarithmic returns sampled at a lower frequency (for which the impact of market microstructure noise and rounding is expected to be negligible).

The microstructure noise variance estimator is

$$\widehat{\sigma}_\varepsilon^2 = \frac{1}{n-2}\sum_{j=1}^{n-2}(p_{j+1}-p_j)(p_{j+2}-p_{j+1}),$$

when the latter quantity is positive. Otherwise, the estimate is set to zero.

We now turn to the spot volatilities. In the spirit of Fan and Wang (2008), we use the following kernel estimator:

$$\widehat{\sigma}_i^2 = \frac{\pi}{2}\frac{\displaystyle\sum_{j=1}^{M-1} K\left(\frac{i-j}{h}\right)|\tilde{r}_j||\tilde{r}_{j+1}| I_{\{\tilde{r}_j^2 \le \theta_j\}} I_{\{\tilde{r}_{j+1}^2 \le \theta_{j+1}\}}}{\displaystyle\Delta\sum_{j=1}^{M-1} K\left(\frac{i-j}{h}\right) I_{\{\tilde{r}_j^2 \le \theta_j\}} I_{\{\tilde{r}_{j+1}^2 \le \theta_{j+1}\}}}, \quad i = 1, \dots, M, \tag{C.1}$$

where the threshold θ_j, for $j = 1, \dots, M$, is chosen as in Corsi, Pirino, and Renò (2010) with $c_\theta = 5$. We set $h = 25$. The function $K(\cdot)$ is a double-exponential kernel

$$K(x) = \frac{1}{2}e^{-|x|}.$$

Consistent with the logic in Corsi, Pirino, and Renò (2010), the bipower variation term $|\tilde{r}_j||\tilde{r}_{j+1}|$, combined with the threshold θ_j, provides a jump-robust volatility estimator with satisfactory finite sample properties.

EXIT's own threshold ξ_n is set proportional to spot volatility, according to the expression

$$\xi_{n,i} = \alpha \cdot \widehat{\sigma}_i \cdot \sqrt{\Delta} \cdot (\Delta)^{1/100}. \tag{C.2}$$

Define, now, the *error function* as

$$\mathrm{erf}(x) = \frac{2}{\sqrt{\pi}}\int_0^x e^{-t^2}\,dt.$$

For $i = 1, \dots, M$, write

$$\mathcal{P}_i = \mathrm{erf}\left(\frac{\xi_{n,i}}{\sqrt{2\left(\Delta\widehat{\sigma}_i^2 + \widehat{\sigma}_\varepsilon^2\right)}}\right), \tag{C.3}$$

where $\Delta\widehat{\sigma}_i^2$ is an asymptotically-vanishing finite-sample correction (justified by the proof of Theorem 2).

EXIT is computed as

$$\mathrm{EXIT} = \frac{1}{M}\sum_{i=1}^{M}\left(1_{\{|\tilde{r}_i| \le \xi_{n,i}\}} - \mathcal{P}_i\right), \tag{C.4}$$

while its variance V_{EXIT} is estimated by

$$V_{\text{EXIT}} = \frac{1}{M^2} \sum_{i=1}^{M} \left(\mathcal{P}_i - \mathcal{P}_i^2\right). \tag{C.5}$$

The reader will notice, in these expressions, the use of the error function instead of its asymptotic equivalent (as derived in the proofs of Theorem 1 and Theorem 2). The logic of Eq. (C.4) and Eq. (C.5) is immediate once it is recognized that the indicators amount to approximate Bernoulli variates. Finally, the use of the term $\mathcal{P}_i^2$ is asymptotically irrelevant, but our simulations show that it may be empirically important in a finite sample.

APPENDIX S.D: ADDITIONAL SIMULATIONS

This section evaluates EXIT's finite sample performance further. We accommodate stochastic volatility, intraday effects, microstructure noise, and rounding of the simulated prices.

