Econometrica Supplementary Material

SUPPLEMENT TO "LIMITED INFORMATION AND ADVERTISING IN THE U.S. PERSONAL COMPUTER INDUSTRY": B. MISCELLANEOUS

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This supplement consists of three sections. The first section presents a proof showing that the fixed-point algorithm described in Section 5.1 of the main paper is a contraction mapping. The second section contains parameter estimates for alternative models discussed in Section 7 of the main paper. The third section illustrates why the full information and limited information models will (most likely) result in different estimates for price elasticities of demand.

1. CONTRACTION MAPPING

IN THIS SUPPLEMENTAL SECTION, I show that the function used in the fixed-point algorithm is a contraction mapping. The proof parallels the proof for the full information case, see Berry, Levinsohn, and Pakes (1995, Appendix I) (BLP) for more detail. Variable definitions are given in the main paper.

Following BLP, I define

$$f(\delta_j) \equiv \delta_j + \ln(S_j^{\text{obs}}) - \ln(s_j(\delta)),$$

where some of the arguments of s_j are suppressed for ease of exposition. To prove that f is a contraction mapping, I must show that $\forall j, m$,

$$\partial f_j(\delta)/\partial \delta_m \geq 0, \tag{1}$$

and $\forall j$,

$$\sum_{m=1}^{J} \partial f_j(\delta)/\partial \delta_m < 1. \tag{2}$$

For the limited information model we can write

$$s_j = \int \sum_{\mathcal{S}_j \in \mathcal{C}_j} \prod_{l \in \mathcal{S}_j} \phi_{il} \prod_{k \notin \mathcal{S}_j} (1 - \phi_{ik}) \mathbb{P}_j(\mathcal{S}_j)\, dG_{y,D}(y, D)\, dG_\nu(\nu)\, dG_\kappa(\kappa),$$

where

$$\mathbb{P}_j(\mathcal{S}_j) = \frac{\exp\{\delta_j + \mu_{ij}\}}{y_i{}^\alpha + \sum_{r \in \mathcal{S}_j} \exp\{\delta_r + \mu_{ir}\}}.$$

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A direct computation verifies that for all m,

$$\frac{\partial f_j(\delta)}{\partial \delta_m} = \frac{1}{s_j} \int \sum_{\mathcal{S}_j \in \mathcal{C}_j} \prod_{l \in \mathcal{S}_j} \phi_{il} \prod_{k \notin \mathcal{S}_j} (1 - \phi_{ik}) \times \mathbb{P}_j(\mathcal{S}_j) \mathbb{P}_j^m(\mathcal{S}_j)\, dG_{y,D}(y, D)\, dG_\nu(\nu)\, dG_\kappa(\kappa), \tag{3}$$

where we defined

$$\mathbb{P}_j^m(\mathcal{S}_j) = \begin{cases} \dfrac{\exp\{\delta_m + \mu_{im}\}}{y_i^\alpha + \sum_{r \in \mathcal{S}_j} \exp\{\delta_r + \mu_{ir}\}} & \text{when } m \in \mathcal{S}_j, \\ 0 & \text{when } m \notin \mathcal{S}_j. \end{cases}$$

(Note that for $m = j$, $\mathbb{P}_j^m(\mathcal{S}_j) = \mathbb{P}_j(\mathcal{S}_j)$ since j is always in $\mathcal{S}_j$.) All derivatives in (3) are positive, hence (1) is satisfied. Moreover,

$$\sum_m \mathbb{P}_j^m(\mathcal{S}_j) = \frac{\sum_{r \in \mathcal{S}_j} \exp\{\delta_r + \mu_{ir}\}}{y_i^\alpha + \sum_{r \in \mathcal{S}_j} \exp\{\delta_r + \mu_{ir}\}} < 1,$$

so (2) is satisfied.

2. PARAMETER ESTIMATES OF BENCHMARK MODELS

This section provides parameter estimates for the benchmark models presented in the main paper. Table I presents estimate for a full information BLP model. Table II presents estimates for a model where advertising impacts demand directly (referred to as the uninformative model in the main text).

3. LIMITED INFORMATION ELASTICITIES

This section illustrates why the full information and limited information models will (most likely) result in different estimates for price elasticities of demand. I consider a simplification, but one which exhibits the properties of the models employed in this literature. The conditional indirect utility of consumer i from product j is given by

$$u_{ij} = -\alpha p_j + x_j \beta + \xi_j + \epsilon_{ij}.$$

Note that consumer heterogeneity enters the model only through the additive random shock, ϵ_{ij}. If the draws are independent across products and consumers, then this model exhibits the well-known independence of irrelevant alternatives (IIA) property. Traditional full information models would yield cross-price derivatives $\partial s_j / \partial p_k = \alpha s_j s_k$.

