Optimal Fiscal and Monetary Policy with Investment Spillovers and Endogenous Private Information*

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Abstract

How should firms be incentivized to invest efficiently (e.g., in AI or the supply of smart inputs), when such investments come with spillovers and their profitability depends on uncertain aggregate economic conditions? We show that, under flexible prices, firms can be induced to collect information about aggregate fundamentals efficiency and then use it in society's best interest through a subsidy that resembles a Pigou's correction but accounts for the non-verifiability of firms' acquisition and usage of information. The same fiscal policy also induces efficiency in information acquisition and usage when prices are sticky, under an appropriate monetary policy that induces firms to disregard their endogenous private information when setting prices and only use it for investment purposes.

Keywords: endogenous information, investment spillovers, optimal fiscal and monetary policy, Pigouvian corrections

JEL classification: D21, D62, D83, E60, E62

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1 Introduction

New technologies typically come with spillovers which influence firms' investment decisions. In fact, one of the key questions many firms face these days is whether to switch to new AI-based technologies immediately, or wait – taking advantage of the positive spillovers created by those firms investing early – and switch at a later stage, when the adoption cost is lower. There is no doubt that the new technologies are superior and that most firms will eventually adopt them. However, whether it is best for an individual firm to invest early or late often depends on aggregate economic conditions that are uncertain but which the firm can collect information about by prior to making its decision.

Similarly, the choice to supply intermediate goods – tools and machinery – in a traditional or "smart" (Industry 4.0) specification comes with analogous challenges. A smart component helps regulating important aspects of the production process of final goods (such as its temperature and humidity) and may facilitate the detection of malfunctions in the utilization of other intermediate goods.[1] As a result, the contribution of each intermediate good to the production of the final good increases with the aggregate amount of the intermediate goods supplied in their smart specification. The decision over which specification to favor depends on firms' expectation about aggregate economic fundamentals responsible for the demand of the final goods as well as their expectations about other firms' decisions of whether to supply intermediate goods in traditional or smart specification.

In such contexts, how should a benevolent government use fiscal incentives and monetary policy to incentivize firms to collect and use information in society's best interest? This question is at the center of an active policy debate as many countries are devoting significant resources to incentivize firms to switch to new technologies, develop "smart" inputs, provide critical infrastructure, and, more broadly, invest in sectors, products, and production processes believed to be of strategic national importance.

In this paper, we develop a flexible framework that permits us to capture some of the key trade-offs that firms face in a broad class of investment problems with spillovers and endogenous private information such as those mentioned above. We show that, in the absence of nominal rigidities (namely when firms make their investment decisions under imperfect in-

[1] A 2020 (June 23^{rd}) report by *The Economist Intelligence Unit* on the Internet of Things (https://www.eiu.com/n/the-internet-of-things-applications-for-industry/) illustrates a number of circumstances in which smart components play an important role. For instance, they contribute to reducing waste – e.g., through sensors optimizing energy use based on the level of activity, or reducing the spoilage of products in transit by monitoring temperatures – and increase productivity – e.g., through sensors that process information on fundamental aspects of the production process and the supply chain optimizing them, hence increasing efficiency and reducing costs across the board (also for the functions within the production and distribution processes that involve the usage of traditional components only).

formation about aggregate economic conditions but set prices and hire labor under complete information), efficient decisions can be induced by combining familiar revenue subsidies correcting for firms' market power with additional subsidies to the investing firms. The latter subsidies need to be appropriately designed to induce firms to invest when, and only when, investment is socially efficient, given firms' limited private information about the relevant fundamentals.

We also show that, if information was dispersed but exogenous, these additional subsidies would often be simple, namely they could be made invariant in aggregate fundamentals and other firms' decisions. When, instead, firms must also be incentivized to collect information in society's best interest and the cost of information acquisition is unknown to the government, it is essential to condition these subsidies on aggregate fundamentals and total investment (both revealed ex-post, i.e., at the end of the relevant time window). Such richer subsidies operate as a Pigouvian correction, realigning the private value of investment to its social counterpart, by inducing firms to internalize the externality associated with the spillovers generated by their investment decisions. Importantly, these Pigouvian-like policies also realign the private value of acquiring more precise information to its social counterpart, accounting for the fact that neither the acquisition nor the usage of information is verifiable. That, when information is complete and firms' activities are verifiable, Pigouvian subsidies/taxes correct externalities and induce efficient allocations is known. The paper's contribution is in showing that a specific version of these policies also create the right incentives for information acquisition and its subsequent utilization when neither of the two activities is verifiable.

Finally, we show that, when prices are sticky, that is, firms set them under dispersed (and endogenous) information about aggregate fundamentals, the same fiscal policies described above remain optimal but must be paired with a monetary policy that induces firms to disregard their endogenous private information when setting prices, and only use it for investment purposes.

In our model, the key externality originates in investment spillovers. We expect Pigouvian policies similar to those discussed in the paper to induce efficiency in information acquisition and usage also in the presence of other externalities such as those associated with pollution and/or the adoption of "greener" technologies.

Related literature. Optimal policy under endogenous private information has been studied in both the macroeconomics and microeconomics literature. See, among others, Angeletos and La'O (2020), and Angeletos, Iovino, and La'O (2020) for fiscal and monetary policy over the business cycle with dispersed endogenous information, and Bergemann and Välimäki (2002) for how to use Vickrey-Clarke-Groves (VCG) transfers to incentivize information acquisition prior to participating in a mechanism. Our contribution is in introducing investment spillovers and showing how they interact with the acquisition of private information in a stylized but standard general-equilibrium model, and investigating how the interaction shapes optimal fiscal and monetary policy.

The paper is also related to the literature on corrective taxation in the presence of various types of externalities, as pioneered by Pigou (1920)—see also Baumol (1972). This is a conspicuous literature that is too broad to summarize here. See Sandmo (1975) for one of the earlier applications to environmental economics, and Barrage (2020) for recent developments within the same literature. See also Bovenberg and Goulder (1996) for one of the early general-equilibrium analysis of Pigouvian policies, Romer (1986), Barro (1990), and more recently Chan et al. (2009), Grossman et al. (2013), Heutel (2012), and Jeanne and Korinek (2019) for the growth and business cycle implications of these policies. Our contribution is in endogenizing private information about relevant economic fundamentals affecting the profitability of the relevant investment decisions and showing how an appropriate combination of fiscal and monetary policy can correct for inefficiencies in both the acquisition and usage of information, both when prices are flexible and when they are sticky.

To isolate the novel effects, we abstract from the familiar learning externalities that arise when firms learn from the behavior of other firms, for example by observing the behavior of firms investing earlier, as in the literature on observational and social learning pioneered by Banerjee (1992) and Bikhchandani et al (1992)—see also Wolitzky (2018) for a recent contribution in which firms learn from the outcomes instead of the decisions of their predecessor. Learning externalities also arise when prices aggregate information. See Grossman and Stiglitz (1980) for one of the early contributions, Angeletos and Werning (2006) for information aggregation preceding financial crises, and Pavan, Sundaresan, and Vives (2023) for the design of taxes in markets in which traders compete in schedules and private information is endogenous. None of these papers investigates how to correct the inefficiencies (in information acquisition and usage) that arise in the presence of investment spillovers when private information is endogenous, which is the focus of the present paper. The closest paper to ours in this literature is Lemoine (2023) who studies climate change policies in the presence of pollution externalities when financial markets aggregate private information. We share with this paper the focus on how to correct for direct payoff-relevant externalities under dispersed information. Contrary to this paper, we do not consider information aggregation and instead endogenize the acquisition of private information.

Outline. The rest of the paper is organized as follows. Section 2 introduces the model, abstracting from nominal rigidities. Section 3 contains the key results about the structure of optimal fiscal policy. Section 4 introduces nominal rigidities (sticky prices) and discusses how the fiscal policies in Section 3 remain optimal when paired with an appropriate monetary policy. Section 5 concludes. All proofs omitted in the main text are in 6. In the online Supplement, we show that the results for the version of the model discussed in Subsection 2.2 in which the intermediate goods can be supplied in a "smart" or "traditional" specification are identical to those for the version of the model introduced in Subsection 2.1 and analyzed in the rest of the paper. We also show how the results extend to a richer family of economies in which consumers have preferences that are non-linear over the consumption of the final good.

2 The Model

We start by describing a parsimonious but fairly flexible model of investment under uncertainty with endogenous private information. We then show how the model can accommodate for early vs late adoption of new technologies, or the supply of intermediate goods in baseline or "smart" specification. As anticipated above, the model abstracts from learning externalities (which are well-understood) and instead focuses on the interaction between endogenous private information and investment spillovers.

The economy is populated by (i) a measure-1 continuum of firms, each producing a differentiated intermediate good, (ii) a competitive retail sector producing a final good using the intermediate goods as inputs, (iii) a measure-1 continuum of homogenous workers, and (iv) a benevolent government controlling fiscal and monetary policy.

Each firm is run by a single entrepreneur who must decide whether or not to invest. Indexing firms by $i \in [0, 1]$, we denote by $n_i = 1$ the decision by firm i to "invest," and by $n_i = 0$ the decision to "not invest". The interpretation of the decision to invest (alternatively, to not invest) is application-specific. For example, when it comes to technology adoption, "invest" may correspond to the decision to adopt the new superior technology early, whereas the decision "to not invest" corresponds to the decision to adopt the same technology later, as in Subsection 2.1. When it comes to the supply of inputs in different specifications, "invest" may correspond to the decision to supply an intermediate good in its "smart" specification, whereas "not invest" corresponds to the decisions to supply the same input in its traditional specification, as in Subsection 2.2. More broadly, "invest" may stand for a broad range of economic activities for which spillovers play an important role. The main results in the next two sections extend to the case where such an extensive margin is paired with an intensive one, whereby investment is a continuous choice; however, the key insights are easier to appreciate by focusing entirely on the extensive margin, which is what the paper does.

Investing costs $k > 0$, with the cost interpreted to be in terms of the entrepreneur's disutility. What matters for the results is that the cost is not mediated by a market that fully aggregates the entrepreneurs' dispersed information.

Let

$$N = \int n_i di$$

denote the aggregate size of investment, y_i the amount of the intermediate good produced by firm i, and Y the amount of the final good. The production of the intermediate and the final goods depends on the application of interest. We discuss two alternative specifications in Subsections 2.1 and 2.2 below.

The price of the final good is P and the profits of the competitive retail sector are given by

$$\Pi = PY - \int p_i y_i di,$$

where p_i is the price of the intermediate good paid to firm i.

Let $\Theta \in \mathbb{R}_+$ denote the "fundamental" variable, summarizing all exogenous aggregate economic conditions responsible for the profitability of the firms' investment decisions, with $\theta \equiv \log \Theta$. Both the firms and the government commonly believe that θ is drawn from a Normal distribution with mean 0 and precision π_θ. The realization of θ is not observed by the firms at the time they make their investment decisions. Instead, each firm i chooses the precision π_i^x of an additive signal

$$x_i = \theta + \xi_i$$

about θ it privately observes, with ξ_i drawn from a Normal distribution with mean zero and precision π_i^x, independently from θ and independently across i. The cost of information of precision π_i^x is equal to $\mathcal{I}(\pi_i^x)$, with $\mathcal{I}$ continuously differentiable and such that $\mathcal{I}'(0) = 0$, $\mathcal{I}'(\pi_i^x) > 0$ and $\mathcal{I}''(\pi_i^x) \geq 0$ for all $\pi_i^x > 0$.[2] Such a cost can also be interpreted as disutility of effort. The results extend to general/flexible information technologies (see Propositions 2 and 3) but are best illustrated with the Gaussian structure described above.

After selecting π_i^x and receiving information x_i, firm i chooses whether or not to invest. After learning Θ and N, the firm then chooses the price p_i for its intermediate good. Finally, given Θ, N, and the observed demand for its product, firm i employs labor l_i on a competitive market to meet its demand. Labor is supplied by the continuum of measure-one workers.

Consistently with the pertinent literature, we assume that the entrepreneur running each

[2] We denote such a cost with $\mathcal{I}$ (which is meant to be mnemonic for information cost) instead of C to void confusion with the consumption of the final good.

firm i is a member of a representative household whose utility function is given by

$$U = C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon} - \int \mathcal{I}(\pi_i^x) di - \Upsilon,$$

with $\varepsilon > 0$, where $l^{1+\varepsilon}/(1+\varepsilon)$ denotes the disutility of labor, and Υ is a tax paid to the government, expressed in terms of units of consumption of the final good. Because labor is homogenous and exchanged in a competitive market, each worker provides the same amount of labor (i.e., $l_i = l$ for all i). That U is linear in C is not important for the results. In the Supplementary Material, we consider the case where U is iso-elastic in C. In this case, the assumption that each entrepreneur is a member of a representative household implies perfect consumption-risk sharing, as in Angeletos, Iovino and La'O (2016), and Angeletos and La'O (2020).[3]

Being a member of the representative household, each entrepreneur maximizes his firm's market valuation, taking into account that the profits the firm generates are used for the purchase of the final good. This means that each entrepreneur maximizes

$$\mathbb{E}\left[\frac{p_i y_i - W l_i}{P} + T_i \middle| x_i, \pi_i^x\right] - k n_i - \mathcal{I}(\pi_i^x),$$

where W is the nominal wage rate, and T_i is a transfer to the firm in terms of the consumption of the final good based on the firm's revenue $r = p_i y_i / P$, expressed in terms of the consumption of the final good. Naturally, T_i may also depend on whether the firm invested or not.[4]

The representative household collects profits from all firms and wages from all workers, and pays a lump-sum tax Υ to the government. Using the fact that (a) the government budget must be balanced, i.e., $\int T_i di = \Upsilon$, (b) the total labor demand must equal the total labor supply, (c) all entrepreneurs choose the same precision of private information in equilibrium, (d) firms' total revenues coincide with the total expenditure on the final good, and (e) the total consumption of the final good C coincides with its production Y, we have that the government's objective can be expressed as

$$\mathcal{W} = \mathbb{E}\left[C - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x). \tag{1}$$

The government thus maximizes aggregate consumption, net of all the investment costs, the

[3]See Llosa and Venkateswaran (2022) for a recent business-cycle paper in which U is linear in C.

[4]One could also consider other fiscal policies in which the transfers to the firms are a function of employment, profits, or a combination of these and other verifiable variables. Following the pertinent literature, we focus on revenue-based transfers.

labor costs, and the information-acquisition costs. It does so by designing a fiscal and monetary policy (more on this in the next sections).

Summarizing, the timing of events is the following:

1. The government announces its policies;
2. Nature draws θ;
3. each entrepreneur i chooses the precision π_i^x of his private information;
4. each entrepreneur i receives a private signal x_i about θ;
5. entrepreneurs simultaneously choose n_i;
6. after θ and N are publicly revealed, entrepreneurs simultaneously set prices p_i;
7. the competitive retail sector chooses how much of each intermediate good to purchase, taking the prices of the intermediate goods and the price P of the final good as given;
8. given the demand y_i for his intermediate good, entrepreneur i hires l_i units of labor to produce y_i, taking N and θ as given;
9. a representative household comprising all workers and entrepreneurs chooses how much of the final good to buy, taking the price of the final good P as given.

We now discuss two applications which correspond to two different specifications of the firms' production function. In the remaining of the paper, we focus on the first specification. In the Appendix, we verify that the results for the second specification are identical.

2.1 Early vs late technology adoption

Firms must decide whether to adopt a new technology (e.g., AI, or some other process enhancing firms' output) early or late. The new technology is superior to the existing one and all firms eventually adopt it. A faster adoption of the new technology comes with positive spillovers, originating in the combination of usual network effects (for the early adopters) with the development of auxiliary products and services (e.g., knowledge and software) benefitting all firms, including those that, in the early stages, retain the old technology. For example, a firm deciding to postpone a full-scale switch to an AI-based production process may benefit from more firms switching early to AI through the development of AI-based software useful also when operating under the old technology. Importantly, whether firms find it optimal to adopt early or late depends on other firms' choices and on aggregate economic conditions (the fundamentals) unknown to the firms at the time they make their decisions.

We capture this situation by assuming that the amount of the intermediate good produced by firm i is given by

$$y_i = \begin{cases} \gamma\Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 1 \\ \Theta\left(1+\beta N\right)^{\alpha} l_i^{\psi} & \text{if } n_i = 0 \end{cases}, \tag{2}$$

with $\gamma > 1$, $\beta \geq 0$, $\alpha \geq 0$, and $\psi \leq 1$. Under this specification, the fundamental variable $\Theta > 0$ proxies for aggregate economic conditions responsible for the production of firms' intermediate goods. The assumption that $\gamma > 1$ reflects the property that the new technology is superior, in the sense of boosting the output of each firm that adopts it. The parameters α and β control for the returns to scale and the intensity of the production spillovers, respectively. Finally, the parameter ψ controls for the returns to scale of labor. That all firms benefit from a larger adoption N at the early stages, including those that retain the old technology, reflects the type of spillovers described above. That the investment spillovers are the same for all firms simplifies the analysis but is not essential to the results. What matter is that the extra output

$$(\gamma - 1)\Theta\left(1+\beta N\right)^{\alpha} l^{\psi}$$

produced by each investing firms is increasing in N and Θ.

The final good is produced by a competitive retail sector according to the CES technology

$$Y = \left(\int y_i^{\frac{v-1}{v}} di\right)^{\frac{v}{v-1}}, \tag{3}$$

with $v > 1$ denoting the elasticity of substitution between intermediate goods.

There are two periods. Those firms that do not adopt the new technology in the first period do so in the second one, when the adoption cost becomes low enough to make it dominant for all firms to switch. The profits that each firm makes in the second period are invariant in whether the firm adopted early (in the first period) or late (in the second period). This simplifying assumption permits us to turn the dynamic economy under consideration into the static one described above, with all relevant decisions made in the first period.[5] Under this specification, one can drop all period-2 decisions and the associated flow profits and interpret all the relevant variables as pertaining to the first period.[6]

[5] When, instead, firms' early technology adoption has long-lasting effects on firms' production (e.g., in the presence of learning by doing), results similar to those reported below continue to hold but the static externalities must be replaced by their dynamic counterparts, accounting for the effects that early decisions have on continuation profits.

[6] As mentioned above, this specification abstracts from familiar learning externalities that arise when late

The assumptions that firms are differentiated monopolists, that the production function is Cobb-Douglas, and that the technology for producing the final good is iso-elastic, are standard in the pertinent macro literature on optimal fiscal and monetary policy. Assuming the same structure facilitates the comparison with previous work and permits us to isolate the novel effects on optimal fiscal and monetary policy originating in the interaction between (a) investment spillovers and (b) endogenous private information, which is the contribution of the paper. This structure is also known to facilitate the computation of the equilibrium allocations in the presence of nominal rigidities (sticky prices), which we address in Section 4.

