Econometrica Supplementary Material

SUPPLEMENT TO "NONPARAMETRIC IDENTIFICATION OF A CONTRACT MODEL WITH ADVERSE SELECTION AND MORAL HAZARD"

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This supplemental material contains the proofs of the propositions and lemmas stated in Section 2.

PROOF OF PROPOSITION 1: From (8), the Hamiltonian of the optimization problem (P′) is

$$\begin{aligned}\mathcal{H} = \bigg\{ & \int_p^\infty \overline{y}(v)\,dv \\ & + (1+\lambda)\big(p\overline{y}(p) - \psi(e) - \mathrm{E}\big[(\theta - e)c_o(y(p,\varepsilon_d),\varepsilon_c)\big]\big) \\ & - \lambda U(\theta)\bigg\} f(\theta) + \gamma(\theta)(-\psi'(e)),\end{aligned}$$

where $p = p(\theta)$ and $e = e(\theta)$ are the control functions, $U(\theta)$ is the state variable, and $\gamma(\theta)$ is the co-state variable. Hence, applying the Pontryagin principle, the FOC are

$$\begin{aligned}\mathcal{H}_p &= \big\{\lambda\overline{y}(p) + (1+\lambda)p\overline{y}'(p) \\ &\qquad - (1+\lambda)\mathrm{E}\big[(\theta - e)c_{o1}(y(p,\varepsilon_d),\varepsilon_c)y_1(p,\varepsilon_d)\big]\big\} f(\theta) = 0, \\ \mathcal{H}_e &= \big\{-(1+\lambda)\psi'(e) + (1+\lambda)\mathrm{E}\big[c_o(y(p,\varepsilon_d),\varepsilon_c)\big]\big\} f(\theta) \\ &\qquad - \gamma(\theta)\psi''(e) = 0, \\ -\mathcal{H}_U &= \lambda f(\theta) = \gamma'(\theta).\end{aligned}$$

The last equation gives $\gamma(\theta) = \lambda F(\theta)$ using the transversality condition $\gamma(\underline{\theta}) = 0$. Thus, rearranging $\mathcal{H}_p$ and $\mathcal{H}_e$, the solutions $p = p^*(\theta)$ and $e = e^*(\theta)$ are given by (12) and (13). *Q.E.D.*

PROOF OF PROPOSITION 2: Given the price schedule $p^*(\cdot)$ and the transfer function $t^*(\cdot,\cdot)$, we show that the firm will announce its true type θ and will exert the optimal effort $e^*(\theta)$ by verifying the FOC of the firm's problem (F).

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Under A1, this problem becomes

$$(\mathrm{F}^*)\qquad \max_{\tilde\theta,e} \mathrm{E}\big[t^*\big(\tilde\theta,(\theta-e)c_o\big(y(p^*(\tilde\theta),\varepsilon_d),\varepsilon_c\big)\big)\mid\theta\big]-\psi(e)$$
$$=\mathrm{E}\big[t^*\big(\tilde\theta,(\theta-e)c_o\big(y(p^*(\tilde\theta),\varepsilon_d),\varepsilon_c\big)\big)\big]-\psi(e)$$
$$=A(\tilde\theta)+\psi'[e^*(\tilde\theta)]\{\tilde\theta-e^*(\tilde\theta)-(\theta-e)\}-\psi(e),$$

where the first equality follows from the independence between θ and $(\varepsilon_d,\varepsilon_c)$, while the second equality follows from (15). Thus, using (16), the FOCs with respect to $\tilde\theta$ and e are

$$0=\psi'[e^*(\tilde\theta)]e^{*\prime}(\tilde\theta)-\psi'[e^*(\tilde\theta)]+\frac{d\psi'[e^*(\tilde\theta)]}{d\tilde\theta}\{\tilde\theta-e^*(\tilde\theta)-(\theta-e)\}$$
$$+\psi'[e^*(\tilde\theta)][1-e^{*\prime}(\tilde\theta)]$$
$$=\frac{d\psi'[e^*(\tilde\theta)]}{d\tilde\theta}\{\tilde\theta-e^*(\tilde\theta)-(\theta-e)\},$$
$$0=\psi'[e^*(\tilde\theta)]-\psi'(e).$$

It is easy to see that these FOCs are verified if $\tilde\theta=\theta$ and $e=e^*(\theta)$.