For the price dynamics, we simulate a one-factor diffusion model with stochastic volatility. The model is described by the pair of stochastic differential equations

$$\begin{aligned} dp_t^e &= \mu\, dt + \gamma_{t,\tau} c_\sigma \sigma_t\, dW_{p,t}, \\ d\log\sigma_t^2 &= \left(\alpha - \beta \log\sigma_t^2\right) dt + \eta\, dW_{\sigma,t}, \end{aligned} \tag{D.1}$$

where W_p and W_σ are standard Brownian motions with $\operatorname{corr}(dW_p, dW_\sigma) = \rho\, dt$ and σ_t is a stochastic volatility factor. We use the model parameters estimated by Andersen, Benzoni, and Lund (2002) on S&P 500 prices: $\mu = 0.0304$, $\alpha = -0.012$, $\beta = 0.0145$, $\eta = 0.1153$, $\rho = -0.6127$, where the parameters are expressed in daily units and returns are in percentage. We further set $c_\sigma = 2$, which calibrates the daily volatility to nearly 20% in annual terms. In addition, we add a multiplicative intraday effect

$$\gamma_{t,\tau} = \frac{1}{0.1033}\left(0.1271\tau^2 - 0.1260\tau + 0.1239\right),$$

where τ is the fraction of a day elapsed from opening ($\tau = 0$ at the beginning of the day and $\tau = 1$ at the end of the day) and the parameters in $\gamma_{t,\tau}$ have been calibrated on S&P 500 intraday returns with the constraint $\int_0^1 \gamma_{t,\tau}\, d\tau = 1$. The numerical integration of the system (D.1) is performed with the Euler scheme, using a discretization step of one tenth of a second.

Each day, we simulate $n = 240{,}000$ prices, corresponding to six hours and 40 minutes of artificial trading every one tenth of a second. We add a microstructure noise shock to every price leading to

$$\tilde{p}_t = p_t^e + \eta_t,$$

with η_t i.i.d. normally distributed with zero mean and variance σ_η^2. We set $\sigma_\eta^2 = c_\sigma^2 e^{\alpha/\beta}/(7 \times 60 \times 60) = 6.94 \times 10^{-5}$ so that, at the frequency of one tenth of a second, the ratio of the Brownian variance to the microstructure noise variance (the signal-to-noise ratio) is equal to 1.

Finally, we generate illiquidity in the price series using the model described in Assumption 3 with dependent Bernoulli trials $B_{i,n}$ drawn according to the following specification:

$$B_{i,n} = B_{i-1,n} B_{i,n}^{(2)} + \left(1 - B_{i,n}^{(2)}\right) B_{i,n}^{(1)}, \quad i \geq 2, \tag{D.2}$$