2.2 Traditional vs "smart" input supply

Each firm must decide whether to produce the intermediate good in a traditional or in a "smart" (Industry 4.0) specification.[7] A smart specification comes with software that improves the interoperability of the inputs used in the production of the final good. Under this alternative specification, the amount of intermediate good that each firm produces is equal to

$$y_i = l_i^{\psi}, \tag{4}$$

where $l_i \in \mathbb{R}_+$ continues to denote the amount of labor employed by firm i, and $\psi \leq 1$ the labor returns to scale. The cost of producing the intermediate good in its smart specification is k. This cost is over and above the cost of employing labor l_i. Denoting by $n_i = 1$ (alternatively, $n_i = 0$) the decision by firm i to produce the good in its smart (alternatively, traditional) specification, and by $N = \int n_i di$ the aggregate measure of firms producing goods in their "smart" specification, we have that the amount of the final good produced is equal to

$$Y = \Theta \left(1 + \beta N\right)^{\alpha} \left(\int_i ((1 - n_i + \gamma n_i) y_i)^{\frac{\upsilon - 1}{\upsilon}} \, di \right)^{\frac{\upsilon}{\upsilon - 1}}, \tag{5}$$

where $\upsilon > 1$, $\alpha \geq 0$, and $\beta \geq 0$ continue to denote the elasticity of substitution between intermediate goods, the returns to scale, and the intensity of the investment spillovers, respectively, $\gamma > 1$ is the extra output generated by a good supplied in ints smart specification, and $\Theta > 0$ denotes aggregate economic conditions affecting the production of the final good.

Under this alternative specification, fixing the amount y_i of intermediate good produced, the decision by each firm to provide this good in its smart specification increases the amount

adopters learn from the experience of early adopters. Instead, it focuses on how policy (both fiscal and monetary) affects firms' decision to acquire private information about aggregate fundamentals and use it to choose whether to adopt early or late.

[7] See, e.g., Bai et al. (2020).

of the final good produced both directly and by enhancing the interoperability/productivity of each other intermediate good, including those supplied in their traditional specification. As in the alternative version of the production economy described above, the final good is produced in a competitive retail sector, taking its price P and the prices $(p_i)_{i\in[0,1]}$ of all the intermediate goods as given. These prices naturally depend on whether the intermediate goods are supplied in their "smart" or traditional specification.

3 Constrained Efficiency, Equilibrium, and Optimal Fiscal Policy

From now on, we will refer to $n_i = 1$ (alternatively, $n_i = 0$) as firm i's decision to invest (alternatively, to not invest), without committing to a specific interpretation of what investment means. However, to make things concrete, we will assume that the production function takes the form in Subsection 2.1, with the understanding that all the results apply verbatim also to the specification in Subsection 2.2.[8]

Subsection 3.1 characterizes constrained efficiency, whereas Subsection 3.2 characterizes the properties of the equilibrium allocations. Finally, Subsection 3.3 characterizes optimal fiscal policies. Because prices in the economy under consideration are flexible (i.e., are set by the firms after observing θ), money in this economy has only a nominal effect on prices and plays no other role. We thus omit it for the time being, and introduce it only in Section 4, where we consider optimal fiscal and monetary policy in the presence of nominal rigidities.

3.1 Constrained Efficiency

We assume that the government cannot transfer information across agents. This restriction is standard in the literature on optimal fiscal and monetary policy under dispersed information (see, among others, Vives (1988), Angeletos and Pavan (2007), Colombo, Femminis and Pavan (2014), Angeletos, Iovino and La'O (2016), Angeletos and La'O (2020), and Llosa and Venkateswaran (2022)).

The constrained efficient allocation has three parts: the precision of private information, π^{x*}, a rule specifying whether or not firms should invest based on their private information x, and a rule describing how much labor each firm should employ as a function of θ and x (equivalently, θ and the type of technology adopted). These three parts are chosen jointly to maximize ex-ante welfare, $\mathcal{W}$, as given in (1). Lemma 1 focuses on efficient investment

[8]See the discussion in the online Supplement.

decisions. The rule describing the efficient employment of labor is in the proof of Lemma 1, whereas the formula for the efficient precision of private information π^{x*} is in the proof of Lemma 3. The reason for relegating these parts to 6 is that they are useful for comparative statics but not essential to the arguments establishing the key results.

Lemma 1. *Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$, *and assume that* $\gamma^{\varphi} \geq 1+\beta$ *and* $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency in investment decisions requires that each firm with signal* $x > \hat{x}(\pi^x)$ *invests, whereas each firm with signal* $x < \hat{x}(\pi^x)$ *does not.*

Proof. See 6.

The parameters' restrictions in the lemma guarantee that the social value of investing (net of its cost) is increasing in the fundamental θ and in the mass N of firms investing. These monotonicities, in turn, imply that the efficient rule for investment is monotone in the firms' private information. These restrictions are fairly standard. They have a role similar to the one played by the assumption that substitution effects are stronger than income effects in other macro settings. That the efficient investment rule is monotone in signals is not essential for our key results but it facilitates the exposition. In particular, it permits us to fully characterize necessary and sufficient conditions for a fiscal policy to implement the efficient allocation, both when information is exogenous (Lemma 2), and when it is endogenous (Lemma 3). On the other hand, the results in Propositions 1 and 2 below, establishing that Pigouvian corrections eliminate any discrepancy between private and social objectives (and hence induce efficiency in both information acquisition and usage, despite the fact that neither of the two activities is verifiable), apply also to economies in which the constrained-efficient allocation is not monotone.[9]

3.2 Equilibrium

The following definition summarizes the key equilibrium conditions.

Definition 1. A (symmetric) **equilibrium** consists of (1) a precision π^x of private information, (2) an investment strategy $n(x;\pi^x)$, and (3) a pair of price functions $p_1(\theta;\pi^x)$ and $p_0(\theta;\pi^x)$, respectively for firms investing and for those refraining from doing so, such that, when each firm $j \neq i$ chooses a precision of information equal to π^x, decides whether or not to invest according to $n(x;\pi^x)$, and sets its price according to $p_1(\theta;\pi^x)$ and $p_0(\theta;\pi^x)$, each entrepreneur i maximizes his firm's market valuation by doing the same.

[9]It is also easy to see that these results extend to economies in which investment is a continuous choice.

The complete description of the equilibrium allocation also entails the specification of the labor $l_1(\theta; \pi^x)$ and $l_0(\theta; \pi^x)$ demanded respectively by those firms investing and those not investing, the total labor supply $L(\theta; \pi^x)$, the wage $W(\theta; \pi^x)$, and the price $P(\theta; \pi^x)$ of the final good, with all the equilibrium variables naturally conditioning on the fundamentals θ and the endogenous precision of private information π^x. These functions are standard and described concisely below. They are not included in the equilibrium definition so as to highlight the parts that are most relevant for our results.[10]

As usual, the assumption that the retail sector is competitive implies that, in equilibrium, profits are equal to zero (i.e., $\Pi = 0$), and that the price of the final good is equal to

$$P = \left(\int p_i^{1-v} di \right)^{\frac{1}{1-v}}, \tag{6}$$

with the demand for each intermediate good given by

$$y_i = C \left(\frac{P}{p_i} \right)^v, \tag{7}$$

where $C = Y$. Furthermore, because labor is undifferentiated and the labor market is competitive, the supply of labor is given by

$$\frac{W}{P} = l^{\varepsilon}, \tag{8}$$

where the left-hand side is the real wage (that is, the wage in units of consumption of the final good), whereas the right-hand side is the marginal disutility of labor. The labor demand for each entrepreneur i is then given by

$$l_{1i} = \left(\frac{y_i}{\gamma \Theta (1 + \beta N)^{\alpha}} \right)^{1/\psi}, \tag{9}$$

for the investing entrepreneurs, and by

$$l_{0i} = \left(\frac{y_i}{\Theta (1 + \beta N)^{\alpha}} \right)^{1/\psi}, \tag{10}$$

for the non-investing ones. In both cases, the entrepreneur takes N and Θ as given and employs labor to produce the amount of intermediate good y_i demanded. Market clearing in

[10]The dependence of all the equilibrium variables on π^x is meant to highlight the fact that the fraction of investing firms in each state θ depends on π^x. Highlighting the dependence on π^x also facilitates the comparison between the equilibrium and the efficient allocations.

the labor market then implies that

$$\frac{W}{P} = \left(\int l_i di \right)^{\varepsilon}. \tag{11}$$

3.3 Optimal Fiscal Policy

We first characterize (jointly necessary and sufficient) conditions that any optimal fiscal policy satisfies when the precision of private information π^x is exogenous. Next, we characterize additional conditions that any optimal policy must satisfy when information is endogenous. The comparison between the two sets of conditions permits us to illustrate that policies that are optimal under exogenous information need not be optimal when information is endogenous. Along the way, we also show that simple subsidies to the investing firms that are invariant in θ suffice to induce efficiency in the usage of information, but may fail to induce efficiency in the acquisition of information. The latter requires that the subsidies co-move with the marginal effect of more precise private information on the measure of investing firms, which in turn requires conditioning the subsidies on the fundamentals θ. At the end of the section, we discuss how a government that does not know the cost of information can induce efficiency in both information acquisition and usage with even richer subsidies that condition on both the fundamentals θ and the measure of investing firms N.

3.3.1 Exogenous Information

Suppose that the precision of private information is exogenous and equal to π^x. Let $\hat{n}(x;\pi^x)$ denote the rule describing the efficient investment decisions, and $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ the rules describing the efficient labor employment, for the investing and the non-investing firms, respectively. Let $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ denote the efficient production of the intermediate goods for each of the two types of firms. Finally, let $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ denote the prices, respectively for the investing and the non-investing firms, that induce demands equal to $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ and hence employment equal to the efficient levels $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$.

Definition 2. Assume that the precision of private information is exogenous and equal to π^x. The fiscal policy $\bar{T}$ is **optimal** if it implements the efficient usage of information as an equilibrium; that is, if it induces all firms to invest according to the efficient rule $\hat{n}(x;\pi^x)$ and set prices according to the rules $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$.

Let $r = py/P$ denote a representative firm's revenue in terms of the consumption of the final good. Next, let $\hat{C}(\theta;\pi^x)$ and $\hat{N}(\theta;\pi^x)$ denote, respectively, the amount of the final good consumed and the measure of firms investing in state θ when the precision of private information is π^x and all firms make all decisions efficiently. Hereafter, we denote by s the differential in the subsidy paid to an investing firm relative to a non-investing one, when the two firms generate the same revenue. We adopt the convention that s is paid to the investing firms.

The following lemma provides a complete characterization of the policies that, when information is exogenous, implement the efficient use of information.

Lemma 2. *Assume that the precision of private information is exogenous and equal to π^x and that the conditions in Lemma 1 hold. Let*

$$\mathcal{R}(\theta;\pi^x) \equiv \tfrac{v-\psi(v-1)}{v-1} \; \hat{C}\left(\theta;\pi^x\right)^{\frac{1}{v}} \left(\hat{y}_1(\theta;\pi^x)^{\frac{v-1}{v}} - \hat{y}_0(\theta;\pi^x)^{\frac{v-1}{v}}\right) + s\left(\theta;\pi^x\right) - k. \qquad (12)$$

Any optimal fiscal policy $\bar{T}$ pays to each non-investing firm a transfer equal to

$$\bar{T}_0\left(r\right) = \frac{1}{v-1} r,$$

and to each investing firm a transfer equal to

$$\bar{T}_1\left(r,\theta;\pi^x\right) = \frac{1}{v-1} r + s\left(\theta;\pi^x\right),$$

where the additional subsidy $s\left(\theta;\pi^x\right)$ to the investing firms is such that $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right] < 0$ when $x < \hat{x}(\pi^x)$, and $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right] > 0$ when $x > \hat{x}(\pi^x)$, where $\hat{x}(\pi^x)$ is the signal threshold for the efficient investment decision as defined in Lemma 1.

Proof. See 6.

Any fiscal policy implementing the efficient use of information must combine the familiar revenue subsidy $r/(v-1)$ designed to offset firms' market power with an additional subsidy $s\left(\theta;\pi^x\right)$ to the investing firms appropriately designed to satisfy the conditions in the lemma. Naturally, the investing firms expect higher revenues, and hence a higher subsidy $r/(v-1)$. However, this standard subsidy alone is not sufficient to induce firms to invest efficiently. This is because firms do not internalize that, by investing, they increase other firms' output. The additional subsidy $s\left(\theta;\pi^x\right)$ to the investing firms must correct for such an externality. In the proof of the lemma in 6, we show that $\mathcal{R}(\theta;\pi^x)$ is the private benefit of investing, net of its cost. Such a benefit is equal to

$$\mathcal{R}(\theta;\pi^x) = \mathcal{Q}(\theta;\pi^x) - \frac{\alpha\beta\hat{C}\left(\theta;\pi^x\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)} + s\left(\theta;\pi^x\right),$$

where $\mathcal{Q}(\theta;\pi^x)$ is the social benefit, and

$$\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}(\theta;\pi^x)}$$

is the marginal externality created by the investment spillover. The externality coincides with the increase in the production of the final good that obtains if one increases the total mass of firm investing, N, by a small amount $\varepsilon > 0$ around the efficient level $\hat{N}(\theta;\pi^x)$, holding firms' investment and employment decisions fixed. The subsidy $s(\theta;\pi^x)$ must thus be designed to compensate for the fact that firms do not internalize such an externality. Many subsidies $s(\theta;\pi^x)$ accomplish this objective. In fact, because efficiency requires that firms invest when $\mathbb{E}[\mathcal{Q}(\theta;\pi^x)|x,\pi^x] > 0$ and refrain from investing when $\mathbb{E}[\mathcal{Q}(\theta;\pi^x)|x,\pi^x] < 0$, any subsidy that aligns the sign of the expected private benefit $\mathbb{E}[\mathcal{R}(\theta;\pi^x)|x,\pi^x]$ to the sign of the expected social benefit $\mathbb{E}[\mathcal{Q}(\theta;\pi^x)|x,\pi^x]$ does the job. When the conditions in Lemma 1 hold, $\mathbb{E}[\mathcal{Q}(\theta;\pi^x)|x,\pi^x] > 0$ turns from negative to positive at $x = \hat{x}(\pi^x)$. Hence, any subsidy that makes the expected private benefit $\mathbb{E}[\mathcal{R}(\theta;\pi^x)|x,\pi^x]$ turn from negative to positive at $x = \hat{x}(\pi^x)$ induces all firms to invest efficiently. A particularly simple one entails a constant (i.e., state-invariant) subsidy, as shown in the following corollary.

Corollary 1. *Assume that the precision of private information is exogenous and equal to π^x and that the conditions in Lemma 1 hold. A fiscal policy that pays to each firm a standard revenue subsidy equal to $r/(v-1)$ and, in addition, pays to each investing firm an extra (state-invariant) subsidy equal to*

$$\bar{s}_{\pi^x} \equiv \mathbb{E}\left[\left.\frac{\alpha\beta\hat{C}(\theta;\pi^x)}{1+\beta\hat{N}(\theta;\pi^x)}\right|\hat{x}(\pi^x),\pi^x\right] \tag{13}$$

is optimal.

Proof. See 6.

The constant subsidy $\bar{s}_{\pi^x}$ to the investing firms is thus the externality expected by the marginal investor with signal equal to the efficient threshold $\hat{x}(\pi^x)$. The advantage of such a simple policy is that it does not require the government to track the fundamental variable θ. When the government promises to pay to the investing firms a constant subsidy equal to $\bar{s}_{\pi^x}$, a firm with signal equal to $\hat{x}(\pi^x)$ that expects all other firms to invest efficiently and then set prices according to the rules $\hat{p}_1(\theta;\pi^x)$ and $\hat{p}_0(\theta;\pi^x)$ that induce the efficient demands $\hat{y}_1(\theta;\pi^x)$ and $\hat{y}_0(\theta;\pi^x)$ (and hence the efficient employment $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$), is indifferent between investing and not investing. Because

$$\mathcal{Q}(\theta;\pi^x) - \frac{\alpha\beta\hat{C}\left(\theta;\pi^x\right)}{1+\beta\hat{N}\left(\theta;\pi^x\right)}$$

is monotone in θ, under the same expectations, any firm with signal above $\hat{x}(\pi^x)$ finds it optimal to invest, whereas any firm with signal below $\hat{x}(\pi^x)$ finds it optimal not to invest. This means that the the constant subsidy $\bar{s}_{\pi^x}$ to the investing firms, along with the familiar revenue subsidy $r/(v-1)$, aligns the sign of the private benefit $\mathbb{E}\left[\mathcal{R}(\theta;\pi^x)|x,\pi^x\right]$ to its social counterpart $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^x)|x,\pi^x\right]$, and hence implements the efficient allocation.

3.3.2 Endogenous Information

We now turn to the case in which firms' information is endogenous. Let π^{x*} denote the precision of the firms' private information that maximizes welfare (its characterization is in the proof of Lemma 3). In the presence of endogenous information, optimality is defined as follows.

Definition 3. The fiscal policy T^* is **optimal** if it implements the efficient acquisition and usage of information as an equilibrium. That is, if it induces all firms to (1) choose the efficient precision of private information π^{x*}, (2) follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and (3) set prices $\hat{p}_1(\theta;\pi^{x*})$ and $\hat{p}_0(\theta;\pi^{x*})$ that induce demands for the intermediate products equal to $\hat{y}_1(\theta;\pi^{x*})$ and $\hat{y}_0(\theta;\pi^{x*})$ and hence efficient employment $\hat{l}_1(\theta;\pi^{x*})$ and $\hat{l}_0(\theta;\pi^{x*})$.

Let $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^x$ denote the marginal variation in the measure of firms investing at θ that obtains when one varies π^x infinitesimally at $\pi^x=\pi^{x*}$, holding fixed the rule for efficient investment $\hat{n}(x;\pi^{x*})$.

Lemma 3. *Assume that information is endogenous and that the economy satisfies the conditions in Lemma 1. Any optimal fiscal policy T^* pays to each firm that does not invest a transfer equal to*

$$T_0^*\left(r\right)=\frac{1}{v-1}r$$

and to each investing firm a transfer equal to

$$T_1^*\left(r\right)=\frac{1}{v-1}r+s\left(\theta;\pi^{x*}\right),$$

where the additional subsidy $s\left(\theta;\pi^{x*}\right)$ to the investing firms satisfies the condition in Lemma

2, applied to $\pi^x = \pi^{x*}$, and in addition satisfies the following condition

$$\mathbb{E}\left[s\left(\theta;\pi^{x*}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]. \tag{14}$$

Proof. See 6.

