Econometrica Supplementary Material

SUPPLEMENT TO "A COMMENT ON 'CAN RELAXATION OF BELIEFS RATIONALIZE THE WINNER'S CURSE?: AN EXPERIMENTAL STUDY'"

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BY MIGUEL A. COSTA-GOMES AND MAKOTO SHIMOJI

IN THIS NOTE, WE PROVIDE (i) the derivation of the cursed equilibrium and analogy-based expectation equilibrium (which are the same under our specification) and (ii) the derivation of $L1$'s bids with a discrete bid space (which is analogous to ILN's Proposition 3).

APPENDIX A: ANALOGY-BASED EXPECTATION EQUILIBRIUM AND CURSED EQUILIBRIUM

The set of signals is $X = \{0, 1, \dots, 10\}$. Let ω_{x_1,x_2} correspond to the state where player i's signal is x_i ($i = 1, 2$). There are 121 ($= 11 \times 11$) possible states, $\omega_{x_1,x_2} \in \Omega$. The item's value at ω_{x_1,x_2} is $\max\{x_1, x_2\}$. Each state is equally possible. We have

$$\begin{aligned}\mathcal{P}_1 = \big\{&\{\omega_{x_1,0}, \omega_{x_1,1}, \omega_{x_1,2}, \omega_{x_1,3}, \omega_{x_1,4}, \omega_{x_1,5},\\ &\omega_{x_1,6}, \omega_{x_1,7}, \omega_{x_1,8}, \omega_{x_1,9}, \omega_{x_1,10}\}_{x_1\in X}\big\},\\ \mathcal{P}_2 = \big\{&\{\omega_{0,x_2}, \omega_{1,x_2}, \omega_{2,x_2}, \omega_{3,x_2}, \omega_{4,x_2}, \omega_{5,x_2},\\ &\omega_{6,x_2}, \omega_{7,x_2}, \omega_{8,x_2}, \omega_{9,x_2}, \omega_{10,x_2}\}_{x_2\in X}\big\},\end{aligned}$$

where $\mathcal{P}_i$ is the partition of the states from player i's point of view.

Let $\mathcal{A}_i$ be the analogy partitions of the states from player i's point of view ($i = 1, 2$). For the analogy-based expectation equilibrium, we assume that $\mathcal{A}_i = \mathcal{P}_i$ for $i = 1, 2$, that is, the *private information analogy partition* (Jehiel and Koessler (2008, p. 538)). This is visualized in Figure S.1. As Jehiel and Koessler (2008, p. 539) and Eyster and Rabin (2005, p. 1634) noted, this specification coincides with the *fully* cursed equilibrium (i.e., $\chi = 1$, in Eyster and Rabin (2005) χ-cursed equilibrium). Thus, the analogy-based expectation equilibrium that we construct is also a fully cursed equilibrium.

Note that $E[X^{\max} \mid 10] = 10$. For $x_i \in X \setminus \{10\}$, the expected value of the item is

$$\begin{aligned}E\big[X^{\max} \mid x_i\big] &= \left(\frac{1}{11}\right)10 + \cdots + \left(\frac{1}{11}\right)(x_i + 1) + \left(\frac{x_i + 1}{11}\right)x_i\\ &= \left(\frac{1}{11}\right)\sum_{l=x_i+1}^{10} l + \left(\frac{x_i + 1}{11}\right)x_i\end{aligned}$$