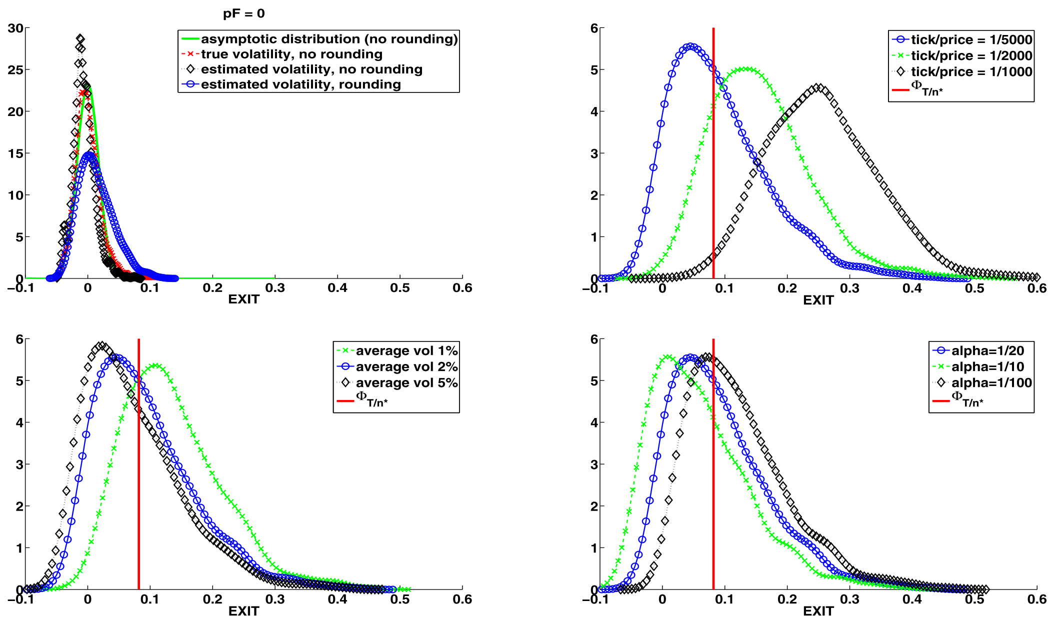

FIGURE S.D.1.—Monte Carlo results. Top-left panel: estimated probability of flat trading under *the null*. Top-right panel: sensitivity to the level of rounding *under the alternative*. Bottom-left panel: sensitivity to volatility *under the alternative*. Bottom-right panel: sensitivity to the threshold parameter α *under the alternative*.

with $B_{1,n} = B_{1,n}^{(1)}$, where the $B_{i,n}^{(1)}$'s are i.i.d. Bernoulli variates with probability $p^F = 0.1137$ and the $B_{i,n}^{(2)}$'s are i.i.d. Bernoulli variates independent of $B_{i,n}^{(1)}$ with probability $p^R = 0.999$. The parameters of the model (p^F and p^R) are calibrated on the same S&P 500 futures data used in Section 6. Note that $\mathbb{E}[B_{i,n}] = \mathbb{E}[B_{i,n}^{(1)}] = p^F$. We use an initial value $P_0 = \$50$ and round prices to the nearest cent (\$0.01).

The estimation target is $\Phi_{T/n^*} = 0.0818$ with $n^* = 84$. In order to compute EXIT, we apply the procedure described in Section 5 with $\alpha = 1/20$ and optimize the MSE with respect to T/n.

We evaluate the performance of EXIT under *the null*, as well as its sensitivity, under *the alternative*, to (1) rounding, (2) the average volatility, and (3) the value of α used in Eq. (24). Under the null (first panel of Figure S.D.1), EXIT is correctly centered, even when replacing the true volatility with the estimator described in Eq. (C.1) and when prices are rounded to generate price discreteness. The second panel of Figure S.D.1 shows that EXIT becomes more biased when price discreteness is larger. It is important to remark that, in the S&P 500 data set we use, the average value of the ratio $tick/P$ is $\approx 1/7000$. The third panel shows that the impact of volatility on the measure is small. So is the impact of the choice of α. However, our simulations indicate that—for the assumed parameter values—a value of α much larger, or much smaller, than $1/20$ would be detrimental in practice. This is easily understood. Consistent with theory, a small α is needed to identify price idleness. A relatively larger α is required to control the finite sample impact of rounding. This said, one could treat α as a choice variable and employ the MSE criterion provided in Section 5.1 to trade off the two effects explicitly.

APPENDIX S.E: GENERATED REGRESSORS

Denote yearly excess returns $\mathrm{R}^e_{t:t+1}$ by y_t and yearly (past) $\mathrm{EXIT}_{t-1:t}$ by $\widehat{x}_{n,t-1}$. In agreement with our asymptotic design, assume $\mathrm{EXIT}_{t-1:t}$ is an estimate of p^F over the previous year (where the latter quantity is defined here by x_{t-1}). Suppose $\max_{1\leq t\leq T}|x_{t-1}-\widehat{x}_{n,t-1}| = O_p(\gamma_{T,n})$, where n is the number of high-frequency observations used to estimate EXIT over one year, T is the number of years, and $\gamma_{T,n}\to 0$ as $T, n\to\infty$ *jointly*. Ignoring the intercept for notational simplicity, we are estimating the regression

$$y_t = \beta\widehat{x}_{n,t-1} + \xi_t,$$

but the true model is

$$y_t = \beta x_{t-1} + u_t.$$

Therefore, $\xi_t = \beta(x_{t-1}-\widehat{x}_{n,t-1}) + u_t$. The percentage estimation error can now be written as