The lemma provides a complete characterization of the policies that induce efficiency in both information acquisition and information usage. Relative to the case in which information is exogenous (with precision π^{x*}), the subsidy to the investing firms must satisfy an additional restriction on the co-movement between the subsidy $s\left(\theta;\pi^{x*}\right)$ and the marginal effect $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ of more precise private information on aggregate investment under the efficient allocation. The restriction is necessary to realign the private benefit from acquiring more precise information to its social counterpart. Under the conditions of Lemma 1, the externality $\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)/[1+\beta\hat{N}\left(\theta;\pi^{x*}\right)]$ increases with the state θ. The marginal variation $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ in the measure of investing firms due to more precise private information is also monotone in θ (it is negative for $\theta<\hat{x}(\pi^{x})$ and positive for $\theta>\hat{x}(\pi^{x})$). The subsidy $s\left(\theta;\pi^{x*}\right)$ must thus change with the state θ, so that the co-movement between $s\left(\theta;\pi^{x*}\right)$ and the marginal variation $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$ is the same as that between the externality $\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)/[1+\beta\hat{N}\left(\theta;\pi^{x*}\right)]$ and $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$.

As a result of the additional restriction, policies that are optimal under exogenous information need not be optimal when information is endogenous. For example, the simple policy of Corollary 1, specialized to $\pi^x=\pi^{x*}$, under which the government pays a constant subsidy $\bar{s}_{\pi^{x*}}$ to the investing firms in addition to the familiar revenue subsidy $r/(v-1)$, fails to induce efficiency in information acquisition. Hence it is not optimal when information is endogenous. This is because a constant subsidy equal to the externality expected by the marginal investor with signal $\hat{x}(\pi^{x*})$ does not induce the right co-movement between the subsidy $s(\theta;\pi^{x*})$ and the (state-dependent) marginal effect of more precise private information on aggregate investment $\partial\hat{N}\left(\theta;\pi^{x*}\right)/\partial\pi^{x}$, which is necessary to realign the private benefit of information acquisition to its social counterpart. Conversely, a policy that pays, in each state θ, a subsidy to the investing firms equal to the state-specific externality from the investment spillover satisfies the co-movement condition in (14), and hence induces efficiency in both information acquisition and information usage.

Proposition 1. *Irrespective of whether the economy satisfies the conditions in Lemma 1, the* *fiscal policy of Lemma 3 with a state-contingent subsidy to the investing firms equal to*

$$s(\theta;\pi^{x*}) = \frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)} \tag{15}$$

is optimal.

Proof. Suppose that all other firms (1) acquire information of precision π^{x*}, (2) invest when, and only when, it is socially efficient to do so (i.e., invest when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right] > 0$ and not invest when $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right] \geq 0$), and (3) set the prices $\hat{p}_1(\theta;\pi^{x*})$ and $\hat{p}_0(\theta;\pi^{x*})$ that induce the efficient employment and production decisions. Then, in each state θ, irrespective of the precision π^x of its private information, each firm finds it optimal to set a price equal to $\hat{p}_1(\theta;\pi^{x*})$ when investing, and equal to $\hat{p}_0(\theta;\pi^{x*})$ when not investing. Furthermore, the private value $\mathbb{E}\left[\mathcal{R}(\theta;\pi^{x*})|x,\pi^{x}\right]$ to investing coincides with the social value $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x}\right]$ for any x (see the proof of Lemma 2 in 6 for the formal arguments). These properties hold irrespective of whether the precision π^x selected by the firm coincides with the efficient level π^{x*}. They also hold irrespective of whether the economy satisfies the conditions in Lemma 1, the sole role of which is to guarantee that, when $\pi^x = \pi^{x*}$, the social benefit $\mathbb{E}\left[\mathcal{Q}(\theta;\pi^{x*})|x,\pi^{x*}\right]$ of investing turns from negative to positive at $x = \hat{x}(\pi^{x*})$. The same properties also imply that the gross value that the firm assigns to acquiring information coincides with the social value. Because the private cost of information also coincides with the social one, the above results imply that acquiring information of precision π^{x*} and then using the information efficiently (both when it comes to choosing whether or not to invest and setting the prices) is individually optimal for each firm expecting all other firms to do the same. Q.E.D.

As anticipated above, the state-contingent subsidy in (15) operates as a Pigouvian correction that induces each firm to internalize the effect of its investment choice on the production of the final good when all other firms acquire and use information efficiently. To see this, let Λ denote the cross-sectional distribution of firms' investment and employment decisions (n_i, l_i). Let $C_N\left(\theta,\Lambda\right)$ denote the marginal change in the production of the final good that obtains when, holding θ and Λ fixed, one changes N in all firms' production functions by a small $\varepsilon > 0$, starting from $N = N_\Lambda$, where N_Λ is aggregate investment under the distribution Λ. Next, let $\hat{\Lambda}(\theta;\pi^{x*})$ denote the cross-sectional distribution of firms' investment and employment decisions (n_i, l_i) under the efficient allocation. Then one has that

$$C_N\left(\theta,\hat{\Lambda}(\theta;\pi^{x*})\right) = \frac{\alpha\beta\hat{C}\left(\theta;\pi^{x*}\right)}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}.$$

That is, the state-dependent subsidy in (15) coincides with the marginal change in the production of the final good that obtains as a result of a marginal change in N, evaluated at $N = \hat{N}(\theta; \pi^{x*})$, holding all firms' investment and employment decisions fixed at the efficient level. Such a policy is thus reminiscent of familiar Pigouvian corrections for complete-information economies. Importantly, these corrections also induce firms to collect and use information efficiently even when firms' decisions (i.e., how much they invest in information acquisition and how they use their information) is not verifiable.

The Pigouvian policy of Proposition 1 is not the unique one implementing the efficient allocation. Other state-contingent policies do the job. One of the limitations of many of these policies (including the one in Proposition 1) is that they require the government to know what type of information the firms can collect (equivalently, the cost of different information structures). This knowledge is necessary to compute $\hat{C}(\theta; \pi^{x*})$ and $\hat{N}(\theta; \pi^{x*})$, and hence the state-contingent subsidy $s(\theta; \pi^{x*})$ in (15), but may not be available in some economies of interest. When this is the case, efficiency in both information acquisition and usage can still be induced by conditioning the subsidy to the investing firms directly on C and N. Alternatively, it can be obtained by conditioning the subsidy s on the cross-sectional distribution of firms' investment and employment decisions, as the next proposition shows.

Proposition 2. *Assume that the government does not know what type of information the firms can collect (equivalently, the cost of different information structures). Efficiency in both information acquisition and usage can be induced through a fiscal policy that pays to the non-investing firms a transfer equal to*

$$T_0^{\#}(r) = \frac{1}{\upsilon - 1} r,$$

and to the investing firms a transfer equal to

$$T_1^{\#}(r, \theta, \Lambda) = \frac{1}{\upsilon - 1} r + C_N(\theta, \Lambda),$$

where Λ is the ex-post cross-sectional distribution of firms' investment and employment decisions (n_i, l_i), and where $C_N(\theta, \Lambda)$ is the marginal change in the production of the final good that obtains as a result of a marginal change in N holding all firms' investment and employment decisions fixed at the level specified by Λ.

Proof. Suppose that all other firms (1) acquire information efficiently (with information acquisition taking the form of a private signal $q : \Theta \to \Delta(\mathcal{S})$ mapping θ into a distribution over a Polish space $\mathcal{S}$ of signal realizations that, without loss of generality can be taken to coincide with $[0, 1]$), (2) use information efficiently to make their investment decisions, and

(3) in each state θ, given aggregate investment N, set prices so as to induce the efficient employment (and hence production) decisions. Then, each firm has enough knowledge about the economy to compute the efficient allocation, and has incentives to follow the same efficient policies as any other firm. In fact, the revenue subsidy $r/(v-1)$ guarantees that each firm, no matter its investment decision, after learning θ, has the right incentives to set the price for its intermediate good at a level that induces the efficient demand for its product, and hence the efficient employment decisions (see the proof of Lemma 2 in 6 where the result is established without using the specific properties of the firms' information structure). Furthermore, when in each state θ the extra subsidy to the investing firms takes the form of the marginal externality $C_N(\theta, \Lambda)$ exerted by N on the production of the final good (holding all firms' information, investment, and pricing rules fixed), the marginal value that each firm assigns to investing coincides with the government's value in each state (see the proof of Lemma 2 in 6). The above properties imply that the private value of information acquisition coincides with the social one, no matter the cost of each experiment q. Hence, all firms have the right incentives to acquire and use information efficiently when expecting all other firms to do the same. Q.E.D.

The result in Proposition 2 illustrates the power of the Pigouvian logic. When the policy maker announces that investing firms will receive a subsidy equal to the *ex-post* (marginal) externality $C_N(\theta, \Lambda)$ that each firm's investment choice exerts on the production of the final good, it re-aligns firms' (marginal) incentives with their social counterpart, not just at the interim stage but also ex-post. The government can then delegate to firms the computation of the efficient allocation, while guaranteeing that, in equilibrium, they acquire and use information efficiently.

One can also show that the power of the Pigouvian logic extends to economies in which firms are heterogeneous in their cost of acquiring information and/or in their investment cost. It also extends to economies in which investment features an intensive instead of an extensive margin, i.e., firms decide how much to invest, with the latter decision taking a continuum of possible values. This is because there are no discrepancies between private and social marginal costs. As a result, the subsidy in Proposition 2, by aligning each firm's private benefit to investment with its social counterpart induces efficiency in both information acquisition and usage, irrespective of whether investment is a discrete or a continuous choice and of any heterogeneity across firms.

Propositions 1 and 2 complement each other. Proposition 1 shows that, when the government knows the cost of different information structures, efficiency in both information acquisition and usage can be induced with a fiscal policy that conditions the subsidy s to the investing firms only on the fundamental state θ — no further contingencies are necessary. Proposition 2, instead, shows that, when the cost is unknown to the government, efficiency in information acquisition and usage requires expanding the contingencies in the optimal subsidy by conditioning on the cross-sectional distribution of investment and employment decisions.

The policies of Propositions 1 and 2 also resemble VCG transfers, but with the correction operating at the margin instead of the levels.[11] While the VCG transfers eliminate the wedge between the private and the social objectives by making firms' profits (net of the transfers) proportional to their contribution to total welfare, the policies in Propositions 1 and 2 eliminate the wedge between the marginal private and social benefit of varying the firms' decisions.[12]

4 Sticky Prices and Optimal Monetary Policy

We now extend the analysis by introducing nominal rigidities. We do so by assuming that firms set prices under their endogenous private information before observing the realization of the fundamental variable θ. Such nominal rigidities introduce a role for monetary policy, in the spirit of Correia, Nicolini, and Telles (2008), and Angeletos and La'O (2020). The purpose of the extension is twofold: it permits us to investigate the extent to which the insights from the previous section are robust to the introduction of nominal rigidities; it also permits us to investigate how monetary and fiscal policy must be combined to incentivize firms to acquire and use information efficiently in the presence of investment spillovers.

To capture the role of these nominal rigidities in the simplest possible terms, we introduce a cash-in-advance constraint. The government provides the representative household with an amount of money M, and the maximal expenditure on the purchase of the final good cannot exceed M, that is

$$PY \leq M.$$

The timing of events is the same as in Section 2, with the exception that prices are set under dispersed information about θ (i.e., with each p_i based on x_i instead of θ), and that the supply of money is state-dependent and governed by a monetary policy $M(\cdot)$. Each firm knows the monetary policy but does not observe the realized money supply at the time it sets the price for its intermediate good. This economy is consistent with most of the assumptions that are

[11] See Bergemann and Välimäki (2002) for the role of VCG payments in mechanism design with endogenous information acquisition.

[12] In our economy with a continuum of infinitesimal firms, VCG payments do not work, as the contribution of each firm's decisions to total welfare is zero.

typically made in the pertinent literature.

The presence of price rigidities has no implications for the efficient allocation, which continues to be characterized by the conditions in the proof of Lemmas 1 and 3. The analysis of the equilibrium allocation, instead, must be amended to account for price rigidity. In this economy, the demands for the intermediate products, as well as the labor demands, continue to satisfy the same conditions as in Subsection 3.2. In particular, equilibrium in the labor market requires that Condition (11) holds.

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and employment, respectively, of each investing firm. The corresponding functions for the non-investing firms are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$. Because prices are set under (endogenous) imperfect information about θ, the firms' labor demands $l_1(x,\theta;\pi^x)$ and $l_0(x,\theta;\pi^x)$ depend not only on θ and π^x but also on x.

Definition 4. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$, and a pair of price functions $p_1(x;\pi^x)$ and $p_0(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then invests according to $n(x;\pi^x)$ and sets its price according to $p_1(x;\pi^x)$ and $p_0(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

As in Section 3, the above equilibrium definition abstracts from other conditions (for wages, labor demand and supply, price of the final good) that are standard to isolate the novel and most relevant parts.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition 5. The monetary policy $M^*(\cdot)$ and the fiscal policy $T^*(\cdot)$ are **optimal** if, jointly, they implement the efficient acquisition and usage of information as an equilibrium. That is, they induce all firms to (1) acquire information of precision π^{x*}, (2) follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and (3) set prices (under dispersed information) according to rules $\hat{p}_1(x;\pi^{x*})$ and $\hat{p}_0(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to the efficient levels $\hat{y}_1(\theta;\pi^{x*})$ and $\hat{y}_0(\theta;\pi^{x*})$ and hence result in firms employing labor according to the efficient rules $\hat{l}_1(\theta;\pi^{x*})$ and $\hat{l}_0(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the amount of money supplied to the representative household in state θ when all firms are expected to acquire information of precision π^x. The policy $\hat{M}(\cdot;\pi^x)$ is designed so that, when all firms make their investment decisions according to the efficient rule $\hat{n}(x;\pi^x)$ and set prices according to $\hat{p}_1(x;\pi^x)$ and $\hat{p}_0(x;\pi^x)$, the resulting employment decisions coincide with the efficient ones $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ for an economy with private information of precision π^x.

The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma 4. *Assume that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot;\pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot;\pi^x)$, implements the efficient use of information as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms making the same investment decision to set the same price, irrespective of their information about θ.

Proof. See 6.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ implements the efficient allocation by inducing firms to disregard their private information about the aggregate economic conditions (the fundamental variable θ) when setting their prices, and condition the latter only on their investment decision. That prices do not respond to firms' information about θ, given their investments, is necessary to avoid allocative distortions in the induced employment and production decisions. In fact, given the firms' investments, relative prices must not vary with firms' signals about θ when the latter are imprecise. The monetary policy in Lemma 4 is designed so that, even if firms could condition their prices on θ, thus bypassing the nominal rigidity, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supply in a way that replicates the same allocations sustained when money is constant and prices are flexible.

The result in Lemma 4 may suggest that the monetary authority needs to know the cost of information to compute the optimal money supply in each state θ. However, as anticipated above, this is not the case. In fact, it suffices that the authority observes the cross-sectional distribution of employment and investment decisions for it to be able to compute the amount of money that needs to be supplied.

Lemma 4 in turn permits us to establish the following result.

Proposition 3. *All the results about the structure of the optimal fiscal policy in the previous section for the case of flexible prices carry over to the economy with price rigidities under consideration.*

Proof. See 6.

The proof in 6 first shows that, when information is exogenous and of precision π^x, any fiscal policy that induces efficiency in information usage must induce firms to set prices that, given the firms' investments, are invariant in the firms' signals. The only policies that satisfy this property take the form $T_0(r) = r/(v-1)$ and $T_1(r, \theta; \pi^x) = r/(v-1) + s(\theta; \pi^x)$, as in Lemma 2. It then shows that, under any such fiscal policy, when the monetary policy is the one in Lemma 4, all firms have incentives to set prices that induce them to hire the efficient amount of labor in each state. Building on these observations, the proof then shows that, when the monetary policy takes the form in Lemma 4, the net private benefit that each firm with signal x expects from investing continues to be given by $\mathbb{E}[\mathcal{R}(\theta; \pi^x)|x, \pi^x]$, as in the case of flexible prices. This property, in turn, implies that the extra subsidy $s(\theta; \pi^x)$ to the investing firms must satisfy the conditions in Lemma 2 and, when information is endogenous, the additional Condition (14) in Lemma 3.

The above result in turn implies that the Pigouvian fiscal policy of Proposition 1, in which the extra subsidy to the investing firms takes the form

$$s(\theta; \pi^{x*}) = \frac{\alpha\beta\hat{C}(\theta; \pi^{x*})}{1 + \beta\hat{N}(\theta; \pi^{x*})},$$

when paired with the monetary policy of Lemma 4 (specialized to $\pi^x = \pi^{x*}$), continues to realign the private value from investing with its social counterpart, state by state. Once this realignment is established, the value that firms assign to information acquisition coincides with the social value, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same, as in the economy with flexible prices. Similar arguments imply that, when the fiscal or monetary authorities do not know the cost of information acquisition, it remains possible to implement the efficient acquisition and usage of information but it becomes necessary to expand the contingencies in the policies, by conditioning the policies on the cross-sectional distribution of firms' investment and employment decisions.

5 Conclusions

We investigate optimal fiscal and monetary policy in economies in which firms face endogenous uncertainty about aggregate economic conditions affecting the profitability of their investment decisions (e.g., in AI-based technologies, or in smart intermediate products), and where the output they produce is affected by investment spillovers. We show that firms can be incentivized to acquire information efficiency and then use it in society's best interest through a fiscal policy that, in addition to correcting for firms' market power, provides the investing firms with a subsidy that makes them internalize the effects of their investments on the production of intermediate and final goods. This result shows how the power of Pigouvian corrections extends to economies in which neither the collection nor the usage of information is verifiable. The same fiscal policy induces efficiency in information acquisition and usage when firms set prices under dispersed information (nominal rigidities), provided that it is accompanied by a monetary policy that makes firms disregard their endogenous private information when setting prices and only use it for investment decisions.

We expect results similar to those discussed in the present paper to obtain in economies in which externalities originate in pollution, and/or spillovers from investments in human capital.