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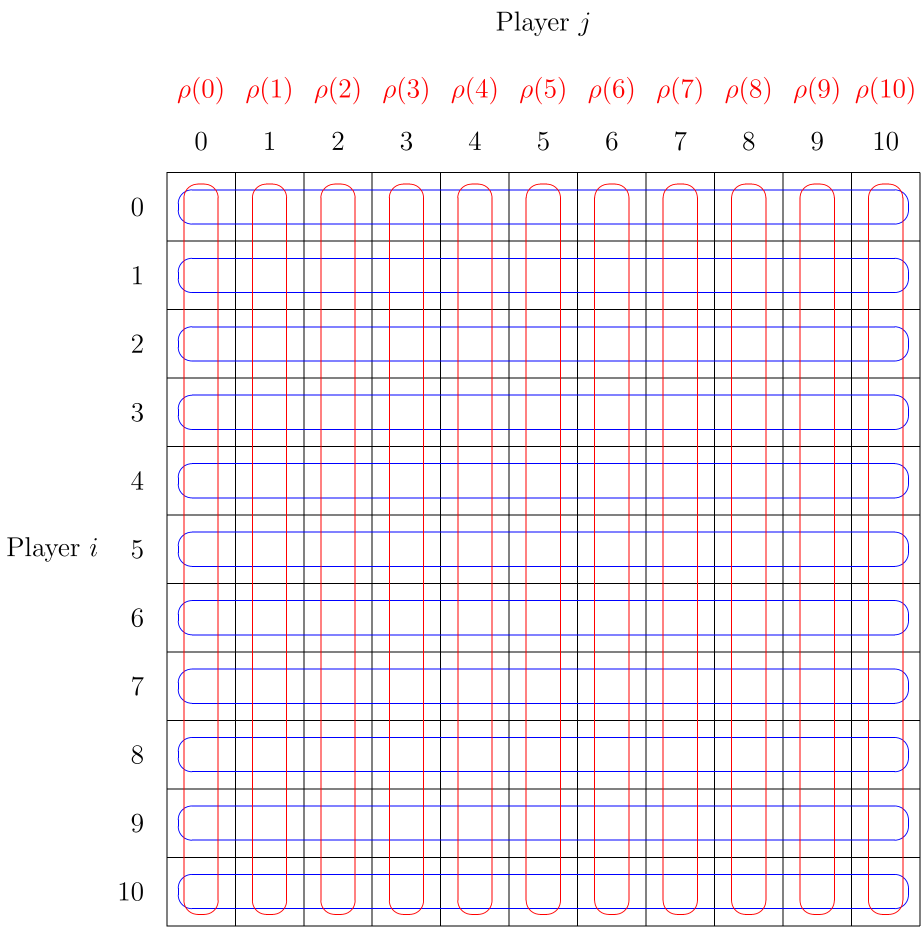

FIGURE S.1.—Partitions of states.

$$= \left(\frac{1}{11}\right)\left(55 - \frac{x_i(x_i+1)}{2}\right) + \frac{x_i(x_i+1)}{11}$$

$$= \frac{x_i(x_i+1)+110}{22}.$$

Table S.I shows the value of $E[X^{\max} \mid x_i]$ for each $x_i \in X$.

Suppose that player i with x_i chooses $b^l(x_i) = \frac{\lfloor 100E[X^{\max}|x_i]\rfloor}{100}$ with probability $\rho(x_i) \in (0,1)$ and $b^h(x_i) = \frac{\lceil 100E[X^{\max}|x_i]\rceil}{100}$ with $1-\rho(x_i)$, where

$$\rho(x_i) = \frac{b^h(x_i) - E\big[X^{\max} \mid x_i\big]}{b^h(x_i) - b^l(x_i)} \in (0,1)$$

TABLE S.I

x_i, $E[X^{\max} \mid x_i]$, AND EQUILIBRIUM STRATEGY

x_i	0	1	2	3	4	5	6	7	8	9	10
$E[X^{\max} \mid x_i]$	5	$\frac{56}{11}$	$\frac{58}{11}$	$\frac{61}{11}$	$\frac{65}{11}$	$\frac{70}{11}$	$\frac{76}{11}$	$\frac{83}{11}$	$\frac{91}{11}$	$\frac{100}{11}$	10
$b^l(x_i)$	5	5.09	5.27	5.54	5.90	6.36	6.90	7.54	8.27	9.09	10
$b^h(x_i)$	5	5.10	5.28	5.55	5.91	6.37	6.91	7.55	8.28	9.10	10
$\rho(x_i)$	–	$\frac{10}{11}$	$\frac{8}{11}$	$\frac{5}{11}$	$\frac{1}{11}$	$\frac{7}{11}$	$\frac{1}{11}$	$\frac{5}{11}$	$\frac{8}{11}$	$\frac{10}{11}$	–

for each $x_i \in X$.[1] Table S.I shows their values for each $x_i \in X$. It is important to note that for each x_i, (i) $b^l(x_i) < E[X^{\max} \mid x_i] < b^h(x_i)$ and (ii) $\rho(x_i)b^l(x_i) + (1-\rho(x_i))b^h(x_i) = E[X^{\max} \mid x_i]$. Let

$$B^p = \{b \in B \mid \text{there exists } x_i \text{ such that } b = b^l(x_i) \text{ or } b = b^h(x_i)\}.$$

Given the strategy specified above, Table S.II shows *the strategy for player j perceived by player i*, $\bar{\rho}$. Note that $\bar{\rho}$ is independent of the state.