It remains to show that $[p^*(\cdot),t^*(\cdot,\cdot),e^*(\cdot),U^*(\cdot)]$ solves the FOC of problem (P). In view of the discussion surrounding problem (P′), it suffices to show that the transfer function $t^*(\cdot,\cdot)$ satisfies (6) and (7), where $[p^*(\cdot),e^*(\cdot),U^*(\cdot)]$ solves the FOC of problem (P′). The preceding statement shows that the transfer function $t^*(\cdot,\cdot)$ satisfies (7). It remains to show that $t^*(\cdot,\cdot)$ also satisfies (6). Using (15), the right-hand side of (6) is

$$A(\theta)+\psi'[e^*(\theta)]\big\{\theta-e^*(\theta)-(\theta-e^*(\theta))\big\}-\psi[e^*(\theta)]$$
$$=A(\theta)-\psi[e^*(\theta)]=U^*(\theta)$$

by (14) and (16), as desired. *Q.E.D.*

PROOF OF LEMMA 1: From the problem (F), the second partial derivative of the firm's objective function with respect to e is

$$\int U_{33}(\tilde\theta,\theta,e,\varepsilon_d,\varepsilon_c)\,dG(\varepsilon_d,\varepsilon_c)=\int t_{22}(\cdot)c_o^2(\cdot)\,dG(\varepsilon_d,\varepsilon_c)-\psi''(e),$$

where we have omitted the arguments of the functions to simplify the notation. When the transfer function $t(\cdot,\cdot)$ is weakly decreasing and concave in realized cost c so that $t_2(\cdot)\le 0$ and $t_{22}(\cdot)\le 0$, it follows from $\psi''(\cdot)>0$ that the firm's objective function is *strictly* concave in e for any $(\tilde\theta,\theta)$. Hence, the effort $e(\tilde{\theta}, \theta)$, which solves the FOC (3), is uniquely defined and corresponds to a global maximum of the problem (FE).

Next, we show that $0 \leq e_2(\theta, \theta) < 1$. This can be seen by differentiating the FOC (3) that defines $e(\tilde{\theta}, \theta)$ with respect to θ. This gives

$$0 = [1 - e_2(\tilde{\theta}, \theta)]\mathrm{E}[t_{22}(\cdot)c_o^2(\cdot)] + \psi''[e(\tilde{\theta}, \theta)]e_2(\tilde{\theta}, \theta).$$

Rearranging and evaluating at $\tilde{\theta} = \theta$ give

$$e_2(\theta, \theta)\big\{\mathrm{E}[t_{22}(\cdot)c_o^2(\cdot)] - \psi''[e(\theta)]\big\} = \mathrm{E}[t_{22}(\cdot)c_o^2(\cdot)].$$

Thus the expectation term is nonpositive whenever the transfer function $t(\cdot, \cdot)$ is weakly decreasing and concave in realized cost. Because $\psi''(\cdot) > 0$ by A2(iii), it follows that $0 \leq e_2(\theta, \theta) < 1$. *Q.E.D.*

PROOF OF LEMMA 2: As noted before A3, the local SOC (19) is satisfied as soon as $e^{*\prime}(\cdot) \leq 0$. We show that $e^{*\prime}(\cdot) < 0$. By definition, $[p^*(\cdot), e^*(\cdot)]$ satisfies the FOC (12) and (13), which can be written as

$$p^*(\theta)\overline{y}'[p^*(\theta)] = (\theta - e^*(\theta))\overline{c}_o'[p^*(\theta)] - \mu\overline{y}[p^*(\theta)],$$
$$\psi'[e^*(\theta)] = \overline{c}_o[p^*(\theta)] - \mu\frac{F(\theta)}{f(\theta)}\psi''[e^*(\theta)],$$

where we have used A1, the definition of $\overline{c}_o(\cdot)$, and the expression found earlier for $\overline{c}_o'(\cdot)$. Differentiating (12) and (13) with respect to θ and rearranging equations give

$$\text{(S.1)} \qquad Ae^{*\prime}(\theta) + Bp^{*\prime}(\theta) = A,$$
$$Ce^{*\prime}(\theta) - Ap^{*\prime}(\theta) = D,$$

where

$$A = \overline{c}_o'[p^*(\theta)],$$
$$B = (1+\mu)\overline{y}'[p^*(\theta)] + p^*(\theta)\overline{y}''[p^*(\theta)] - (\theta - e^*(\theta))\overline{c}_o''[p^*(\theta)]$$
$$= (1-\mu)\overline{V}''[p^*(\theta)] - (\theta - e^*(\theta))\overline{c}_o''[p^*(\theta)],$$
$$C = \psi''[e^*(\theta)] + \mu\frac{F(\theta)}{f(\theta)}\psi'''[e^*(\theta)],$$
$$D = -\mu\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\psi''[e^*(\theta)]$$

with $\mu = \lambda/(1+\lambda)$. Under A1 and A2, note that $A < 0$, $B < 0$, and $C > 0$. Solving for $e^{*\prime}(\theta)$ gives

$$e^{*\prime}(\theta)\left(C + \frac{A^2}{B}\right) = D + \frac{A^2}{B}.$$

Thus, $e^{*\prime}(\cdot) < 0$ if $-C < A^2/B < -D$, that is, if

$$\text{(S.2)} \qquad -\left(\psi''[e^*(\theta)] + \mu\frac{F(\theta)}{f(\theta)}\psi'''[e^*(\theta)]\right) < \frac{\{\overline{c}_o'[p^*(\theta)]\}^2}{(1-\mu)\overline{V}''[p^*(\theta)] - (\theta - e^*(\theta))\overline{c}_o''[p^*(\theta)]} < \mu\frac{d}{d\theta}\left(\frac{F(\theta)}{f(\theta)}\right)\psi''[e^*(\theta)].$$