Our analysis can be extended in several directions. To isolate the novel effects from the familiar learning externalities that are present when late adopters learn from early ones and/or where financial markets imperfectly aggregate private information, we consider a static general-equilibrium economy in which all the relevant production decisions occur simultaneously and there is no information aggregation. In future work, it would be interesting to extend the analysis to combine the externalities from investment spillovers discussed in the present paper with the learning ones as, e.g., in Dasgupta (2007), but in a setting with endogenous private information. It would also be interesting to enrich the model to allow for partial information aggregation in financial markets and study how inefficiencies in investment and production decisions interact with those in the trading of financial assets (see also Angeletos, Lorenzoni, and Pavan (2023), and Pavan, Sundaresan and Vives (2022) for models with some of these ingredients, but without spillovers).

Finally, it would be interesting to extend the analysis to economies in which firms expand the set of available products over time and strategically choose when to replace existing products with new ones, thus contributing to the understanding of how governments can increase the efficiency of the innovation diffusion process.

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6 Appendix

Proof of Lemma 1. Fix π^x and drop it from all expressions to ease the notation. Efficiency requires that any two firms making the same investment decision employ the same amount of labor. Letting $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing and the non-investing firms respectively, we have that the planner's problem can be written as

$$\max_{n(x),l_1(\theta),l_0(\theta)} \int_\theta C(\theta)d\Omega(\theta) - k\int_\theta N(\theta)d\Omega(\theta) + \\ -\frac{1}{1+\varepsilon}\int_\theta [l_1(\theta)N(\theta)+l_0(\theta)(1-N(\theta))]^{1+\varepsilon} d\Omega(\theta) + \\ -\int_\theta \mathcal{Q}(\theta)\left(N(\theta) - \int_x n(x)\Phi(x|\theta)\right)d\Omega(\theta),$$

where $\Omega(\theta)$ is the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$ is the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ is the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{v}{v-1}}, \tag{16}$$

with

$$y_1(\theta) = \gamma\Theta(1+\beta N(\theta))^\alpha l_1(\theta)^\psi, \tag{17}$$

and

$$y_0\left(\theta\right) = \Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha} l_0(\theta)^{\psi}. \tag{18}$$

The first-order condition with respect to $l_1(\theta)$ is thus equal to

$$\psi\left(y_1\left(\theta\right)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right)^{\frac{1}{v-1}} \left(\gamma\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}-1} \\ - \left(l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right)^{\varepsilon} = 0.$$

Letting

$$L\left(\theta\right) \equiv l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta)), \tag{19}$$

and using (16) and (17), we have that the first order condition for $l_1(\theta)$ above can be expressed as

$$\psi C(\theta)^{\frac{1}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta) L\left(\theta\right)^{\varepsilon}. \tag{20}$$

Following similar steps, the first-order condition for $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta) L\left(\theta\right)^{\varepsilon}. \tag{21}$$

Jointly, the above first-order conditions – together with (18) and (19) – yield

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon-\psi}} \left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1}{1+\varepsilon-\psi}} \left(\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v\varepsilon}{(v-1)(1+\varepsilon-\psi)}}, \tag{22}$$

and

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{23}$$

Notice that (23) implies that, at the efficient allocation, the total labor demand, as defined in (19), is equal to

$$L\left(\theta\right) = l_0\left(\theta\right)\left[\left(\gamma^{\varphi}-1\right) N\left(\theta\right)+1\right]. \tag{24}$$

The above conditions are both necessary and sufficient given that the planner's problem has a unique stationary point in (l_0, l_1) for any θ.

Differentiating the government's objective with respect to $N(\theta)$, we have that

$$\mathcal{Q}(\theta) = \frac{v}{v-1} C(\theta)^{\frac{1}{v}} \left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + \frac{\alpha\beta}{1+\beta N(\theta)} C(\theta) - k - L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right). \tag{25}$$

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_\theta \mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right) d\theta.$$

Using the fact that $\phi\left(x|\theta\right)\omega\left(\theta\right) = f\left(\theta|x\right)g\left(x\right)$, where $f\left(\theta|x\right)$ is the conditional density of θ given x, and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_\theta \mathcal{Q}(\theta) f\left(\theta|x\right) d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that $n(x) = 1$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ and $n(x) = 0$ if $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$.
Use (20) and (21) to observe that

$$L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right) = \psi C(\theta)^{\frac{1}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right).$$

Replacing the above expression into (25), we have that

$$\mathcal{Q}(\theta) = \left(\frac{v - \psi\left(v-1\right)}{v-1}\right) C(\theta)^{\frac{1}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + \frac{\alpha\beta}{1+\beta N\left(\theta\right)}C(\theta) - k.$$

Using (16), (17), (18), and (23), after some manipulations, we have that

$$C(\theta)^{\frac{1}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) = \left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha} l_0(\theta)^{\psi}\left(\gamma^{\varphi}-1\right), \quad (26)$$

and $C(\theta) = \left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v}{v-1}}\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha} l_0(\theta)^{\psi}$. It follows that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi}{1+\varepsilon-\psi}}\Theta^{\frac{1+\varepsilon}{1+\varepsilon-\psi}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{1+\varepsilon}{\varphi(1+\varepsilon-\psi)}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1+\varepsilon)}{1+\varepsilon-\psi}} \times$$
$$\times\left(\frac{\gamma^{\varphi}-1}{\varphi} + \frac{\alpha\beta\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)}{1+\beta N(\theta)}\right) - k. \quad (27)$$

When the parameters satisfy the conditions in the lemma, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the firms (and hence by the planner) at the time the investment decisions are made. Such a property implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ. This last property, along with the fact that $\mathcal{Q}$ is increasing in θ for any N, implies that the first-best level of N is increasing in θ. This property, in turn, implies that the efficient strategy $\hat{n}\left(x\right)$ is monotone. For any θ and $\hat{x}$, let $\bar{\mathcal{Q}}(\theta|\hat{x})$ denote the function defined in (27) when $N(\theta) = 1 - \Phi(\hat{x}|\theta)$, that is, when firms invest if and only if $x > \hat{x}$. Under the parameters' restrictions in the lemma, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that

$$\lim_{\hat{x}\to-\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x}\to+\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits one and only one solution. Let $\hat{x}$ denote the solution to this equation. Then note that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. We conclude that, under the assumptions in the lemma, there exists a threshold $\hat{x}$ such that the investment rule $\hat{n}(x) = \mathbb{I}(x \geq \hat{x})$, along with the employment functions $\hat{l}_1(\theta)$ and $\hat{l}_0(\theta)$ satisfying the first-order conditions above, constitute a solution to the planner's problem. Q.E.D.

Proof of Lemma 2. As in the proof of Lemma 1, we drop π^x from all formulas to ease the notation. We also drop θ when there is no risk of confusion.
Each investing firm chooses p_1 to maximize

$$\frac{p_1 y_1 - W l_1}{P} + T_1\left(\frac{p_1 y_1}{P}\right), \tag{28}$$

taking W and P as given, accounting for the fact that y_1 is given by (7), with C exogenous to the firm's problem, and with l_1 given by (9). The first-order condition with respect to p_1 is given by

$$(1-v)\,CP^{v-1}p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\,(p_1y_1/P)}{dr}\frac{d\,(p_1y_1)}{dp_1} = 0. \tag{29}$$

Using (7) and (9), we have that

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{30}$$

and

$$\frac{d\,(p_1y_1)}{dp_1} = (1-v)\,CP^{v}p_1^{-v}. \tag{31}$$

Replacing (30) and (31) into (29), using (7), and rearranging terms, we obtain that

$$\frac{1-v}{v}\frac{y_1p_1}{P} + \frac{1}{\psi}\frac{W}{P}l_1 + \frac{1-v}{v}\frac{dT_1\,(p_1y_1/P)}{dr}\frac{y_1p_1}{P} = 0. \tag{32}$$

Next use (2) and (7), along with (23), to observe that, in any equilibrium implementing the efficient allocation, firms must set prices equal to (hereafter we use "hats" to denote variables under the rules inducing the efficient allocation)

$$\hat{p}_1 = \left((\gamma^{\varphi} - 1)\,\hat{N} + 1\right)^{\frac{1}{v-1}} \gamma^{\frac{\varphi}{1-v}}\hat{P}, \tag{33}$$

and

$$\hat{p}_0 = \left((\gamma^{\varphi} - 1) \hat{N} + 1 \right)^{\frac{1}{v-1}} \hat{P}, \tag{34}$$

with

$$\hat{P} = \left(\hat{p}_1^{1-v} \hat{N} + \hat{p}_0^{1-v} \left(1 - \hat{N} \right) \right)^{\frac{1}{1-v}}. \tag{35}$$

Market-clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}.$$

Then use (19) and (23) to note that $\hat{L} = \hat{l}_0 \left[(\gamma^{\varphi} - 1)\hat{N} + 1 \right]$. Next, use (20) to observe that efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon} \hat{l}_1 = 0. \tag{36}$$

Condition (32) then implies that T implements the efficient allocation only if

$$\frac{1}{v} = \frac{v-1}{v} \frac{dT_1 \left(\hat{p}_1 \hat{y}_1 / \hat{P} \right)}{dr}.$$

Because $\hat{p}_1 \hat{y}_1 / \hat{P}$ is state dependent, we thus have that T_1 must be affine and satisfy

$$T_1 (r) = \frac{1}{v-1} r + s, \tag{37}$$

with s invariant in r. Furthermore, one can show that, under the policy (37), the payoff of each investing firm is quasi-concave in its price, which implies that the above first-order condition is also sufficient for the firm to choose $p_1 = \hat{p}_1$.

Similar arguments imply that the transfer to the non-investing firms must be equal to

$$T_0 (r) = \frac{1}{v-1} r \tag{38}$$

for these firms to find it optimal to set $p_0 = \hat{p}_0$.

Next, consider the decision of whether or not to invest. When the policy satisfies (37) and (38), with $s(\theta)$ possibly depending on θ, each firm finds it optimal to invest if $\mathbb{E}\left[\mathcal{R}(\theta) | x\right] > 0$ and to not invest if $\mathbb{E}\left[\mathcal{R}(\theta) | x\right] < 0$, where

$$\mathcal{R}(\theta) \equiv \left(\frac{v - \psi (v-1)}{v-1} \right) \hat{C} (\theta)^{\frac{1}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}} \right) + s (\theta) - k \tag{39}$$

is the extra profit (net of the subsidy) from investing relative to not investing. Now use the proof of Lemma 1 to note that efficiency requires that each firm invests if $\mathbb{E}\left[\mathcal{Q}(\theta)|x\right] > 0$ and does not invest if $\mathbb{E}\left[\mathcal{Q}(\theta)|x\right] < 0$, where $\mathcal{Q}(\theta)$ can be conveniently rewritten as

$$\mathcal{Q}(\theta) = \left(\tfrac{v-\psi(v-1)}{v-1}\right) \hat{C}\left(\theta\right)^{\frac{1}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \tfrac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}(\theta)} - k.$$

When the economy satisfies the conditions of Lemma 1, $\mathbb{E}\left[\mathcal{Q}(\theta)|x\right] > 0$ turns from negative to positive at $x = \hat{x}$. Hence, for the policy defined by (37) and (38) to induce efficiency in investment decisions it is both necessary and sufficient that $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]$ turns from negative to positive at $x = \hat{x}$. Q.E.D.

Proof of Corollary 1. Use the derivations in the proof of Lemma 2 to observe that

$$\mathcal{R}(\theta) = \mathcal{Q}(\theta) - \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}\left(\theta\right)} + s(\theta).$$

Next observe that the function

$$\mathcal{Q}(\theta) - \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}\left(\theta\right)}$$

is non-decreasing in θ under the conditions in Lemma 1. We thus have that, when $s(\theta) = \bar{s}_{\pi^x}$ for all θ, $\mathbb{E}\left[\mathcal{R}(\theta)|x\right]$ turns from negative to positive at $x = \hat{x}$, implying that the fiscal policy T satisfies all the conditions in Lemma 2 and hence is optimal. Q.E.D.

Proof of Lemma 3. The proof is in two parts. Part 1 characterizes the efficient precision of information π^{x*}. Part 2 uses the characterization in part 1 to establish the claim in the lemma.

Part 1. Using the results in Lemma 1, we have that, for any π^x, irrespective of whether the economy satisfies the restrictions in Lemma 1, ex-ante welfare under the efficient allocation is equal to

$$\begin{aligned}\mathcal{W} = &\int_\theta \Theta\left(1+\beta\hat{N}\left(\theta;\pi^x\right)\right)^\alpha \hat{l}_0\left(\theta;\pi^x\right)^\psi \left(\left(\gamma^\varphi - 1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{v}{v-1}} d\Omega\left(\theta\right) + \\ &- k\int_\theta \hat{N}\left(\theta;\pi^x\right) d\Omega\left(\theta\right) - \int_\theta \frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left(\left(\gamma^\varphi - 1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{1+\varepsilon} d\Omega\left(\theta\right) - \mathcal{I}(\pi^x).\end{aligned}$$

Using the envelope theorem, we then have that π^{x*} solves

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]$$
$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \tag{40}$$

The above condition identifies the efficient precision of private information π^{x*}.

Part 2. Suppose that all firms other than i acquire information of precision π^{x*} and consider firm i's problem. Under the policy in the lemma, in each state θ, the price that maximizes firm i's profit coincides with the one that induces the efficient allocation for precision π^{x*}, irrespective of firm i's choice of π_i^x. This price is equal to $\hat{p}_1^*$ if the firm invest and $\hat{p}_0^*$ if the firm does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the functions in (33) and (34), respectively, evaluated at $\pi^x = \pi^{x*}$. Note that we use the combination between "^" and "*" to denote variables under the efficient allocation for precision π^{x*} (this notation applies not only to $\hat{p}_1^*$ and $\hat{p}_0^*$ but to all expressions below).

Dropping θ from the argument of each function to ease the notation, we have that firm i's value function is equal to

$$\bar{\Pi}_i(\pi_i^x)\equiv\sup_{\varsigma:\mathbb{R}\to[0,1]}\Pi_i(\varsigma;\pi_i^x),$$

where

$$\Pi_i(\varsigma;\pi_i^x)\equiv\mathbb{E}\left[\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right]-\mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]$$
$$+\mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x),$$

with $\bar{n}(\pi_i^x;\varsigma)\equiv\int\varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^*=\hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^*=\hat{p}_0^*\hat{y}_0^*/\hat{P}^*$, after investing and not investing, respectively.

Substituting $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}} \hat{y}_f^{*\frac{v-1}{v}}$, $f = 0, 1$, into $\Pi_i(\varsigma; \pi_i^x)$ and using (2), we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right) \hat{l}_0^{*\psi\frac{v-1}{v}}\right] \\ & - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right] \\ & + \mathbb{E}\left[\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly,

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \mathbb{E}\left[\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right) \hat{l}_0^{*\psi\frac{v-1}{v}}\right] \\ & - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right] \\ & + \mathbb{E}\left[\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}. \quad (41)\end{aligned}$$

Replacing

$$\hat{T}_1^* - \hat{T}_0^* = s + \frac{1}{v-1}\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}$$

into (41), we obtain that

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x} =& \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x} l_0^{*\psi\frac{v-1}{v}}\right] \\ & - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right] + \mathbb{E}\left[s\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x}. \quad (42)\end{aligned}$$

Recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$. Using the envelope theorem, we thus have that

$$\begin{aligned}\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial\Pi_i(\hat{n}^*;\pi^{x*})}{\partial\pi_i^x} =& \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1}{v}} \left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}} \left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right] \\ & - \mathbb{E}\left[\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\hat{N}^*}{\partial\pi^x}\right)\right] + \mathbb{E}\left[s\frac{\partial\hat{N}^*}{\partial\pi^x}\right] - k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right] - \frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x},\end{aligned}$$

where $\partial\hat{N}^*/\partial\pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding $\hat{n}^*$ fixed. For the proposed policy to induce efficiency in information acquisition, it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. This requires that

$$\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}\left(\theta;\pi^{x*}\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)}\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]$$
$$-\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]$$
$$+\mathbb{E}\left[s(\theta)\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}(\theta;\pi^{x*})}{\partial\pi^{x}}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^{x}}, \quad (43)$$

where we reintroduce all the arguments of the various functions to make the result consistent with the claim in the main text.

Comparing (43) with (40) in part 1, we thus have that the policy in Lemma 3 induces the firms to acquire the efficient precision of private information only if, in addition to $s(\theta)$ satisfying the property in Lemma 2, it also satisfies Condition (14). Q.E.D.

Proof of Lemma 4. We drop π^x from all formulas to ease the notation. Using (20) and (21), we have that

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon}=\psi\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$
$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon}=\psi\hat{C}(\theta)^{\frac{1}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}},$$

with $\hat{L}\left(\theta\right)$ defined by (19). The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, efficiency requires that the prices set by any two firms making the same investment decision coincide, which means that they must be independent of the signal x, conditional on the investment decision. Let $\hat{p}_1$ be the (state-invariant) price set by the investing firms and $\hat{p}_0$ the price set by the non-investing firms. Let $\hat{P}(\theta)$ denote the price of the final good in state θ when all firms follow the efficient rules. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon}=\psi\hat{C}(\theta)\left(\hat{P}\left(\theta\right)/\hat{p}_1\right)^{v-1}, \quad (44)$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon}=\psi\hat{C}(\theta)\left(\hat{P}\left(\theta\right)/\hat{p}_0\right)^{v-1}, \quad (45)$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1}=\left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (23), implies that $\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}} \hat{p}_0$. The price of the final good is then equal to

$$\hat{P}(\theta) = \left((\gamma^{\varphi} - 1) \hat{N}(\theta) + 1 \right)^{\frac{1}{1-v}} \hat{p}_0. \tag{46}$$

Combining (45) with the cash-in-advance constraint $M = PC$, we have that, in each state θ,

$$\hat{l}_0(\theta) \hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta) \hat{P}(\theta)^{v-2} \hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta) \hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta) \left((\gamma^{\varphi} - 1) \hat{N}(\theta) + 1 \right)^{\frac{v-2}{1-v}} \hat{p}_0^{-1},$$

where we also used (46) to express $\hat{P}(\theta)$ as a function of $\hat{N}(\theta)$ and $\hat{p}_0$. Finally, using (24), we obtain that, in each state θ, the money supply must be given by

$$\hat{M}(\theta) = \frac{1}{\psi} \hat{l}_0(\theta)^{1+\varepsilon} \left((\gamma^{\varphi} - 1) \hat{N}(\theta) + 1 \right)^{\frac{(1+\varepsilon)(v-1)-1}{v-1}} \hat{p}_0.$$

It is immediate to verify that the same conclusion can be obtained starting from (44). Because $\hat{p}_0$ can be taken to be arbitrary, the result in the lemma obtains by setting $m = \frac{1}{\psi} \hat{p}_0$. Q.E.D.