$$\begin{aligned}
\left|\frac{\widehat{\beta}-\beta}{\beta}\right| &= \frac{\frac{1}{T}\left|\sum \widehat{x}_{n,t-1}\xi_t\right|}{\frac{\beta}{T}\sum \widehat{x}^2_{n,t-1}} \\
&= \frac{\frac{1}{T}\left|\sum \widehat{x}_{n,t-1}\big[\beta(x_{t-1}-\widehat{x}_{n,t-1})+u_t\big]\right|}{\frac{\beta}{T}\sum \widehat{x}^2_{n,t-1}} \\
&\leq \frac{\frac{1}{T}\sum\left|\widehat{x}_{n,t-1}(x_{t-1}-\widehat{x}_{n,t-1})\right|}{\frac{1}{T}\sum \widehat{x}^2_{n,t-1}} + \frac{\frac{1}{T}\left|\sum \widehat{x}_{n,t-1}u_t\right|}{\frac{\beta}{T}\sum \widehat{x}^2_{n,t-1}} \\
&\leq \underbrace{\frac{O_p(\gamma_{T,n})\frac{1}{T}\sum|\widehat{x}_{n,t-1}|}{\frac{1}{T}\sum \widehat{x}^2_{n,t-1}}}_{\mathrm{I}_{T,n}} + \underbrace{\frac{\frac{1}{T}\left|\sum \widehat{x}_{n,t-1}u_t\right|}{\frac{\beta}{T}\sum \widehat{x}^2_{n,t-1}}}_{\mathrm{II}_{T,n}}.
\end{aligned}$$

The quantity $\mathrm{I}_{T,n}$ is the component of the percentage estimation error in $\widehat{\beta}$ due to measurement error in the regressor. The quantity $\mathrm{II}_{T,n}$ is, instead, classical. If $\gamma_{T,n}\to 0$, the use of generated regressors does not lead to inconsistencies. If $\sqrt{T}\gamma_{T,n}\to 0$, the limiting distribution is also unaffected. In both cases, we require having a large enough number of high-frequency observations n relative to the number of periods T. Thus, when using high-frequency observations, the generated regressor problem can, in general, be controlled asymptotically.

Its finite sample impact is, of course, data-dependent. To this extent, we quantify the percentage in-sample deviation induced by measurement error in the slope estimates (i.e., $\mathrm{I}_{T,n}$) by evaluating $\frac{\max_{1\leq t\leq T}(x_{t-1}-\widehat{x}_{n,t-1})\frac{1}{T}\sum\widehat{x}_{n,t-1}}{\frac{1}{T}\sum\widehat{x}^2_{n,t-1}}$. Assume the estimation error $(x_{t-1}-\widehat{x}_{n,t-1})$ is i.i.d. normal over each year. (Our limiting results imply that normality is satisfied for a large n. Here, we use normality for the purpose of a back-of-the-envelope calculation.)

Thus, it is known that $\max_{1\leq t\leq T}(x_{t-1}-\widehat{x}_{n,t-1}) \leq \sqrt{\log T \max_{1\leq t\leq T}\mathrm{var}_n(x_{t-1}-\widehat{x}_{n,t-1})}$. For each year in the sample, $\mathrm{var}_n(x_{t-1}-\widehat{x}_{n,t-1})$ can be estimated using Eq. (C.5) in Part C. When doing so, we find that the percentage measurement error-induced deviation in $\widehat{\beta}$ (i.e., $\mathrm{I}_{T,n}$) associated with the 1-year horizon is around 0.08% and, therefore, virtually negligible. Aggregation over multiple years renders the estimation error in EXIT even less influential, since T (the number of observations in the regression) decreases and n (the number of high-frequency observations used to evaluate the regressor, i.e., EXIT) increases (thereby leading to a reduction in $\mathrm{var}_n(x_{t-1}-\widehat{x}_{n,t-1})$). Identical considerations (and similar numbers) apply to the use of yearly variance (and aggregates of yearly variance) as a regressor.

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