Because $-B \geq -(1-\mu)\overline{V}''[p^*(\theta)] > 0$, A3(i) ensures that

$$-\psi''[e^*(\theta)] < \frac{\{\overline{c}_o'[p^*(\theta)]\}^2}{(1-\mu)\overline{V}''[p^*(\theta)] - (\theta - e^*(\theta))\overline{c}_o''[p^*(\theta)]},$$

which implies the first inequality in (S.2) by A2. By A2(iii) and A3(ii), we have $D < 0$, while $B < 0$ thereby implying the second inequality in (S.2). Lastly, because $e^{*\prime}(\theta) + p^{*\prime}(\theta)B/A = 1$ by (S.1) with $A < 0$ and $B < 0$, it follows from $e^{*\prime}(\cdot) < 0$ that $p^{*\prime}(\cdot) > 0$, as desired. *Q.E.D.*

PROOF OF PROPOSITION 3: Recalling that $e(\tilde{\theta}, \theta)$ is the optimal level of effort for a firm with type θ, the firm's expected utility (4) from announcing $\tilde{\theta}$ is

$$U(\tilde{\theta}, \theta) = A(\tilde{\theta}) + \psi'[e^*(\tilde{\theta})]\big\{\tilde{\theta} - e^*(\tilde{\theta}) - (\theta - e(\tilde{\theta}, \theta))\big\} - \psi[e(\tilde{\theta}, \theta)]$$

(see the optimization problem (F*) in the proof of Proposition 2). To show that $\tilde{\theta} = \theta$ provides a global maximum, we first show that $U_{12}(\tilde{\theta}, \theta) > 0$ for any $(\tilde{\theta}, \theta)$. Using $U_{12}(\tilde{\theta}, \theta) = -\psi''[e(\tilde{\theta}, \theta)]e_1(\tilde{\theta}, \theta)$, this is equivalent to showing $e_1(\tilde{\theta}, \theta) < 0$, where $e(\tilde{\theta}, \theta)$ solves the FOC (3), which can be written under A1 as $0 = \psi'[e^*(\tilde{\theta})] - \psi'[e(\tilde{\theta}, \theta)]$ from the FOC of problem (F*). Differentiating this FOC with respect to $\tilde{\theta}$ gives $e_1(\tilde{\theta}, \theta)\psi''(\cdot) = \psi''(\cdot)e^{*\prime}(\cdot)$. Because $e^{*\prime}(\cdot) < 0$ by Lemma 2, the right-hand side is strictly negative under A2. Hence $e_1(\tilde{\theta}, \theta) < 0$, implying $U_{12}(\cdot, \cdot) > 0$ as desired. Second, we apply the argument in Appendix A1.4 in Laffont and Tirole (1993) with $\phi(\beta, \hat{\beta}) = U(\tilde{\theta}, \theta)$. Hence, $\tilde{\theta} = \theta$ provides the global maximum of $U(\tilde{\theta}, \theta)$.

To prove the second part, let $\bar{t}(\theta) \equiv \mathrm{E}[t^*(\theta, (\theta - e^*(\theta))c_o(y(p^*(\theta), \varepsilon_d), \varepsilon_c))]$ so that $\bar{t} = \bar{t}(\theta)$. Note that $\mathcal{E}(\theta) \equiv \theta - e^*(\theta)$ is strictly increasing in θ because $d(\theta - e^*(\theta))/d\theta = [1 - e^{*\prime}(\theta)] > 0$ and $e^{*\prime}(\cdot) < 0$. Thus $\theta = \mathcal{E}^{-1}(\mathcal{E})$, where $\mathcal{E}$ is the firm's cost inefficiency. We want to show that $\bar{t}(\theta) = \bar{t}[\mathcal{E}^{-1}(\mathcal{E})]$ is strictly decreasing in $\mathcal{E}$. From (15) and A1, we have $\bar{t}(\theta) = A(\theta)$. Hence, using (16),

$$\frac{d\bar{t}}{d\mathcal{E}} = \frac{A'(\theta)}{\mathcal{E}'(\theta)} = -\frac{\psi'[e^*(\theta)]}{1 - e^{*\prime}(\theta)]} < 0.$$

Thus, the expected transfer is strictly decreasing in $\mathcal{E}$, as desired. *Q.E.D.*

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