Proof of Proposition 3. The proof is in two parts. Part 1 shows that, when information is exogenous and the monetary policy is the one in Lemma 4 (which, by virtue of the lemma, is the only one that can induce efficiency in information usage), any optimal fiscal policy must take the form $T_0(r) = r/(v-1)$ and $T_1(r) = r/(v-1) + s$, for some s that is invariant in r. The reason why this result is not implied by Lemma 2 and requires a separate proof is that the information upon which the firms set their prices is different from the one considered in Lemma 2; this implies that, in principle, the way the government provides incentives to the firms may be different from what established for flexible prices. Part 2 then uses the result in Part 1 to establish the conclusions in the proposition.

Part 1. Fix the precision of private information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions below when there is no risk of confusion. Consider first the pricing decision of an investing firm. The firm sets p_1 to maximize

$$\mathbb{E}\left[\frac{p_1 y_1 - W l_1}{P} + T_1(r_1) \middle| x \right], \tag{47}$$

where $r_1 = p_1 y_1 / P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C \left(\frac{P}{p_1} \right)^{v}, \tag{48}$$

and that the amount of labor that the firm will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma \Theta \left(1 + \beta N\right)^{\alpha}} \right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (47) with respect to p_1 is given by

$$\mathbb{E}\left[\left(1-v\right) CP^{v-1} p_1^{-v} - \frac{W}{P}\frac{dl_1}{dp_1} + \frac{1}{P}\frac{dT_1\left(r_1\right)}{dr}\frac{d(p_1 y_1)}{dp_1} \middle| x \right] = 0. \tag{49}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi}\frac{l_1}{p_1}, \tag{50}$$

$$\frac{d\left(p_1 y_1\right)}{dp_1} = \left(1-v\right) CP^{v} p_1^{-v},$$

and (48), we have that (49) can be rewritten as

$$\mathbb{E}\left[\left(1-v\right) \frac{y_1}{P} + \frac{W}{P}\frac{v}{\psi}\frac{l_1}{p_1} + \frac{dT_1\left(r_1\right)}{dr}\frac{\left(1-v\right) y_1}{P} \middle| x \right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1-v}{v}\frac{y_1 p_1}{P} + \frac{1}{\psi}\frac{W}{P} l_1 + \frac{1-v}{v}\frac{dT_1\left(r_1\right)}{dr}\frac{y_1 p_1}{P} \middle| x \right] = 0. \tag{51}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to make their investment decisions and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend on the signals x only through the effect that the latter has on firms' investment decisions, as in the proof of Lemma 4. Consistently with the notation used above, we add "hats" to all relevant variables to highlight that these are computed under the efficient rules. Observe that market clearing in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}, \tag{52}$$

and recall that, as established in the Proof of Lemma 1, $\hat{L} = \hat{l}_0 \left[\left(\gamma^{\varphi} - 1\right) \hat{N} + 1 \right]$. Also, observe that efficiency requires that

$$-\psi \hat{C}^{\frac{1}{v}} \hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon} \hat{l}_1 = 0.$$

Accordingly, using Condition (51), we have that each investing firm finds it optimal to set the price $\hat{p}_1$ that sustains the efficient allocation only if

$$\mathbb{E}\left[\frac{1-v}{v}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\bigg]=0, \tag{53}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (48), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (53) as

$$\mathbb{E}\left[\frac{1-v}{v}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\bigg]=0,$$

or, equivalently, as

$$\mathbb{E}\left[\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\right|x\bigg]=0.$$

It follows that, to induce the firm to set the efficient price $\hat{p}_1$ irrespective of his signal x, the fiscal policy must satisfy $dT_1\left(r_1\right)/dr=1/(v-1)$ for all r_1. Furthermore, one can verify that, when $dT_1\left(r_1\right)/dr=1/(v-1)$ for all r_1, the firm's payoff is quasi-concave in p_1, which implies that setting the price $p_1=\hat{p}_1$ is indeed optimal for all x. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient rules and

$$T_1(r)=\frac{r}{v-1}+s=\frac{1}{v-1}\left(\frac{p_1y_1}{P}\right)+s,$$

where s is invariant in r, the firm's objective (47) is equal to

$$\mathbb{E}\left[\frac{v}{v-1}\frac{p_1y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}l_1+s\right|x\bigg].$$

Using (48) and (50), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right|x\bigg],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{1}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\right|x\bigg].$$

From the analysis above, when $p_1=\hat{p}_1$, $y_1=\hat{y}_1$ and $l_1=\hat{l}_1$ in each state θ. Furthermore, irrespective of x, the derivative of the firm's objective function with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[-v\frac{\hat{y}_1}{\hat{P}} + \frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1} \middle| x \right] = 0. \tag{54}$$

Using (54), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the non-investing firms, we have that any fiscal policy that induces efficiency in information usage must pay to each non-investing firm a transfer equal to $T_0(r_0)$ such that $dT_0(r_0)/dr = 1/(v-1)$, and that any such policy indeed induces these firms to set a price equal to $\hat{p}_0$ irrespective of the signals x. Thus, we conclude that any policy inducing efficiency in information usage must have the structure

$$T_0\left(r\right) = \frac{1}{v-1}r, \tag{55}$$

and

$$T_1\left(\theta, r\right) = \frac{1}{v-1}r + s(\theta), \tag{56}$$

where we reintroduce the dependence of s on θ in light of the analysis below.

Part 2. Observe that, under any monetary and fiscal policy that implement the efficient allocation, the real revenues, i.e., the revenues expressed in terms of the consumption of the final good, must be the same as under flexible prices. This follows from the fact that the equilibrium in the market for intermediate goods implies that

$$\hat{y}_f = \hat{C}\left(\frac{\hat{P}}{\hat{p}_f}\right)^v,$$

for $f = 0, 1$, which means that $\hat{p}_f/\hat{P}$ – and hence $\hat{r}_f = (\hat{p}_f\hat{y}_f)/\hat{P}$ – is uniquely pinned down by the efficient allocation. Because the transfers to the firms are in terms of real revenues, and because real wages are also uniquely pinned down by the efficient allocation (as one can see from (52)), the value of investing and of acquiring information must coincide with their counterparts under flexible prices. In turn, this implies that the subsidy to the investing firms $s(\theta)$ must satisfy the same conditions as in Lemma 2 when information is exogenous, and those in Lemma 3 when information is endogenous. Finally, that the conclusions in Propositions 1 and 2 hold follows directly from the same arguments as in the proofs of these propositions. Q.E.D.

Optimal Fiscal and Monetary Policy with Investment Spillovers and Endogenous Private Information

Online Supplement

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Abstract

This document contains two sections. Section S.1 establishes that the results for the version of the model in Subsection 2.2 in the main text (where firms produce intermediate goods in a "smart" or "traditional" specification) are equivalent to those for the version of the model in Subsection 2.1 analyzed in the rest of the paper. Section S.2 contains an extension to a family of economies in which the firms' managers, and hence the representative household, are risk averse with a diminishing marginal utility for the consumption of the final good. All numbered items in this document contain the prefix "S". Any numbered reference without the prefix "S" refers to an item in the main text.

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S.1 Equivalence between Specifications in Subsections 2.1 and 2.2 in main text

Lemma 1. To see that Lemma 1 in the main text holds under the production function of Subsection 2.2 in the main text, note that, in each state θ, the amount of the final good produced is equal to (we dropped the dependence of the various functions on π_x to ease the notation)

$$Y(\theta) = \Theta(1+\beta N(\theta))^{\alpha}\left(N(\theta)(\gamma y_1(\theta))^{\frac{v-1}{v}} + (1-N(\theta))\, y_0(\theta)^{\frac{v-1}{v}}\right)^{\frac{v}{v-1}}. \qquad \text{(S.1)}$$

Because $C(\theta) = Y(\theta)$, using Condition (4) in the main text, we have that the consumption of the final good in each state θ is equal to

$$C(\theta) = \left(N(\theta)\left(\gamma\Theta(1+\beta N(\theta))^{\alpha} l_1(\theta)^{\psi}\right)^{\frac{v-1}{v}} + (1-N(\theta))\left(\Theta(1+\beta N(\theta))^{\alpha} l_0(\theta)^{\psi}\right)^{\frac{v-1}{v}}\right)^{\frac{v}{v-1}},$$

which coincides with the expression under the specification of Subsection 2.1 – see Condition A.1 in the proof of Lemma 1 in the main text. It is then easy to see that all the arguments in the proof of Lemma 1 in the main text apply also to the production specification in Subsection 2.2.

Equilibrium Price of the Final Good. To derive the equilibrium price of the final good under the production specification in Subsection 2.2, recall that the final good is produced in a competitive market in which profits are equal to

$$\Pi = PY - \int p_i y_i di,$$

where Y is given by Condition (S.1) above. Note that, for each intermediate input i, the price y_i naturally depends on whether the good is provided in its smart or traditional specification. Letting p_1 denote the price for the goods provided in the smart specification and p_0 the price for the goods provided in the traditional specification, we have that the first-order conditions for the maximization of Π yield

$$p_1 = P\left(\frac{y_1}{Y}\right)^{-\frac{1}{v}}\left(\gamma\Theta(1+\beta N)^{\alpha}\right)^{\frac{v-1}{v}} \qquad p_0 = P\left(\frac{y_0}{Y}\right)^{-\frac{1}{v}}\left(\Theta(1+\beta N)^{\alpha}\right)^{\frac{v-1}{v}}, \qquad \text{(S.2)}$$

where we dropped the arguments of all the functions to ease the notation. The demands for the intermediate goods supplied in their smart specification are then given by

$$y_1 = \gamma^{v-1} \left(\frac{p_1}{p_0} \right)^{-\frac{1}{v}} y_0.$$

Using again Condition (S.1) above, we thus have that the amount of the final good produced in each state θ is equal to

$$Y = C\Theta \left(1 + \beta N\right)^{\alpha} \left(Np_1^{1-v}\gamma^{v-1} + (1-N)\, p_0^{1-v}\right)^{\frac{v}{v-1}} \frac{y_0}{p_0^{-v}},$$

which in turn implies that the price of the final good is equal to

$$P = \frac{\left(Np_1^{1-v}\gamma^{v-1} + (1-N)\, p_0^{1-v}\right)^{\frac{1}{v-1}}}{\Theta \left(1 + \beta N\right)^{\alpha}}.$$

This condition is the analog of Condition (A.20) in the main text.

Optimal policies. To verify that the optimal policies under the production specification of Subsection 2.2 coincides with those under the specification of Subsection 2,1, we show that the extra profit (net of the subsidy) $\mathcal{R}(\theta)$ that each firm makes by choosing the smart specification takes the same form as in the proof of Lemma 2 in the main text.

Given W and P, each firm providing its input in the smart specification chooses p_1 to maximize[1]

$$\frac{p_1 y_1 - W l_1}{P} + T_1 \left(\frac{p_1 y_1}{P} \right), \tag{S.3}$$

where

$$y_1 = \left(\gamma \Theta \left(1 + \beta N\right)^{\alpha}\right)^{v-1} C \left(\frac{p_1}{P} \right)^{-v}, \tag{S.4}$$

and $l_1 = y_1^{1/\psi}$. After some algebra, the first-order condition of the above maximization problem for p_1 yields

$$\frac{1-v}{v} \frac{y_1 p_1}{P} + \frac{1}{\psi} \frac{W}{P} l_1 + \frac{1-v}{v} \frac{dT_1 \left(p_1 y_1 / P\right)}{dr} \frac{y_1 p_1}{P} = 0, \tag{S.5}$$

which is the same as Condition (A.17) in the main text.

Next, use (S.2), (S.4), and the fact $y_i = l_i^{\psi}$ to verify that, in any equilibrium implementing the efficient allocation, firms must set prices equal to

$$\hat{p}_1 = \Theta \left(1 + \beta N\right)^{\alpha} \left(\left(\gamma^{\varphi} - 1\right) \hat{N} + 1 \right)^{\frac{1}{v-1}} \gamma^{\left(1 + \frac{\varphi}{1-v}\right)} \hat{P}, \tag{S.6}$$

[1] We drop π^x and θ from all the formulas to ease the notation.

and

$$\hat{p}_0 = \Theta\left(1+\beta N\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}+1\right)^{\frac{1}{v-1}}\hat{P}, \tag{S.7}$$

with

$$\hat{P} = \frac{\left(\hat{N}\hat{p}_1^{1-v}\gamma^{v-1}+\left(1-\hat{N}\right)\hat{p}_0^{1-v}\right)^{\frac{1}{1-v}}}{\Theta\left(1+\beta N\right)^{\alpha}}. \tag{S.8}$$

Equilibrium in the labor market requires that

$$\frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}$$

where

$$\hat{L} = \hat{l}_1\hat{N} + \hat{l}_0(1-\hat{N})$$

Furthermore, as shown in the proof of Lemma 1 in the main text, efficiency requires that $\hat{l}_1 = \gamma^{\varphi}\hat{l}_0$ and that

$$-\psi\hat{C}^{\frac{1}{v}}\hat{y}_1^{\frac{v-1}{v}} + \hat{L}^{\varepsilon}\hat{l}_1 = 0. \tag{S.9}$$

Condition (S.5) then implies that T implements the efficient allocation only if

$$T_1\left(r\right) = \frac{1}{v-1}r + s, \tag{S.10}$$

and

$$T_0\left(r\right) = \frac{1}{v-1}r \tag{S.11}$$

exactly as under the production specification of Subsection 2.1 (see the proof of Lemma 2 in the main text).

Using again (S.4) above, we have that

$$\frac{y_1 p_1}{P} = y_1^{\frac{v-1}{v}}Y^{\frac{1}{v}}\left(\gamma\Theta\left(1+\beta N\right)^{\alpha}\right)^{\frac{v-1}{v}}.$$

Hence, when the labor market clears, the extra profit (net of the subsidy) from producing inputs in their smart specification (relative to the profits of producing them in their traditional specification) is equal to

$$\mathcal{R} = \left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\frac{v-1}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + s\left(\theta\right) - k.$$

Using Condition (S.1) above along with the fact that $y_i = l_i^{\psi}$, the above expression can be rewritten as

$$\mathcal{R}=\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\left(\hat{N}\left(\theta\right)\gamma^{\frac{v-1}{v}}\hat{l}_1\left(\theta\right)^{\psi\frac{v-1}{v}}+\left(1-\hat{N}\left(\theta\right)\right)\hat{l}_0\left(\theta\right)^{\psi\frac{v-1}{v}}\right)^{\frac{1}{v}}\cdot \tag{S.12}$$
$$\cdot\theta\left(1+\beta\hat{N}\right)^{\alpha}\left(\gamma\hat{l}_1\left(\theta\right)^{\psi\frac{v-1}{v}}-\hat{l}_0\left(\theta\right)^{\psi\frac{v-1}{v}}\right)+s\left(\theta\right)-k.$$

Finally, use Condition (2) and (3) in the main text to observe that the formula for $\mathcal{R}$ at the end of the proof of Lemma 2 in the main text coincides with the one in (S.12). Hence Lemma 2 continues to hold under the specification of Subsection 2.2.

That Lemmas 1 and 2 hold under the specification of Subsection 2.2 implies that all the other results in Sections 3 and 4 in the main text for the specification of Subsection 2.1 hold verbatim also for the specification of Subsection 2.2.

S.2 Richer Economies with Risk-Averse Managers

Consider the following economy in which the firms' managers are risk averse and set prices under imperfect information about the underlying fundamentals. Consistently with the rest of the pertinent literature, we assume that each manager is a member of a representative household, whose utility function is given by

$$U=\frac{C^{1-R}}{1-R}-kN-\frac{l^{1+\varepsilon}}{1+\varepsilon}-\int\mathcal{I}(\pi_i^x)di,$$

where $R\geq 0$ is the coefficient of relative risk aversion in the consumption of the final good (the case $R=0$ corresponds to what examined in Section 4 in the main text). The assumption that all managers are members of the same representative household is meant to capture the existence of a rich set of financial instruments that make the market complete in the sense of allowing the managers to fully insure against idiosyncratic consumption risk. The latter property, in turn, isolates the frictions (and associated inefficiencies) that originate in the interaction between (a) investment spillovers and (b) endogenous private information at the time of the investment decisions from the more familiar inefficiencies that originate in the lack of insurance possibilities.

As in the baseline model, each agent provides the same amount of labor (i.e., $l_i=l$ for all i), which is a consequence of the assumption that labor is homogenous and exchanged in a competitive market. Being a member of the representative household, each manager maximizes her firm's market valuation taking into account that the profits the firm generates will be used for the purchase of the final good. This means that each manager maximizes

$$\mathbb{E}\left[C^{-R}\left(\frac{p_i y_i - W l_i}{P} + T\right)\middle| x_i; \pi_i^x\right] - k n_i - \mathcal{I}(\pi_i^x),$$

where C^{-R} is the representative household's marginal utility of consumption of the final good.

The representative household is endowed with an amount M of money provided by the government as a function of θ before the markets open (but after firms make their investment and pricing decisions). The household faces a cash-in-advance constraint according to which the maximal expenditure on the purchase of the final good cannot exceed M, that is,

$$PY \leq M.$$

The representative household collects profits from all firms and wages from all workers and uses them to repay M to the government at the end of the period. The government maximizes the ex-ante utility of the representative household, which is given by

$$\mathcal{W} = \mathbb{E}\left[\frac{C^{1-R}}{1-R} - kN - \frac{l^{1+\varepsilon}}{1+\varepsilon}\right] - \mathcal{I}(\pi^x),$$

by means of a monetary policy $M(\cdot)$ and a fiscal policy $T(\cdot)$, subject to the constraint that the tax deficit be non-positive in each state.

The timing of events is the same as in Section 4 in the main text (note, in particular, that prices are set under dispersed information about θ, that is, each p_i is based on x_i instead of θ). This richer economy is consistent with most of the assumptions typically made in the pertinent literature.

S.2.1 Efficient Allocation

The following proposition characterizes the efficient allocation in this economy.