TABLE I

FULL INFORMATION NO ADVERTISING PARAMETER ESTIMATES[a]

Variable	BLP Coefficient	BLP Standard Error	Bajari–Benkard Large Shares Coefficient	Bajari–Benkard Large Shares Standard Error
Price coefficient				
ln(income − price)	1.1980**	(0.5130)	1.9074**	(0.3488)
Mean utility coefficients				
Constant	−32.4815**	(13.5997)	−9.3776**	(0.8890)
CPU speed (MHz)	12.1745**	(2.2525)	28.0316**	(3.1201)
Pentium	2.2631	(2.9031)	0.6132*	(0.5970)
Laptop	3.0241*	(0.8242)	0.9654**	(0.1742)
Acer	2.2559	(12.7105)	0.3635	(0.9125)
Apple	7.3454**	(0.6321)	0.4761**	(0.1558)
Compaq	8.7814**	(3.2137)	1.1281**	(0.0871)
Dell	1.2345*	(0.6980)	0.7226*	(0.4545)
Gateway	9.9450*	(5.1786)	1.7742*	(1.1622)
Hewlett–Packard	4.5117*	(2.3775)	2.6007*	(1.5305)
IBM	6.1112**	(0.6909)	0.9373**	(0.0746)
Micron	1.1279	(2.2789)	0.0345	(0.1969)
Packard–Bell	6.6300*	(3.3207)	0.9319**	(0.4520)
Standard deviations				
Constant	0.2429	(0.9822)	0.3754	(1.9628)
CPU speed (MHz)	0.2878**	(0.0566)	0.1047**	(0.0412)
Pentium	0.7168*	(0.3617)	0.7051**	(0.2108)
Laptop	0.3158**	(0.1425)	1.1943**	(0.3961)
Interactions				
CPU speed* household size	0.6967**	(0.2925)	0.2435**	(0.0255)
Pentium* income > $100,000	0.7495*	(0.3893)	0.9040*	(0.4893)
Laptop* 30 < age < 50	−0.2052	(0.5434)	1.4386*	(1.1192)
Laptop* white male	0.3913*	(0.2015)	0.9048	(1.9959)
Marginal cost				
Constant	12.6836**	(0.3503)	7.1642**	(0.4113)
ln(CPU speed)	1.2788*	(0.6788)	0.6473*	(0.6183)
Pentium	0.8888**	(0.1854)	0.2142**	(0.0240)
Laptop	−0.5078**	(0.1347)	0.4135**	(0.1570)
Quarterly trend	−0.1009**	(0.0432)	−0.0489**	(0.0071)

[a] ** indicates t-stat > 2; * indicates t-stat > 1. The BLP model includes micromoments. The Bajari–Benkard model includes only those products which sold more than 5000 units.

TABLE II

FULL INFORMATION UNINFORMATIVE ADVERTISING PARAMETER ESTIMATES[a]

				Interactions With Demographic Variables					
Variable	Coefficient	Standard Error	Standard Deviation	Household Size	Income < \$60,000	Income > \$100,000	30 < age < 50	High School Graduate	White Male
Demand side parameters									
Constant	−16.3836**	(6.7999)							
CPU speed (MHz)	18.5052**	(4.5050)	0.5352**	0.9336**	—	—	—	—	—
			(0.2262)	(0.4387)					
Pentium	4.3071	(8.7092)	0.0649**	—	—	−1.9431*	—	—	—
			(0.0289)			(1.6543)			
Laptop	−1.8485*	(0.9696)	0.1562**	—	—	—	−2.8122*	—	1.5265*
			(0.0778)				(2.7168)		(1.5109)
ln(income − price)	1.3962**	(0.6839)							
Acer	2.6190	(12.7105)							
Apple	7.1964**	(3.1603)							
Compaq	3.9684*	(2.4103)							
Dell	−3.5496*	(2.6175)							
Gateway	4.0329	(4.1429)							
Hewlett–Packard	−5.6777*	(2.9198)							
IBM	3.8068**	(1.5545)							
Micron	6.1322*	(5.4693)							
Packard–Bell	−2.8169*	(1.5094)							
Group advertising	0.9456**	(0.4530)							
(Group advertising)^2	0.0328**	(0.0160)							
Magazine	7.5328**	(3.1603)							
Newspaper	−0.0726	(0.4387)							
Radio	−5.3824*	(2.8625)							

(*Continues*)