Given $\bar{\rho}$, the expected payoff for player i with x_i from $b_i \in B$ is computed as follows: If player i chooses $b_i \in B \setminus B^p$,

$$\begin{aligned}&\left(\frac{1}{11}\right)\sum_{x_j \in X}\sum_{b<b_i, b\in B^p}\bar{\rho}(b)\{\max\{x_i, x_j\} - b\}\\ &\quad= \sum_{b<b_i, b\in B^p}\bar{\rho}(b)\{E[X^{\max} \mid x_i] - b\},\end{aligned}$$

TABLE S.II

STRATEGY OF PLAYER j PERCEIVED BY PLAYER i

b	5	5.09	5.10	5.27	5.28	5.54	5.55	5.90	5.91	6.36
$\bar{\rho}(b)$	$\frac{1}{11}$	$\frac{10}{121}$	$\frac{1}{121}$	$\frac{8}{121}$	$\frac{3}{121}$	$\frac{5}{121}$	$\frac{6}{121}$	$\frac{1}{121}$	$\frac{10}{121}$	$\frac{7}{121}$
b	6.37	6.90	6.91	7.54	7.55	8.27	8.28	9.09	9.10	10
$\bar{\rho}(b)$	$\frac{4}{121}$	$\frac{1}{121}$	$\frac{10}{121}$	$\frac{5}{121}$	$\frac{6}{121}$	$\frac{8}{121}$	$\frac{3}{121}$	$\frac{10}{121}$	$\frac{1}{121}$	$\frac{1}{11}$

[1]That is, we choose two numbers in B closest to $E[X^{\max} \mid x_i]$.

while if player i chooses $b_i \in B^p$,

$$\left(\frac{1}{11}\right)\sum_{x_j\in X}\Bigg[\sum_{b<b_i, b\in B^p}\bar{\rho}(b)\big\{\max\{x_i, x_j\} - b\big\}$$
$$+\bar{\rho}(b_i)\big\{\max\{x_i, x_j\} - b_i\big\}\left(\frac{1}{2}\right)\Bigg]$$
$$= \sum_{b<b_i, b\in B^p}\bar{\rho}(b)\big\{E\big[X^{\max} \mid x_i\big] - b\big\} + \bar{\rho}(b_i)\big\{E\big[X^{\max} \mid x_i\big] - b_i\big\}\left(\frac{1}{2}\right).$$

Note (i) that $b_i = 5$ is a best response for $x_i = 0$, (ii) $b_i = 10$ is a best response for $x_i = 10$, and (iii) that every $b_i < b^l(x_i)$ and $b_i > b^h(x_i)$ cannot be a best response for each $x_i \in X \setminus \{0, 10\}$, meaning that the only remaining bids are $\{b^l(x_i), b^h(x_i)\}$ for each $x_i \in X \setminus \{0, 10\}$.[2] Given $x_i \in \{0, 10\}$, the expected payoff from $b_i = b^l(x_i)$ is

$$\text{(S.1)}\quad \sum_{b<b^l(x_i), b\in B^p}\bar{\rho}(b)\big\{E\big[X^{\max} \mid x_i\big] - b\big\}$$
$$+\bar{\rho}\big(b^l(x_i)\big)\big\{E\big[X^{\max} \mid x_i\big] - b^l(x_i)\big\}\left(\frac{1}{2}\right),$$

while the expected payoff from $b_i = b^h(x_i)$ is

$$\text{(S.2)}\quad \sum_{b<b^h(x_i), b\in B^p}\bar{\rho}(b)\big\{E\big[X^{\max} \mid x_i\big] - b\big\}$$
$$+\bar{\rho}\big(b^h(x_i)\big)\big\{E\big[X^{\max} \mid x_i\big] - b^h(x_i)\big\}\left(\frac{1}{2}\right).$$

Then, we have

$$(\text{S.2}) - (\text{S.1})$$
$$= \bar{\rho}\big(b^l(x_i)\big)\big\{E\big[X^{\max} \mid x_i\big] - b^l(x_i)\big\}\left(\frac{1}{2}\right)$$
$$+\bar{\rho}\big(b^h(x_i)\big)\big\{E\big[X^{\max} \mid x_i\big] - b^h(x_i)\big\}\left(\frac{1}{2}\right)$$
$$= \left(\frac{1}{22}\right)\big[\rho(x_i)\big\{E\big[X^{\max} \mid x_i\big] - b^l(x_i)\big\}$$