Proposition S.1. *(1) Let* $\varphi \equiv \frac{v-1}{v-\psi(v-1)}$ *and* $\bar{R} \equiv 1 - \frac{(v-1)(1+\varepsilon)}{(1+\varepsilon)v+\varepsilon\psi(1-v)}$. *Assume that* $\gamma^{\varphi} \geq 1+\beta$, $\psi < \min\left\{1, \frac{1+\varepsilon}{\varepsilon(v-1)}\right\}$, *and* $0 \leq R \leq \bar{R}$. *For any precision of private information* π^x, *there exists a threshold* $\hat{x}(\pi^x)$ *such that efficiency requires that* $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$. *The threshold* $\hat{x}(\pi^x)$, *along with the functions* $\hat{N}(\theta;\pi^x)$, $\hat{l}_1(\theta;\pi^x)$, *and* $\hat{l}_0(\theta;\pi^x)$, *satisfy the following* *properties:*

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta;\pi^{x}\right)\right)^{\alpha}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x}\right)+1\right)^{\frac{1}{\varphi}}\right)^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$

$$\left.\times\left(\frac{\gamma^{\varphi}-1}{\varphi\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x}\right)+1\right)}+\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x}\right)}\right)\middle|\hat{x}(\pi^{x}),\pi^{x}\right]=k,$$

$$\hat{N}\left(\theta;\pi^{x}\right)=1-\Phi\left(\hat{x}(\pi^{x})|\theta;\pi^{x}\right),$$

$$\hat{l}_0(\theta;\pi^{x})=\psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta\hat{N}\left(\theta;\pi^{x}\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\times$$

$$\times\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}},\quad \text{(S.13)}$$

and

$$\hat{l}_1(\theta;\pi^{x})=\gamma^{\varphi}\hat{l}_0(\theta;\pi^{x}),\quad \text{(S.14)}$$

where $\Theta\equiv\exp(\theta)$.

(2) The efficient acquisition of private information is implicitly defined by the solution to

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}+\frac{v}{v-1}\frac{\left(\gamma^{\varphi}-1\right)}{\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]+$$

$$+\mathbb{E}\left[\hat{l}_0(\theta;\pi^{x*})^{1+\varepsilon}\left[\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right]^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi^{x}}.$$

The restriction $0\leq R\leq\bar{R}$ guarantees that the marginal utility of consuming the final good does not decrease 'too quickly' with C. Along with the other restrictions in the proposition, which are the same as in Lemma 1 in the main text, this property implies that the efficient investment strategy is monotone. When, instead, $R>\bar{R}$, a higher value of θ may entail a low enough marginal utility of consumption to induce the planner to ask some firms receiving a high signal to refrain from investing. As we clarify below, our key results extend to this case, but the exposition is less transparent.

S.2.2 Equilibrium Allocation

Firms make their investment decisions and set the price for their intermediate goods under dispersed information about θ. Given these choices, they acquire labor l to meet their demands, after observing θ and aggregate investment N. In this richer economy, the equilibrium price of the final good and the demands for the intermediate products continue to be given by the same conditions as in the main text. Likewise for the labor demands. Because labor is undifferentiated and the labor market is competitive, the supply of labor is then given by

$$\frac{W}{P}C^{-R} = l^{\varepsilon},$$

where the right-hand side is the marginal disutility of labor, whereas the left-hand side is the marginal utility of expanding the consumption of the final good by W/P units, starting from a level of consumption equal to C. Market clearing in the labor market then requires that

$$\frac{W}{P}C^{-R} = \left(\int l_i di\right)^{\varepsilon}.$$

Let $p_1(x;\pi^x)$ and $l_1(x,\theta;\pi^x)$ denote the equilibrium price and labor demand, respectively, of each investing firm. The corresponding functions for the firms that do not invest are $p_0(x;\pi^x)$ and $l_0(x,\theta;\pi^x)$.[2]

The above equilibrium conditions are standard. The following definition identifies the components of the equilibrium allocation that are most relevant for our analysis.

Definition S.1. Given the monetary policy $M(\cdot)$ and the fiscal policy $T(\cdot)$, an **equilibrium** is a precision π^x of private information, along with an investment strategy $n(x;\pi^x)$ and a pair of price functions $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$ such that, when each firm $j \neq i$ chooses a precision of information equal to π^x and then invests according to $n(x;\pi^x)$ and sets its price according to $p_0(x;\pi^x)$ and $p_1(x;\pi^x)$, each firm i maximizes its market valuation by doing the same.

The following definition clarifies what it means that $M(\cdot)$ and $T(\cdot)$ are optimal.

Definition S.2. The monetary policy $M^*(\cdot)$ along with the fiscal policy $T^*(\cdot)$ are **optimal** if they implement the efficient acquisition and usage of information as an equilibrium. That is, if they induce all firms to choose the efficient precision of information π^{x*}, follow the efficient investment rule $\hat{n}(x;\pi^{x*})$, and set prices according to rules $\hat{p}_0(x;\pi^{x*})$ and $\hat{p}_1(x;\pi^{x*})$ that, when followed by all firms, induce in each state θ demands for the intermediate products equal to

[2] As in the baseline model, the dependence of these functions on π^x reflects the fact that, in each state θ, the measure of investing firms N depends on the precision π^x of firms' information.

$\hat{y}_0(\theta;\pi^{x*})$ and $\hat{y}_1(\theta;\pi^{x*})$ and result in firms employing labor according to the efficient schedules $\hat{l}_0(\theta;\pi^{x*})$ and $\hat{l}_1(\theta;\pi^{x*})$.

For any precision of private information π^x (possibly different from π^{x*}), and any θ, let $\hat{M}(\theta;\pi^x)$ denote the optimal money supply in state θ. The following lemma characterizes the monetary policy $\hat{M}(\cdot;\pi^x)$.

Lemma S.1. *Suppose that the precision of private information is exogenously fixed at π^x for all firms. Any monetary policy $\hat{M}(\cdot;\pi^x)$ that, together with some fiscal policy $\hat{T}(\cdot)$, implements the efficient use of information (for precision π^x) as an equilibrium is of the form*

$$\hat{M}(\theta;\pi^x) = m\hat{l}_0(\theta;\pi^x)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

for all θ, where m is an arbitrary positive constant. The monetary policy $\hat{M}(\cdot;\pi^x)$ induces all firms that make the same investment decision to set the same price, irrespective of their information about θ.

As in other economies with nominal rigidities, the monetary policy $\hat{M}(\cdot;\pi^x)$ induces firms to disregard their private information about the fundamentals, and set prices based only on their investment decision. That prices do not respond to firms' information about θ, given the firms' investments, is necessary to avoid allocative distortions in the induced employment and productions decisions. Relative prices must not vary with firms' signals about θ when the latter signals are imprecise. The monetary policy in Lemma S.1 is designed so that, even if firms could condition their prices on θ, they would not find it optimal to do so. Under the proposed policy, variations in employment and production decisions in response to changes in fundamentals are sustained by adjusting the money supplied in a way that replicates the same allocations sustained when the supply of money is constant and prices are flexible.

Lemma S.1, in turn, permits us to establish the following result.

Proposition S.2. *Irrespective of whether the economy satisfies the conditions in Proposition S.1, the fiscal policy*

$$T_0^*\left(r\right) = \frac{1}{v-1}r,$$

and

$$T_1^*\left(\theta,r\right) = \frac{\alpha\beta\hat{C}(\theta;\pi^{x*})}{1+\beta\hat{N}(\theta;\pi^{x*})} + \frac{1}{v-1}r,$$

along with the monetary policy

$$M^*(\theta) = m\hat{l}_0(\theta;\pi^{x*})^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

are optimal.

The monetary policy in the proposition (which belongs to the family in Lemma S.1, specialized to $\pi^x = \pi^{x*}$) neutralizes the effects of price rigidity by replicating the same allocations as under flexible prices. When paired with the fiscal policy in the proposition, it guarantees that, if firms were constrained to acquire information of precision π^{x*}, they would follow the efficient rule $\hat{n}(x; \pi^{x*})$ to make their investment decisions and then set prices $\hat{p}_0(x; \pi^x)$ and $\hat{p}_1(x; \pi^x)$ that induce the efficient labor demands, and hence the efficient production of the intermediate and final goods. This is accomplished through a fiscal policy that, in addition to offsetting firms' market power with a familiar revenue subsidy $r/(v-1)$, realigns the private value of investing with the social value through an additional subsidy to the investing firms that operates as a Pigouvian correction. As in the baseline economy, the subsidy

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta; \pi^{x*})}{1 + \beta\hat{N}(\theta; \pi^{x*})}$$

makes each firm internalize the marginal effect of investment on the production of the final good, in each state θ. Once this realignment is established, the value that firms assign to acquiring information coincides with its social counterpart, inducing all firms to acquire the efficient amount of private information when expecting other firms to do the same.

S.3 Proofs

Proof of Proposition S.1. The proof is in two parts, each corresponding to the two claims in the proposition.

Part 1. Fix the precision of private information π^x and then drop it from all expressions to ease the notation. Let $n(x)$ denote the probability that a firm receiving signal x invests, and $l_1(\theta)$ and $l_0(\theta)$ the amount of labor employed by the investing firms and by those deciding not to invest, respectively. The planner's problem can be written as

$$\max_{n(x), l_1(\theta), l_0(\theta)} \int_\theta \frac{C(\theta)^{1-R}}{1-R} d\Omega(\theta) - k\int_\theta N(\theta) d\Omega(\theta) + \\ - \frac{1}{1+\varepsilon}\int_\theta \left[l_1(\theta)N(\theta) + l_0(\theta)(1-N(\theta))\right]^{1+\varepsilon} d\Omega(\theta) + \\ - \int_\theta \mathcal{Q}(\theta)\left(N(\theta) - \int_x n(x)\,\Phi(x|\theta)\right) d\Omega(\theta),$$

where $\Omega(\theta)$ denotes the cumulative distribution function of θ (with density $\omega(\theta)$), $\Phi(x|\theta)$

the cumulative distribution function of x given θ (with density $\phi(x|\theta)$), $\mathcal{Q}(\theta)$ the multiplier associated with the constraint $N(\theta) = \int_x n(x)\, d\Phi(x|\theta)$, and

$$C(\theta) = \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}} (1 - N(\theta)) \right)^{\frac{v}{v-1}}, \tag{S.15}$$

with

$$y_1(\theta) = \gamma \Theta (1 + \beta N(\theta))^{\alpha} l_1(\theta)^{\psi}, \tag{S.16}$$

and

$$y_0(\theta) = \Theta (1 + \beta N(\theta))^{\alpha} l_0(\theta)^{\psi}. \tag{S.17}$$

Using (S.15) and (S.16), the first-order condition of the planner's problem with respect to $l_1(\theta)$ can be written as

$$\psi C(\theta)^{-R} \left(y_1(\theta)^{\frac{v-1}{v}} N(\theta) + y_0(\theta)^{\frac{v-1}{v}} (1 - N(\theta)) \right)^{\frac{1}{v-1}} (\gamma \Theta (1 + \beta N(\theta))^{\alpha})^{\frac{v-1}{v}} l_1(\theta)^{\psi \frac{v-1}{v} - 1}$$
$$- (l_1(\theta) N(\theta) + l_0(\theta)(1 - N(\theta)))^{\varepsilon} = 0.$$

Letting

$$L(\theta) \equiv l_1(\theta) N(\theta) + l_0(\theta)(1 - N(\theta)), \tag{S.18}$$

and using (S.15) and (S.16), we have that the above first-order condition reduces to

$$\psi C(\theta)^{\frac{1 - vR}{v}} y_1(\theta)^{\frac{v-1}{v}} = l_1(\theta) L(\theta)^{\varepsilon}. \tag{S.19}$$

Following similar steps, the first-order condition with respect to $l_0(\theta)$ yields

$$\psi C(\theta)^{\frac{1 - vR}{v}} y_0(\theta)^{\frac{v-1}{v}} = l_0(\theta) L(\theta)^{\varepsilon}. \tag{S.20}$$

Using (S.16) and (S.17), the ratio between (S.19) and (S.20) can be written as

$$\gamma^{\frac{v-1}{v}} \left(\frac{l_1(\theta)}{l_0(\theta)} \right)^{\psi \frac{v-1}{v}} = \frac{l_1(\theta)}{l_0(\theta)},$$

which implies that

$$l_1(\theta) = \gamma^{\varphi} l_0(\theta). \tag{S.21}$$

Notice that (S.21) entails that, at the efficient allocation, the total labor demand, as defined in (S.18), is equal to

$$L(\theta) = l_0(\theta) \left[(\gamma^{\varphi} - 1) N(\theta) + 1 \right]. \tag{S.22}$$

Using (S.16) and (S.17), we can also write aggregate consumption as

$$C(\theta) = \Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\left(\gamma^{\frac{v-1}{v}} l_1(\theta)^{\psi\frac{v-1}{v}} N(\theta) + l_0(\theta)^{\psi\frac{v-1}{v}}\left(1-N(\theta)\right)\right)^{\frac{v}{v-1}}.$$

Using (S.21), we can rewrite the latter expression as

$$C(\theta) = \Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha} l_0(\theta)^{\psi}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{v}{v-1}}. \tag{S.23}$$

Next, use (S.21) and (S.23) to rewrite (S.20) as

$$\psi\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-vR}{v}} l_0(\theta)^{\psi\frac{1-vR}{v}}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}} \times$$
$$\times\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{v-1}{v}} l_0(\theta)^{\psi\frac{v-1}{v}} = l_0(\theta)L\left(\theta\right)^{\varepsilon},$$

which, using (S.22), can be expressed as

$$\psi\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}}$$
$$= l_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\varepsilon}.$$

From the derivations above, we have that the efficient labor demands are given by

$$l_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}}\left(\Theta\left(1+\beta N\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}, \tag{S.24}$$

and by (S.21).

Note that $l_0(\theta) > 0$ for all θ. Also note that the above conditions are both necessary and sufficient given that the planner's problem has a unique critical point in (l_0, l_1) for each θ.

Next, consider the derivative of the planner's problem with respect to $N(\theta)$. Ignoring that $N(\theta)$ must be restricted to be in $[0, 1]$, we have that

$$\mathcal{Q}(\theta) \equiv C(\theta)^{-R}\frac{dC(\theta)}{dN\left(\theta\right)} - k - L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right).$$

The derivative $dC(\theta)/dN(\theta)$ is computed holding the functions $l_1(\theta)$ and $l_0(\theta)$ fixed, and varying the proportion of investing firms and the amounts that each firm produces (for given investment decision) when N changes.

Lastly, consider the effect on welfare of changing $n(x)$ from 0 to 1, which is equal to

$$\Delta(x) \equiv \int_{\theta}\mathcal{Q}(\theta)\phi\left(x|\theta\right)\omega\left(\theta\right)d\theta.$$

Using the fact that $\phi(x|\theta)\,\omega(\theta) = f(\theta|x)\,g(x)$, where $f(\theta|x)$ is the conditional density of θ given x and $g(x)$ is the marginal density of x, we have that

$$\Delta(x) \overset{sgn}{=} \int_\theta \mathcal{Q}(\theta) f(\theta|x)\, d\theta = \mathbb{E}[\mathcal{Q}(\theta)|x].$$

Hence, efficiency requires that all firms receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] > 0$ invest, whereas all those receiving a signal x such that $\mathbb{E}[\mathcal{Q}(\theta)|x] < 0$ refrain from investing.
Next, use (S.15) to observe that

$$C(\theta)^{-R}\tfrac{dC(\theta)}{dN(\theta)} = \tfrac{v}{v-1}C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] +$$

$$+C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))\right],$$

and (S.16) and (S.17) to note that

$$y_1(\theta)^{-\frac{1}{v}}\tfrac{\partial y_1(\theta)}{\partial N(\theta)}N(\theta) + y_0(\theta)^{-\frac{1}{v}}\tfrac{\partial y_0(\theta)}{\partial N(\theta)}(1-N(\theta))$$

$$= \tfrac{\alpha\beta}{1+\beta N(\theta)}\left(y_1(\theta)^{\frac{v-1}{v}}N(\theta) + y_0(\theta)^{\frac{v-1}{v}}(1-N(\theta))\right) = \tfrac{\alpha\beta}{1+\beta N(\theta)}C(\theta)^{\frac{v-1}{v}},$$

where the last equality uses again (S.15).
Finally, using (S.19) and (S.20), we have that

$$\psi C(\theta)^{\frac{1-vR}{v}}\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) = L(\theta)^{\varepsilon}\left(l_1(\theta) - l_0(\theta)\right).$$

We conclude that

$$\mathcal{Q}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)C(\theta)^{\frac{1-vR}{v}}\left[y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right] + C(\theta)^{1-R}\frac{\alpha\beta}{1+\beta N(\theta)} - k.$$

Using (S.16), (S.17), (S.21), and (S.23), after some manipulations, we have that

$$\begin{aligned} C(\theta)^{\frac{1-vR}{v}}&\left(y_1(\theta)^{\frac{v-1}{v}} - y_0(\theta)^{\frac{v-1}{v}}\right) = \\ &= \left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{1-vR}{v-1}}\left(\Theta(1+\beta N(\theta))^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}(\gamma^{\varphi}-1). \end{aligned} \tag{S.25}$$

Using (S.23), we also have that

$$C(\theta)^{1-R} = \left((\gamma^{\varphi}-1)N(\theta)+1\right)^{\frac{v(1-R)}{v-1}}\left(\Theta(1+\beta N(\theta))^{\alpha}\right)^{1-R} l_0(\theta)^{\psi(1-R)}.$$

It follows that

$$\mathcal{Q}(\theta) = \left(\left(\gamma^{\varphi} - 1 \right) N\left(\theta \right) + 1 \right)^{\frac{v(1-R)}{v-1}} \left(\Theta \left(1 + \beta N\left(\theta \right) \right)^{\alpha} \right)^{1-R} l_0(\theta)^{\psi(1-R)} \times$$

$$\times \left(\frac{\gamma^{\varphi} - 1}{\varphi[\left(\gamma^{\varphi} - 1 \right) N\left(\theta \right) + 1]} + \frac{\alpha\beta}{1 + \beta N\left(\theta \right)} \right) - k.$$

Next, recall that the optimal labor demand for the non-investing firms is given by (S.24). Replacing the expression for $l_0(\theta)$ into that for $\mathcal{Q}(\theta)$, we obtain that

$$\mathcal{Q}(\theta) = \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi} - 1 \right) N(\theta) + 1 \right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))} - 1} \left(1 + \beta N(\theta) \right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$

$$\times \left(\frac{\gamma^{\varphi} - 1}{\varphi} + \frac{\alpha\beta \left(\left(\gamma^{\varphi} - 1 \right) N(\theta) + 1 \right)}{1 + \beta N(\theta)} \right) - k.$$