TABLE II—*Continued*

Variable	Coefficient	Standard Error	Standard Deviation	Interactions With Demographic Variables: Household Size	Income < \$60,000	Income > \$100,000	30 < age < 50	High School Graduate	White Male
Television	2.6127*	(1.5094)	0.0880*	0.0382	0.0152**	0.0021	−0.0177*	0.0290	−0.0724*
			(0.0792)	(0.1580)	(0.0074)	(0.0055)	(0.0094)	(0.1509)	(0.0439)
Magazine and newspaper			0.6122*	−0.6658**	−0.1630**	−0.0248*	0.7535*	0.2328	−0.8555*
			(0.3167)	(0.3187)	(0.3178)	(0.0125)	(0.6232)	(0.8034)	(0.7299)
ln marginal cost of production parameters									
Constant	7.5037**	(0.7005)							
ln(CPU speed)	0.2486**	(0.0185)							
Pentium	−0.4403**	(0.2039)							
Laptop	1.1417**	(0.5387)							
Quarterly trend	−0.1874**	(0.0886)							
ln marginal cost of advertising parameters									
Constant	4.6904**	(2.3076)							
Price of advertising	1.0000**	(0.0197)							
Non-home sector marginal revenue parameters									
Constant	1.2943*	(1.1699)							
Non-home sector price	1.0252**	(0.1648)							
CPU speed	0.0169**	(0.0083)							
Non-PC sales	5.1320*	(2.6860)							

[a] ** indicates t-stat > 2; * indicates t-stat > 1. Standard errors are given in parentheses.

Consider a limited information framework in a market consisting of three products, each sold by a different firm (and an outside good) and one individual. Denote the probability the consumer is aware of a product by ϕ_j and let $\delta_j = -\alpha p_j + x_j\beta + \xi_j$. The market share of product 1 is

$$s_1 = \phi_1 \exp(\delta_1)\left\{\frac{(1-\phi_2)(1-\phi_3)}{1+\exp(\delta_1)} + \frac{(1-\phi_2)\phi_3}{1+\sum_{k=\{1,3\}}\exp(\delta_k)} + \frac{\phi_2(1-\phi_3)}{1+\sum_{k=\{1,2\}}\exp(\delta_k)} + \frac{\phi_2\phi_3}{1+\sum_{k=\{1,2,3\}}\exp(\delta_k)}\right\}.$$

Similarly for the other two products. For ease of exposition, consider the situation in which the consumer is aware of product 3. The resulting derivatives with respect to price are

$$\frac{\partial s_1}{\partial p_3} = \alpha\phi_1\phi_3 \exp(\delta_1+\delta_3) \times \left[\frac{(1-\phi_2)}{(1+\sum_{k=\{1,3\}}\exp(\delta_k))^2} + \frac{\phi_2}{(1+\sum_{k=\{1,2,3\}}\exp(\delta_k))^2}\right] \tag{4}$$

and

$$\frac{\partial s_2}{\partial p_3} = \alpha\phi_2\phi_3 \exp(\delta_2+\delta_3) \times \left[\frac{(1-\phi_1)}{(1+\sum_{k=\{2,3\}}\exp(\delta_k))^2} + \frac{\phi_1}{(1+\sum_{k=\{1,2,3\}}\exp(\delta_k))^2}\right]. \tag{5}$$

Under full information, if the market shares for products 1 and 2 are approximately the same, then these two products will have similar cross-price derivatives with respect to any third product. As equations (4) and (5) show, only if ϕ_1 and ϕ_2 are approximately the same and δ_1 is close to δ_2 will products 1 and 2 have similar cross-price derivatives with respect to any other product in the limited information framework.

The substitution patterns are not as restrictive as in traditional models because cross-price derivatives depend on δ and ϕ, which are functions of product and consumer characteristics. Furthermore, price elasticities of demand generated under limited information will be functions of the characteristics of all products offered as well as consumer attributes. To reiterate, if the only source of consumer heterogeneity in the indirect utility function is from an additive independent and identically distributed shock, the limited information substitution patterns are driven by differences across products and consumers, and hence not as restrictive as in traditional models.

To calculate the price elasticities of demand, we compute the (numerical) derivative, multiply by actual price, and divide by actual shares. Again for ease of exposition, consider the simplified market presented above where we examine the effect of a change in the price for product 3 on the market share of product 2. The limited information model will result in more inelastic cross-price elasticities when

$$\frac{\partial s_2^{\text{limited}}}{\partial p_3} < \frac{\partial s_2^{\text{full}}}{\partial p_3}.$$

When limited information is important, the implied values of ϕ_1, ϕ_2, ϕ_3 will be lower. Equation (5) suggests that which model produces more inelastic demand curves depends not only on the mean utility (which is a function of ξ), but also on the implied values of ϕ_j for every product. The lower are the ϕ_1, ϕ_2, ϕ_3 terms, the more likely the limited information model will produce more inelastic elasticities.

Allowing consumer tastes to vary with product attributes (as in BLP and others) softens the restrictive substitution patterns in traditional models, but does not change the fundamental results presented here. This supplemental section supports the notion that the limited information model (i) will result in more inelastic estimated product elasticities (for certain values of the parameters when limited information is important) and (ii) will rely more on differences in consumer attributes and product attributes (as well as differences in information across households) to explain substitution patterns than will traditional models of full information.

REFERENCE

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