[2]More generally, any $b_i < 5.09$ is a best response for $x_i = 0$ and any $b_i > 9.10$ is a best response for $x_i = 10$.

$$
\begin{aligned}
&+ \big(1-\rho(x_i)\big)\big\{E\big[X^{\max} \mid x_i\big] - b^h(x_i)\big\}\big] \\
&= \left(\frac{1}{22}\right)\big\{E\big[X^{\max} \mid x_i\big] - \big[\rho(x_i)b^l(x_i) + \big(1-\rho(x_i)\big)b^h(x_i)\big]\big\} \\
&= 0,
\end{aligned}
$$

where the last equality comes from the fact that $\rho(x_i)b^l(x_i) + (1-\rho(x_i)) \times b^h(x_i) = E[X^{\max} \mid x_i]$ for each $x_i \in X$. This shows that both $b^l(x_i)$ and $b^h(x_i)$ are best responses for each $x_i \in X \setminus \{0, 10\}$. The strategy specified above hence constitutes an analogy-based expectation equilibrium with private information analogy partition, and hence is also a fully cursed equilibrium.

APPENDIX B: ILN'S PROPOSITION 3 WITH A DISCRETE BID SPACE

We derive *random* $L1$'s best responses for *BL/SBF* under the assumption that *random* $L0$ randomizes uniformly over $B^0 = \{0, 0.01, \dots, 9.99, 10\} \subset B$. Given signal $x_i \in X$, the following expression is the expected payoff for player i by bidding $b_i \in B$ (note that we use "cents" instead of "dollars"):

$$
\begin{aligned}
&\left(\frac{1}{11}\right)\sum_{x_j \in X}\left[\left(\frac{1}{|B^0|}\right)\sum_{b<b_i, b\in B^0}\big\{\max\{x_i, x_j\} - b\big\}\right. \\
&\qquad \left. + \left(\frac{1}{|B^0|}\right)\big\{\max\{x_i, x_j\} - b_i\big\}\left(\frac{1}{2}\right)\right] \\
&= \left(\frac{1}{|B^0|}\right)\sum_{b<b_i, b\in B^0}\big\{E\big[X^{\max} \mid x_i\big] - b\big\} \\
&\qquad + \left(\frac{1}{|B^0|}\right)\big\{E\big[X^{\max} \mid x_i\big] - b_i\big\}\left(\frac{1}{2}\right) \\
&= \left(\frac{b_i}{|B^0|}\right)E\big[X^{\max} \mid x_i\big] - \left(\frac{1}{|B^0|}\right)\left(\frac{(b_i-1)b_i}{2}\right) \\
&\qquad + \left(\frac{1}{|B^0|}\right)\big\{E\big[X^{\max} \mid x_i\big] - b_i\big\}\left(\frac{1}{2}\right) \\
&= \left(\frac{b_i + \frac{1}{2}}{|B^0|}\right)E\big[X^{\max} \mid x_i\big] - \left(\frac{1}{|B^0|}\right)\left[\frac{(b_i-1)b_i}{2} + \frac{b_i}{2}\right] \\
&= \left(\frac{1}{|B^0|}\right)\left\{\left(b_i + \frac{1}{2}\right)E\big[X^{\max} \mid x_i\big] - \left(\frac{b_i^2}{2}\right)\right\}.
\end{aligned}
$$

Given that $b_i \in B^0$ and the expression above is quadratic, the expected payoff is maximized at either $E[X^{\max} \mid x_i] - 0.01$ or $E[X^{\max} \mid x_i] + 0.01$ (in dollars). We numerically computed the values of the expected payoff at these points, and selected the bid with the highest expected payoff as the best response.

REFERENCES

EYSTER, E., AND M. RABIN (2005): "Cursed Equilibrium," *Econometrica*, 73, 1623–1672. [1]

JEHIEL, P., AND F. KOESSLER (2008): "Revisiting Games of Incomplete Information With Analogy-Based Expectations," *Games and Economic Behavior*, 62, 533–557. [1]

School of Economics and Finance, University of St. Andrews, Castlecliffe, The Scores, St. Andrews, KY16 9AZ, U.K.; miguel.costa-gomes@st-andrews.ac.uk

and

Dept. of Economics and Related Studies, University of York, Heslington, York, YO10 5DD, U.K.; makoto.shimoji@york.ac.uk.

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