Note that, when the parameters satisfy the conditions in the proposition, $\mathcal{Q}$ is increasing in both N (for given θ) and in θ (for given N). That, for any θ, $\mathcal{Q}(\theta)$ is increasing in N implies that welfare is convex in N under the first best, i.e., when θ is observable by the planner at the time the investment decisions are made. In turn, such a property implies that the first-best choice of N is either $N = 0$ or $N = 1$, for all θ. This observation, along with the fact that $\mathcal{Q}(\theta)$ is increasing in θ for any N then implies that the first-best level of N is increasing in θ. These properties, in turn, imply that the optimal investment policy is monotone. For any $\hat{x}$, let

$$\bar{N}(\theta|\hat{x}) \equiv 1 - \Phi(\hat{x}|\theta)$$

denote the measure of investing firms at θ when firms follow the monotone rule $n(x) = \mathbb{I}(x > \hat{x})$. Then let

$$\bar{\mathcal{Q}}(\theta|\hat{x}) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}} \Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi} - 1 \right) \bar{N}(\theta|\hat{x}) + 1 \right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))} - 1} \left(1 + \beta \bar{N}(\theta|\hat{x}) \right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}} \times$$

$$\times \left(\frac{\gamma^{\varphi} - 1}{\varphi} + \frac{\alpha\beta \left(\left(\gamma^{\varphi} - 1 \right) \bar{N}(\theta|\hat{x}) + 1 \right)}{1 + \beta \bar{N}(\theta|\hat{x})} \right) - k$$

denote the function $\mathcal{Q}(\theta)$ characterized above, specialized to $N(\theta) = \bar{N}(\theta|\hat{x})$.
Observe that, under the parameters' restrictions in the proposition, $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}]$ is continuous, strictly increasing in $\hat{x}$, and such that

$$\lim_{\hat{x} \to -\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] < 0 < \lim_{\hat{x} \to +\infty} \mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}].$$

Hence, the equation $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|\hat{x}] = 0$ admits exactly one solution. Letting $\hat{x}$ denote the solution to this equation, we have that $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] < 0$ for $x < \hat{x}$, and $\mathbb{E}[\bar{\mathcal{Q}}(\theta|\hat{x})|x] > 0$ for $x > \hat{x}$. We conclude that, under the assumptions in the proposition, there exists a threshold $\hat{x}(\pi^x)$ such that the investment strategy $\hat{n}(x;\pi^x) = \mathbb{I}(x \geq \hat{x}(\pi^x))$ along with the employment strategies $\hat{l}_1(\theta;\pi^x)$ and $\hat{l}_0(\theta;\pi^x)$ in the proposition satisfy all the first-order conditions of the planner's problem. The threshold $\hat{x}(\pi^x)$ solves

$$\mathbb{E}\left[\psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left((\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times\right.$$
$$\left.\times\left(\frac{\gamma^\varphi-1}{\varphi}+\frac{\alpha\beta\left((\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right)}{1+\beta\hat{N}(\theta;\pi^x)}\right)\right|\hat{x}(\pi^x),\pi^x\Bigg]=k,$$

with $\hat{N}(\theta;\pi^x) = 1-\Phi(\hat{x}(\pi^x)|\theta;\pi^x)$.

Finally note that, irrespective of whether the parameters satisfy the conditions in the proposition (recall that these conditions guarantee that $\hat{n}(x;\pi^x)$ is monotone), any solution to the planner's problem must be such that the functions $\hat{l}_0(\theta;\pi^x)$ and $\hat{l}_1(\theta;\pi^x)$ satisfy Conditions (S.13) and (S.14) in the proposition and $\hat{n}(x;\pi^x) = \mathbb{I}(\mathbb{E}[\hat{\mathcal{Q}}(\theta;\pi^x)|x,\pi^x] > 0)$, where

$$\hat{\mathcal{Q}}(\theta;\pi^x) \equiv \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left((\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\times$$
$$\times\left(\frac{\gamma^\varphi-1}{\varphi}+\frac{\alpha\beta\left((\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right)}{1+\beta\hat{N}(\theta;\pi^x)}\right)-k,$$

with $\hat{N}(\theta;\pi^x) = \int_\theta \hat{n}(x;\pi^x)\,d\Phi(x|\theta,\pi^x)$.

Part 2. For any precision of private information π^x, use Conditions (S.22) and (S.23) in part (1) to write ex-ante welfare as

$$\mathbb{E}[\mathcal{W}|\pi^x] =$$
$$= \frac{1}{1-R}\int_\theta \Theta^{1-R}\left(1+\beta\hat{N}(\theta;\pi^x)\right)^{\alpha(1-R)}\hat{l}_0(\theta;\pi^x)^{\psi(1-R)}\left((\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right)^{\frac{v}{v-1}(1-R)}d\Omega(\theta)+$$
$$-k\int_\theta \hat{N}(\theta;\pi^x)\,d\Omega(\theta)-\int_\theta\frac{\hat{l}_0(\theta;\pi^x)^{1+\varepsilon}}{1+\varepsilon}\left[(\gamma^\varphi-1)\hat{N}(\theta;\pi^x)+1\right]^{1+\varepsilon}d\Omega(\theta)-\mathcal{I}(\pi^x).$$

Using the envelope theorem, we have that the marginal effect of a variation in the precision of private information on welfare is given by

$$\frac{d\mathbb{E}\left[\mathcal{W}|\pi^x\right]}{d\pi^x} =$$

$$= \mathbb{E}\left[\hat{C}\left(\theta;\pi^x\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^x\right)}+\frac{v\left(\gamma^{\varphi}-1\right)}{\left(v-1\right)\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+$$

$$-k\mathbb{E}\left[\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]+\mathbb{E}\left[\hat{l}_0\left(\theta;\pi^x\right)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^x\right)+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}\left(\theta;\pi^x\right)}{\partial\pi^x}\right]-\frac{d\mathcal{I}(\pi_x)}{d\pi_x}.$$

The result in part 2 then follows from the fact that, at the optimum, the above derivative must be equal to zero. Q.E.D.

Proof of Lemma S.1. We drop π^x from all formulas to ease the notation. Using (S.19) and (S.20), we have that

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}},$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}}.$$

The Dixit and Stiglitz demand system implies that $y_i = C\left(P/p_i\right)^v$. Hence, the prices set by any two investing firms coincide, implying that they are independent of the signal x. Let $\hat{p}_1$ be the (state-invariant) price set by the investing firms, and $\hat{p}_0$ that set by the non-investing firms. Let $\hat{P}(\theta)$ denote the price of the final good when all firms follow the efficient policies. Efficiency requires that such prices satisfy

$$\hat{l}_1(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_1\right)^{v-1}, \tag{S.26}$$

$$\hat{l}_0(\theta)\hat{L}\left(\theta\right)^{\varepsilon} = \psi\hat{C}(\theta)^{1-R}\left(\hat{P}\left(\theta\right)/\hat{p}_0\right)^{v-1}, \tag{S.27}$$

from which we obtain that

$$\frac{\hat{p}_0}{\hat{p}_1} = \left(\frac{\hat{l}_1(\theta)}{\hat{l}_0(\theta)}\right)^{\frac{1}{v-1}},$$

which, using (S.21), implies that

$$\hat{p}_1 = \gamma^{\frac{\varphi}{1-v}}\hat{p}_0.$$

The price of the final good is then equal to

$$\hat{P}\left(\theta\right) = \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1}{1-v}}\hat{p}_0. \tag{S.28}$$

Combining the cash-in-advance constraint $M = PC$ with (S.27), we then have that

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)^{1-R}\hat{P}(\theta)^{v+R-2}\,\hat{p}_0^{1-v},$$

and therefore

$$\hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon} = \psi \hat{M}(\theta)^{1-R}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{v+R-2}{1-v}}\,\hat{p}_0^{R-1},$$

where we also used (S.28). Finally, using Condition (S.22), we obtain that

$$\hat{M}(\theta)^{1-R} = \frac{1}{\psi}\hat{l}_0(\theta)^{1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}(\theta)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{v-1}}\,\hat{p}_0^{1-R}.$$

It is immediate to verify that the same conclusion can be obtained starting from (S.26). Because $\hat{p}_0^{1-R}$ can be taken to be arbitrary, the result in the lemma obtains by setting $m^{1-R} = \frac{1}{\psi}\hat{p}_0^{1-R}$. Q.E.D.

Proof of Proposition S.2. The proof is in two parts and establishes a more general result than the one in the proposition. Part 1 fixes the precision of information and identifies a condition on the fiscal policy $T(\cdot)$ that guarantees that, when $T(\cdot)$ is paired with the monetary policy of Lemma S.1, and the economy satisfies the parameters' restrictions of Proposition S.1, firms have incentives to use information efficiently when the latter is exogenous. Part 2 identifies an additional restriction on the fiscal policy that, when combined with the condition in part 1, guarantees that, when the economy satisfies the parameters' restrictions of Proposition S.1, agents have also incentives to acquire information efficiently. The arguments in parts 1 and 2 also allow us to establish that, irrespective of whether or not the economy satisfies the parameters' restrictions of Proposition S.1, when $M(\cdot)$ and $T(\cdot)$ are the specific policies of Proposition S.2, any firm that expects all other firms to acquire and use information efficiently has incentives to do the same.

Part 1. We fix the precision of information π^x and drop it to ease the notation. We also drop θ from the arguments of the various functions when there is no risk of confusion.
Consider first the pricing decision of an investing firm. The firm sets p_1 to maximize

$$\mathbb{E}\left[C^{-R}\left(\frac{p_1 y_1 - W l_1}{P} + T_1(r_1)\right)\middle|\, x\right], \tag{S.29}$$

where $r_1 = p_1 y_1/P$, taking C, W, and P as given, and accounting for the fact that the demand for its product is given by

$$y_1 = C\left(\frac{P}{p_1}\right)^{v}, \tag{S.30}$$

and that the amount of labor that it will need to procure is given by

$$l_1 = \left(\frac{y_1}{\gamma \Theta \left(1 + \beta N\right)^{\alpha}} \right)^{\frac{1}{\psi}}.$$

The first-order condition for the maximization of (S.29) with respect to p_1 is given by

$$\mathbb{E}\left[C^{-R} \left(\left(1 - v\right) C P^{v-1} p_1^{-v} - \frac{W}{P} \frac{dl_1}{dp_1} + \frac{1}{P} \frac{dT_1\left(r_1\right)}{dr} \frac{d(p_1 y_1)}{dp_1} \right) \middle| x \right] = 0. \quad \text{(S.31)}$$

Using

$$\frac{dl_1}{dp_1} = -\frac{v}{\psi} \frac{l_1}{p_1}, \quad \text{(S.32)}$$

$$\frac{d\left(p_1 y_1\right)}{dp_1} = \left(1 - v\right) C P^{v} p_1^{-v},$$

and (S.30), we have that (S.31) can be rewritten as

$$\mathbb{E}\left[C^{-R} \left(\left(1 - v\right) \frac{y_1}{P} + \frac{W}{P} \frac{v}{\psi} \frac{l_1}{p_1} + \frac{dT_1\left(r_1\right)}{dr} \frac{\left(1 - v\right) y_1}{P} \right) \middle| x \right] = 0.$$

Multiplying all the addenda by p_1/v, we have that

$$\mathbb{E}\left[\frac{1 - v}{v} C^{-R} \frac{y_1 p_1}{P} + \frac{1}{\psi} C^{-R} \frac{W}{P} l_1 + \frac{1 - v}{v} C^{-R} \frac{dT_1\left(r_1\right)}{dr} \frac{y_1 p_1}{P} \middle| x \right] = 0. \quad \text{(S.33)}$$

Suppose that all other firms follow policies that induce the efficient allocations, meaning that they follow the rule $\hat{n}(x)$ to determine whether or not to invest, and then set prices $\hat{p}_0$ and $\hat{p}_1$ that depend only on the investment decision but not on the signal x, as in the proof of Lemma S.1. Hereafter, we add 'hats' to all relevant variables to highlight that these are computed under the efficient policies.

Observe that market clearing in the labor market requires that

$$\hat{C}^{-R} \frac{\hat{W}}{\hat{P}} = \hat{L}^{\varepsilon}, \quad \text{(S.34)}$$

and recall that, as established in the Proof of Proposition S.1,

$$\hat{L} = \hat{l}_0 \left[\left(\gamma^{\varphi} - 1\right) \hat{N} + 1 \right].$$

Also, consider that efficiency requires that

$$-\psi\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\hat{L}^{\varepsilon}\hat{l}_1=0.$$

Accordingly, using Condition (S.33), we have that each investing firm finds it optimal to set the price $\hat{p}_1$ only if

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{\frac{1-vR}{v}}\hat{y}_1^{\frac{v-1}{v}}+\frac{1-v}{v}C^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\Bigg]=0, \tag{S.35}$$

where $\hat{r}_1=\hat{p}_1\hat{y}_1/\hat{P}$. Using again (S.30), we have that $\hat{y}_1^{-\frac{1}{v}}=\hat{C}^{-\frac{1}{v}}\frac{\hat{p}_1}{\hat{P}}$, which allows us to rewrite Condition (S.35) as

$$\mathbb{E}\left[\frac{1-v}{v}\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}+\frac{1-v}{v}\hat{C}^{-R}\frac{dT_1\left(\hat{r}_1\right)}{dr}\hat{r}_1\right|x\Bigg]=0,$$

or, equivalently,

$$\mathbb{E}\left[\hat{C}^{-R}\frac{\hat{y}_1\hat{p}_1}{\hat{P}}\left(\frac{1}{v}+\frac{1-v}{v}\frac{dT_1\left(\hat{r}_1\right)}{dr}\right)\right|x\Bigg]=0.$$

It follows that, when $dT_1\left(\hat{r}_1\right)/dr=1/(v-1)$, the first-order condition of the firm's optimization problem with respect to its price is satisfied. Furthermore, one can verify that, under the proposed fiscal policy, the firm's payoff is quasi-concave in p_1, which implies that setting a price $p_1=\hat{p}_1$ is indeed optimal for the firm. To see that the firm's payoff is quasi-concave in p_1 note that, when all other firms follow the efficient policies and

$$T_1(r)=\frac{r}{v-1}+s=\frac{1}{v-1}\left(\frac{p_1y_1}{P}\right)+s,$$

where s may depend on θ but is invariant in r, the firm's objective (S.29) is equal to

$$\mathbb{E}\left[\hat{C}^{-R}\left(\frac{v}{v-1}\frac{p_1y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}l_1+s\left(\theta\right)\right)\right|x\Bigg].$$

Using (S.30) and (S.32), we have that the first derivative of the firm's objective with respect to p_1 is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{y_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{l_1}{p_1}\right)\right|x\Bigg],$$

whereas the second derivative is

$$\mathbb{E}\left[\frac{\hat{C}^{-R}}{p_1}\left(v^2\frac{y_1}{\hat{P}}-\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\left(\frac{v}{\psi}+1\right)\frac{l_1}{p_1}\right)\middle| x\right].$$

From the analysis above, we have that $y_1 = \hat{y}_1$ and $l_1 = \hat{l}_1$ in each state θ when $p_1 = \hat{p}_1$. Furthermore, irrespective of x, the derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is

$$\mathbb{E}\left[\hat{C}^{-R}\left(-v\frac{\hat{y}_1}{\hat{P}}+\frac{\hat{W}}{\hat{P}}\frac{v}{\psi}\frac{\hat{l}_1}{\hat{p}_1}\right)\middle| x\right]=0. \tag{S.36}$$

Using (S.36), we then have that the second derivative of the firm's payoff with respect to p_1, evaluated at $p_1 = \hat{p}_1$, is negative. Because the firm's objective function has a unique critical point at $p_1 = \hat{p}_1$, we conclude that the firm's payoff is quasi-concave in p_1. Applying similar arguments to the non-investing firms, we have that a fiscal policy that pays to each non-investing firm a transfer equal to $T_0(r) = r/(v-1)$ induces these firms to set the price $\hat{p}_0$ irrespective of the signal x.

Next, consider the firms' investment choice. Hereafter, we reintroduce θ in the notation. When

$$T_0\left(r\right)=\frac{1}{v-1}r, \tag{S.37}$$

and

$$T_1\left(\theta,r\right)=s(\theta)+\frac{1}{v-1}r, \tag{S.38}$$

no matter the shape of the function $s(\theta)$, each firm anticipates that, by investing, it will set a price $\hat{p}_1$, hire $\hat{l}_1(\theta)$, and produce $\hat{y}_1(\theta)$ in each state θ, whereas, by not investing, it will set a price $\hat{p}_0$, hire $\hat{l}_0(\theta)$, and produce $\hat{y}_0(\theta)$. Let

$$\hat{\mathcal{R}}(\theta)\equiv\hat{C}\left(\theta\right)^{-R}\left(\hat{r}_1(\theta)-\hat{r}_0(\theta)-\frac{\hat{W}(\theta)}{\hat{P}\left(\theta\right)}\left(\hat{l}_1(\theta)-\hat{l}_0(\theta)\right)+T_1\left(\theta,\hat{r}_1(\theta)\right)-T_0\left(\hat{r}_0(\theta)\right)\right)-k,$$

where $\hat{r}_1(\theta)$ and $\hat{r}_0(\theta)$ are the firm's (real) revenues when the firm follows the efficient policies, respectively, after investing and not investing. Each firm receiving signal x finds it optimal to invest if

$$\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]\geq 0,$$

and not to invest if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]\leq 0$. Recall from (S.30) that the Dixit and Stiglitz demand system implies that $\hat{p}_f = \hat{P}\left(\theta\right)\hat{C}\left(\theta\right)^{\frac{1}{v}}\hat{y}_f(\theta)^{-\frac{1}{v}}$, so that $\hat{r}_f\left(\theta\right)=\hat{C}\left(\theta\right)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}}$, for $f=0,1$.

Also, recall that market clearing in the labor market implies that

$$\frac{\hat{W}(\theta)}{\hat{P}(\theta)}\hat{C}(\theta)^{-R} = \hat{L}(\theta)^{\varepsilon}.$$

Hence, $\hat{\mathcal{R}}(\theta)$ can be rewritten as

$$\hat{\mathcal{R}}(\theta) = \hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) - \hat{L}(\theta)^{\varepsilon}\left(\hat{l}_1(\theta) - \hat{l}_0(\theta)\right) + \\ + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Using the fact that the efficient allocation satisfies the following two conditions (see the proof of Proposition S.1)

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_1(\theta)^{\frac{v-1}{v}} = \hat{l}_1(\theta)\hat{L}(\theta)^{\varepsilon},$$

and

$$\psi\hat{C}(\theta)^{\frac{1-vR}{v}}\hat{y}_0(\theta)^{\frac{v-1}{v}} = \hat{l}_0(\theta)\hat{L}(\theta)^{\varepsilon},$$

we have that $\hat{\mathcal{R}}(\theta)$ can be further simplified as follows:

$$\hat{\mathcal{R}}(\theta) = (1-\psi)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}\left(T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta))\right) - k.$$

Next, use (S.30) to note that

$$\hat{r}_f(\theta) = \hat{C}(\theta)^{\frac{1}{v}}\hat{y}_f(\theta)^{\frac{v-1}{v}},$$

for $f = 0, 1$. It follows that

$$T_1(\theta, \hat{r}_1(\theta)) - T_0(\hat{r}_0(\theta)) = s(\theta) + \frac{1}{v-1}\hat{C}(\theta)^{\frac{1}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right).$$

Accordingly, $\hat{\mathcal{R}}(\theta)$ can be written as

$$\hat{\mathcal{R}}(\theta) = \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) + \hat{C}(\theta)^{-R}s(\theta) - k. \tag{S.39}$$

Recall from the proof of Proposition S.1 that efficiency requires that each firm invests if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] > 0$ and does not invest if $\mathbb{E}\left[\hat{\mathcal{Q}}(\theta)|x\right] < 0$, where $\hat{\mathcal{Q}}(\theta)$ is given by

$$\hat{\mathcal{Q}}(\theta) \equiv \left(\frac{v-\psi(v-1)}{v-1}\right)\hat{C}(\theta)^{\frac{1-vR}{v}}\left[\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right] + \hat{C}(\theta)^{1-R}\frac{\alpha\beta}{1+\beta\hat{N}(\theta)} - k.$$

Hence, we conclude that the proposed policy induces all firms to follow the efficient investment rule $\hat{n}(x)$ if $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \geq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \geq 0$, and $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right] \leq 0$ whenever $\mathbb{E}[\hat{\mathcal{Q}}(\theta)|x] \leq 0$.

As shown in the proof of Proposition S.1 (see Equations (S.25) and (S.24), respectively),

$$\hat{C}(\theta)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1-vR}{v-1}} \left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{1-R} \hat{l}_0(\theta)^{\psi(1-R)}\left(\gamma^{\varphi}-1\right),$$

and

$$\hat{l}_0(\theta) = \psi^{\frac{1}{1+\varepsilon+\psi(R-1)}} \left(\Theta\left(1+\beta\hat{N}\left(\theta\right)\right)^{\alpha}\right)^{\frac{1-R}{1+\varepsilon+\psi(R-1)}} \left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta\right)+1\right)^{\frac{1+\varepsilon-v(R+\varepsilon)}{(v-1)(1+\varepsilon+\psi(R-1))}}.$$

Using the last two expressions, we have that the first addendum in (S.39) can be rewritten as

$$\left(\frac{v-\psi\left(v-1\right)}{v-1}\right)\hat{C}\left(\theta\right)^{\frac{1-vR}{v}} \left(\hat{y}_1(\theta)^{\frac{v-1}{v}} - \hat{y}_0(\theta)^{\frac{v-1}{v}}\right) =$$
$$= \psi^{\frac{\psi(1-R)}{1+\varepsilon+\psi(R-1)}}\Theta^{\frac{(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\left(\gamma^{\varphi}-1\right)N(\theta)+1\right)^{\frac{(1-R)(1+\varepsilon)}{\varphi(1+\varepsilon+\psi(R-1))}-1}\left(1+\beta N(\theta)\right)^{\frac{\alpha(1-R)(1+\varepsilon)}{1+\varepsilon+\psi(R-1)}}\left(\frac{\gamma^{\varphi}-1}{\varphi}\right).$$

When the economy satisfies the conditions in Proposition S.1, the above expression is increasing in N (for given θ) and in θ (for given N). In this case, when the second addendum $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ in (S.39) is non-decreasing in θ, then $\hat{\mathcal{R}}(\theta)$ is non-decreasing in θ, implying that $\mathbb{E}\left[\hat{\mathcal{R}}(\theta)|x\right]$ is non-decreasing in x. As in the baseline model, we thus have that, when the economy satisfies the parameters' restrictions in Proposition S.1, a subsidy $s(\theta)$ to the investing firms satisfying conditions (a) and (b) below guarantees that firms find it optimal to follow the efficient rule $\hat{n}(x)$:

(a) $\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)$ non-decreasing in θ;

(b)

$$\mathbb{E}\left[\hat{C}\left(\theta\right)^{-R}s\left(\theta\right)\Big|\,\hat{x}\right] = \mathbb{E}\left[\frac{\alpha\beta\hat{C}(\theta)^{1-R}}{1+\beta\hat{N}\left(\theta\right)}\Bigg|\,\hat{x}\right].$$

The analysis above also reveals that, when the fiscal policy takes the form in (S.37) and (S.38) with

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta)}{1+\beta\hat{N}\left(\theta\right)},$$

for all θ, and the monetary policy takes the form in Lemma S.1, then irrespective of whether or not the economy satisfies the conditions in Proposition S.1, each firm expecting all other firms to follow the efficient investment rule $\hat{n}(x)$, and setting prices according to $\hat{p}_0$ and $\hat{p}_1$ (thus inducing the efficient employment decisions), finds it optimal to do the same.

Part 2. We now show that, when the economy satisfies the conditions in Proposition S.1, the fiscal policy in (S.37) and (S.38), when paired with the monetary policy

$$M^*(\theta) = m\hat{l}_0\left(\theta;\pi^{x*}\right)^{\frac{1+\varepsilon}{1-R}}\left(\left(\gamma^{\varphi}-1\right)\hat{N}\left(\theta;\pi^{x*}\right)+1\right)^{\frac{(1+\varepsilon)(v-1)+R-1}{(v-1)(1-R)}},$$

implement the efficient acquisition and usage of information if and only if the subsidy $s(\theta)$ to the innovating firms, in addition to properties (a) and (b) in part 1, is such that

$$\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right]=\mathbb{E}\left[\hat{C}\left(\theta;\pi^{x*}\right)^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}\left(\theta;\pi^{x*}\right)}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^{x}}\right].$$

To see this, suppose that all firms other than i acquire information of precision π^{x*} and follow the efficient investment and pricing rules. Consider firm i's problem. As shown above, irrespective of the information acquired by the firm, under the proposed fiscal and monetary policies, the firm finds it optimal to set a price equal to $\hat{p}_1^*$ after investing and equal to $\hat{p}_0^*$ if it does not invest, where $\hat{p}_1^*$ and $\hat{p}_0^*$ are given by the values of $\hat{p}_1$ and $\hat{p}_0$, respectively, when the precision of private information is π^{x*}.

Let

$$\hat{N}^*(\theta) \equiv \hat{N}\left(\theta;\pi^{x*}\right),$$

$$\hat{l}_0^*(\theta) \equiv \hat{l}_0(\theta;\pi^{x*}),$$

$$\hat{l}_1^*(\theta) \equiv \hat{l}_1(\theta;\pi^{x*}),$$

$$\hat{y}_1^*(\theta) \equiv \gamma\Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_1^*(\theta)^{\psi},$$

$$\hat{y}_0^*(\theta) \equiv \Theta\left(1+\beta\hat{N}^*(\theta)\right)^{\alpha}\hat{l}_0^*(\theta)^{\psi},$$

$$\hat{C}^*(\theta) = \hat{Y}^*(\theta) \equiv \left(\hat{y}_1^*(\theta)^{\frac{v-1}{v}}\hat{N}^*(\theta)+\hat{y}_0^*(\theta)^{\frac{v-1}{v}}\left(1-\hat{N}^*(\theta)\right)\right)^{\frac{v}{v-1}},$$

$$\hat{W}^*(\theta) \equiv \hat{W}(\theta;\pi^{x*}),$$

and

$$\hat{P}^*\left(\theta\right) \equiv \left(\hat{p}_1^{*1-v}\hat{N}^*\left(\theta\right)+\hat{p}_0^{*1-v}(1-\hat{N}^*\left(\theta\right)\right)^{\frac{1}{1-v}}.$$

Dropping the state θ from the argument of each function, as well as all the arguments of the fiscal policy, so as to ease the exposition, we have that firm i's market valuation (i.e., its payoff) is equal to

$$\bar{\Pi}_i(\pi_i^x) \equiv \sup_{\varsigma:\mathbb{R}\to[0,1]} \Pi_i(\varsigma;\pi_i^x),$$

where

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) \equiv & \, \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{r}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{r}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & + \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x),\end{aligned}$$

with $\bar{n}(\pi_i^x;\varsigma) \equiv \int \varsigma(x)d\Phi(x|\theta,\pi_i^x)$ denoting the probability that firm i invests when using the strategy $\varsigma:\mathbb{R}\to[0,1]$, and $\hat{T}_1^*$ and $\hat{T}_0^*$ denoting the transfers received when generating (real) revenues $\hat{r}_1^* = \hat{p}_1^*\hat{y}_1^*/\hat{P}^*$ and $\hat{r}_0^* = \hat{p}_0^*\hat{y}_0^*/\hat{P}^*$, respectively in case it invests and in case it does not invest.

Using (S.30), we have that $\hat{r}_f^* = \hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}$ for $f=0,1$. Hence,

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) = & \mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\hat{y}_1^{*\frac{v-1}{v}}\bar{n}(\pi_i^x;\varsigma)+\hat{y}_0^{*\frac{v-1}{v}}\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & - \mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\hat{l}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{l}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] \\ & + \mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right] - k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right] - \mathcal{I}(\pi_i^x).\end{aligned}$$

Using

$$\hat{y}_1^* = \gamma\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_1^{*\psi}, \tag{S.40}$$

$$\hat{y}_0^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}, \tag{S.41}$$

and

$$\hat{l}_1^* = \gamma^{\varphi}\hat{l}_0^*, \tag{S.42}$$

we have that

$$\begin{aligned}\Pi_i(\varsigma;\pi_i^x) =&\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\&-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\bar{n}(\pi_i^x;\varsigma)+1\right)\hat{l}_0^*\right]+\\&+\mathbb{E}\left[\hat{C}^{*-R}\left(\hat{T}_1^*\bar{n}(\pi_i^x;\varsigma)+\hat{T}_0^*\left(1-\bar{n}(\pi_i^x;\varsigma)\right)\right)\right]-k\mathbb{E}\left[\bar{n}(\pi_i^x;\varsigma)\right]-\mathcal{I}(\pi_i^x).\end{aligned}$$

Accordingly, the marginal effect of a change in π_i^x on firm i's objective is given by

$$\begin{aligned}\frac{\partial\Pi_i(\varsigma;\pi_i^x)}{\partial\pi_i^x}=&\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\left(\gamma^{\varphi}-1\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+\\&-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left(\left(\gamma^{\varphi}-1\right)\hat{l}_0^*\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right)\right]+\\&+\mathbb{E}\left[\hat{C}^{*-R}\left(\frac{\hat{T}_1^*-\hat{T}_0^*}{\hat{P}^*}\right)\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial\bar{n}(\pi_i^x;\varsigma)}{\partial\pi_i^x}\right]-\frac{\partial\mathcal{I}(\pi_i^x)}{\partial\pi_i^x},\quad\text{(S.43)}\end{aligned}$$

where $\partial\bar{n}(\pi_i^x;\varsigma)/\partial\pi_i^x$ is the marginal effect of varying π_i^x on the probability that the firm invests at θ, holding fixed the rule ς.

Next, recall again that, for $f=0,1$,

$$\hat{r}_f^*\equiv\frac{\hat{p}_f^*\hat{y}_f^*}{\hat{P}^*}=\hat{C}^{*\frac{1}{v}}\hat{y}_f^{*\frac{v-1}{v}}.$$

Using (S.40) and (S.41), we have that

$$\hat{r}_1^*-\hat{r}_0^*=\hat{C}^{*\frac{1}{v}}\Theta^{\frac{v-1}{v}}\left(1+\beta\hat{N}^*\right)^{\alpha\frac{v-1}{v}}\left(\gamma^{\frac{v-1}{v}}\hat{l}_1^{*\psi\frac{v-1}{v}}-\hat{l}_0^{*\psi\frac{v-1}{v}}\right).$$

Therefore, using (S.42) and the structure of the proposed fiscal policy, we have that

$$\hat{T}_1^*-\hat{T}_0^*=s+\frac{1}{v-1}\hat{C}^{*\frac{1}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}\left(\gamma^{\varphi}-1\right)\hat{l}_0^{*\psi\frac{v-1}{v}}.$$

Substituting this expression in (S.43), we obtain that

$$\frac{\partial \Pi_i(\varsigma;\pi_i^x)}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right)\right]+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-k\mathbb{E}\left[\frac{\partial \bar{n}(\pi_i^x;\varsigma)}{\partial \pi_i^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x}.$$

Next recall that, when $\pi_i^x = \pi^{x*}$, the optimal investment strategy is the efficient one, i.e., $\varsigma = \hat{n}^*$, where $\hat{n}^*(x) \equiv \hat{n}(x;\pi^{x*})$ is the efficient investment choice for a firm receiving signal x after acquiring information of precision π^{x*}. Using the envelope theorem, we thus have that

$$\frac{d\bar{\Pi}_i(\pi^{x*})}{d\pi_i^x} = \frac{\partial \Pi_i(\hat{n}^*;\pi^{x*})}{\partial \pi_i^x} = \frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-\frac{\partial \mathcal{I}(\pi_i^x)}{\partial \pi_i^x},$$

where $\partial\hat{N}^*/\partial\pi^x$ is the marginal change in the measure of investing firms that obtains when one changes π^x at $\pi^x = \pi^{x*}$, holding the strategy $\hat{n}^*$ fixed. Note that, in writing the expression above, we use the fact that, when $\varsigma = \hat{n}^*$, $\bar{n}(\pi_i^x;\varsigma) = \hat{N}^*$, which implies that

$$\frac{\partial \bar{n}(\pi_i^{x*};\hat{n}^*)}{\partial \pi_i^x} = \frac{\partial \hat{N}^*}{\partial \pi^x}.$$

For the fiscal policy to induce efficiency in information acquisition (when paired with the monetary policy in the proposition), it must be that $d\bar{\Pi}_i(\pi^{x*})/d\pi_i^x = 0$. Given the derivations above, this requires that

$$\frac{v}{v-1}\mathbb{E}\left[\hat{C}^{*\frac{1-vR}{v}}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{v-1}{v}}(\gamma^{\varphi}-1)\frac{\partial \hat{N}^*}{\partial \pi^x}\hat{l}_0^{*\psi\frac{v-1}{v}}\right]+$$
$$-\mathbb{E}\left[\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*}\left((\gamma^{\varphi}-1)\,\hat{l}_0^*\frac{\partial \hat{N}^*}{\partial \pi^x}\right)\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial \hat{N}^*}{\partial \pi^x}\right]-k\mathbb{E}\left[\frac{\partial \hat{N}^*}{\partial \pi^x}\right]=\frac{\partial \mathcal{I}(\pi^{x*})}{\partial \pi^x}. \quad \text{(S.44)}$$

Next, use (S.34) and (S.42) to note that

$$\hat{C}^{*-R}\frac{\hat{W}^*}{\hat{P}^*} = \left(\hat{l}_1^*\hat{N}^* + \hat{l}_0^*\left(1-\hat{N}^*\right)\right)^{\varepsilon} = \hat{l}_0^{*\varepsilon}\left((\gamma^{\varphi}-1)\,\hat{N}^*+1\right)^{\varepsilon}.$$

Hence, using the fact that $\hat{C}^{*\frac{1-vR}{v}} = \hat{C}^{*1-R}\hat{C}^{*\frac{1-v}{v}}$, along with the fact that, as shown in the proof of Proposition S.1,

$$\hat{C}^* = \Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\hat{l}_0^{*\psi}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\frac{v}{v-1}},$$

we have that

$$\hat{C}^{*\frac{1-vR}{v}} = \hat{C}^{*1-R}\left(\Theta\left(1+\beta\hat{N}^*\right)^{\alpha}\right)^{\frac{1-v}{v}}\hat{l}_0^{*\psi\frac{1-v}{v}}\frac{1}{\left(\gamma^{\varphi}-1\right)\hat{N}^*+1}.$$

It follows that (S.44) is equivalent to

$$\mathbb{E}\left[\frac{v\left(\gamma^{\varphi}-1\right)\hat{C}^{*1-R}}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$
$$-\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]+$$
$$+\mathbb{E}\left[\hat{C}^{*-R}s\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{\partial\mathcal{I}(\pi^{x*})}{\partial\pi^x}. \quad \text{(S.45)}$$

Recall that the efficient precision of private information π^{x*} solves

$$\mathbb{E}\left[\hat{C}^{*1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}^*}+\frac{v\left(\gamma^{\varphi}-1\right)}{(v-1)\left(\left(\gamma^{\varphi}-1\right)\hat{N}^*+1\right)}\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]$$
$$+\mathbb{E}\left[\hat{l}_0^{*1+\varepsilon}\left(\left(\gamma^{\varphi}-1\right)N^*+1\right)^{\varepsilon}\left(\gamma^{\varphi}-1\right)\frac{\partial\hat{N}^*}{\partial\pi^x}\right]-k\mathbb{E}\left[\frac{\partial\hat{N}^*}{\partial\pi^x}\right]=\frac{d\mathcal{I}(\pi^{x*})}{d\pi_x}. \quad \text{(S.46)}$$

Comparing (S.45) with (S.46), we have that, for the policy T to implement the efficient acquisition and usage of information (when paired with the monetary policy in the proposition, which, by virtue of Lemma S.1, is the only monetary policy that can induce efficiency in both information usage and information acquisition), the subsidy s to the investing firms must satisfy the following condition

$$\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{-R}s(\theta)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right]=\mathbb{E}\left[\hat{C}(\theta;\pi^{x*})^{1-R}\left(\frac{\alpha\beta}{1+\beta\hat{N}(\theta;\pi^{x*})}\right)\frac{\partial\hat{N}\left(\theta;\pi^{x*}\right)}{\partial\pi^x}\right],$$

where we reintroduce the arguments of the various functions.

Finally, note that, independently of whether the economy satisfies the conditions in Proposition S.1, when the subsidy to the investing firms is equal to

$$s(\theta) = \frac{\alpha\beta\hat{C}(\theta; \pi^{x*})}{1 + \beta\hat{N}(\theta; \pi^{x*})}$$

in each state, then, as shown in part 1, the private value $\mathcal{R}$ that each firm assigns to investing coincides with the social value $\mathcal{Q}$ in each state, implying that the firm finds it optimal to acquire the efficient amount of private information and then uses it efficiently when expecting all other firms to do the same. This establishes the claim in the proposition. Q.E.D.